

1. સાબિત કરો કે, $\frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \frac{1 + \sin A}{\cos A}$. ગણતરી માટેનું સૂચન : $\sec^2 A - \tan^2 A = 1$ ત્રિકોણમિતિય નિત્યસમ અને $a^2 - b^2 = (a + b)(a - b)$ નો ઉપયોગ કરો.

$$\begin{aligned}
 \Rightarrow \text{ડા.બા.} &= \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1} \\
 &= \frac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{(\tan A - \sec A + 1)} [\because \sec^2 A - \tan^2 A = 1] \\
 &= \frac{(\tan A + \sec A) - (\sec A + \tan A)(\sec A - \tan A)}{(1 - \sec A + \tan A)} \\
 &= \frac{(\sec A + \tan A)(1 - \sec A + \tan A)}{1 - \sec A + \tan A} \\
 &= \sec A + \tan A = \frac{1}{\cos A} + \frac{\sin A}{\cos A} \\
 &= \frac{1 + \sin A}{\cos A} = \text{જ.બા.}
 \end{aligned}$$

2. જો $a \cos \theta + b \sin \theta = m$ અને $a \sin \theta - b \cos \theta = n$ હોય, તો સાબિત કરો કે, $a^2 + b^2 = m^2 + n^2$.

$$\begin{aligned}
 \Rightarrow a \cos \theta + b \sin \theta &= m \quad \dots\dots\dots(i) \\
 \text{અને } a \sin \theta - b \cos \theta &= n \text{ આપેલ છે. } \dots\dots\dots(ii) \\
 \text{પરિણામ (i) અને (ii) નો વર્ગ કરી સરવાળો કરો.} \\
 m^2 + n^2 &= (a \cos \theta + b \sin \theta)^2 + (a \sin \theta - b \cos \theta)^2 \\
 &= a^2 \cos^2 \theta + b^2 \sin^2 \theta + 2ab \sin \theta \cdot \cos \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta - 2ab \sin \theta \cdot \cos \theta \\
 \therefore m^2 + n^2 &= a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\sin^2 \theta + \cos^2 \theta) \\
 \therefore m^2 + n^2 &= a^2 + b^2 \\
 \text{આમ, માંગેલ પરિણામ સાબિત થાય છે.}
 \end{aligned}$$

3. કિંમત મેળવો : $\tan 22^\circ 30'$.

$$\begin{aligned}
 \Rightarrow \text{આપણે જાણીએ છીએ કે,} \\
 \tan\left(\frac{\theta}{2}\right) &= \frac{\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} = \frac{2\sin\left(\frac{\theta}{2}\right) \cdot \cos\left(\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\theta}{2}\right)} \\
 \Rightarrow \tan \frac{\theta}{2} &= \frac{\sin \theta}{1 + \cos \theta} \\
 \text{હવે } \theta &= 45^\circ \text{ લેતાં,} \\
 \therefore \tan(22^\circ 30') &= \frac{\sin(45^\circ)}{1 + \cos(45^\circ)} \\
 &= \frac{\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2} + 1}
 \end{aligned}$$

4. સાબિત કરો : $\sin 4A = 4\sin A \cos^3 A - 4\cos A \sin^3 A$.

$$\begin{aligned}
 \Rightarrow \text{ડા.બા.} &= \sin(4A) \\
 &= 2 \sin(2A) \cdot \cos(2A) \\
 &= 2 (2\sin A \cdot \cos A)(\cos^2 A - \sin^2 A) \\
 &= 4 \sin A \cdot \cos^3 A - 4\cos A \sin^3 A = \text{જ.બા.}
 \end{aligned}$$

$$[\because \cos 2A = \cos^2 A - \sin^2 A \text{ અને } \sin 2A = 2\sin A \cdot \cos A]$$

$$\therefore \text{ડા.બી.} = \text{જ.બી.}$$

5. જો $\tan \theta + \sin \theta = m$ અને $\tan \theta - \sin \theta = n$ હોય તો સાબિત કરો કે, $m^2 - n^2 = 4 \sin \theta \tan \theta$.

$$\Rightarrow \tan \theta + \sin \theta = m \quad \dots\dots(i)$$

$$\text{અને } \tan \theta - \sin \theta = n \text{ આપેલ છે.} \quad \dots\dots(ii)$$

$$\text{હવે } m + n = \tan \theta + \sin \theta + \tan \theta - \sin \theta$$

$$m + n = 2 \tan \theta \quad \dots\dots(iii)$$

$$\text{તેમજ } m - n = \tan \theta + \sin \theta - \tan \theta + \sin \theta$$

$$m - n = 2 \sin \theta \quad \dots\dots(iv)$$

પરિણામ (iii) અને (iv) પરથી,

$$(m + n)(m - n) = 4\sin \theta \cdot \tan \theta$$

$$m^2 - n^2 = 4\sin \theta \cdot \tan \theta$$

જે માંગેલ પરિણામ છે.

6. જો $\tan(A + B) = p$ અને $\tan(A - B) = q$, તો બતાવો કે, $\tan 2A = \frac{p + q}{1 - pq}$.

$$\Rightarrow \text{અહીં } \tan(A + B) = p \quad \dots\dots(i)$$

$$\text{અને } \tan(A - B) = q \quad \dots\dots(ii) \text{ આપેલ છે.}$$

$$2A = A + A$$

$$\therefore 2A = (A + B) + (A - B)$$

$$\tan(2A) = \tan [A + B + A - B]$$

$$= \frac{\tan(A + B) + \tan(A - B)}{1 - \tan(A + B) \tan(A - B)} \left[\because \tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \right]$$

$$= \frac{p + q}{1 - pq} \quad (\because \text{પરિણામ (i) અને (ii) પરથી})$$

7. જો $\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$ હોય, તો બતાવો કે, $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$.

$$\Rightarrow \cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta \text{ આપેલ છે.}$$

$$\therefore (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 = 0$$

$$\therefore \cos^2 \alpha + \cos^2 \beta + 2\cos \alpha \cos \beta - \sin^2 \alpha - \sin^2 \beta - 2\sin \alpha \sin \beta = 0$$

$$\therefore \cos^2 \alpha - \sin^2 \alpha + \cos^2 \beta - \sin^2 \beta = 2(\sin \alpha \sin \beta - \cos \alpha \cos \beta)$$

$$\therefore \cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$$

જે માંગેલ પરિણામ છે.

8. જો $\frac{\sin(x + y)}{\sin(x - y)} = \frac{a + b}{a - b}$ હોય, તો $\frac{\tan x}{\tan y} = \frac{a}{b}$ મેળવો.

$$\Rightarrow \frac{\sin(x + y)}{\sin(x - y)} = \frac{a + b}{a - b} \text{ આપેલ છે.}$$

હવે બંને બાજુ યોગ-વિયોગ પ્રમાણ લેતાં,

$$\therefore \frac{\sin(x + y) + [\sin(x - y)]}{\sin(x + y) - \sin(x - y)} = \frac{a + b + a - b}{a + b - a + b}$$

$$\therefore \frac{2\sin\left(\frac{x + y + x - y}{2}\right) \cdot \cos\left(\frac{x + y - x + y}{2}\right)}{2\cos\left(\frac{x + y + x - y}{2}\right) \cdot \sin\left(\frac{x + y - x + y}{2}\right)} = \frac{2a}{2b}$$

$$\left[\because \sin x + \sin y = 2\sin \frac{x + y}{2} \cdot \cos \frac{x - y}{2} \text{ અને } \sin x - \sin y = 2\cos \frac{x + y}{2} \cdot \sin \frac{x - y}{2} \right]$$

$$\therefore \frac{\sin x \cdot \cos y}{\cos x \cdot \sin y} = \frac{a}{b}$$

$$\therefore \frac{\tan x}{\tan y} = \frac{a}{b} \text{ જે માંગેલ પરિણામ છે.}$$

9. જો $\cot \theta + \tan \theta = 2\operatorname{cosec} \theta$ હોય તો θ મેળવો.

➡ $\cot \theta + \tan \theta = 2 \operatorname{cosec} \theta$ આપેલ છે.

$$\therefore \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta} = \frac{2}{\sin \theta}$$

$$\therefore \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cdot \cos \theta} = \frac{2}{\sin \theta}$$

$$\therefore \frac{1}{\cos \theta} = 2 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\therefore \cos \theta = \frac{1}{2} \Rightarrow \cos \theta = \cos \frac{\pi}{3}$$

$$\therefore \theta = 2n\pi \pm \frac{\pi}{3}; \quad n \in \mathbb{Z}$$

10. જો $2\sin^2 \theta = 3\cos \theta$, જ્યાં $0 \leq \theta \leq 2\pi$ હોય તો θ મેળવો.

➡ આપેલ સમીકરણ $2\sin^2 \theta = 3\cos \theta$

$$\therefore 2 - 2\cos^2 \theta = 3\cos \theta$$

$$\therefore 2\cos^2 \theta + 3\cos \theta - 2 = 0$$

$$\therefore 2\cos^2 \theta + 4\cos \theta - \cos \theta - 2 = 0$$

$$\therefore 2\cos \theta (\cos \theta + 2) - 1 (\cos \theta + 2) = 0$$

$$\therefore (\cos \theta + 2)(2\cos \theta - 1) = 0$$

$$\therefore \cos \theta = -2 \text{ જે શક્ય નથી. } [\because -1 \leq \cos \theta \leq 1]$$

$$\therefore 2\cos \theta = 1$$

$$\therefore \cos \theta = \frac{1}{2}$$

$$\therefore \cos \theta = \cos \frac{\pi}{3}$$

$$\therefore \theta = \frac{\pi}{3}$$

$$\text{તેમજ } \cos \theta = \cos \left(2\pi - \frac{\pi}{3} \right)$$

$$\therefore \cos \theta = \cos \frac{5\pi}{3}$$

$$\therefore \theta = \frac{5\pi}{3}$$

આમ, θ ની કિંમત $\frac{\pi}{3}$ અને $\frac{5\pi}{3}$ થાય.

11. જો $\sec x \cos 5x + 1 = 0$, જ્યાં $0 < x \leq \frac{\pi}{2}$ હોય, તો x મેળવો.

➡ $\sec x \cos 5x + 1 = 0$ આપેલ છે.

$$\therefore \frac{\cos 5x}{\cos x} + 1 = 0 \therefore \cos 5x + \cos x = 0$$

$$\therefore 2\cos \left(\frac{5x + x}{2} \right) \cdot \cos \left(\frac{5x - x}{2} \right) = 0 \quad \left[\because \cos x + \cos y = 2\cos \frac{x+y}{2} \cdot \cos \frac{x-y}{2} \right]$$

$$\therefore 2\cos 3x \cdot \cos 2x = 0$$

$$\therefore \cos 3x = 0 \text{ અથવા } \cos 2x = 0$$

$$\therefore \cos 3x = \cos \frac{\pi}{2} \text{ અથવા } \cos 2x = \cos \frac{\pi}{2}$$

$$\therefore 3x = \frac{\pi}{2} \text{ અથવા } 2x = \frac{\pi}{2}$$

$$\therefore x = \frac{\pi}{6} \text{ અથવા } x = \frac{\pi}{4}$$

આમ, x ની કિંમતો $\frac{\pi}{6}$ અને $\frac{\pi}{4}$ થાય.

12. જો $\frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha} = y$ હોય, તો સાબિત કરો કે, $\frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}$ નું મૂલ્ય પણ y છે.

➡ $\frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha} = y$ આપેલ છે.

$$\begin{aligned} \text{હવે, } & \frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha} \\ &= \frac{(1 - \cos\alpha + \sin\alpha)}{(1 + \sin\alpha)} \cdot \frac{(1 + \cos\alpha + \sin\alpha)}{(1 + \cos\alpha + \sin\alpha)} \\ &= \frac{\{(1 + \sin\alpha) - \cos\alpha\}}{(1 + \sin\alpha)} \cdot \frac{\{(1 + \sin\alpha) + \cos\alpha\}}{(1 + \cos\alpha + \sin\alpha)} \\ &= \frac{(1 + \sin\alpha)^2 - \cos^2\alpha}{(1 + \sin\alpha)(1 + \sin\alpha + \cos\alpha)} \\ &= \frac{(1 + \sin^2\alpha + 2\sin\alpha) - \cos^2\alpha}{(1 + \sin\alpha)(1 + \sin\alpha + \cos\alpha)} \\ &= \frac{1 + \sin^2\alpha + 2\sin\alpha - 1 + \sin^2\alpha}{(1 + \sin\alpha)(1 + \sin\alpha + \cos\alpha)} \\ &= \frac{2\sin^2\alpha + 2\sin\alpha}{(1 + \sin\alpha)(1 + \sin\alpha + \cos\alpha)} \\ &= \frac{2\sin\alpha(1 + \sin\alpha)}{(1 + \sin\alpha)(1 + \sin\alpha + \cos\alpha)} \\ &= \frac{2\sin\alpha}{1 + \sin\alpha + \cos\alpha} = y \end{aligned}$$

આમ, માંગેલ પરિણામ સાબિત થાય છે.

13. જો $m \sin\theta = n \sin(\theta + 2\alpha)$ હોય, તો સાબિત કરો કે, $\tan(\theta + \alpha) \cot\alpha = \frac{m + n}{m - n}$.

➡ જો $m \sin\theta = n \sin(\theta + 2\alpha)$ આપેલ છે.

બંને બાજુ યોગ વિયોગ પ્રમાણ લેતાં,

$$\begin{aligned} \therefore \frac{\sin(\theta + 2\alpha) + \sin\theta}{\sin(\theta + 2\alpha) - \sin\theta} &= \frac{m + n}{m - n} \\ \therefore \frac{2\sin\left(\frac{\theta + 2\alpha + \theta}{2}\right) \cdot \cos\left(\frac{\theta + 2\alpha - \theta}{2}\right)}{2\cos\left(\frac{\theta + 2\alpha + \theta}{2}\right) \cdot \sin\left(\frac{\theta + 2\alpha - \theta}{2}\right)} &= \frac{m + n}{m - n} \end{aligned}$$

$$\begin{aligned} \left[\because \sin x + \sin y &= 2\sin\left(\frac{x + y}{2}\right) \cdot \cos\left(\frac{x - y}{2}\right) \text{ અને } \sin x - \sin y = 2\cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right) \right] \\ \therefore \frac{\sin(\theta + \alpha) \cdot \cos\alpha}{\cos(\theta + \alpha) \cdot \sin\alpha} &= \frac{m + n}{m - n} \\ \therefore \tan(\theta + \alpha) \cdot \cot\alpha &= \frac{m + n}{m - n} \end{aligned}$$

જે માંગેલ પરિણામ છે.

14. જો $\cos(\alpha + \beta) = \frac{4}{5}$ અને $\sin(\alpha - \beta) = \frac{5}{13}$ હોય, તથા α એ 0 તથા $\frac{\pi}{4}$ વચ્ચે હોય તો $\tan(2\alpha)$ ની કિંમત મેળવો.

➡ $\cos(\alpha + \beta) = \frac{4}{5}$ અને $\sin(\alpha - \beta) = \frac{5}{13}$ આપેલ છે.

$$\text{હવે } \sin(\alpha + \beta) = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \pm \frac{3}{5}$$

$$\therefore \sin(\alpha + \beta) = \frac{3}{5}$$

$$\text{અને } \cos(\alpha - \beta) = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \pm \frac{12}{13}$$

$$\therefore \cos(\alpha - \beta) = \frac{12}{13}$$

$$\begin{aligned} \text{હવે, } \tan(\alpha + \beta) &= \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} \left[\alpha \text{ એ } 0 \text{ અને } \frac{\pi}{4} \text{ વચ્ચે છે.} \right] \\ &= \frac{\frac{3}{5}}{\frac{12}{13}} = \frac{3}{4} \end{aligned}$$

$$\text{અને } \tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12}$$

$$\begin{aligned} \text{હવે, } \tan 2\alpha &= \tan(\alpha + \beta + \alpha - \beta) \\ &= \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)} \\ &\left[\because \tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y} \right] \\ &= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \cdot \frac{5}{12}} = \frac{\frac{9+5}{12}}{\frac{16-5}{16}} = \frac{14 \times 16}{12 \times 11} = \frac{56}{33} \end{aligned}$$

15. જો $\tan x = \frac{b}{a}$ હોય, તો $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$ નું મૂલ્ય મેળવો. ગણતરી માટેનું સૂત્ર : સૌ પ્રથમ આપેલ પદાવલીનું સંમેયીકરણ કરો અને $\cos 2x = \cos^2 x - \sin^2 x$ સૂત્રનો ઉપયોગ કરો.

➡ અહીં $\tan x = \frac{b}{a}$ આપેલ છે.

$$\begin{aligned} \therefore \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} &= \frac{\sqrt{(a+b)^2} + \sqrt{(a-b)^2}}{\sqrt{(a-b)(a+b)}} \\ &= \frac{(a+b) + (a-b)}{\sqrt{a^2 - b^2}} = \frac{2a}{\sqrt{a^2 - b^2}} \\ &= \frac{2a}{a\sqrt{1 - \left(\frac{b}{a}\right)^2}} \left[\because \frac{b}{a} = \tan x \right] \\ &= \frac{2}{\sqrt{1 - \tan^2 x}} = \frac{2\cos x}{\sqrt{\cos^2 x - \sin^2 x}} \quad [\because \cos 2x = \cos^2 x - \sin^2 x] \\ &= \frac{2\cos x}{\sqrt{\cos 2x}} \end{aligned}$$

16. સાબિત કરો : $\cos \theta \cos\left(\frac{\theta}{2}\right) - \cos 3\theta \cos\left(\frac{9\theta}{2}\right) = \sin 7\theta \sin 8\theta$.

$$\begin{aligned} \text{ડા.બા.} &= \cos \theta \cos\left(\frac{\theta}{2}\right) - \cos 3\theta \cos\left(\frac{9\theta}{2}\right) \\ &= \frac{1}{2} \left[2\cos \theta \cdot \cos\left(\frac{\theta}{2}\right) - 2\cos 3\theta \cdot \cos\left(\frac{9\theta}{2}\right) \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} \left[\cos\left(\theta + \frac{\theta}{2}\right) + \cos\left(\theta - \frac{\theta}{2}\right) - \cos\left(3\theta + \frac{9\theta}{2}\right) - \cos\left(3\theta - \frac{9\theta}{2}\right) \right] \\
&= \frac{1}{2} \left[\cos\left(\frac{3\theta}{2}\right) + \cos\left(\frac{\theta}{2}\right) - \cos\left(\frac{15\theta}{2}\right) - \cos\left(\frac{3\theta}{2}\right) \right] \\
&= \frac{1}{2} \left[\cos\left(\frac{\theta}{2}\right) - \cos\left(\frac{15\theta}{2}\right) \right] \\
&= -\frac{1}{2} \left[2\sin\left(\frac{\theta + 15\theta}{2}\right) \cdot \sin\left(\frac{\theta - 15\theta}{2}\right) \right] \left[\because \cos x - \cos y = -2\sin\frac{x+y}{2} \cdot \sin\frac{x-y}{2} \right] \\
&= -\frac{1}{2} [2\sin(8\theta) \cdot \sin(-7\theta)] \\
&= + (\sin 8\theta \cdot \sin 7\theta) = \text{જ.બ.} \\
\therefore \text{જ.બ.} &= \text{જ.બ.}
\end{aligned}$$

17. જો $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$ હોય તો સાબિત કરો કે, $\sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$.

➡ અહીં $\tan\theta = \frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$ આપેલ છે.

$$\therefore \tan\theta = \frac{\cos\alpha (\tan\alpha - 1)}{\cos\alpha (\tan\alpha + 1)}$$

$$\therefore \tan\theta = \frac{\tan\alpha - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{4} \cdot \tan\alpha} \quad \left[\because \tan\frac{\pi}{4} = 1 \right]$$

$$\therefore \tan\theta = \tan\left(\alpha - \frac{\pi}{4}\right)$$

$$\therefore \theta = \alpha - \frac{\pi}{4} \Rightarrow \alpha = \theta + \frac{\pi}{4}$$

$$\begin{aligned}
\therefore \sin\alpha + \cos\alpha &= \sin\left(\theta + \frac{\pi}{4}\right) + \cos\left(\theta + \frac{\pi}{4}\right) \\
&= \sin\theta \cdot \cos\frac{\pi}{4} + \cos\theta \cdot \sin\frac{\pi}{4} + \cos\theta \cdot \cos\frac{\pi}{4} - \sin\theta \cdot \sin\frac{\pi}{4} \\
&= \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta \left[\because \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \right] \\
&= \frac{2}{\sqrt{2}} \cdot \cos\theta = \sqrt{2}\cos\theta
\end{aligned}$$

18. જો $\sin\theta + \cos\theta = 1$ હોય તો θ નું સામાન્ય મૂલ્ય મેળવો. ગણતરી માટેનું સૂચન : $\sin\theta = \sin\alpha$ હોય તો $\theta = n\pi + (-1)^n \cdot \alpha$ સમીકરણનો ઉકેલ થાય.

➡ આપેલ છે કે, $\sin\theta + \cos\theta = 1$

બંને બાજુ વર્ગ કરતાં,

$$\sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta = 1$$

$$\therefore 1 + 2\sin\theta \cdot \cos\theta = 1 \quad [\because \sin 2x = 2\sin x \cos x]$$

$$\therefore \sin 2\theta = 0 \Rightarrow 2\theta = n\pi + (-1)^n \cdot 0, \quad n \in \mathbb{Z}$$

$$\therefore \theta = \frac{n\pi}{2}$$

અન્ય રીત :

$$\sin\theta + \cos\theta = 1$$

$$\therefore \frac{1}{\sqrt{2}} \cdot \sin\theta + \frac{1}{\sqrt{2}} \cdot \cos\theta = \frac{1}{\sqrt{2}}$$

$$\therefore \sin\theta \cdot \cos\frac{\pi}{4} + \cos\theta \cdot \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad \left[\because \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} \right]$$

$$\therefore \sin\left(\theta + \frac{\pi}{4}\right) = \sin\frac{\pi}{4} \quad [\because \sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y]$$

$$\therefore \theta + \frac{\pi}{4} = n\pi + (-1)^n \frac{1}{4}; \quad n \in \mathbb{Z}$$

$$\therefore \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}; \quad n \in \mathbb{Z}$$

19. સમીકરણ $\tan\theta = -1$ અને $\cos\theta = \frac{1}{\sqrt{2}}$ નું સમાધાન કરતી θ ની કિંમત મેળવો.

➡ અહીં $\tan\theta = -1$ (i)

અને $\cos\theta = \frac{1}{\sqrt{2}}$ (ii) આપેલા સમીકરણો છે.

સમીકરણ (i) પરથી $\tan\theta = -\tan\left(\frac{\pi}{4}\right)$

$$\therefore \tan\theta = \tan\left(2\pi - \frac{\pi}{4}\right) \Rightarrow \tan\theta = \tan\left(\frac{7\pi}{4}\right)$$

$$\therefore \theta = \frac{7\pi}{4}$$

સમી. (ii) પરથી, $\cos\theta = \frac{1}{\sqrt{2}} \therefore \cos\theta = \cos\frac{\pi}{4}$

$$\therefore \cos\theta = \cos\left(2\pi - \frac{\pi}{4}\right) \Rightarrow \cos\theta = \cos\frac{7\pi}{4}$$

$$\therefore \theta = \frac{7\pi}{4}$$

અહીં θ ની મૂળભૂત સામાન્ય કિંમત $\theta = 2n\pi + \frac{7\pi}{4}$ થાય. જ્યાં, $n \in \mathbb{Z}$.