

Class XI Session 2024-25
Subject - Applied Mathematics
Sample Question Paper - 9

Time Allowed: 3 hours

Maximum Marks: 80

General Instructions:

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there is some internal choice in some questions.
2. Section A has 18 MCQ's and 02 Assertion Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer(VSA) questions of 2 marks each.
4. Section C has 6 Short Answer(SA) questions of 3 marks each.
5. Section D has 4 Long Answer(LA) questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (04 marks each) with sub parts.
7. Internal Choice is provided in 2 questions in Section-B, 2 questions in Section-C, 2 Questions in Section-D. You have to attempt only one alternatives in all such questions.

Section A

1. The decimal equivalent of the binary number 10101 is [1]
a) 12 b) 31
c) 22 d) 21
2. The geometric mean and harmonic mean of two non-negative observations are 10 and 8, respectively. Then, what is the arithmetic mean of the observations? [1]
a) 12.5 b) 9
c) 4 d) 25
3. Section 87A refers to: [1]
a) tax rebate b) investment in NSC
c) investment in ELSS d) interest on a home loan
4. Mean of the first n terms of the A.P. $a + (a + d) + (a + 2d) + \dots$ is [1]
a) $a + \frac{(n-1)d}{2}$ b) $\frac{a+nd}{2}$
c) $a + nd$ d) $a + (n - 1)d$
5. If $A = \{-1, 2, 5, 8\}$, $B = \{0, 1, 3, 6, 7\}$ and R be the relation "is one less than" from A to B, then R as a set of ordered pairs is [1]
a) $\{(0, 1), (2, 3), (6, 7)\}$ b) $\{(-1, 0), (2, 3), (5, 6)\}$

- c) $\{(-1, 0), (2, 1), (8, 7)\}$ d) $\{(1, 2), (2, 3), (5, 6), (6, 7), (7, 8)\}$
6. If $\log_{0.2} x = 3$, then value of x is [1]
 a) 0.08 b) 9
 c) 0.6 d) 0.008
7. Let $A = \{a, b, c\}$, then the range of the relation $R = \{(a, b), (a, c), (b, c)\}$ defined on A is [1]
 a) $\{b, c\}$ b) $\{c\}$
 c) $\{a, b\}$ d) $\{a, b, c\}$
8. The slope of line, whose equation is $5x + 6y = 7$ is [1]
 a) $\frac{6}{5}$ b) -5
 c) $\frac{5}{6}$ d) $-\frac{5}{6}$
9. If CASUAL represents FXVRDI, then PEOPLE in coded language represents [1]
 a) SHRSQH b) SBRMPB
 c) SBRMOB d) SHRSOH
10. The mean deviation of the data 2, 9, 9, 3, 6, 9, 4 from the mean is [1]
 a) 3.57 b) 2.57
 c) 2.23 d) 3.23
11. If $\log 0.0007392 = -3.1313$, then $\log 73.92$ is [1]
 a) 1.8687 b) 2.8687
 c) 1.1313 d) 2.1313
12. Sanjay buys land for 200000 and agrees to pay an equal amount at the end of each year for 3 years. If the money [1]
 is worth 8%, then the amount of each instalment is: (Given $(1.08)^{-3} = 0.7938$).
 a) ₹ 78895.12 b) ₹ 75323.48
 c) ₹ 75428.56 d) ₹ 77594.56
13. Deduction of Medical Insurance Premium is allowed under section _____. [1]
 a) 80C b) 80TTA
 c) 80E d) 80D
14. If $\frac{\log x}{a-b} = \frac{\log y}{b-c} = \frac{\log z}{c-a}$, then xyz is equal to: [1]
 a) 2 b) 0
 c) 1 d) -1
15. The probability that in the toss of two dice we obtain the sum 7 or 11 is: [1]
 a) $\frac{1}{18}$ b) $\frac{2}{9}$
 c) $\frac{23}{108}$ d) $\frac{1}{6}$
16. What sum must be invested at the end of each year to provide funds for the replacement of a machine costing ₹ [1]
 8000 at the end of 3 years, if money is worth 5% effective? (Given $(1.05)^3 = 1.1576$)

a) ₹ 3042.07

b) ₹ 2737.67

c) ₹ 2538.07

d) ₹ 2832.67

17. Which of the following is not a binary number?

[1]

a) 11E

b) 000

c) 1111

d) 101

18. If $A = \{x : x^2 - 5x + 6 = 0\}$, $B = \{2, 4\}$, $C = \{4, 5\}$ then $A \times (B \cap C)$ is

[1]

a) $\{(4, 2), (4, 3)\}$

b) $\{(2, 2), (3, 3), (4, 4), (5, 5)\}$

c) $\{(2, 4), (3, 4), (4, 4)\}$

d) $\{(2, 4), (3, 4)\}$

19. **Assertion (A):** If $P(A) = 0.6$ and $P(B) = 0.4$ and $P(A|B) = 0.4$, then $P(A \cup B) = 0.72$.

[1]

Reason (R): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** $\left(\sum_{r=0}^{100} {}^{500-r}C_3\right) + {}^{400}C_4 = {}^{501}C_4$

[1]

Reason (R): ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Which day of the week was on 21st October 1948?

[2]

22. Which of the two conclusions is/are true on the basis of given statements:

[2]

Statements I: Some rats are cats

II: Some cats are not dogs

Conclusions I: Some cats are rats

II: Some dogs are rats

OR

Find the values of the letter and give a reason for the steps involved.

$$\begin{array}{r} 2 \text{ A } B \\ + A \text{ B } 1 \\ \hline B \text{ 1 } 8 \end{array}$$

23. The marks obtained by 15 students in a monthly test are: 11, 09, 07, 03, 18, 21, 13, 15, 18, 04, 06, 17, 22, 13, 15

[2]

i. Find the average marks of 15 students.

ii. Find the average of their marks when the marks of each student are increased by 2.

24. Differentiate the function with respect to x: $3e^{-3x} \log(1 + 2)$

[2]

OR

Differentiate $3^x + x^3 + 4x - 5$ with respect to x.

25. A bag contains 4 white and 2 black balls. Another contains 3 white and 5 black balls. If one ball is drawn from each bag, find the probability that both are black.

[2]

Section C

26. If ${}^nP_4 : {}^nP_2 = 12$, find n . [3]

OR

A class consists of 60 boys and 40 girls. In how many ways can a president, vice president, secretary and treasurer can be chosen if the secretary must be a boy, the treasurer must be a girl and a student may not hold more than one office?

27. Rohit is the husband of Vanshika. Sumita is the sister of Rohit. Anushka is the sister of Vanshika. How Anushka is related to Rohit? [3]

28. Discuss the continuity of the junction $f(x)$ at $x = \frac{1}{2}$, where [3]

$$f(x) = \begin{cases} \frac{1}{2} - x & ; 0 \leq x < \frac{1}{2} \\ 1 & ; x = \frac{1}{2} \\ \frac{3}{2} - x & ; \frac{1}{2} < x \leq 1 \end{cases}$$

29. A family in Agra, U.P consumes 78 SCM of gas in 60 days. The GST is included in the gas charges and the minimum charge is 10% of the gas consumption charges. The PNG rate in Agra are as follows: [3]

Units of Consumption (in SCM)	Price Per Unit
up to 45 SCM/60 days	₹ 29.50
above 45 SCM/60 days	₹ 42.61

Calculate the bimonthly PNG bill of the family.

30. Find the present value of a regular annuity of ₹ 1000 payable for 3 years at 12% per annum compounded annually? [3]

31. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group. [3]

Section D

32. Find the sum of 30 terms of the sequence 7, 7.7, 7.77, 7.777,... [5]

OR

The AM between two positive numbers a and b ($a > b$) is twice their GM. Prove that $a : b = (2 + \sqrt{3}) : (2 - \sqrt{3})$.

33. Find the domain of the function $f(x) = \frac{1}{\log(1-x)} + \sqrt{x+3}$ [5]

34. Find the coefficient of variation for the following data: [5]

CI	60 - 70	50 - 60	40 - 50	30 - 40	20 - 30	10 - 20
Frequencies	3	6	10	12	15	6

OR

Calculate Karl Pearson's coefficient of skewness from the following data:

Profits: (₹ in lacs)	Below 20	40	60	80	100
No. of companies:	8	20	50	64	70

35. A retailer buys a TV from a manufacturer for ₹25000. He marks the price of the TV 20% above his cost price and sells it to a consumer at 10% discount on the marked price. If the sales are intra-state and rate of GST is 12%, find: [5]

- i. the marked price of the TV.
- ii. Consumer's cost price of TV inclusive of tax (under GST).

iii. GST paid by the retailer to the Central and State Governments.

Section E

36. **Read the text carefully and answer the questions:** [4]

The girder of a railway bridge is a parabola with its vertex at the highest point, 10 metres above the ends and span is 100 metres.

- (a) Find the equation of bridge.
- (b) Find the height of the bridge at 20 metres from the mid-point.
- (c) Find the coordinates of the focus of parabola.

OR

Find the length of latus rectum.

37. **Read the text carefully and answer the questions:** [4]

Data of all the previous cricket matches are stored to analyze the average batting score of various batsmen. The scores of a batsman in ten innings are:

38, 70, 48, 34, 42, 55, 63, 46, 54, 44



- (a) What is the median of the data?
- (b) What is the mean deviation about the median of the given scores?
- (c) If the scores 38 and 34 are replaced by 68 and 74 what will be the mean of the data?

OR

Difference between maximum value of data and minimum value of data is called?

38. **Read the text carefully and answer the questions:** [4]

In a survey of 200 students it was found that 120 had got grade A in Mathematics, 90 had got grade A in Physics and 70 had got grade A in Chemistry, 50 had got grade A in Mathematics and Chemistry, 40 had got grade A in Mathematics and Physics, 30 had got grade A in Physics and Chemistry and 10 had got grade A in all three subjects one student is chosen randomly as a class representative.

- (a) The probability that the chosen class representative had got grade A in at least one of the subject which is?
- (b) The probability that the chosen class representative had got grade A in at least two of the subjects which is?
- (c) The probability that the chosen class representative had got grade 'A' in Mathematics or Chemistry is?

OR

Read the text carefully and answer the questions: [4]

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let: E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E: represent completing the construction work on time.

- (a) What is the probability that all the workers are present for the job?

- (b) What is the probability that construction will be completed on time?
- (c) What is the probability that many workers are not present given that the construction work is completed on time?

Solution

Section A

1. (d) 21
Explanation: $10101 = 1 \times 2^4 + 0 + 1 \times 2^2 + 0 + 1 \times 2^0 = 16 + 4 + 1 = 21$
2. (a) 12.5
Explanation: Given, geometric mean (G) = 10 and harmonic mean (H) = 8
Let A be the arithmetic mean, then $G^2 = AH$
 $\Rightarrow A = \frac{G^2}{H}$
 $\Rightarrow A = \frac{(10)^2}{8} = \frac{100}{8} = 12.5$
3. (a) tax rebate
Explanation: tax rebate
4. (a) $a + \frac{(n-1)d}{2}$
Explanation: Mean = $\frac{\frac{n}{2}[2a+(n-1)d]}{n} = \frac{[2a+(n-1)d]}{2} = a + \frac{(n-1)d}{2}$
5. (b) $\{(-1, 0), (2, 3), (5, 6)\}$
Explanation: $\{(-1, 0), (2, 3), (5, 6)\}$
6. (d) 0.008
Explanation: $\log_{0.2} x = 3$
 $\log_{0.2} (0.2)^3 = 3$
 $\log_{(0.2)} 0.008 = 3$
 $\therefore x = 0.008$
7. (a) {b, c}
Explanation: Since the range is represented by the y- coordinate of the ordered pair (x, y). Therefore, the range of the given relation is { b, c }.
8. (d) $-\frac{5}{6}$
Explanation: equation is $5x + 6y = 7$
Slope = $\frac{\text{coefficient of } x}{\text{coefficient of } y} = \frac{-5}{6}$
9. (c) SBRMOB
Explanation: CASUAL \leftrightarrow FXVRDI
 $C \xrightarrow{+3} F; A \xrightarrow{-3} X; S \xrightarrow{+3} V; U \xrightarrow{-3} R; A \xrightarrow{+3} D; L \xrightarrow{-3} I$
 $PEOPLE \xrightarrow{+3} P \xrightarrow{+3} E; C \xrightarrow{-3} B; O \xrightarrow{+3} R; P \xrightarrow{-3} M; L \xrightarrow{+3} O; E \xrightarrow{-3} B$
 \therefore SBRMOB
10. (b) 2.57
Explanation: M.D. $(\bar{x}) = \frac{|x_i - \bar{x}|}{n} = \frac{4+3+3+3+0+3+2}{7} = 2.57$
11. (a) 1.8687
Explanation: $\log 0.0007392 = -3.1313 = -4 + 4 - 3.1313 = -4 + 0.8687$
 $\Rightarrow \log 0.0007392 = 4.8687$

$$\Rightarrow \text{mantissa of digits } 7392 = 0.8687$$

$$\therefore \log 73.92 = 1.8687 \quad (\because \text{Characteristic of } \log 73.92 = 1)$$

12.

(d) ₹ 77594.56

Explanation: Present value = ₹ 200000, time = 3 years, $i = 0.08$ Let each instalment be of ₹ R.

\therefore It forms an annuity.

$$\therefore ₹ 200000 = R \left[\frac{1 - (1.08)^{-3}}{0.08} \right]$$

$$\Rightarrow R = \frac{200000 \times 0.08}{1 - 0.7938} = \frac{200000 \times 0.08}{0.2062} = ₹ 77594.56$$

13.

(d) 80D

Explanation: Deduction on Section **80D** in Income Tax Act. You are allowed to claim a deduction up to Rs. 25,000 per budgetary year for medical insurance premium instalments.

14.

(c) 1

Explanation: 1

15.

(b) $\frac{2}{9}$

Explanation: By throwing 7 we mean a sum of 7.

Now the first die may appear in 6 different ways and according to any one way in which the first appearance, the second can appear in 6 ways,

The two dice, therefore, may appear in $6 \times 6 = 36$ ways. favourable ways of getting a sum of 7 are (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3).

Thus the sum of 7 may appear in 6 different ways. i.e p_1 (probability of getting the sum of 7) = $\frac{6}{36} = \frac{1}{6}$

The sum of 11 may appear as 5 + 6, 6 + 5 i.e. in 2 different ways.

$$\therefore p_2 \text{ (probability of getting the sum of 11)} = \frac{2}{36} = \frac{1}{18}$$

$$\text{Hence the probability of 7 or 11} = \frac{1}{6} + \frac{1}{18} = \frac{4}{18} = \frac{2}{9}$$

16.

(c) ₹ 2538.07

Explanation: Let R be the sum invested at the end of every year for 3 year which amounts to ₹ 8000 and $i = 0.05$

$$\therefore R \left[\frac{(1.05)^3 - 1}{0.05} \right] = 8000$$

$$\Rightarrow R = \frac{8000 \times 0.05}{(1.1576 - 1)} = \frac{8000 \times 0.05}{0.1576} = ₹ 2538.07$$

17.

(a) 11E

Explanation: A binary number can have only two possible digits 0 and 1. Option 11E, there is an alphabet E present which makes it invalid binary number. Alphabets are only allowed in the hexadecimal number system.

18.

(d) $\{(2, 4), (3, 4)\}$

$$\text{Explanation: } x^2 - 5x + 6 = 0$$

$$\Rightarrow x^2 - 2x - 3x + 6 = 0$$

$$\Rightarrow x(x - 2) - 3(x - 2) = 0$$

$$\Rightarrow (x - 3)(x - 2) = 0$$

$$\therefore A = \{2, 3\}; \text{ Also, } B = \{2, 4\}, C = \{4, 5\}$$

$$\text{Now, } B \cap C = \{4\}$$

$$\therefore A \times B \cap C = \{2, 3\} \times \{4\}$$

$$= \{(2, 4), (3, 4)\}$$

19.

(a) Both A and R are true and R is the correct explanation of A.

$$\text{Explanation: } P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$0.4 = \frac{P(A \cap B)}{0.4}$$

$$\Rightarrow P(A \cap B) = 0.16$$

$$\text{Now, } (A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 0.6 + 0.4 - 0.16$$

$$= 1 - 0.28 = 0.72$$

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation: $\sum_{r=0}^{100} {}^{500-r}C_3 + {}^{400}C_4$

$$= {}^{500}C_3 + {}^{499}C_3 + \dots + {}^{401}C_3 + {}^{400}C_3 + {}^{400}C_4$$

$$= {}^{500}C_3 + {}^{499}C_3 + \dots + {}^{401}C_3 + {}^{401}C_4$$

$$= {}^{500}C_3 + {}^{499}C_3 + \dots + {}^{402}C_4$$

Similarly,

$${}^{500}C_3 + {}^{500}C_4$$

$${}^{501}C_4 = \text{RHS}$$

Section B

21. Date is 21st October 1948.

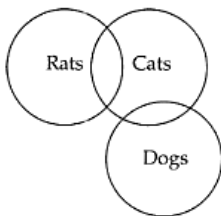
Last two digits of the year	48
Quotient on dividing 48 by 4	12
Date (21st)	21
Month (October)	07
Year (1948)	00
Total	88

Remainder on dividing 88 by 7 is 4.

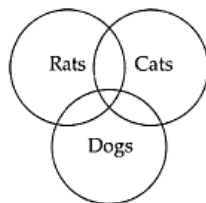
4 is associated with Thursday.

\therefore Day was Thursday.

22. For the given statements; two possible Venn diagrams are shown below:



(i)



(ii)

Since conclusion, I can be deduced from both the Venn diagrams, so the conclusion I is true. But conclusion II cannot be deduced from Venn diagram (i), so conclusion II is false.

OR

$$\begin{array}{r} 2 \ A \ B \\ + A \ B \ 1 \\ \hline B \ 1 \ 8 \end{array}$$

We have to find the value of A And B

Since, B + 1 we get 8, i.e., the number whose ones digit is 8

\therefore The only possible value of B is 7.

So, the question has been decoded as

$$\begin{array}{r} 2 \ A \ 7 \\ + A \ 7 \ 1 \\ \hline 7 \ 1 \ 8 \end{array}$$

Now, A + 7 we get 1, i.e. the number whose ones digit is 1

Clearly, 4 + 7 = 11

\therefore the value of A = 4.

So, the question has been decoded as

$$\begin{array}{r} 2 \ 4 \ 7 \\ +4 \ 7 \ 1 \\ \hline 7 \ 1 \ 8 \end{array}$$

Hence $A = 4$ and $B = 7$

23. i. The sum of marks of all 15 students = $11 + 9 + 7 + 3 + 18 + 21 + 13 + 15 + 18 + 4 + 6 + 17 + 22 + 13 + 15 = 192$

$$\therefore \text{Average marks} = \frac{192}{15} = 12.8$$

- ii. When the marks of each student are increased by 2, then the sum of their marks increases by 15×2 i.e. by 30

$$\therefore \text{The new sum of marks of all students} = 192 + 30 = 222$$

$$\therefore \text{The new average marks} = \frac{222}{15} = 14.8$$

24. Let $y = 3e^{-3x} \log(1+x)$

Differentiate it with respect to x we get,

$$\begin{aligned} \frac{dy}{dx} &= 3 \frac{d}{dx} [e^{-3x} \log(1+x)] \\ &= 3 \left\{ e^{-3x} \frac{1}{1+x} + \log(1+x)(-3e^{-3x}) \right\} \text{ [Using product rule and chain rule]} \\ &= 3 \frac{e^{-3x}}{1+x} - 3e^{-3x} \log(1+x) \\ &= 3e^{-3x} \left\{ \frac{1}{1+x} - 3 \log(1+x) \right\} \\ \text{So, } \frac{d}{dx} [3e^{-3x} \log(1+x)] &= 3e^{-3x} \left\{ \frac{1}{1+x} - 3 \log(1+x) \right\} \end{aligned}$$

OR

$$\text{Let, } f(x) = 3^x + x^3 + 4x - 5$$

$$\therefore \frac{d}{dx} f(x) = f'(x)$$

$$\Rightarrow f'(x) = \frac{d}{dx} (3^x + x^3 + 4x - 5)$$

$$= 3^x \log_e 3 + 3x^2 + 4 \times 1 - 0$$

$$= 3^x \log_e 3 + 3x^2 + 4.$$

25. Consider the following events:

W_1 = Drawing a white ball from the first bag, W_2 = Drawing a white ball from second bag.

B_1 = Drawing a black ball from first bag, B_2 = Drawing a black ball from second bag.

$$\text{Clearly, } P(W_1) = \frac{4}{6}, P(B_1) = \frac{2}{6}, P(W_2) = \frac{3}{8} \text{ and } P(B_2) = \frac{5}{8}$$

Therefore, required probability is given by,

$$P(\text{both balls are black}) = P[(\text{black ball from 1st bag}) \text{ and } (\text{black ball from 2nd bag})]$$

$$= P(B_1 \cap B_2)$$

$$= P(B_1) P(B_2) \text{ [} \because B_1 \text{ and } B_2 \text{ are independent events]}$$

$$= \frac{2}{6} \times \frac{5}{8} = \frac{5}{24}$$

Section C

26. Given, $\frac{{}^n P_4}{{}^n P_2} = 12$

$$\Rightarrow {}^n P_4 = 12 {}^n P_2$$

$$\Rightarrow \frac{n!}{(n-4)!} = 12 \frac{n!}{(n-2)!}$$

$$\Rightarrow \frac{n-2!}{(n-4)!} = 12$$

$$\Rightarrow \frac{(n-2)(n-3)(n-4)!}{(n-4)!} = 12$$

$$\Rightarrow n^2 - 5n + 6 = 12$$

$$\Rightarrow n^2 - 5n - 6 = 0$$

$$\Rightarrow n^2 - 6n + n - 6 = 0$$

$$\Rightarrow n(n-6) + 1(n-6) = 0$$

$$\Rightarrow (n-6)(n+1) = 0$$

$$\Rightarrow n = -1 \text{ or } n = 6$$

$\therefore n$ cannot be negative

$$\therefore n = 6$$

OR

The treasurer can be selected from the 40 girls in ${}^{40}C_1$ ways, i.e., in 40 ways.

The secretary can be selected from the 60 boys in ${}^{60}C_1$ ways, i.e., 60 ways.

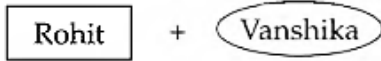
We now have $100 - 2 = 98$ students left for two positions – president and vice president.

The two can be selected in ${}^{98}C_2$ ways and can be arranged in 2 ways i.e. the first person selected can be either the president or the vice president.

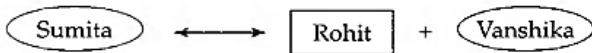
Hence this can be done $2 \times {}^{98}C_2$ ways. (OR we can say that the president can be selected in ${}^{98}C_1$ (i.e. 98 ways) and vice president can be selected in $({}^{98-1}C_1)$ ways = 97 ways).

Total ways of selection = $40 \times 60 \times 98 \times 97 = 22814400$

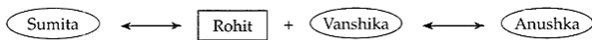
27. Rohit is the husband of Vanshika



Sumita is the sister of Rohit



Anushka is the sister of Vanshika



So Anushka is Rohit's wife's sister

Anushka is the sister-in-law of Rohit.

28. We observe that:

$$(L.H.L \text{ at } x = \frac{1}{2}) = \lim_{x \rightarrow 1/2^-} f(x) = \lim_{x \rightarrow \frac{1}{2}} (\frac{1}{2} - x) \left[\because f(x) = \frac{1}{2} - x \text{ for } 0 \leq x < \frac{1}{2} \right]$$

$$= \frac{1}{2} - \frac{1}{2} = 0 \text{ [Using direct substitution method]}$$

$$\text{and, (R.H.L at } x = 1/2) = \lim_{x \rightarrow 1/2^+} f(x) = \lim_{x \rightarrow 1/2} (3/2 - x) \dots \left[\because f(x) = \frac{3}{2} - x \text{ for } \frac{1}{2} < x \leq 1 \right]$$

$$= \frac{3}{2} - \frac{1}{2} = 1 \dots [\text{Using direct substitution method}]$$

$$\text{Clearly, } \lim_{x \rightarrow 1/2^-} f(x) \neq \lim_{x \rightarrow 1/2^+} f(x)$$

Hence, $f(x)$ is not continuous at $x = \frac{1}{2}$. Clearly, $f(x)$ has discontinuity of first kind at $x = \frac{1}{2}$.

29. Here, the consumption of gas is given to be 78 SCM for 60 days.

According to the given tariff plan:

$$\text{Gas consumption charges} = ₹[(45 \times 29.50) + (33 \times 42.61)]$$

$$= ₹(1327.50 + 1406.13) = ₹2733.63$$

The minimum charges is 10% of gas consumption charges

$$\therefore \text{Minimum charges} = 10\% \text{ of } ₹2733.63$$

$$= ₹ 273.36$$

$$\therefore \text{Bimonthly PNG bill of the family} = \text{Gas consumption charges} + \text{Minimum charges}$$

$$= ₹ 2733.63 + ₹ 273.36$$

$$= ₹ 3006.99$$

30. Given $R = ₹ 1000$, $i = \frac{12}{100} = 0.12$ and $n = 3$

$$P = R \left[\frac{1 - (1 + i)^{-n}}{i} \right] \Rightarrow P = 1000 \left[\frac{1 - (1 + 0.12)^{-3}}{0.12} \right]$$

$$\Rightarrow P = \frac{1000}{0.12} [1 - (1.12)^{-3}] \dots (i)$$

$$\text{Let } x = (1.12)^{-3}$$

Taking log on both sides, we get

$$\log x = -3 \log 1.12 = -3 \times 0.0492 = -0.1476$$

$$\Rightarrow x = \text{antilog}(-0.1476) = \text{antilog}(1.8524)$$

$$\Rightarrow x = 0.7119$$

Substituting the value of x in equation (i), we get

$$P = \frac{1000}{0.12} [1 - 0.7119] = \frac{1000}{0.12} \times 0.2881 = 2400.83$$

31. Let F and S denote the no. of students who know French and Spanish, respectively.

$$\text{Given, } n(F) = 225, n(S) = 100, n(F \cap S) = 45$$

Using identity,

$$\begin{aligned} n(F \cup S) &= n(F) + n(S) - n(F \cap S) \\ &= 225 + 100 - 45 \\ &= 325 - 45 \\ &= 280 \end{aligned}$$

Section D

$$\begin{aligned} 32. & 7 + 7.7 + 7.77 + 7.777 + \dots + 30 \text{ terms} \\ &= 7 [1 + 1.1 + 1.11 + 1.111 + \dots 30 \text{ terms}] \\ &= \frac{7}{9} [9 + 9.9 + 9.99 + 9.999 + \dots 30 \text{ terms}] \\ &= \frac{7}{9} [(10 - 1) + (10 - 0.1) + (10 - 0.01) + (10 - 0.001) + \dots 30 \text{ terms}] \\ &= \frac{7}{9} [10 \times 30 - \{1 + 0.1 + 0.01 + 0.001 + \dots 30 \text{ terms}\}] \\ &= \frac{7}{9} [300 - \frac{1\{1 - (0.1)^{30}\}}{1 - 0.1}] \\ &= \frac{7}{9} [300 - \frac{10}{9} \{1 - (0.1)^{30}\}] \end{aligned}$$

OR

Here, it is given: Arithmetic mean is twice of geometric mean.

By using Formula:

$$\text{Arithmetic mean between } a \text{ and } b = \frac{a+b}{2}$$

$$\text{Geometric mean between } a \text{ and } b = \sqrt{ab}$$

$$AM = 2(GM)$$

$$\frac{a+b}{2} = 2\sqrt{ab}$$

$$a + b = 4\sqrt{ab}$$

Squaring both side, we get

$$\Rightarrow (a + b)^2 = 16ab \dots\dots(i)$$

$$\text{We know that } (a - b)^2 = (a + b)^2 - 4ab$$

From eqn.(i)

$$\Rightarrow (a - b)^2 = 16ab - 4ab$$

$$\Rightarrow (a - b)^2 = 12ab \dots\dots(ii)$$

Dividing eq.. (i) and (ii), we get

$$\frac{(a+b)^2}{(a-b)^2} = \frac{16ab}{12ab}$$

Taking square root both side

$$\Rightarrow \frac{a+b}{a-b} = \frac{2}{\sqrt{3}}$$

Applying componendo and dividendo

$$\frac{a+b+a-b}{a+b-a+b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

Hence proved.

$$33. \text{ Let } f = g + h, \text{ then } g(x) = \frac{1}{\log(1-x)} \text{ and } h(x) = \sqrt{x+3}.$$

For D_g , $g(x)$ must be a real number

$$\Rightarrow \frac{1}{\log(1-x)} \text{ must be a real number}$$

$$\Rightarrow 1 - x > 0 \text{ and } 1 - x \neq 1 \Rightarrow x < 1 \text{ and } x \neq 0$$

$$\Rightarrow D_g = (-\infty, 0) \cup (0, 1).$$

For D_h , $h(x)$ must be a real number

$$\Rightarrow \sqrt{x+3} \text{ must be a real number} \Rightarrow x + 3 \geq 0$$

$$\Rightarrow x \geq -3$$

$$\Rightarrow D_h = [-3, \infty)$$

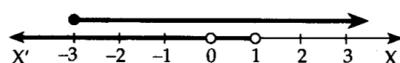


Fig. 9.19.

as $f = g + h$, so $D_f = D_g \cap D_h = [-3, 0) \cup (0, 1).$

34. Table for mean and standard deviation

Here A = 35 (say), h = 10

CI	x_i	f_i	$d_i = \frac{x_i - 65}{10}$	$f_i d_i$	$f_i d_i^2$
60 - 70	65	3	3	9	27
50 - 60	55	6	2	12	24
40 - 50	45	10	1	10	10
30 - 40	35	12	0	0	0
20 - 30	25	15	-1	-15	15
10 - 20	15	6	-2	-12	24
		$\sum_{i=1}^6 f_i = 52$		$\sum_{i=1}^6 f_i d_i = 4$	$\sum_{i=1}^6 f_i d_i^2 = 100$

$$\text{Mean} = A + \frac{\sum_{i=1}^6 f_i d_i}{\sum_{i=1}^6 f_i} \times h = 35 + \frac{4}{52} \times 10 = 35 + 0.77 = 35.77$$

$$\sigma = 10 \sqrt{\frac{100}{52} - \left(\frac{4}{52}\right)^2} = 10 \sqrt{1.92 - 0.0059} = 10 \sqrt{1.9141} = 10 \times 1.384 = 13.84$$

$$\therefore \text{Coefficient of variation} = \frac{\sigma}{\text{Mean}} \times 100 = \frac{13.84}{35.77} \times 100 = 38.69$$

OR

The given frequency distribution is a cumulative frequency distribution. This can be written as a frequency distribution as follows:

Profits: (₹ in lacs)	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
No. of companies:	8	12	30	14	6

We shall now prepare a table for the computation of mean, mode and standard deviation as given below:

Profits (₹ in lacs)	No. of companies f_i	x_i	$u_i = \frac{x_i - 50}{20}$ A = 50, h = 20	$f_i u_i$	$f_i u_i^2$
0 - 20	8	10	-2	-16	32
20 - 40	12	30	-1	-12	12
40 - 60	30	50	0	0	0
60 - 80	14	70	1	14	14
80 - 100	6	90	2	12	24
		N = 70		$\Sigma f_i u_i = -2$	$\Sigma f_i u_i^2 = 82$

Computation of Mean: We have, N = 70, $\Sigma f_i u_i = -2$, A = 50, and h = 20

$$\therefore \bar{X} = A + h \left(\frac{\Sigma f_i u_i}{N} \right) \Rightarrow \bar{X} = 50 + 20 \times \frac{-2}{70} = 49.42$$

Computation of S.D.: We have,

N = 70, $\Sigma f_i u_i = -2$, $\Sigma f_i u_i^2 = 82$, A = 50 and h = 20

$$\therefore \sigma = h \sqrt{\frac{1}{N} \Sigma f_i u_i^2 - \left(\frac{1}{N} \Sigma f_i u_i \right)^2} = 20 \sqrt{\frac{82}{70} - \left(\frac{-2}{70} \right)^2} = 20 \sqrt{\frac{82 \times 70 - 4}{70^2}}$$

$$\Rightarrow \sigma = 20 \times \frac{\sqrt{5736}}{70} = \frac{20 \times 75.73}{70} = 21.63$$

Computation of Mode: Since the maximum frequency '30' is of class 40-60. So, 40-60 is the modal class such that l = 40, h = 20, f = 30, $f_1 = 12$ and $f_2 = 14$.

Now,

$$\text{Mode} = l + \frac{f - f_1}{2f - f_1 - f_2} \times h$$

$$\Rightarrow \text{Mode} = 40 + \frac{30 - 12}{60 - 12 - 14} \times 20 = 40 + \frac{18}{34} \times 20 = 40 + 10.58 = 50.58$$

Compilation of Karl Pearson's coefficient of skewness: We have,

$$S_{kp} = \frac{\text{Mean} - \text{Mode}}{\text{S.D.}} = \frac{49.42 - 50.58}{21.63} = -0.0536$$

35. The cost price of the TV which the retailer pays to the manufacturer = ₹25000

i. As the retailer marks the price of TV 20% above his cost price,

$$\therefore \text{the marked price of the TV} = ₹\left(1 + \frac{20}{100}\right) \times 25000$$

$$= ₹\left(\frac{120}{100} \times 25000\right) = ₹30000$$

ii. As the sales are intra-state and the rate of GST is 12%, so GST comprises of CGST at 6% and SGST at 6%

As the retailer sells the TV to a consumer at 10% discount on the marked price, selling price of the TV by the retailer

$$= ₹\left(1 - \frac{10}{100}\right) \times 30000 = ₹\left(\frac{9}{10} \times 30000\right) = ₹27000$$

Amount of GST collected by retailer from consumer (or paid by consumer to retailer):

$$\text{CGST} = 6\% \text{ of } ₹27000 = ₹\left(\frac{6}{100} \times 27000\right) = ₹1620$$

$$\text{SGST} = 6\% \text{ of } ₹27000 = ₹1620$$

\therefore Consumer's cost price of TV inclusive of tax (under GST)

= Cost price of TV to consumer + GST paid by consumer

= ₹27000 + CGST paid by consumer + SGST paid by consumer

$$= ₹27000 + ₹1620 = ₹28620$$

$$= ₹30240$$

iii. Amount of GST collected by manufacturer from retailer:

$$\text{CGST} = 6\% \text{ of } ₹25000 = ₹\left(\frac{6}{100} \times 25000\right) = ₹1500$$

$$\text{SGST} = 6\% \text{ of } ₹25000 = ₹1500$$

Amount of input GST of the retailer:

$$\text{CGST} = ₹1500, \text{SGST} = ₹1500$$

Amount of output GST of the retailer:

$$\text{CGST} = ₹1620, \text{SGST} = ₹1620$$

GST paid by the retailer to the Central Government

= Output CGST - input CGST

$$= ₹1620 - ₹1500 = ₹120$$

GST paid by the retailer to the State Government

= Output SGST - input SGST

$$= ₹1620 - ₹1500 = ₹120$$

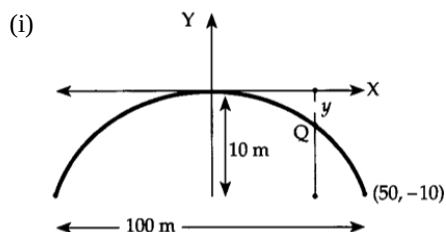
\therefore Total GST paid by the retailer to the Central and State Governments

$$= ₹120 + ₹120 = ₹240$$

Section E

36. Read the text carefully and answer the questions:

The girder of a railway bridge is a parabola with its vertex at the highest point, 10 metres above the ends and span is 100 metres.



The equation of the parabola in standard form is $x^2 = -4ay$.

Coordinates of one end of the girder is (50, -10).

As it lies on parabola, $\therefore (50)^2 = -4a(-10)$

$$\Rightarrow a = \frac{125}{2}$$

$$\therefore \text{Parabola is } x^2 = -4 \times \frac{125}{2}y \Rightarrow x^2 = -250y$$

(ii) Let Q be a point as the girder which is 20 m from the middle. Let the coordinates of Q be (20, -y).

As it lies on the parabola,

$$(20)^2 = -250(y)$$

$$\Rightarrow y = -\frac{20^2}{250} = -\frac{400}{250} = -\frac{8}{5} = -1.6$$

$$\therefore \text{Height} = 10 - |y| = 10 - 1.6 = 8.4\text{m}$$

(iii) Here, $a = \frac{125}{2}$ and focus of $x^2 = -4ay$ is (0, -a),

$$\therefore \text{Focus is } \left(0, -\frac{125}{2}\right)$$

OR

The length of latus-rectum = $4a$ units

$$= 4 \times \frac{125}{2} = 250m$$

37. Read the text carefully and answer the questions:

Data of all the previous cricket matches are stored to analyze the average batting score of various batsmen. The scores of a batsman in ten innings are:

38, 70, 48, 34, 42, 55, 63, 46, 54, 44



- (i) Arranging the data in ascending order = 34, 38, 42, 44, 46, 48, 54, 55, 63, 70

Median = A.M. of 5th and 6th observation

$$= \frac{46+48}{2} = 47$$

(ii)

x_i	$ d_i = x_i - 47 $
38	9
70	23
48	1
34	13
42	5
55	8
63	16
46	1
54	7
44	3
Total	$\sum d_i = 86$

$$\text{Mean Deviation} = \frac{1}{n} \times \sum |d_i| = \frac{86}{10} = 8.6$$

- (iii) Sum of new scores = 564

$$\text{New mean} = \frac{564}{10} = 56.4$$

OR

It is called range of the data.

38. Read the text carefully and answer the questions:

In a survey of 200 students it was found that 120 had got grade A in Mathematics, 90 had got grade A in Physics and 70 had got grade A in Chemistry, 50 had got grade A in Mathematics and Chemistry, 40 had got grade A in Mathematics and Physics, 30 had got grade A in Physics and Chemistry and 10 had got grade A in all three subjects one student is chosen randomly as a class representative.

- (i) Number of students who had got grade A in at least one of the subject

$$= 40 + 30 + 10 + 40 + 20 + 30 = 170$$

P (grade 'A' in at least one of the subject)

$$= \frac{170}{200} = \frac{17}{20}$$

- (ii) Number of students who had got grade A in at least two of the subjects

$$= 40 + 30 + 10 + 20 = 100$$

P (grade 'A' in at least two of the subjects)

$$= \frac{100}{200} = \frac{1}{2}$$

- (iii) Number of students who had got grade A in Mathematics or Chemistry = 140

P (grade 'A' in mathematics or chemistry)

$$= \frac{100}{200} = \frac{7}{10}$$

OR

Read the text carefully and answer the questions:

A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0.65. The probability that many are not present and still the work gets completed on time is 0.35. The probability that work will be completed on time when all workers are present is 0.80.

Let: E_1 : represent the event when many workers were not present for the job;

E_2 : represent the event when all workers were present; and

E: represent completing the construction work on time.

$$(i) P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$$

$$\begin{aligned} (ii) P(E) &= P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right) \\ &= 0.65 \times 0.35 + 0.35 \times 0.8 \\ &= 0.35 \times 1.45 \\ &= 0.51 \end{aligned}$$

$$(iii) P\left(\frac{E_1}{E}\right) = \frac{P(E_1) \cdot P\left(\frac{E}{E_1}\right)}{P(E_1) \cdot P\left(\frac{E}{E_1}\right) + P(E_2) \cdot P\left(\frac{E}{E_2}\right)} = \frac{0.65 \times 0.35}{0.51} = 0.45$$