

CBSE Test Paper 02
Chapter 13 Probability

1. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$. find $P(B|A)$
 - a. $\frac{2}{3}$
 - b. $\frac{3}{5}$
 - c. $\frac{4}{5}$
 - d. $\frac{1}{3}$
2. Let A and B be independent events with $P(A)=0.3$ and $P(B)=0.4$. Find $P(A \cap B)$
 - a. 0.15
 - b. 0.10
 - c. 0.14
 - d. 0.12
3. Find the mean number of heads in three tosses of a fair coin.
 - a. 1.2
 - b. 1.4
 - c. 1.5
 - d. 1.0
4. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?
 - a. $X = 6, 5, 2, 0$
 - b. $X = 6, 4, 2, 0$
 - c. $X = 6, 3, 2, 1$
 - d. $X = 6, 4, 2, 1$
5. If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, Find $P(\text{not } A \text{ and not } B)$.
 - a. $\frac{1}{8}$
 - b. $\frac{2}{35}$
 - c. $\frac{3}{35}$
 - d. $\frac{3}{5}$
6. In an experiment, an outcome having highest probability is called _____ outcome.
7. If X follows binomial distribution with parameters $n = 5$, p and $P(X = 2) = 9$, $P(X =$

- 3), then $p = \underline{\hspace{2cm}}$.
8. The probability of drawing two clubs from a well shuffled pack of 52 cards is $\underline{\hspace{2cm}}$.
9. Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?
10. The random variable X can take only the values of 0, 1, 2. Give that $P(X = 0) = P(X = 1) = p$ and that $E(X^2) = E[X]$, find the value of p.
11. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that:
- both balls are red.
 - first ball is black and second is red.
 - one of them is black and other is red.
12. Suppose 10,000 tickets are sold in a lottery each for Re 1. For first position there is prize of Rs 3000. For second position, there is prize of Rs. 2000. For third position there are three prizes each of Rs 500. If you buy one ticket, what is your expectation?
13. A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event 'number is even' and B be the event 'number is red'. Are A and B independent?
14. Determine $P(E|F)$: A dice is thrown three times. E : 4 appears on the third toss, F : 6 and 5 appears respectively on first two tosses.
15. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other mean of transport are respectively $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{12}$ if he comes by train, bus and scooter respectively, but he comes by other means of transport, that he will not be late. When he arrives he is late. What is the probability that he comes by train.
16. Two biased dice are thrown together. For the first die $P(6) = \frac{1}{2}$, the other scores being equally likely while for the second die, $P(1) = \frac{2}{5}$ and the other scores are equally likely. Find the probability distribution of 'the number of one seen'.
17. A black and a red die are rolled. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.
18. From a lot of 15 bulbs which include 5 defectives, a sample of 4 bulbs is drawn one by one with replacement. Find the probability distribution of number of defective bulbs. Hence, find the mean of the distribution.

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Solution

1. a. $\frac{2}{3}$

Explanation: If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \frac{7}{11} = \frac{6}{11} + \frac{5}{11} - P(A \cap B)$$

$$\Rightarrow P(A \cap B) = \frac{4}{11}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{4/11}{6/11} = \frac{2}{3}$$

2. d. 0.12

Explanation: Let A and B be independent events with $P(A) = 0.3$ and $P(B) = 0.4$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$\Rightarrow P(A \cap B) = 0.3 \times 0.4 = 0.12$$

3. c. 1.5

Explanation: Let X is the random variable of “number of heads “ $X = 0, 1, 2, 3$.

$$P(X=0) = P(\bar{H}\bar{H}\bar{H}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

$$P(X=1) = P(H\bar{H}\bar{H} \text{ or } \bar{H}H\bar{H} \text{ or } \bar{H}\bar{H}H) = 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$$

$$P(X=2) = P(HH\bar{H} \text{ or } H\bar{H}H \text{ or } \bar{H}HH) = 3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{8}$$

$$P(X=3) = P(HHH) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

Therefore, the probability distribution is:

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Therefore, Mean of Heads is :

$$E(X) = \sum_{i=1}^n X_i P(X_i) = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = \frac{12}{8} = 1.5$$

4. b. $X = 6, 4, 2, 0$

Explanation: Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. When a coin is tossed 6 times, we have 64 outcomes which consists of : (i) 6 heads and 0 tails (ii) 5 heads and 1 tail (iii) 4 heads and 2 tails (iv) 3 heads and 3 tails (v) 2 heads

and 4 tails (vi) 1 head and 5 tails (vii) 0 head and 6 tails . Let X represents the difference between the number of heads and tails.

1. $\Rightarrow X = 6 - 0 = 6$
2. $\Rightarrow X = 5 - 1 = 4$
3. $\Rightarrow X = 4 - 2 = 2$
4. $\Rightarrow X = 3 - 3 = 0$
5. $\Rightarrow X = 4 - 2 = 2$
6. $\Rightarrow X = 5 - 1 = 4$
7. $\Rightarrow X = 6 - 0 = 6$.

Therefore , $X = 6 , 4 , 2 , 0$.

5. d. $\frac{3}{8}$

Explanation: Since A and B are independent events, not A and not B are also independent events.

$$P(\overline{A}\overline{B}) = P(\overline{A})P(\overline{B}) = (1 - \frac{1}{4})(1 - \frac{1}{2}) = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

6. most likely

7. $\frac{1}{10}$
8. $\frac{1}{17}$

9. Given, $P(E) = \frac{3}{5}$ and $P(F) = \frac{3}{10}$

$$\therefore P(E \cap F) = \frac{1}{5}$$

$$\text{Now } P(E) \cdot P(F) = \frac{3}{5} \times \frac{3}{10} = \frac{9}{50}$$

$$\Rightarrow P(E \cap F) \neq P(E) \cdot P(F)$$

Therefore, E and F are not independent events.

10. Since, $X = 0, 1, 2$ and $P(X)$ at $X = 0$ and 1 is p , let at $X = 2$, $P(X)$ is x .

$$\Rightarrow p + p + x = 1$$

$$\Rightarrow x = 1 - 2p$$

We get, the following distribution.

X	0	1	2
P(X)	p	p	1 - 2p

$$\therefore E(X) = \sum XP(X)$$

$$\begin{aligned}
&= 0 \cdot p + 1 \cdot p + 2(1 - 2p) \\
&= p + 2 - 4p = 2 - 3p \\
\text{And } E(X^2) &= \sum X^2 P(X) \\
&= 0 \cdot p + 1 \cdot p + 4 \cdot (1 - 2p) \\
&= p + 4 - 8p = 4 - 7p \\
\text{Also, given that } E(X^2) &= E[X] \\
\Rightarrow 4 - 7p &= 2 - 3p \\
\Rightarrow 4p &= 2 \Rightarrow p = \frac{1}{2}
\end{aligned}$$

11. S = (10 black balls, 8 red balls) $\Rightarrow n(S) = 18$

Let drawing of a red ball be a success.

$\therefore A = \{8 \text{ red balls}\} \Rightarrow n(A) = 8$

$$P(A) = \frac{n(A)}{n(S)} = \frac{8}{18} = \frac{4}{9}$$

$$\text{And } P(\bar{A}) = 1 - P(A) = 1 - \frac{4}{9} = \frac{5}{9}$$

- i. P (both are red ball) = P (A). P (B) = $\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$
- ii. P (first is black ball and second is red) = $P(\bar{A}) \cdot P(A) = \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}$
- iii. P (one of them is black and other is red) = $P(\bar{A}) \cdot P(A) + P(A) \cdot P(\bar{A}) = \frac{5}{9} \times \frac{4}{9} + \frac{4}{9} \times \frac{5}{9} = \frac{40}{81}$

12. Let x is the random variable for the prize.

X	0	500	2000	3000
P(X)	$\frac{9995}{10000}$	$\frac{3}{10000}$	$\frac{1}{10000}$	$\frac{1}{10000}$

$$\begin{aligned}
\text{Since, } E(X) &= \sum XP(X) \\
\therefore E(X) &= 0 \times \frac{9995}{10000} + \frac{1500}{10000} + \frac{2000}{10000} + \frac{3000}{10000} \\
&= \frac{1500+2000+3000}{10000} \\
&= \frac{6500}{10000} = \frac{13}{20} = \text{Rs } 0.65
\end{aligned}$$

13. S = $\left\{ \frac{1,2,3}{\text{red}} \frac{4,5,6}{\text{green}} \right\} \Rightarrow n(s) = 6$

The number is even = A = {2, 4, 6} $\Rightarrow n(A) = 3$

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

The number is red = B = $\left\{ \frac{1,2,3}{red} \right\} \Rightarrow n(B) = 3$

$$P(B) = \frac{n(B)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

$$(A \cap B) = \{2\} \Rightarrow n(A \cap B) = 1$$

$$\therefore \frac{n(A \cap B)}{n(S)} = \frac{1}{6}$$

$$\Rightarrow P(A \cap B) = \frac{1}{6}$$

$$\text{Now } P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Therefore, $P(A \cap B) \neq P(A) \cdot P(B)$, i.e., A and B are not independent.

14. Since a dice has six faces. Therefore $n(S) = 6 \times 6 \times 6 = 216$

$$E = (1, 2, 3, 4, 5, 6) \times (1, 2, 3, 4, 5, 6) \times (4)$$

$$F = (6) \times (5) \times (1, 2, 3, 4, 5, 6)$$

$$\Rightarrow n(F) = 1 \times 1 \times 6 = 6$$

$$P(F) = \frac{n(F)}{n(S)} = \frac{6}{216}$$

$$\therefore E \cap F = (6, 5, 4)$$

$$n(E \cap F) = 1$$

$$\therefore P(E \cap F) = \frac{n(E \cap F)}{n(S)} = \frac{1}{216}$$

$$\text{And } P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/216}{6/216} = \frac{1}{6}$$

15. let E be the event that the doctor visits the patient late and let T_1, T_2, T_3, T_4 , be the event that the doctor comes by train, bus, scooter and other means of Transport respectively.

$$P(T_1) = \frac{3}{10}, P(T_2) = \frac{1}{5}, P(T_3) = \frac{1}{10}, P(T_4) = \frac{2}{5}$$

$$P\left(\frac{E}{T_1}\right) = \frac{1}{4}, P\left(\frac{E}{T_2}\right) = \frac{1}{3}, P\left(\frac{E}{T_3}\right) = \frac{1}{12}, P\left(\frac{E}{T_4}\right) = 0$$

$$P\left(\frac{T_1}{E}\right) = \frac{P(T_1)P\left(\frac{E}{T_1}\right)}{P(T_1)P\left(\frac{E}{T_1}\right) + P(T_2)P\left(\frac{E}{T_2}\right) + P(T_3)P\left(\frac{E}{T_3}\right) + P(T_4)P\left(\frac{E}{T_4}\right)}$$

$$= \frac{\frac{3}{10} \times \frac{1}{4}}{\frac{3}{10} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{3} + \frac{1}{10} \times \frac{1}{12} + \frac{2}{5} \times 0} = \frac{1}{2}$$

16. For first die, $P(6) = \frac{1}{2}$ and $P(6') = \frac{1}{2}$

$$\Rightarrow P(1) + P(2) + P(3) + P(4) + P(5) = \frac{1}{2}$$

$$\Rightarrow P(1) = \frac{1}{10} \text{ and } P(1') = \frac{9}{10} [\because P(1) = P(2) = P(3) = P(4) = P(5)]$$

For second die, $P(1) = \frac{2}{5}$ and $P(1) = 1 - \frac{2}{5} = \frac{3}{5}$

Let X = Number of one's seen

For X = 0, $P(X = 0) = P(1') \cdot P(1') = \frac{9}{10} \cdot \frac{3}{5} = \frac{27}{50} = 0.54$

$P(X = 1) = P(1') \cdot P(1) + P(1) \cdot P(1) = \frac{9}{10} \cdot \frac{2}{5} + \frac{1}{10} \cdot \frac{3}{5}$
 $= \frac{18}{50} + \frac{3}{50} = \frac{21}{50} = 0.42$

$P(X = 2) = P(1) \cdot P(1) = \frac{1}{10} \cdot \frac{2}{5} = \frac{2}{50} = 0.04$

Hence, the required probability distribution is as below.

X	0	1	2
P(X)	0.54	0.42	0.04

17. Let the first observation be from the black die and second from the red die.

When two dice (one black die and another red) are rolled, the sample space S has $6 \times 6 = 36$ number of elements.

Let A : obtaining a sum greater than 9 = {4,6),(5,5),(5,6),(6,4),(6,5),(6,6)}

and B: black die resulted in a 5 = {(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)}

$\therefore A \cap B = \{(5,5),(5,6)\}$

The conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5, is given by $P(A/B)$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{2}{6} = \frac{1}{3}$$

18. It is given that out of 15 bulbs, 5 are defective.

\therefore Number of non-defective bulbs

$$= 15 - 5 = 10$$

4 bulbs are drawn from a lot with replacement.

Let p = P (drawn defective bulb)

$$= \frac{5}{15} = \frac{1}{3}$$

and q = P (drawn non-defective bulb)

$$= \frac{10}{15} = \frac{2}{3}$$

Let X be the random variable which denotes the defective bulbs. So, X may take values 0, 1, 2, 3 or 4.

Using binomial distribution, we have

$$P(X = r) = {}^nC_r (p)^r (q)^{n-r}$$

Therefore,

$$P(X = 0) = {}^4C_0 p^0 q^4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

$$P(X = 1) = {}^4C_1 p^1 q^3 = 4 \left(\frac{1}{3}\right) \left(\frac{2}{3}\right)^3 = \frac{32}{81}$$

$$P(X = 2) = {}^4C_2 p^2 q^2 = 6 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 = \frac{24}{81}$$

$$P(X = 3) = {}^4C_3 p^3 q^1 = 4 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right) = \frac{8}{81}$$

$$\text{and } P(X = 4) = {}^4C_4 p^4 q^0 = 1 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^0 = \frac{1}{81}$$

Therefore, the required probability distribution is as follows

X	P(X)	X P(X)
0	$\frac{16}{81}$	0
1	$\frac{32}{81}$	$\frac{32}{81}$
2	$\frac{24}{81}$	$\frac{48}{81}$
3	$\frac{8}{81}$	$\frac{24}{81}$
4	$\frac{1}{81}$	$\frac{4}{81}$
		$\Sigma X \cdot P(X) = \frac{108}{81}$

$$\begin{aligned} \therefore \text{Mean} &= \Sigma X \cdot P(X) \\ &= \frac{108}{81} = \frac{12}{9} = \frac{4}{3} = 1.33 \end{aligned}$$