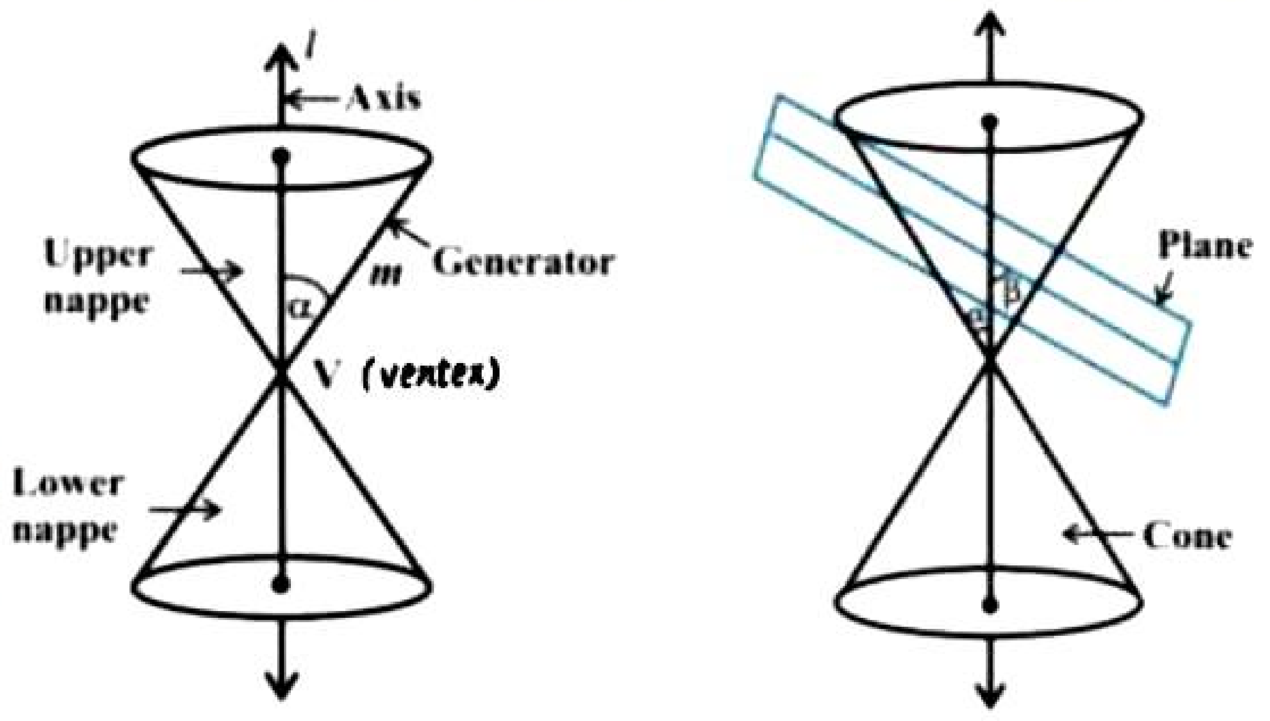
Conic Sections

Sections of a cone :

The intersection of a plane with a cone, the section so obtained is called a conic section.



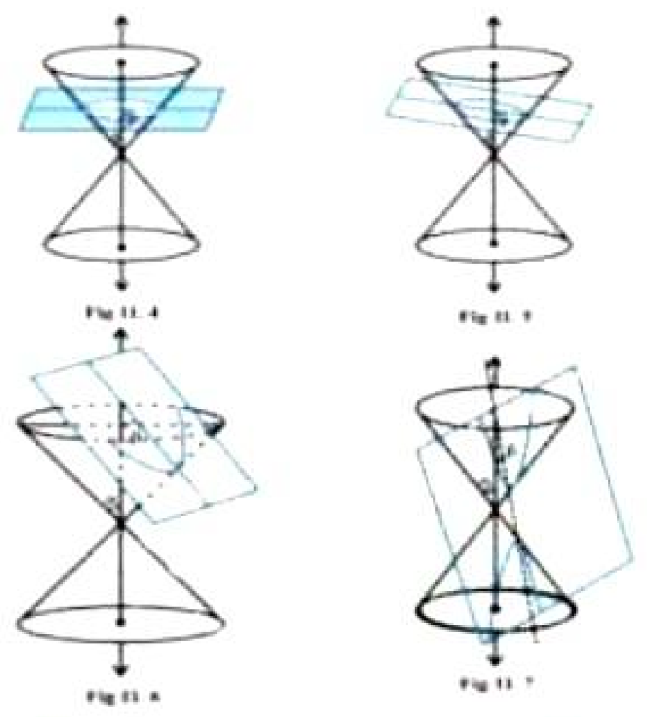
Circle, ellipse, parabola and hyperbola : When the plane cuts nappe of the cone, we have the following situations :

(a) When $\beta = 90^\circ$, the section is a circle.

(b) When $\alpha < \beta < 90^\circ$, the section is an ellipse.

(c) When $\beta = \alpha$; the section is a parabola.

(d) When $0 \leq \beta < \alpha$; the plane cuts through both the nappes and the curves of intersection is a hyperbola.

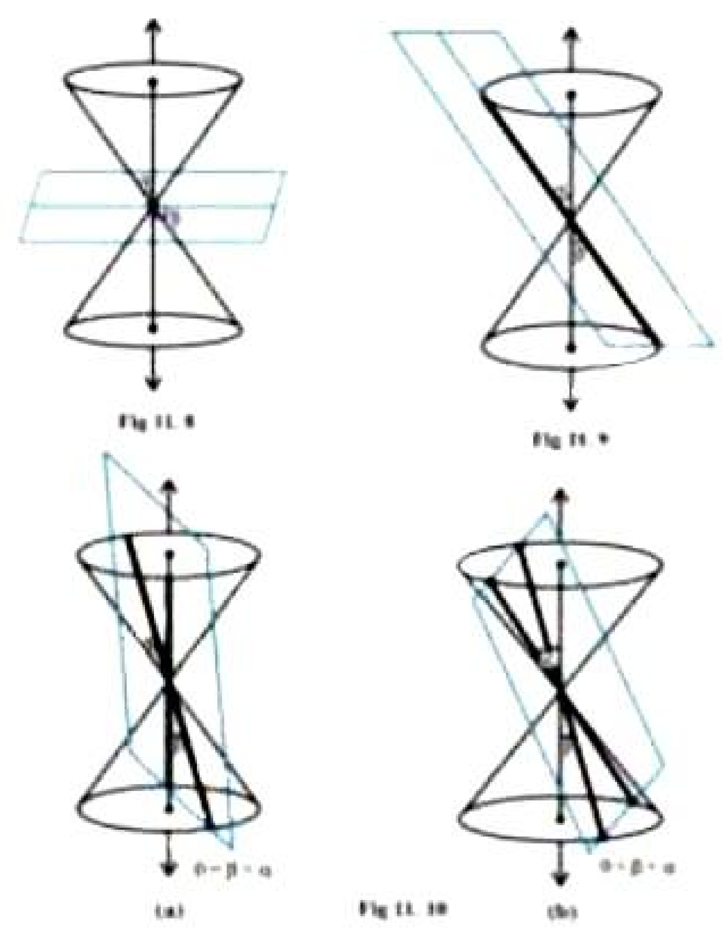


Degenerated conic sections : When the plane cuts at the vertex of the cone, we have the following different cases :

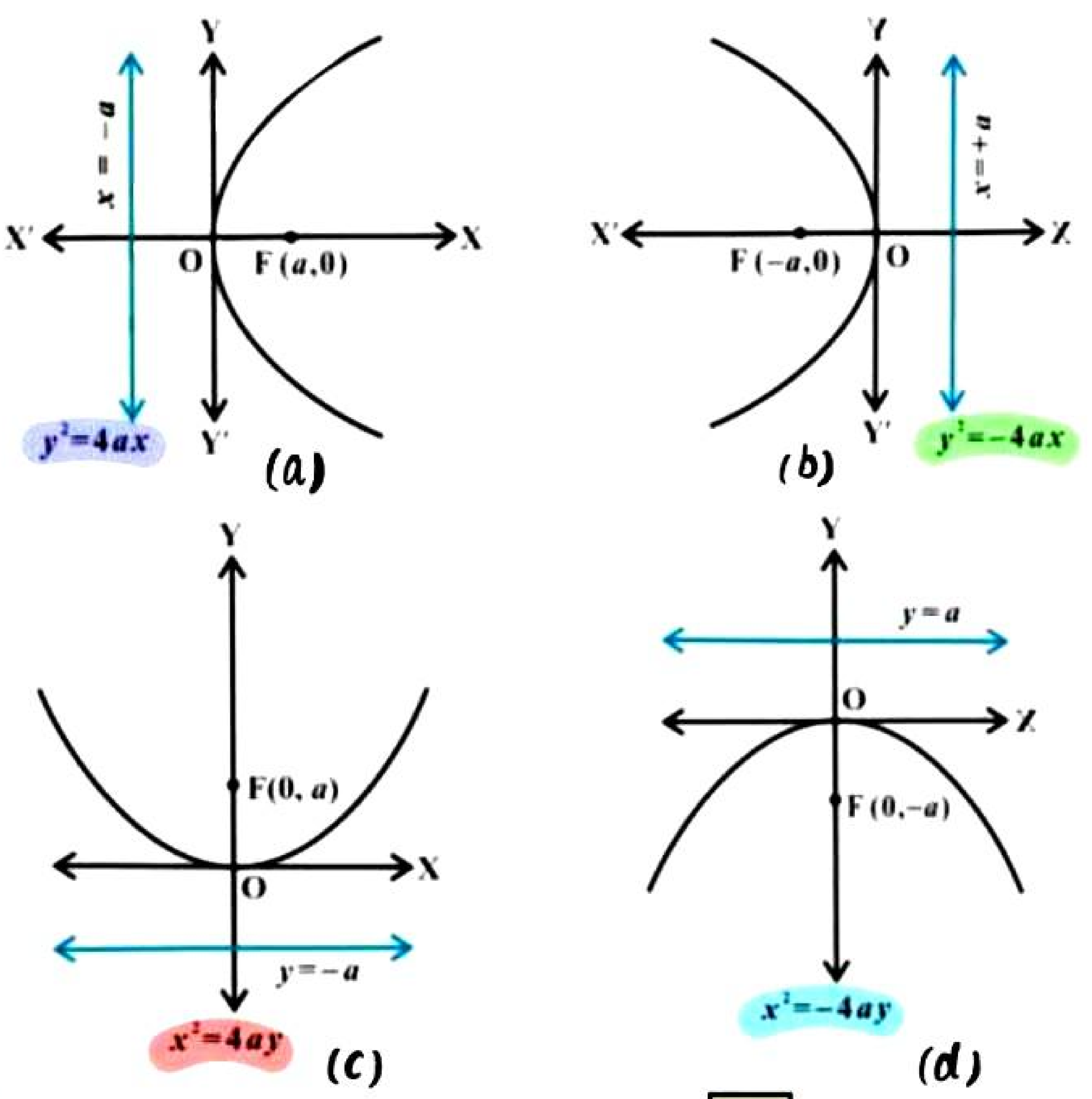
(a) When $\alpha < \beta \leq 90^\circ$, then the section is a point.

(b) When $\beta = \alpha$, the plane contains a generator of the cone and the section is a straight line. It is the degenerated case of a parabola.

(c) When $0 \leq \beta < \alpha$, the section is a pair of intersecting straight lines. It is degenerated case of a hyperbola.



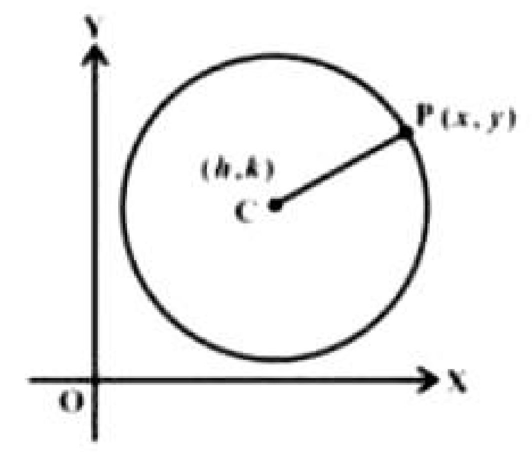
Standard equation of parabola



Circle equation $\boxed{(x-h)^2 + (y-k)^2 = r^2}$

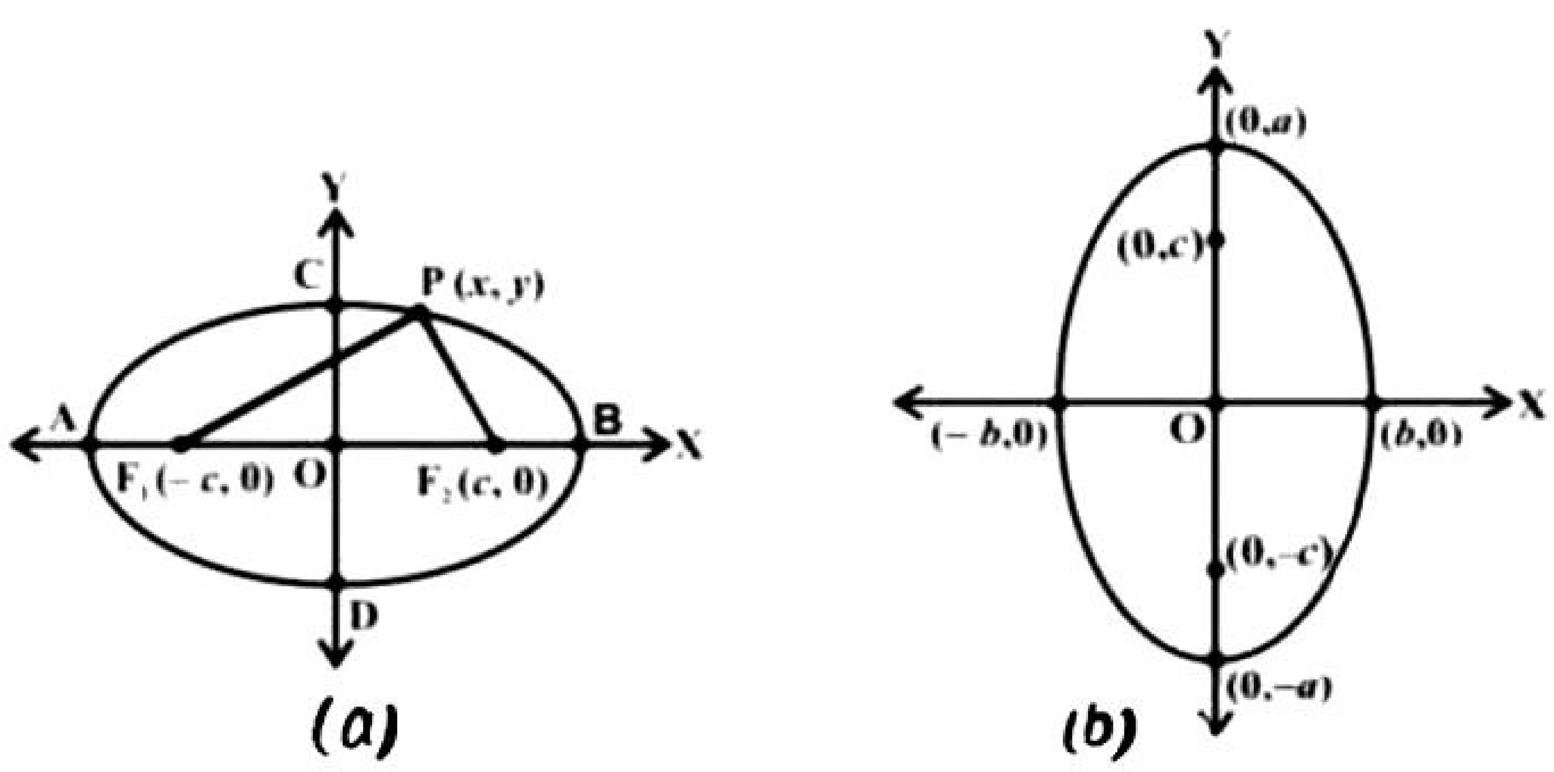
centre at (h, k)

circle radius $= r$



Standard equations of an ellipse

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$



Latus rectum of parabola $\boxed{4a}$

Latus rectum of ellipse $\boxed{\frac{2b^2}{a}}$

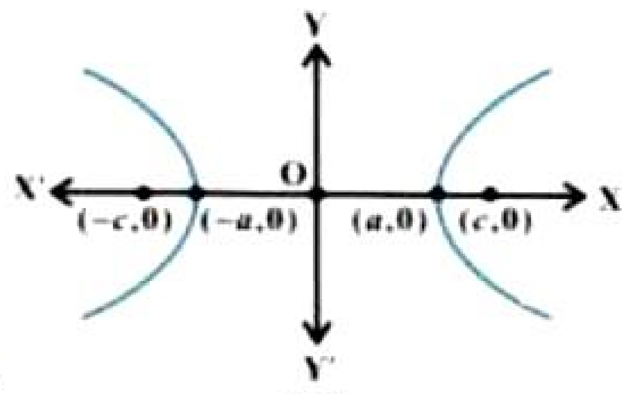
The eccentricity of an ellipse $\boxed{e = \frac{c}{a}}$ distance from the centre

Relationship between semi-major axis, semi minor axis and the distance of the focus from the centre of the ellipse.

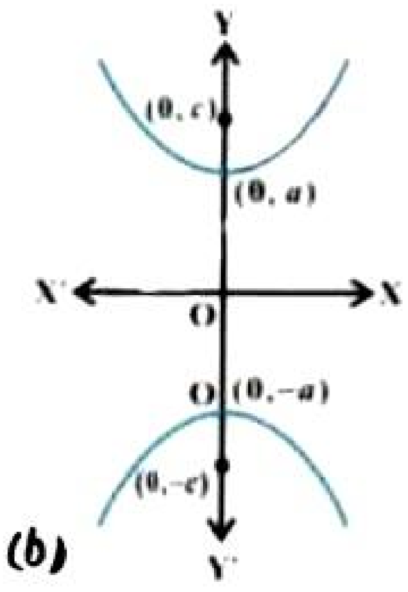
$\boxed{a^2 = b^2 + c^2}$ OR $\boxed{c = \sqrt{a^2 - b^2}}$

Standard equation of hyperbola

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$



(a)



(b)

Latus rectum of hyperbola $\boxed{\frac{2b^2}{a}}$

The eccentricity of an hyperbola $\boxed{e = \frac{c}{a}}$ distance from the centre

Note : A hyperbola in which $a = b$ is called an equilateral hyperbola.