Exercise 2.7

Chapter 2 Derivatives Exercise 2.7 1E

The position of a particle is given by the following function.

$s = f(t) = t^3 - 12t^2 + 36t$.

Where, t is measured in seconds and s in meters.

(a)

The velocity function is the derivative of the position function. Mathematically expressed as follows:

$$\begin{aligned} v(t) &= \frac{ds}{dt} \\ &= \frac{d}{dt}\left(t^3 - 12t^2 + 36t\right) \\ &= 3t^2 - 24t + 36 \end{aligned}$$

Thus, the velocity for the position function $s = t^3 - 12t^2 + 36t$ at time t is

$\boxed{v(t) = 3t^2 - 24t + 36 \text{ ft/s}}$.

(b)

The velocity after 3 s is the instantaneous velocity when $t = 3$. Substitute the values in the velocity function. Solve as follows:

$$\begin{aligned} v(3) &= 27 - 72 + 36 \\ &= -9 \end{aligned}$$

Thus, the velocity of the particle after 3 s is $\boxed{-9 \text{ ft/s}}$.

(c)

The particle is at rest when the velocity $v(t) = 0$.

So, equate the velocity function $v(t) = 3t^2 - 24t + 36$ to zero and solve for t.

$$\begin{aligned} 3t^2 - 24t + 36 &= 0 \\ 3\left(t^2 - 8t + 12\right) &= 0 \\ t^2 - 8t + 12 &= 0 \\ t^2 - 2t - 6t + 12 &= 0 \\ (t-2)(t-6) &= 0 \\ t = 2 \text{ or } t &= 6 \end{aligned}$$

Thus, particle is at rest $\boxed{\text{after 2 s and after 6 s}}$.

(d)

The particle moves in the positive direction when $v(t) > 0$, this is calculated as follows:

$$3t^2 - 24t + 36 > 0$$
$$3(t-2)(t-6) > 0$$

The inequality $3(t-2)(t-6) > 0$ is true when both factors are positive; this gives two different conditions that are calculated as follows:

$$\begin{aligned} t-2 > 0 \quad &\text{and} \quad t-6 > 0 \\ t > 2 \quad &\text{and} \quad t > 6 \end{aligned}$$

The common set of values for *t* that satisfy the inequality $t > 2$ and $t > 6$ together is the inequality $t > 6$.

Also, the inequality $3(t-2)(t-6) > 0$ is true when both factors are negative; this gives two different conditions that are calculated as follows:

$$\begin{aligned} t-2 < 0 \quad &\text{and} \quad t-6 < 0 \\ t < 2 \quad &\text{and} \quad t < 6 \end{aligned}$$

The common set of values for *t* that satisfy the inequality $t < 2$ and $t < 6$ is the inequality $t < 2$.

Thus, the particle moves in the positive direction, in the time intervals $\boxed{t < 2 \text{ and } t > 6}$.

(e)

From the information in part (d), the particle moves in the positive direction in the time intervals $t < 2$ and $t > 6$, and it moves in the negative direction when $2 < t < 6$.

This implies that the particle is moving in the positive direction and in the negative direction. Hence, calculate the distance travelled in the intervals $[0, 2]$, $[2, 6]$, and $[6, 8]$ separately.

The distance traveled during the interval $[0, 2]$ is calculated as follows:

$$\begin{aligned} |f(2) - f(0)| &= \left|(2)^3 - 12(2)^2 + 36(2) - \left((0)^3 - 12(0)^2 + 36(0)\right)\right| \\ &= |8 - 48 + 72 - 0| \\ &= |32| \\ &= 32 \end{aligned}$$

The distance traveled during the interval $[2, 6]$ is calculated as follows:

$$\begin{aligned} |f(6) - f(2)| &= \left|(6)^3 - 12(6)^2 + 36(6) - \left((2)^3 - 12(2)^2 + 36(2)\right)\right| \\ &= |216 - 432 + 216 - (8 - 48 + 72)| \\ &= |0 - 32| \\ &= 32 \end{aligned}$$

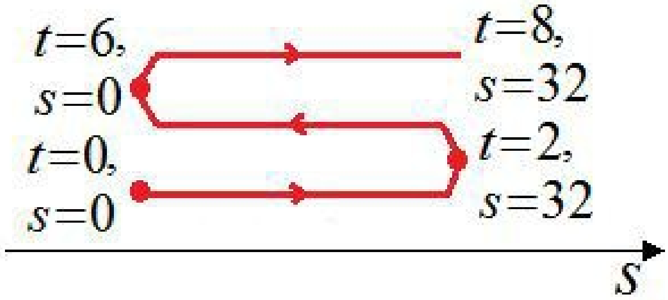
The distance traveled during the interval $[6,\ 8]$ is calculated as follows:

$$\begin{aligned}|f(8)-f(6)| &= \left|(8)^3-12(8)^2+36(8)-\left((6)^3-12(6)^2+36(6)\right)\right| \\ &= |512-768+288-(216-432+216)| \\ &= |32-0| \\ &= 32\end{aligned}$$

Thus, the total distance traveled during the first 8 s is $32+32+32=\boxed{96\text{ ft}}$.

(f)

From the information from parts (d) and (e), make a schematic sketch of the motion of the particle back and forth along a line (the *s*-axis) as follows:



(g)

The acceleration is the second derivative of the position function.

$$\begin{aligned}a(t) &= \frac{d^2s}{dt^2} \\ &= \frac{dv}{dt} \\ &= \frac{d}{dt}\left(3t^2-24t+36\right) \\ &= 6t-24\end{aligned}$$

Thus, the acceleration for the position function $s=t^3-12t^2+36t$ at time t is $\boxed{a(t)=6t-24}$.

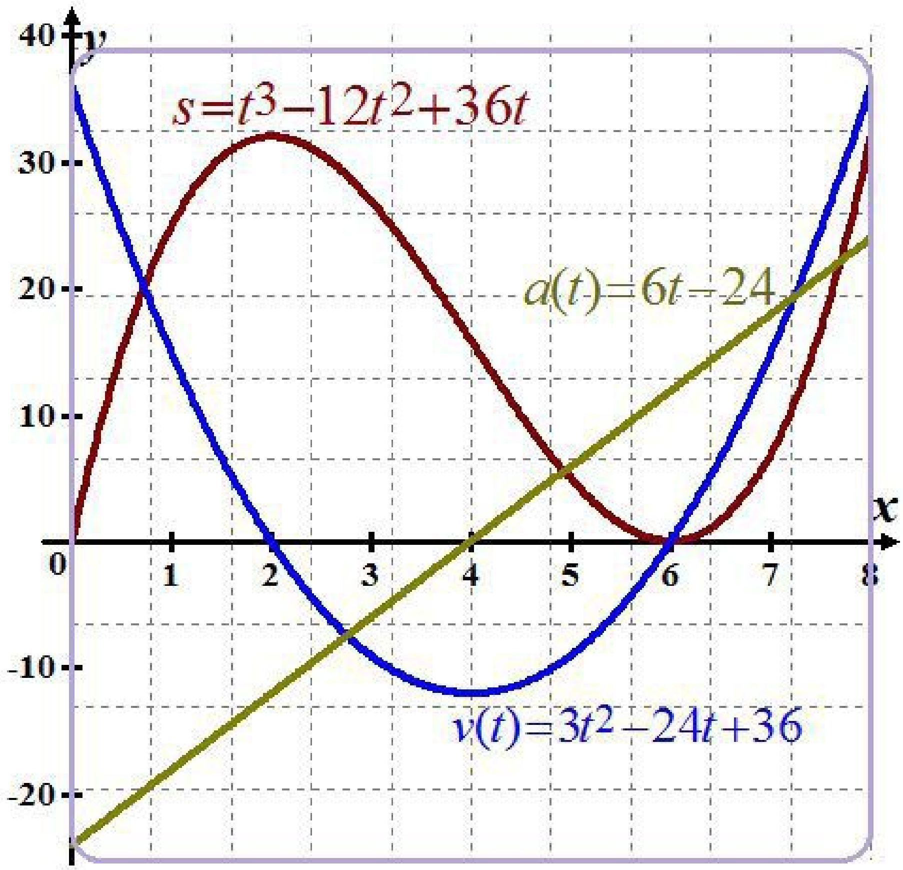
The acceleration after 3 s is the instantaneous acceleration when $t = 3$. Substitute the respective values in the second derivative of the position function as follows:

$$\begin{aligned} a(3) &= 6(3) - 24 \\ &= -6 \end{aligned}$$

Thus, the acceleration of the particle after 3 s is $\boxed{-6 \text{ ft/s}^2}$.

(h)

Sketch the graphs of the functions of position $s = t^3 - 12t^2 + 36t$, velocity $v(t) = 3t^2 - 24t + 36$, and acceleration $a(t) = 6t - 24$ as follows:



(i)

The particle speeds up when *v* and *a* have the same sign.

From the above figure, the functions *v* and *a* are both negative, when $2 < t < 4$, and the functions *v* and *a* are both positive when $t > 6$.

Thus, the particle speeds up when $\boxed{2 < t < 4 \text{ or } t > 6}$.

It slows down when *v* and *a* have opposite signs.

From the above figure, observe that the function *v* and *a* have opposite signs, when $0 \le t < 2$, and the functions *v* and *a* have opposite signs when $4 < t < 6$.

Thus, the particle is slows down when $\boxed{0 \le t < 2 \text{ or } 4 < t < 6}$.

Chapter 2 Derivatives Exercise 2.7 2E

Consider the following position function:

$$f(t) = 0.01t^4 - 0.04t^3$$

Let $s = f(t), t \ge 0$

(a)

The objective is to find the velocity at time t.

The derivative of the position function is velocity function.

Since $s = 0.01t^4 - 0.04t^3$

Differentiate with respect to t.

$$\begin{aligned} v(t) &= \frac{ds}{dt} \\ &= \frac{d}{dt}\left(0.01t^4 - 0.04t^3\right) \\ &= \frac{d}{dt}\left(0.01t^4\right) - \frac{d}{dt}\left(0.04t^3\right) \\ &= 0.04t^3 - 0.12t^2 \quad \left(\frac{d}{dx}\left(x^n\right) = nx^{n-1}\right) \end{aligned}$$

(b)

The objective is to find the velocity after 3sec.

That means to find the instantaneous velocity at $t = 3\,\text{sec}$.

$$\begin{aligned} v(3) &= \left.\frac{ds}{dt}\right|_{t=3} \\ &= 0.04(3)^3 - 0.12(3)^2 \\ &= 1.08 - 1.08 \\ &= 0 \end{aligned}$$

Therefore, the velocity after 3 sec is, $\boxed{v(3) = 0\text{m}/\text{s}}$.

(c)

The objective is to find the particle at rest.

To find t, using the velocity function is equals to zero.

$$\frac{dv}{dt} = 0$$

$$0.04t^3 - 0.12t^2 = 0$$

$$t^2\left(0.04t - 0.12\right) = 0$$

$$t^2 = 0 \text{ or } 0.04t - 0.12 = 0$$

$$t = 0,\ t = \frac{0.12}{0.04}$$

$$t = 0\text{sec}, t = 3\,\text{sec}.$$

Therefore, the particle is at rest after $\boxed{0\text{sec}}$ and after $\boxed{3\text{sec}}$.

(d)

The objective is to determine the particle is moving in the positive direction.

The particle is moving in the positive direction means, $v(t) > 0$

$$0.04t^3 - 0.12t^2 > 0$$

$$t^2\left(0.04t - 0.12\right) > 0$$

$$t > 0 \text{ or } t > 3$$

This result is true for both factors are positive $t > 0 \text{ or } t > 3$ and this result is true for the both are negative $t < 3$.

Thus the particle is move in the positive direction when $t > 3$ and the particle is move in the negative direction when $0 < t < 3$.

(e)

The objective is to find the total distance travelled by the particle during the first 8sec.

To calculate the distances travelled during the time intervals are $[0,2.667],[2.667,5.334]$, and $[5.334,8]$ respectively.

The first distance travelled by the time interval $[0,2.667]$.

Substitute 0 for t in the position function $f(t)$.

$$f(0)=0$$

Substitute 2.667 for t in the position function $f(t)$.

$$\begin{aligned} f(2.667) &= 0.01(2.667)^4 - 0.04(2.667)^3 \\ &= -0.253 \end{aligned}$$

Now find the first distance,

$$\begin{aligned} |f(2.667)-f(0)| &= |-0.253-0| \\ &= 0.253\text{m} \end{aligned}$$

The distance travelled by the time interval $[2.667,5.334]$.

Substitute 5.334 for t in the position function $f(t)$.

$$\begin{aligned} f(5.334) &= 0.01(5.334)^4 - 0.04(5.334)^3 \\ &= 2.0245 \end{aligned}$$

Now find the distance,

$$\begin{aligned} |f(5.334)-f(2.667)| &= |2.0245+0.253| \\ &= 2.2775\text{m} \end{aligned}$$

The distance travelled by the time interval $[5.334,8]$.

Substitute 8for t in the position function $f(t)$.

$$\begin{aligned} f(8) &= 0.01(8)^4 - 0.04(8)^3 \\ &= 20.48 \end{aligned}$$

Now find the distance,

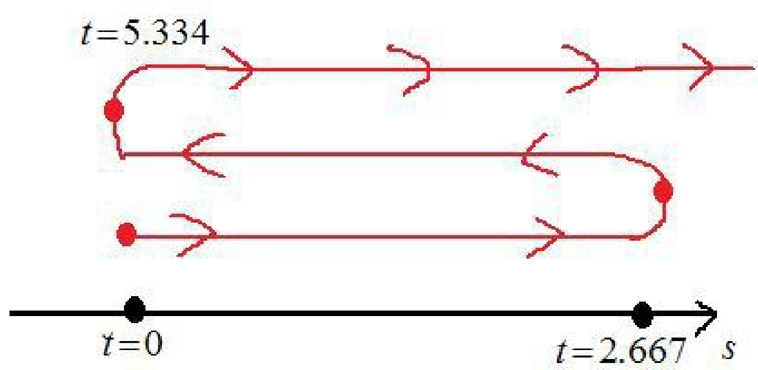
$$\begin{aligned} |f(5.334)-f(2.667)| &= |20.48+0.253| \\ &= 20.733\text{m} \end{aligned}$$

The total distance is, $0.253+2.2775+20.733=23.2635\text{m}$

Therefore, the total distance is, $\boxed{23.2635\text{m}}$

(f)

The objective is to draw the diagram to illustrate the motion of the particle.



(g)

The objective is to find the acceleration at time t after 3s.

The derivative of the velocity function is acceleration.

Since $v(t) = 0.04t^3 - 0.12t^2$

Differentiate with respect to t.

$$\begin{aligned} a(t) &= \frac{dv}{dt} \\ &= \frac{d^2 s}{dt^2} \\ &= \frac{d}{dt}\left(0.04t^3 - 0.12t^2\right) \\ &= 0.12t^2 - 0.24t \end{aligned}$$

Hence, the acceleration is, $a(t) = 0.12t^2 - 0.24t$.

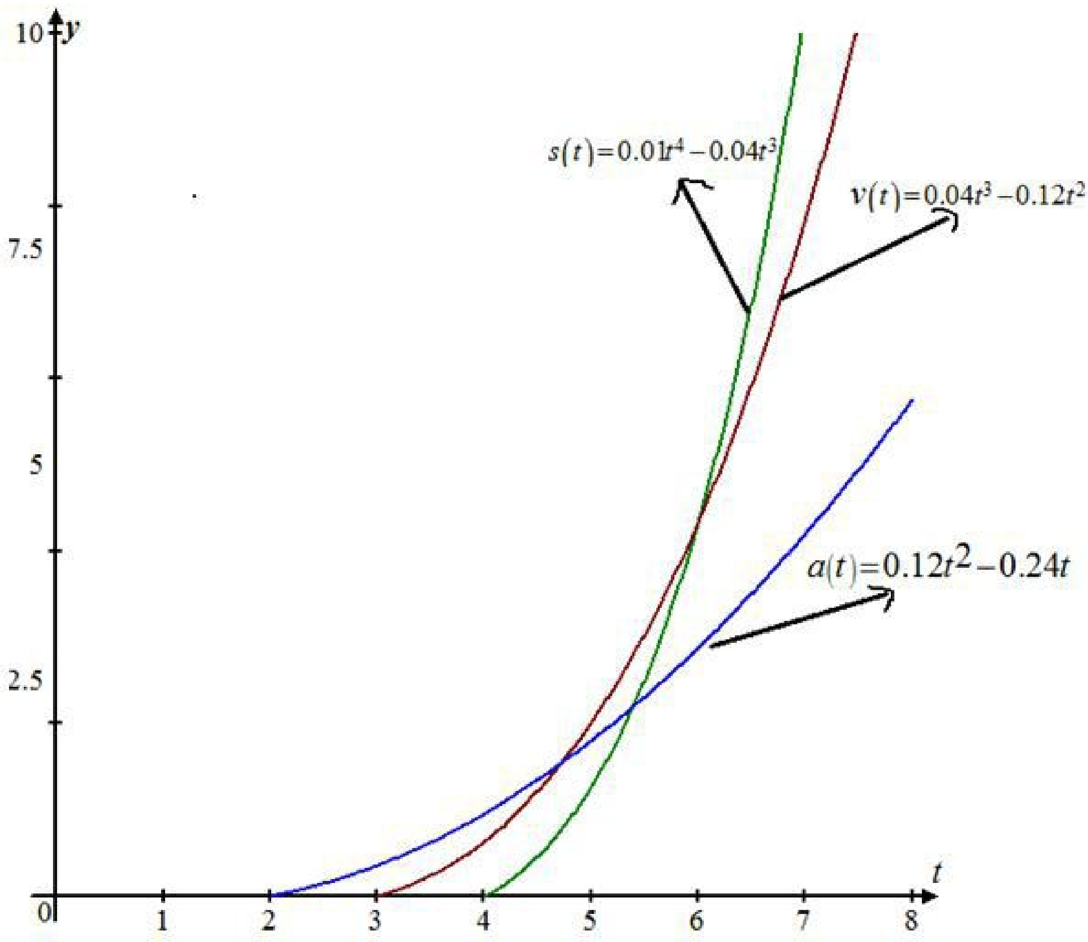
When $t = 3\text{sec}$,

$$\begin{aligned} a(3) &= 0.12(3)^2 - 0.24(3) \\ &= 0.36\text{m/s}^2 \end{aligned}$$

Therefore, the acceleration at time after 3sec is, $\boxed{0.36\text{m/s}^2}$.

(h)

Draw the position, velocity and acceleration functions in the interval $0 \le t \le 8$.



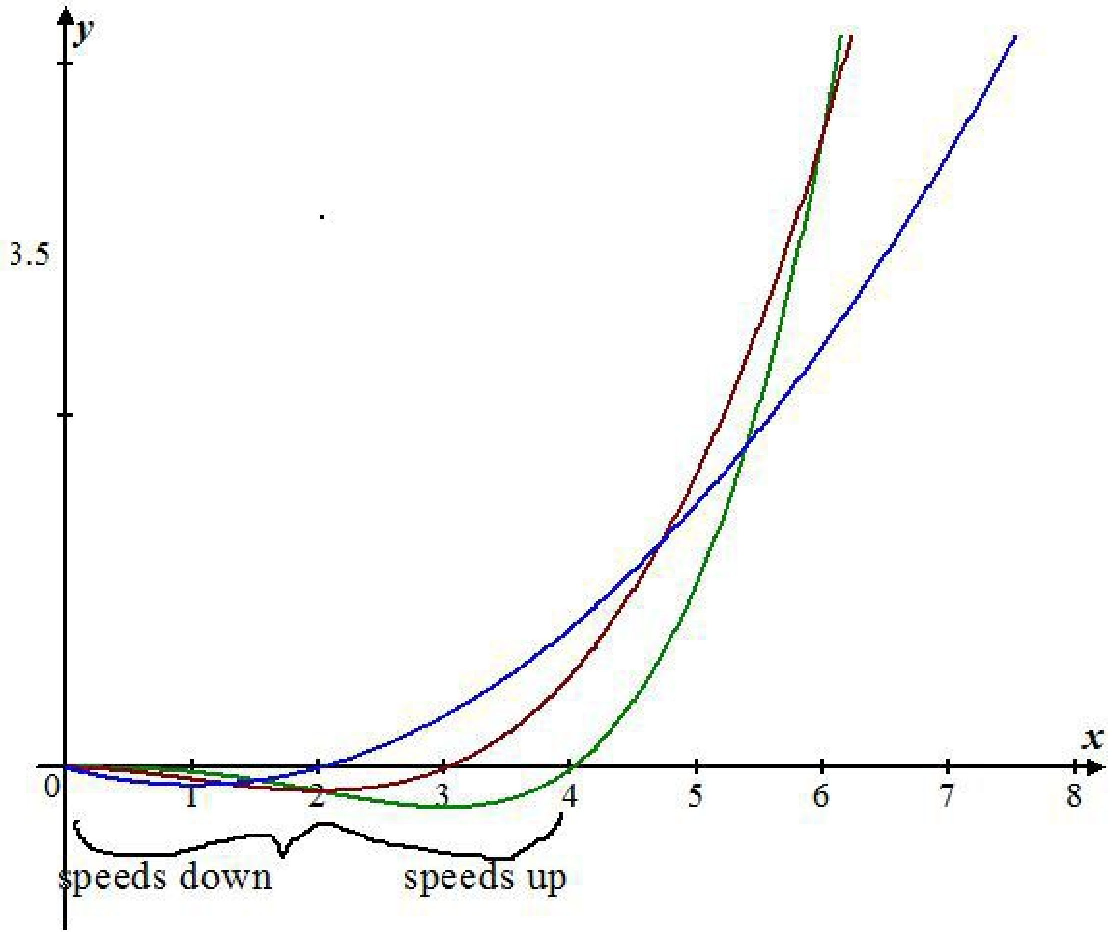
(i)

The objective is to determine the particle is speeding up and slowing down.

The particle is speeding up and the velocity v and acceleration a are same sign.

From the above figure the particle happens when $1 < t < 3$ and the particle slows down when the velocity v and acceleration a have opposite signs, when $0 \le t < 1$ and when $2 < t < 3$.

Draw the following graph:



Chapter 2 Derivatives Exercise 2.7 3E

The particle moves according to a law of motion $f(t) = \cos\left(\frac{\pi t}{4}\right), t \le 10$.

(a)

To find the velocity of the particle, apply the derivative to the function $f(t)$.

The velocity of the particle at time t is,

$$f(t) = \cos\left(\frac{\pi t}{4}\right)$$
$$f'(t) = \frac{-\pi}{4}\sin\left(\frac{\pi t}{4}\right) \quad \left(\text{since } \frac{d}{dx}\cos x = -\sin x\right)$$

Hence, the velocity of the particle at time t is $\boxed{f'(t) = \frac{-\pi}{4}\sin\left(\frac{\pi t}{4}\right)}$.

(b)

To find the velocity after 3 seconds, substitute the value $t = 3\sec$ into the velocity.

The velocity of the particle after 3 seconds is,

$$f'(t) = \frac{-\pi}{4}\sin\left(\frac{\pi t}{4}\right) \left(\text{since } \frac{d}{dx}\cos x = -\sin x\right)$$
$$f'(3) = \frac{-\pi}{4}\sin\left(\frac{3\pi}{4}\right)$$
$$= \frac{-\pi}{4}\cdot\frac{\sqrt{2}}{2}$$
$$= \frac{-\pi\sqrt{2}}{8}$$

Hence, the velocity after 3 seconds is $\boxed{f'(3) = \frac{-\pi\sqrt{2}}{8} \text{ ft/sec}}$.

(c)

The particle will be at rest when the velocity of the particle equals to zero.

$$f(t) = \cos\left(\frac{\pi t}{4}\right)$$
$$f'(t) = \frac{-\pi}{4}\sin\left(\frac{\pi t}{4}\right) \left(\text{since } \frac{d}{dx}\cos x = -\sin x\right)$$
$$0 = \frac{-\pi}{4}\sin\left(\frac{\pi t}{4}\right)$$
$$t = 0, 4, 8 \text{ sec}$$

Hence, the particle will be at rest at $\boxed{t = 0, 4, 8 \text{ sec}}$.

(d)

To find the particle direction, set the velocity greater than 0 and find the interval.

$$f'(t) > 0$$
$$\frac{-\pi}{4}\sin\left(\frac{\pi t}{4}\right) > 0$$
$$4 < t < 8$$

Hence, the particle moves in the positive direction for $\boxed{4 < t < 8}$.

(e)

To find the total distance travelled during the first 8 seconds, we should use the particle moves in the positive direction.

The particle moves the positive direction for $4 < t < 8$ and moves in the negative direction for $t \in (0, 4)$.

Distance travelled in the interval $[0, 4]$ is,

$$|f(4) - f(0)| = \left|\cos\left(\frac{4\pi}{4}\right) - \cos\left(\frac{\pi \cdot 0}{4}\right)\right|$$
$$= 2 \text{ feet}$$

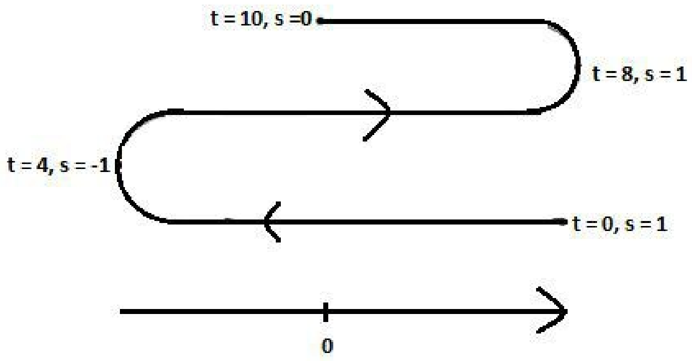
Distance travelled in the interval $[4, 8]$ is,

$$|f(4) - f(8)| = \left|\cos\left(\frac{8\pi}{4}\right) - \cos\left(\frac{\pi \cdot 4}{4}\right)\right|$$
$$= 2 \text{ feet}$$

The total distance travelled during the first 8 seconds is $2 \text{ feet} + 2 \text{ feet} = 4 \text{ feet}$.

(f)

The below diagram will illustrate the motion of the particle.



(g)

To find the acceleration, apply the derivative to the velocity.

The acceleration of the particle at time t is,

$$f(t) = \cos\left(\frac{\pi t}{4}\right)$$
$$f'(t) = \frac{-\pi}{4}\sin\left(\frac{\pi t}{4}\right) \left(\text{since } \frac{d}{dx}\cos x = -\sin x\right)$$
$$f''(t) = \frac{-\pi^2}{16}\cos\left(\frac{\pi t}{4}\right)$$

Hence, the velocity of the particle at time t is $\boxed{f''(t) = \frac{-\pi^2}{16}\cos\left(\frac{\pi t}{4}\right)}$.

The acceleration of the particle after 3 sec is,

$$f(t) = \cos\left(\frac{\pi t}{4}\right)$$
$$f''(t) = \frac{-\pi^2}{16}\cos\left(\frac{\pi t}{4}\right)$$
$$f''(3) = \frac{-\pi^2}{16}\cos\left(\frac{\pi \cdot 3}{4}\right)$$
$$= \frac{\sqrt{2}\cdot\pi^2}{32}$$

Hence, the acceleration of the particle after 3 sec is $\boxed{\frac{\sqrt{2}\cdot\pi^2}{32}\ \text{ft/s}^2}$.

(h)

The graph with input commands as shown below.

Input:

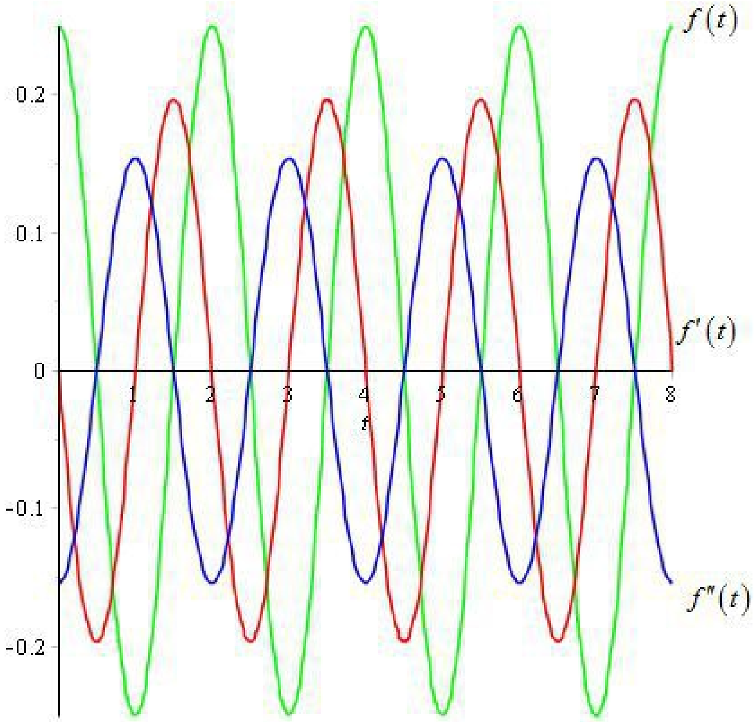
> $with(plots);$

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

> $plots[multiple]\left(plot, \left[\frac{\cos(\mathrm{Pi}\cdot t)}{4}, t = 0..8, color = green\right], \left[-\frac{\mathrm{Pi}}{4}\cdot\frac{\sin(\mathrm{Pi}\cdot t)}{4}, t = 0..8, color = red\right], \left[-\frac{\pi^2}{16}\cdot\frac{\cos(\mathrm{Pi}\cdot t)}{4}, t = 0..8, color = blue\right]\right)$

$$> plots[multiple]\left(plot, \left[\frac{\cos(\text{Pi}\cdot t)}{4}, t=0..8, color=green\right], \left[-\frac{\text{Pi}}{4}\cdot\frac{\sin(\text{Pi}\cdot t)}{4}, t=0..8, color=red\right], \left[-\frac{\pi^2}{16}\cdot\frac{\cos(\text{Pi}\cdot t)}{4}, t=0..8, color=blue\right]\right)$$

Output:



(i)

The particle is speeding up if both velocity and acceleration have same sign.

From the graph, we can say that both have same sign in the interval

$\boxed{0<t<2, 4<t<6 \text{ and } 8<t<10}$.

Particle is slowing down if both velocity and acceleration have opposite sign.

From the graph, we can say that both have opposite sign in the interval $\boxed{2<t<4 \text{ and } 6<t<8}$

Chapter 2 Derivatives Exercise 2.7 4E

The particle moves according to a law of motion $f(t)=\frac{t}{(1+t^2)}$.

(a)

To find the velocity of the particle, apply the derivative to the function $f(t)$.

The velocity of the particle at time t is,

$$f(t)=\frac{t}{(1+t^2)}$$

$$f'(t)=\frac{1-t^2}{(1+t^2)^2} \text{ (Use the quotient Rule)}$$

Hence, the velocity of the particle at time t is $\boxed{f'(t)=\frac{1-t^2}{(1+t^2)^2}}$.

(b)

To find the velocity after 3 seconds, substitute the value $t = 3\,\text{sec}$ into the velocity.

The velocity of the particle after 3 seconds is,

$$f'(t) = \frac{1-t^2}{\left(1+t^2\right)^2}$$

$$f'(3) = \frac{1-(3)^2}{\left(1+(3)^2\right)^2}$$

$$= \frac{-2}{25}$$

Hence, the velocity after 3 seconds is $\boxed{f'(3) = \frac{-2}{25}\ \text{ft/sec}}$.

(c)

The particle will be at rest when the velocity of the particle equals to zero.

$$f'(t) = \frac{1-t^2}{\left(1+t^2\right)^2}$$

$$0 = \frac{1-t^2}{\left(1+t^2\right)^2}$$

$$t = -1 \text{ and } 1 \text{ sec}$$

Hence, the particle will be at rest at $\boxed{t = -1 \text{ and } 1 \text{ sec}}$.

(d)

To find the particle direction, set the velocity greater than 0 and find the interval.

$$f'(t) > 0$$

$$\frac{1-t^2}{\left(1+t^2\right)^2} > 0$$

$$-1 < t < 1$$

Hence, the particle moves in the positive direction for $\boxed{-1 < t < 1}$.

(e)

To find the total distance travelled during the first 8 seconds, we should use the particle moves in the positive direction.

The particle moves the positive direction for $-1 < t < 1$ and moves in the negative direction for $t \in (0, -1)$.

Distance travelled in the interval $[0, -1]$ is,

$$\left|f(-1) - f(0)\right| = \left|\frac{-1}{\left(1+(-1)^2\right)} - \frac{0}{\left(1+(0)^2\right)}\right|$$

$$= \frac{1}{2}$$

Distance travelled in the interval $[-1, 1]$ is,

$$\left|f(1) - f(-1)\right| = \left|\frac{1}{\left(1+(1)^2\right)} - \frac{-1}{\left(1+(-1)^2\right)}\right|$$

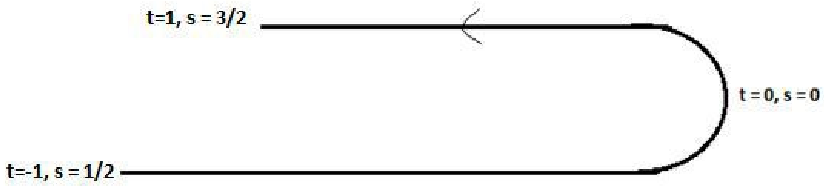
$$= \left|\frac{1}{2} + \frac{1}{2}\right|$$

$$= 1$$

The total distance travelled during the first 8 seconds is $\frac{1}{2}$ feet $+ 1$ feet $= \frac{3}{2}$ feet.

(f)

The below diagram will illustrate the motion of the particle.



(g)

To find the acceleration, apply the derivative to the velocity.

The acceleration of the particle at time t is,

$$f(t)=\frac{t}{\left(1+t^2\right)}$$

$$f'(t)=\frac{1-t^2}{\left(1+t^2\right)^2} \text{ (Use the quotient Rule)}$$

$$f''(t)=\frac{2t\left(t^2-3\right)}{\left(1+t^2\right)^3}$$

Hence, the velocity of the particle at time t is $\boxed{f''(t)=\frac{2t\left(t^2-3\right)}{\left(1+t^2\right)^3}}$.

The acceleration of the particle after 3 sec is,

$$f(t)=\frac{t}{\left(1+t^2\right)}$$

$$f''(t)=\frac{2t\left(t^2-3\right)}{\left(1+t^2\right)^3}$$

$$f''(3)=\frac{6(9-3)}{(1+9)^3}$$

$$=\frac{9}{250}$$

Hence, the acceleration of the particle after 3 sec is $\boxed{\frac{9}{250}\text{ ft/s}^2}$.

(h)

The graph with input commands as shown below.

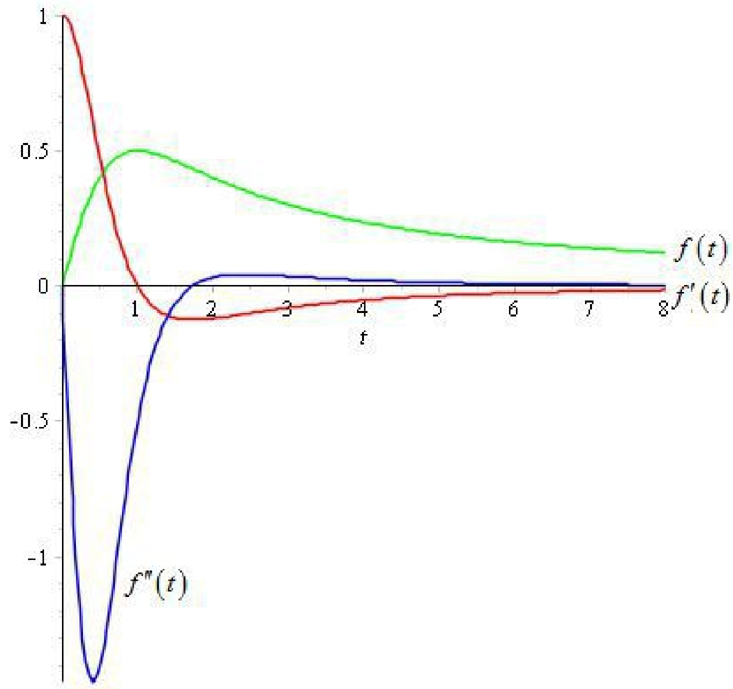
Input:

```
> with(plots);
```

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

$$> \; plots[multiple]\left(plot, \left[\frac{t}{(1+t^2)}, t=0..8, color=green\right], \left[\frac{1-t^2}{(1+t^2)^2}, t=0..8, color=red\right], \left[\frac{2t\cdot(t^2-3)}{(t^2+1)^3}, t=0..8, color=blue\right]\right)$$

Output:



(i)

The particle is speeding up if both velocity and acceleration have same sign.

From the graph, both have same sign in the interval $\boxed{1<t<4}$.

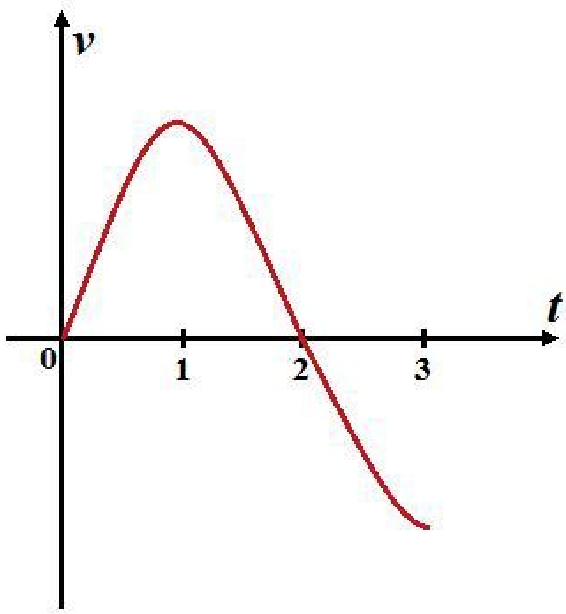
Particle is slowing down if both velocity and acceleration have opposite sign.

From the graph, both have opposite sign in the interval $\boxed{0<t<1 \text{ or } 4<t<8}$.

Chapter 2 Derivatives Exercise 2.7 5E

(a)

The graph of the velocity function of a particle is shown in the below figure, where, *t* is measured in seconds.



From the above figure, the velocity *v* is positive on the interval $(0,2)$ and negative on the interval $(2,3)$.

Since, the acceleration a is positive (negative) when the slope of the tangent line is positive (negative), and the slope in the above graph is positive on the interval $(0,1)$, and negative on the interval $(1,3)$. So, the acceleration (a) is positive on the interval $(0,1)$, and negative on the interval $(1,3)$.

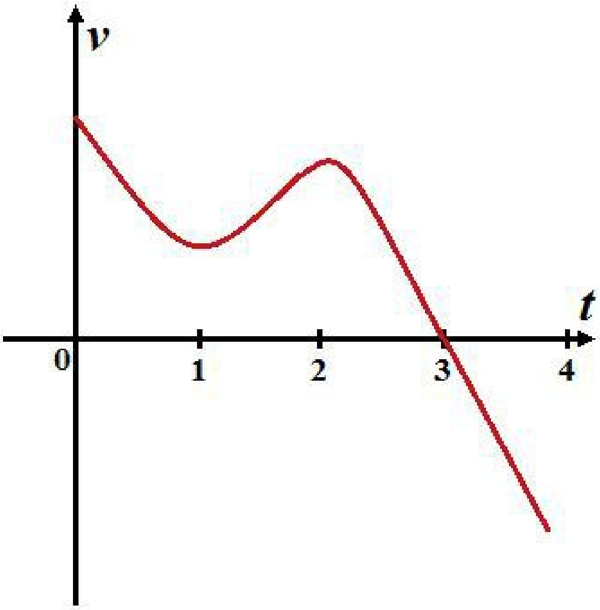
The particle is speeding up when *v* and *a* have the same sign, that is, on the interval $(0,1)$ when $v>0$ and $a>0$, and on the interval $(2,3)$ when $v<0$ and $a<0$.

The particle is slowing down when *v* and *a* have opposite signs, that is, on the interval $(1,2)$ when $v>0$ and $a<0$.

Therefore, the particle is speeding up when $0<t<1$ or $2<t<3$; slowing down when $1<t<2$.

(b)

The graph of the velocity function of a particle is shown in the below figure, where, *t* is measured in seconds.



From the above figure, the velocity *v* is positive on the interval $(0,3)$ and negative on the interval $(3,4)$.

Since, the acceleration a is positive (negative) when the slope of the tangent line is positive (negative), and the slope in the above graph is positive on the interval $(1,2)$, and negative on the intervals $(0,1)$ and $(2,4)$. So, the acceleration (a) is positive on the interval $(1,2)$, and negative on the interval $(0,1)$ and $(2,4)$.

The particle is speeding up when *v* and *a* have the same sign, that is, on the interval $(1,2)$ when $v>0$ and $a>0$, and on the interval $(3,4)$ when $v<0$ and $a<0$.

The particle is slowing down when *v* and *a* have opposite signs, that is, on the interval $(0,1)$ and $(2,3)$ when $v>0$ and $a<0$.

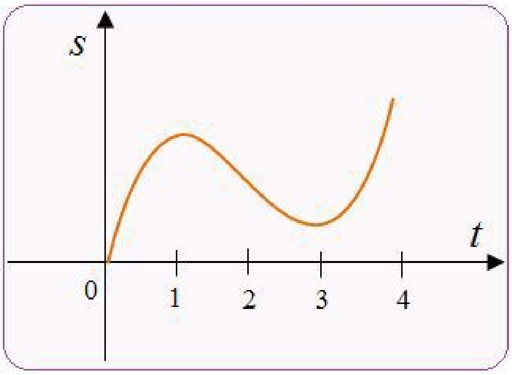
Therefore, the particle is speeding up when $1<t<2$ or $3<t<4$; slowing down when $0<t<1$ or $2<t<3$.

Chapter 2 Derivatives Exercise 2.7 6E

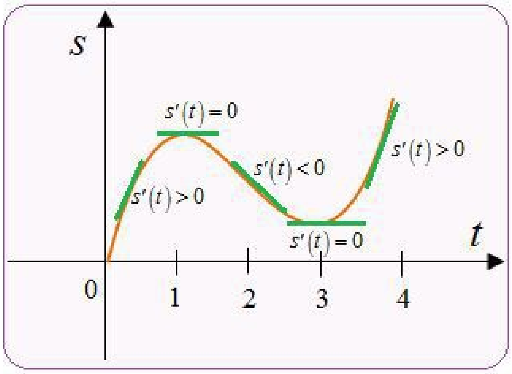
The objective is to determine when each particle speeding up and when it is slowing down for each of the position functions.

(a)

The graph of the position function of a particle is:



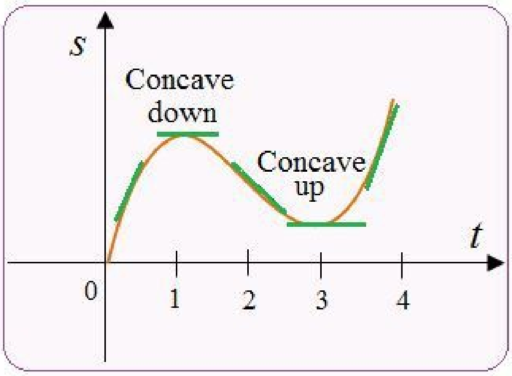
Sketch the slopes of the curve as:



From the graph, observe that $s'(t)=0$ at $t=1,3$ and the position function is increasing on the intervals $(0,1)$ and $(3,4)$ so the velocity function $s'(t)>0$ on these intervals.

The position function is decreasing on the interval $(1,3)$ so $s'(t)<0$ on $(1,3)$.

For concavity sketch the graph as:



The graph is concave up on the interval $(2,4)$ as the acceleration $s''(t)>0$ on the interval $(2,4)$ and downwards on the interval $(0,2)$ as the acceleration $s''(t)<0$ on the interval $(0,2)$.

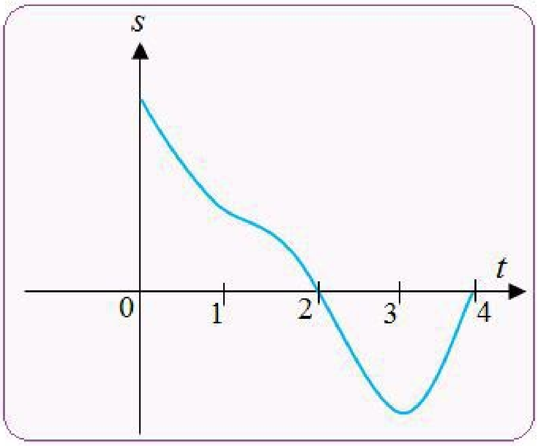
As the velocity and acceleration have same sign on the intervals $(1,2)$ and $(3,4)$ so the particle speeds up on these intervals.

The velocity and accelerations have opposite signs on the intervals $(0,1),(2,3)$ so the particle slowdowns on these intervals.

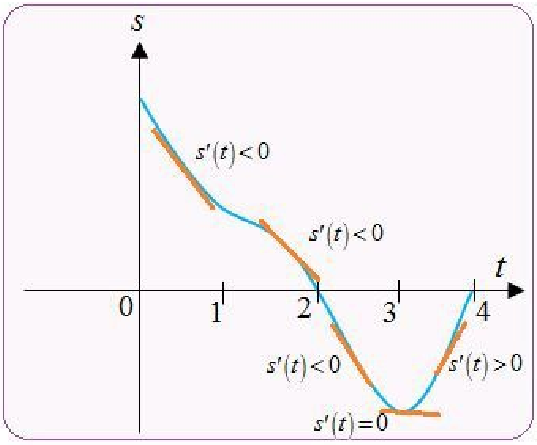
Therefore, the speeds up on $\boxed{(1,2) \text{ and } (3,4)}$ and slow down on $\boxed{(0,1) \text{ and } (2,3)}$.

(b)

The graph of the position function of a particle is:



Sketch the slopes of the curve as:



From the graph, observe that $s'(t)=0$ at $t=3$ and the position function is increasing on the intervals $(3,4)$ so the velocity function $s'(t)>0$ on $(3,4)$.

The position function is decreasing on the interval $(0,3)$ so $s'(t)<0$ on $(0,2),(2,3)$.

Here, $s'(t)$ is not defined at $t=2$.

The graph is concave up on the interval $(0,1)$ and $(2,4)$ as the acceleration $s''(t)>0$ on the interval $(0,1)$ and $(2,4)$ and downwards on the interval $(1,2)$ as the acceleration $s''(t)<0$ on the interval $(1,2)$.

As the velocity and acceleration have same sign on the intervals $(1,2)$ and $(3,4)$ so the particle speeds up on these intervals.

The velocity and accelerations have opposite signs on the intervals $(0,1),(2,3)$ so the particle slowdowns on these intervals.

Therefore, the speeds up on $\boxed{(1,2) \text{ and } (3,4)}$ and slow down on $\boxed{(0,1) \text{ and } (2,3)}$.

Chapter 2 Derivatives Exercise 2.7 7E

The height of a projectile shot vertically upward from a point 2 meter above ground level with an initial velocity of 24.5 meter per sec is $h=2+24.5t-4.9t^2$.

(a)

To find the velocity, integral the height with respect to t,

$$h(t)=2+24.5t-4.9t^2$$
$$h'(t)=24.5-9.8t$$

Hence, the velocity is $\boxed{V(t)=24.5-9.8t}$.

The velocity after 2 seconds is,

$$V(t) = 24.5 - 9.8t$$
$$V(2) = 24.5 - 9.8(2)$$
$$= 4.9$$

Hence, the velocity after 2 seconds is $\boxed{V(2) = 4.9 \text{ m/s}}$.

The velocity after 4 seconds is,

$$V(t) = 24.5 - 9.8t$$
$$V(4) = 24.5 - 9.8(4)$$
$$= -14.7$$

Hence, the velocity after 2 seconds is $\boxed{V(4) = -14.7 \text{ m/s}}$.

(b)

The projectile reach its maximum height when $v(t) > 0$.

$$V(t) > 0$$
$$24.5 - 9.8t > 0$$
$$24.5 > 9.8t$$
$$t < 2.5$$

The projectile reaches its maximum height at $\boxed{t = 2.5 \text{ seconds}}$.

(c)

To find the maximum height, substitute the value $t = 2.5$ **seconds** into the height function.

$$h = 2 + 24.5t - 4.9t^2$$
$$= 2 + 24.5(2.5) - 4.9(2.5)^2$$
$$= 2 + 61.25 - 30.625$$
$$= 32.625 \text{ meter}$$

Hence, the maximum height is $\boxed{h = 32.625 \text{ meter}}$.

(d)

It hit the ground when the height equals to zero.

$$h = 2 + 24.5t - 4.9t^2$$
$$0 = 2 + 24.5t - 4.9t^2$$

Apply the quadratic formula,

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(24.5) \pm \sqrt{(24.5)^2 - 4(2)(-4.9)}}{2(-4.9)}$$
$$= \frac{-(24.5) \pm \sqrt{639.45}}{2(-4.9)}$$
$$= 5.08 \text{ sec}$$

Hence, it hit the ground when the time is at 5.08 seconds.

(e)

To find the velocity when it hit the ground, substitute the value $t = 5.08$ sec into the equation.

$$V(t) = 24.5 - 9.8t$$
$$V(5.08) = 24.5 - 9.8(5.08)$$
$$= 24.5 - 49.784$$
$$= -25.3 \text{ m/s}$$

Hence, the velocity when it hit the ground is $\boxed{V(5.08) = -25.3 \text{ m/s}}$.

(A)

First we have to get the velocity at time t

Here $S = 80t - 16t^2$

Then $V(t) = \frac{dS}{dt} = 80 - 32t$

When the ball at the maximum height then its velocity will be 0 that is $V(t) = 0$

$\Rightarrow 80 - 32t = 0$

$\Rightarrow t = \frac{80}{32} \Rightarrow \boxed{t = \frac{5}{2}}$ Seconds

The after $t = \frac{5}{2}$ seconds the height will be

$S = 80 \times \frac{5}{2} - 16 \times \left(\frac{5}{2}\right)^2$

$\Rightarrow S = 200 - 100$

$\Rightarrow \boxed{S = 100 \text{ feets}}$

Then the maximum height reached by ball is 100 ft

(B)

First we have to calculate the time when

S = 96 that is

$96 = 80t - 16t^2$

$\Rightarrow 6 = 5t - t^2$ (Dividing by 16)

$\Rightarrow t^2 - 5t + 6 = 0$

$\Rightarrow t^2 - 3t - 2t + 6 = 0$

$\Rightarrow (t-3)(t-2) = 0$

This is true when t = 3 or t = 2

Now velocity at t = 3

$V(3) = 80 - 32(3)$

$= \boxed{-16 \text{ ft/s}}$

Negative sign shows that the direction of motion is downward

So when ball's way is downward, the velocity is 16 ft/sec while the ball is at 96 if height from ground.

Velocity at t = 2

$V(2) = 80 - 32(2)$

$= \boxed{16 \text{ feet/s}}$

This means when the ball is at 96 feet height from the ground and its way is upward then velocity is $\boxed{16 \text{ feet/s}}$

Chapter 2 Derivatives Exercise 2.7 9E

The height of the rock (where it thrown vertically upward from the surface of Mars with velocity and its height after t seconds) is $h = 15t - 1.86t^2$.

(a)

To find the velocity of the rock after 2 seconds, substitute the time $t = 2\text{seconds}$ in velocity of the rock.

To find the velocity of the rock, differentiate the height equation with respect to t seconds.

The height of the rock is $h = 15t - 1.86t^2$.

The velocity of the rock is,

$$\begin{aligned} v &= \frac{dh}{dt} \\ &= \frac{d\left(15t - 1.86t^2\right)}{dt} \\ &= 15 - 3.72t \end{aligned}$$

Hence, the velocity of the rock is $\boxed{v = 15 - 3.72t}$.

To find the velocity of the rock after 2 seconds, substitute the time $t = 2\text{seconds}$ in velocity of the rock.

The velocity of the rock after 2 seconds is,

$$\begin{aligned} v &= 15 - 3.72t \\ &= 15 - 3.72(2) \\ &= 7.56 \end{aligned}$$

Hence, the velocity of the rock is $\boxed{v(2) = 7.56 \text{ meter/sec}}$.

(b)

To find the velocity of the rock when its height is 25 m on its way up and on its way down, substitute height value in height equation, find the value of time and then substitute the resultant value in its velocity.

Substitute the value $h = 25\,m$ into the height equation,

$$\begin{aligned} h &= 15t - 1.86t^2 \\ 25 &= 15t - 1.86t^2 \end{aligned}$$

$$15t - 1.86t^2 - 25 = 0$$

$$1.86t^2 - 15t + 25 = 0 \qquad (\text{multiple with "}-\text{"})$$

From quadratic formula,

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-15) \pm \sqrt{(-15)^2 - 4(1.86)(25)}}{2(1.86)} \\ &= \frac{15 \pm \sqrt{39}}{3.72} \\ &= 5.711 \text{ or } 2.353 \end{aligned}$$

The velocity at $t = 5.711$ is,

$$v = 15 - 3.72t$$
$$= 15 - 3.72(5.711)$$
$$= -6.24492$$

The negative sign shows that the direction of motion is down ward.

When the rock moves downward, the velocity is 6.24492 feet/second while the rock is at a height of 25 feet above the ground.

The velocity at $t = 2.353$ is,

$$v = 15 - 3.72t$$
$$= 15 - 3.72(2.353)$$
$$= 6.24684$$

When the rock moves upward, the velocity is 6.24684 feet/second while the rock is at a height of 25 feet above the ground.

Chapter 2 Derivatives Exercise 2.7 10E

The particle moves with position function $s = t^4 - 4t^3 - 20t^2 + 20t, t \geq 0$.

(a)

To find the time at 20 meter/second height of the particle, first find the velocity by applying the derivative.

The velocity of the particle is,

$$v = \frac{ds}{dt}$$
$$= \frac{d}{dt}\left(t^4 - 4t^3 - 20t^2 + 20t\right)$$
$$= 4t^3 - 12t^2 - 40t + 20$$

Make the velocity is equals to 20 meter per second,

$$v = 4t^3 - 12t^2 - 40t + 20$$
$$20 = 4t^3 - 12t^2 - 40t + 20$$
$$4t^3 - 12t^2 - 40t = 0$$
$$4t\left(t^2 - 3t - 10\right) = 0$$
$$4t(t-5)(t+2) = 0$$
$$t = 0, 5, -2$$

But, the value of t is positive. So, ignore the negative value.

Hence, the particle has the velocity of 20 meter per second at time $\boxed{t = 0 \sec \text{ and } 5 \sec}$.

(b)

To find the time at acceleration 0, find the derivative of the velocity of the particle.

The acceleration of the particle is,

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{d\left(4t^3 - 12t^2 - 40t + 20\right)}{dt} \\ &= 12t^2 - 24t - 40 \end{aligned}$$

Make the acceleration of the particle equals to 0,

$$\begin{aligned} a &= 12t^2 - 24t - 40 \\ 0 &= 12t^2 - 24t - 40 \\ 0 &= 3t^2 - 6t - 10 \qquad \text{(Divide by 4 on both sides)} \end{aligned}$$

From the quadratic formula,

$$\begin{aligned} t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(-10)}}{2(3)} \\ &= \frac{6 \pm \sqrt{156}}{6} \\ &= 3.082 \text{ or } -1.082 \end{aligned}$$

But, the value of t is positive. So, ignore the negative value.

Hence, the particle has the zero acceleration at $\boxed{t = 3.082 \text{ sec}}$.

Chapter 2 Derivatives Exercise 2.7 11E

(A)

We know that the area of a square is a function of its side length i.e.

$\Rightarrow A(x) = x^2 \quad mm^2$

Where x is measured in mm

The rate of change in area is the derivative of $A(x)$

$$A'(x) = \frac{d}{dx}\left(x^2\right)$$

$$\Rightarrow A'(x) = 2x \quad mm^2 / mm$$

When the side length is $= 15$ mm the derivative of $A(15)$

$$A'(15) = 2.15$$

$$\Rightarrow \boxed{A'(15) = 30} \quad mm^2 / mm$$

This is the rate of which the area is increasing with respect to x as $x \to 15 \quad mm$

(B)

The perimeter of a square is also a function of its side length such that

$$P(x) = 4.x$$

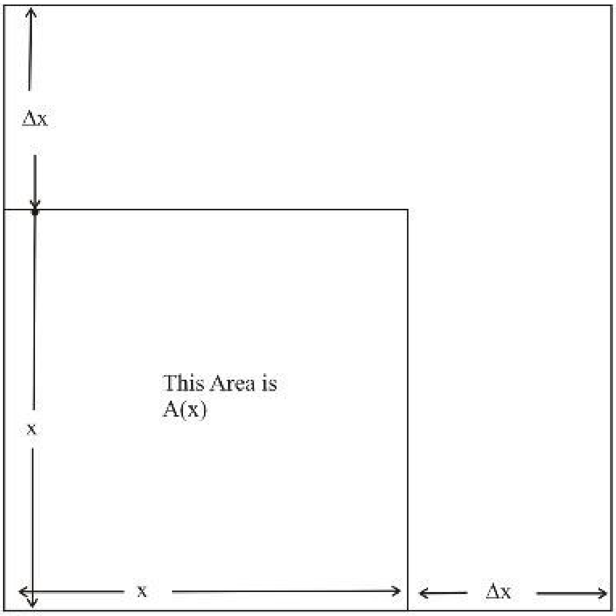
Now we have $A'(x) = 2x$

$$= 2x.2.\frac{1}{2}$$

$$= \frac{1}{2}.(4.x)$$

We have $\boxed{A'(x) = \frac{1}{2}P(x)}$

Hence we have that the rate of change of the area = half of the perimeter



New Area = A(Δx + x)

When Δx is small enough it means $\Delta x \to 0$
Then the rate of change in area

$$\frac{\Delta A}{\Delta x} \approx \lim_{\Delta x \to 0} \frac{\Delta A}{\Delta x}$$

$$\Rightarrow \frac{\Delta A}{\Delta x} \approx \lim_{\Delta x \to 0} \frac{A(\Delta x + x) - A(x)}{\Delta x}$$

$$\Rightarrow \frac{\Delta A}{\Delta x} \approx \lim_{\Delta x \to 0} \frac{(\Delta x + x)^2 - x^2}{\Delta x}$$

$$\Rightarrow \frac{\Delta A}{\Delta x} \approx \lim_{\Delta x \to 0} \frac{x^2 + (\Delta x)^2 + 2(\Delta x).x - x^2}{\Delta x}$$

$$\Rightarrow \frac{\Delta A}{\Delta x} \approx \lim_{\Delta x \to 0} (\Delta x + 2x)$$

$$\Rightarrow \frac{\Delta A}{\Delta x} \approx 2x$$

Thus $\boxed{\Delta A \approx 2x.\Delta x}$

This is the resulting change in area of a square when $\Delta x \to 0$

Chapter 2 Derivatives Exercise 2.7 12E

(A) Side of the cube $= x$

Then volume of cube, $v = x^3$

Thus $\dfrac{dv}{dx} = 3x^2$

When x = 3 mm

$$\left.\frac{dv}{dx}\right|_{x=3} = 3 \times (3)^2$$

$$= \boxed{27 \text{ mm}^3/\text{mm}}$$

It means the volume of the cube is increasing with the rate of $27 \text{ mm}^3/\text{mm}$ when $x = 3$ mm.

(B) We have $\dfrac{dv}{dx} = 3x^2$

Surface area of cube, $S = 6x^2$

Then $\dfrac{1}{2} S = 3x^2$

Or, $\boxed{\dfrac{dv}{dx} = \dfrac{1}{2} S}$

(A)

13E

The area of a circle is a function of r such that

$$A(r) = \pi r^2 \quad \text{--- (1)}$$

(1) Average rate of change of area when r change from 2 to 3

$$\frac{\Delta A(r)}{\Delta r} = \frac{A(3)-A(2)}{3-2}$$
$$= \frac{\pi 3^2 - \pi 2^2}{1}$$
$$= 9\pi - 4\pi$$
$$\boxed{= 5\pi}$$

(2) Average rate of change of area when r changes 2 to 2.5

$$= \frac{A(2.5)-A(2)}{2.5-2} = \frac{\pi(2.5)^2 - \pi(2)^2}{0.5}$$
$$= \frac{6.25\pi - 4\pi}{0.5} = \frac{2.25\pi}{0.5}$$
$$\boxed{= 4.5\pi}$$

(3) Average rate of change of area when r changes from 2 to 2.1

$$= \frac{A(2.1)-A(2)}{2.1-2} = \frac{\pi(2.1)^2 - \pi(2)^2}{0.1}$$
$$= \frac{4.41\pi - 4\pi}{0.1} = \frac{0.41\pi}{0.1}$$
$$\boxed{= 4.1\pi}$$

(B)

The instantaneous rate of change of area $= \frac{dA}{dr}$

We have $A = \pi r^2$

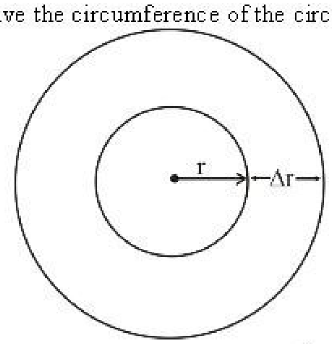
$$\Rightarrow \frac{dA(r)}{dr} = A'(r) = 2\pi r$$

When r = 2 then $A'(2) = 2\pi.2 = 4\pi$

$$\boxed{A'(2) = 4\pi}$$

(C)

We have the circumference of the circle is also the function of radius r such that



New Area = π(r + Δr)²

$$C(r) = 2\pi r \quad \text{--- (2)}$$

And rate of change of the area

$$A'(r) = 2\pi r$$

Hence $\boxed{A'(r) = C(r)}$

When Δr is small enough then rate of change

$$\frac{\Delta A}{\Delta r} \approx \lim_{\Delta r \to 0} \frac{\Delta A}{\Delta r} \approx \lim_{\Delta r \to 0} \frac{A(r+\Delta r) - A(r)}{\Delta r}$$
$$\approx \lim_{\Delta r \to 0} \frac{\pi(r+\Delta r)^2 - \pi r^2}{\Delta r}$$

$$\approx \pi . \lim_{\Delta r \to 0} \frac{r^2 + \Delta r^2 + 2r.\Delta r - r^2}{\Delta r}$$

$$\approx \pi \lim_{\Delta r \to 0} (\Delta r + 2r)$$

$$\Rightarrow \frac{\Delta A}{\Delta r} \approx \pi .2r \qquad \text{Hence } \boxed{\Delta A \approx 2\pi r.\Delta r}$$

Where ΔA is resulting change in area when Δr is very small i.e. $\Delta r \to 0$

Chapter 2 Derivatives Exercise 2.7 14E

We have to get the area as a function of time t

Speed of traveling ripples outward = 60 Cm/s

It means ripple travel 60cm distance in 1 second

Let the ripple covers r cm distance in t seconds

Then distance = Speed × Time

$\boxed{r = 60t}$ cm Where r is the radius of circular ripple

Then

$$\boxed{\frac{dr}{dt} = 60} \ \text{cm}/\text{s}$$

We have area of a circle is

$$A = \pi r^2$$

Then the rate of change of area with respect to time

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$= 2\pi(60).(60t) \qquad \text{[From step 1]}$$

$$= 2 \times 3600\pi t$$

$$\boxed{\frac{dA}{dt} = 7200\pi t \ \text{cm}^2/\text{s}}$$

(A)

The rate of increase of area after 1 s

$$\boxed{\frac{dA}{dt}_{t=1} = 7200\pi \ \text{cm}^2/\text{s}}$$

(B)

The rate of increase of the area after t = 3s is

$$\frac{dA}{dt}_{t=3} = 7200\pi(3)$$

$$\boxed{= 21600\pi \ \text{cm}^2/\text{s}}$$

(C)

The rate of increase of the area after t = 5s is

$$\frac{dA}{dt}_{t=3} = 7200\pi(5)$$

$$\boxed{= 36000\pi \ \text{cm}^2/\text{s}}$$

We can see that area is increasing as time is increasing.

Chapter 2 Derivatives Exercise 2.7 15E

Surface area is the function of radius of the balloon such that

$$S = 4\pi r^2$$

Then the rate of change (increase) of the surface area will be the derivative of S with respect to r

Then the rate of increase of the surface area is

$$S'(r) = \frac{dS}{dr} = \frac{d}{dr}(4\pi r^2)$$

$$= 4\pi .2r = 8\pi r$$

$$\boxed{S'(r) = 8\pi r} \qquad \text{ft}^2/\text{ft}$$

(A)

When r = 1ft then rate of increase of $S(r)$ is

$$S'(1) = 8\pi.1$$

$$\boxed{= 8\pi} \quad \text{ft}^2/\text{ft}$$

(B)

When r = 2 then rate of increase $S(r)$ is

$$S'(2) = 8\pi.2$$

$$\boxed{= 16\pi} \quad \text{ft}^2/\text{ft}$$

(C)

When r = 3 ft then

$$S'(3) = 8\pi.3$$

$$\boxed{= 24\pi} \quad \text{ft}^2/\text{ft}$$

Here we can conclude that the rate of increase of surface area is increasing at the rate of increase of radius.

It means that rate of increase of S increases as radius increases

Chapter 2 Derivatives Exercise 2.7 16E

(A)

The volume of a growing spherical cell is the function of radius r such that

$$V(r) = \frac{4}{3}\pi r^3$$

(1) Average rage of change of V, when r changes from $r = 5\mu m \ to \ 8\mu m$ or

$$= 5\times10^{-6}\,m \ to \ 8\times10^{-6}\,m \qquad \left[1\mu m = 10^{-6}m\right]$$

$$= \frac{V(8\times10^{-6}) - V(5\times10^{-6})}{8\times10^{-6} - 5\times10^{-6}}$$

$$= \frac{\frac{4}{3}\pi(8\times10^{-6})^3 - \frac{4}{3}\pi(5\times10^{-6})^3}{3\times10^{-6}}$$

$$= \frac{10^{-18}}{3\times10^{-6}}\left[\frac{4}{3}\pi 8^3 - \frac{4}{3}\pi 5^3\right]$$

$$= \frac{10^{-18}}{3\times10^{-6}} \times \frac{4}{3}\pi(512-125)$$

$$= \frac{4}{9}\pi(387).10^{-12}\text{m}^3/\text{m}$$

$$= 43.4\pi.10^{-12} \ \text{m}^3/m$$

$$\boxed{= 172\pi\times10^{-12} \ \text{m}^3/m}$$

(2) Average rate of change of V, where r changes from 5 to 6 it means

From $r = 5\times10^{-6}\,m \ to \ 6\times10^{-6}\,m$

$$= \frac{V(6\times10^{-6}) - V(5\times10^{-6})}{6\times10^{-6} - 5\times10^{-6}} \ \text{m}^3/\text{m}$$

$$= \frac{\frac{4}{3}\pi(6\times10^{-6})^3 - \frac{4}{3}\pi(5\times10^{-6})^3}{1\times10^{-6}}$$

$$= \frac{4}{3}\pi\times\frac{10^{-18}}{10^{-6}}\left[6^3 - 5^3\right]$$

$$= \frac{4}{3}\pi\times10^{-12}.(216-125)$$

$$= \frac{4}{3}\pi\times10^{-12}.(91)$$

$$= \boxed{121.33\pi\times10^{-12}} \ m^3/m$$

(3) Average rate of change of V, when r changes from 5×10^{-6} *to* 5.1×10^{-6}

$$= \frac{V\left(5.1\times10^{-6}\right)-V\left(5.0\times10^{-6}\right)}{(5.1-5)10^{-6}}$$

$$= \frac{\frac{4}{3}\pi\left(5.1\times10^{-6}\right)^3-\frac{4}{3}\pi\left(5.0\times10^{-6}\right)^3}{10^{-7}}$$

$$= \frac{4}{3}\pi\times10^{7}\times10^{-18}\times\left((5.1)^3-(5)^3\right)$$

$$= \frac{4}{3}\pi\times10^{-11}\times7.651$$

$$= 10.2\pi\times10^{-11}\ \text{m}^3/m$$

$$\boxed{= 102\pi\times10^{-12} m^3/m}$$

(B) The instantaneous rate of change of V when $r = 5\mu m$ or $r = 5\times10^{-6} m$

$\left[1\mu m = 10^{-6} m\right]$ is the derivative of V with respect to r

$$\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^2\right)$$

$$= \frac{4}{3}.3\pi r^2$$

$$\boxed{\frac{dV}{dr} = 4\pi r^2} \qquad \text{--- (1)}$$

When $r = 5\times10^{-6} m$ the instantaneous rate of change

$$\frac{dV}{dr} = 4\pi\times\left(5\times10^{-6}\right)^2$$

$$= 100\pi\times10^{-6}\times10^{-6}$$

$$= \boxed{100\pi\times10^{-12} m^3/m}$$

(C)

We have surface area is the function of radius r such that

$$S(r) = 4\pi r^2 \qquad \text{--- (2)}$$

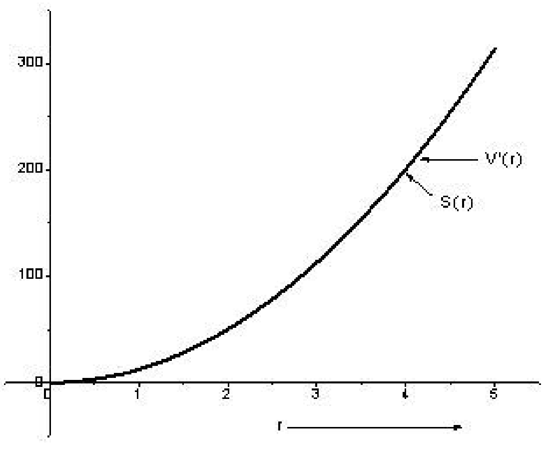
And $\frac{dV}{dr} = 4\pi r^2$ from (1)

Thus we have $\boxed{\frac{dV}{dr} = S(r)}$ this means that rate of change of V is equal to the surface area.

If we draw the graph of S(r) with respect to r and the graph of $V'(r)$ with respect to r on the same axes then we see that both graphs coincide to each other or over lap thus this proves our result.

When radius r increase by Δr then average rate of change in volume

$$= \frac{\Delta V}{\Delta r}$$



It Δr is very small then as $\Delta r \to 0$

We have

$$\frac{\Delta V}{\Delta r} \approx \lim_{\Delta r \to 0} \frac{V(r+\Delta r)-V(r)}{\Delta r}$$

$$\approx \lim_{\Delta r \to 0} \frac{\frac{4}{3}\pi(r+\Delta r)^3 - \frac{4}{3}\pi r^3}{\Delta r}$$

$$\approx \frac{4}{3}\pi \lim_{\Delta r \to 0} \frac{r^3 + \Delta r^3 + 3r^2.\Delta r + 3r(\Delta r)^2 - r^3}{\Delta r}$$

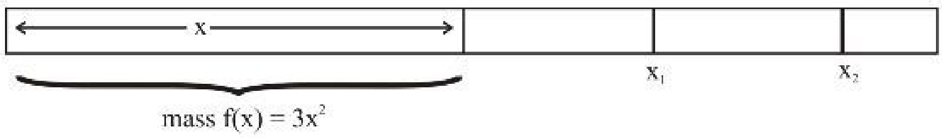
$$\approx \frac{4}{3}\pi \lim_{\Delta r \to 0} \left((\Delta r)^2 + 3r^2 + 3r.\Delta r\right)$$

$$\approx \frac{4}{3}\pi \times 3r^2$$

$$\Rightarrow \frac{\Delta V}{\Delta r} \approx 4\pi r^2$$

Thus resultant change in volume $\Rightarrow \boxed{\Delta V \approx 4\pi r^2.\Delta r}$

Chapter 2 Derivatives Exercise 2.7 17E



Thus mass of the part of the rod that lies between $x_1 and\ x_2$ is given by

$$\Delta m = f(x_2) - f(x_1)$$

So the average density of that part is $= \frac{\Delta m}{\Delta x} = \frac{f(x_2)-f(x_1)}{x_2 - x_1}$

The linear density ρ at x is the limit of these average density as $\Delta x \to 0$

So we have $\rho = \lim_{\Delta x \to 0} \frac{\Delta m}{\Delta x} = \frac{dm}{dx} kg/m$

We have $m = 3x^2 kg$

Then $\boxed{\rho = \frac{dm}{dx} = 6x}$ $\quad kg/m$

(A) When x = 1 m

Then linear density $\rho = 6 \times 1 \boxed{= 6 \quad kg/m}$

(B) Linder density, when x = 2 m

$\rho = 6 \times 2 \quad kg/m$

$\boxed{= 12 \quad kg/m}$

(C) Linear density, when x = 3m

$\Rightarrow \rho = 6 \times 3 \quad kg/m$

$\Rightarrow \boxed{\rho = 18 \quad kg/m}$

This means linear density is increasing as x increases so at the left end (x = 0) the density will lowest and at right end highest

Chapter 2 Derivatives Exercise 2.7 18E

The volume V of water remaining in the tank after t minute as

$$V = 5000\left(1 - \frac{t}{40}\right)^2 \qquad 0 \le t \le 40$$

The rate at which the volume is decreasing $= -\frac{dV}{dt}$

This is the rate at which water is draining from the tank

Then $\frac{dV}{dt} = V'(t) = 5000.2.\left(1 - \frac{t}{40}\right)\left(-\frac{1}{40}\right)$

$$= -10000.\frac{1}{40}\left(1 - \frac{t}{40}\right)$$

$$= -250\left(1 - \frac{t}{40}\right)$$

Then rate of draining is $= -\frac{dV}{dt} = 250\left(1 - \frac{t}{40}\right) {gallons}/{m}$

(A)

Rate of which water is draining form the tank after 5 m

$$-V'(5) = 250\left(1 - \frac{5}{40}\right)$$

$$= 250\left(1 - \frac{1}{8}\right)$$

$$= 250 \times \frac{7}{8}$$

$$= \boxed{218.75 \quad \text{gallons/minute}}$$

(B)

Rate of which water is draining after 10 minute

$$= 250.\left(1 - \frac{10}{40}\right)$$

$$= 250\left(1 - \frac{1}{4}\right)$$

$$= 250 \times \frac{3}{4} = \boxed{187.5 \quad gallons/\text{min}}$$

(C)

Rate which water is draining after 20 min

$$= 250\left(1 - \frac{20}{40}\right)$$

$$= 250\left(1 - \frac{1}{2}\right)$$

$$= 250 \times \frac{1}{2}$$

$$\boxed{= 125 \quad \text{gallons/min}}$$

(D)

Rate at which water is draining after 40 minutes

$$= 250\left(1 - \frac{40}{40}\right)$$

$$= 250(1 - 1)$$

$$\boxed{= 0 \, {gallons}/{\text{min}}}$$

So here we see that the rate of draining is decreasing as time is increase.

At $t = 40$, the rate of draining of water 0 means tank has no water remaining in it

Chapter 2 Derivatives Exercise 2.7 19E

The change Q is the function of time t as

$$Q(t) = t^3 - 2t^2 + 6t + 2$$

Then current at time t

$$I(t) = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t}$$

$$= \frac{dQ}{dt} = Q'(t)$$

Hence $I(t) = \frac{d}{dt}\left(t^3 - 2t^2 + 6t + 2\right)$

$$I(t) = 3t^2 - 4t + 6$$

This is current at time t

(A)

Current at t = 0.5s

$$I(0.5) = 3(0.5)^2 - 4(0.5) + 6$$
$$= 3\times 0.25 - 2.0 + 6$$
$$\boxed{= 4.75\ \text{A}}$$

(B) Current at t = 1s

$$I(1) = 3.1^2 - 4.1 + 6$$
$$= 3 - 4 + 6$$
$$\boxed{= 5\ \text{A}}$$

We have

$$I(t) = 3t^2 - 4t + 6$$

Then $\dfrac{dI(t)}{dt} = \dfrac{d}{dt}(3t^2 - 4t + 6) = 6t - 4$

Now $\dfrac{dI(t)}{dt} = 0$, when $6t - 4 = 0$

$$\Rightarrow t = \frac{4}{6}$$
$$\Rightarrow t = \frac{2}{3}\ \text{sec}$$

Since, there is only one critical number and $\dfrac{d^2I(t)}{dt^2} = 6 > 0$ for all t.

So the function $I(t) = 3t^2 - 4t + 6$ has an absolute minimum at $t = \dfrac{2}{3}$

Thus, at time $\boxed{t = \dfrac{2}{3}\ \text{sec}}$, the current will be lowest.

Chapter 2 Derivatives Exercise 2.7 20E

By Newton's law of gravitation

$$F = \frac{GmM}{r^2} \qquad -(1)$$

Where F is the force exerted by a body of mass m on a body of mass M and G is the gravitational constant. r is the distance between bodies.

(A) We have $F = GmM.r^{-2}$

Differentiating with repeat to r

$$\frac{dF}{dr} = GmM\frac{d}{dr}(r^{-2}) \qquad [F, m, \text{and } M \text{ are constants}]$$
$$\Rightarrow \frac{dF}{dr} = GmM(-2)r^{-3}$$
$$\Rightarrow \boxed{\frac{dF}{dr} = -\frac{2GmM}{r^3}} \qquad \ldots\ldots (2)$$

This means the rate of change of F with respect to r, The negative sign indicate that force is decreasing as r is increasing.

(B) Given that $F'(20000) = -2$ N/km

$$\Rightarrow -\frac{2GmM}{(20000)^3} = -2$$
$$\Rightarrow GmM = (20000)^3$$

Substituting this value in equation (2), we have

$$\frac{dF}{dr} = -\frac{2(20000)^3}{r^3}$$

Now when $r = 10000$ km

Then $F'(10000) = \dfrac{-2(20000)^3}{(10000)^3}$

$$= -2\left(\frac{20000}{10000}\right)^3$$
$$= \boxed{-16\ \text{N/km}}$$

Chapter 2 Derivatives Exercise 2.7 21E

The force F acting on a body with mass m and velocity v is the rate of change of momentum

$F=\frac{d}{dt}(mv)$ where the mass of a particle varies with v is $m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$.

The mass of a particle varies with v is,

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

Multiply with velocity v on both sides,

$$mv=\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$$

Apply derivative with respect to t on both sides,

$$\frac{d}{dt}(mv)=\frac{d}{dt}\left(\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

Apply the quotient rule to the left side of the equation,

$$\frac{d}{dt}(mv)=\frac{d}{dt}\left(\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right)$$

$$F=m_0\frac{d}{dt}\left(\frac{vc}{\sqrt{c^2-v^2}}\right)+\left(\frac{vc}{\sqrt{c^2-v^2}}\right)\frac{d}{dt}(m_0)$$

$$=m_0c\left[\frac{\frac{dv}{dt}\cdot\sqrt{c^2-v^2}-v\cdot\frac{(-2v)}{2\sqrt{c^2-v^2}}\cdot\frac{dv}{dt}}{c^2-v^2}\right]+\left(\frac{vc}{\sqrt{c^2-v^2}}\right)(0)$$

$$=m_0c\left[\frac{\frac{dv}{dt}\cdot(c^2-v^2)+v^2\cdot\frac{dv}{dt}}{(c^2-v^2)^{\frac{3}{2}}}\right]$$

$$=m_0c\left[\frac{c^2\frac{dv}{dt}}{(c^2-v^2)^{\frac{3}{2}}}\right] \quad \ldots\ldots (1)$$

The rate of change of the velocity is acceleration i.e. $\frac{dv}{dt}=a$.

Substitute the value in equation (1),

$$F=m_0c\left[\frac{c^2\frac{dv}{dt}}{(c^2-v^2)^{\frac{3}{2}}}\right]$$

$$=m_0c\cdot\frac{c^2\cdot a}{(c^2-v^2)^{\frac{3}{2}}}$$

$$=\frac{m_0a}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

Hence, we proved i.e. $\boxed{F=\frac{m_0a}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}}}$.

Chapter 2 Derivatives Exercise 2.7 22E

The highest tides in the world occur in the Bay of Fundy on the Atlantic Coast of Canada.

The water depth D (in meters) as a function of the time t (in hours after midnight) on that day is $D(t)=7+5\cos\left[0.503(t-6.75)\right]$.

To find the speed ness of the tide at given time 3:00 AM, first differentiate the above function and then substitute the time value into the equation.

$$D(t)=7+5\cos\left[0.503(t-6.75)\right]$$
$$\frac{dD}{dt}=-2.515\sin\left[0.503(t-6.75)\right]$$

(a)

The speed ness of the tide is,

Substitute the time $t=3$ into the equation,

$$\begin{aligned}\frac{dD}{dt}&=-2.515\sin\left[0.503(t-6.75)\right]\\&=-2.515\sin\left[0.503(3-6.75)\right]\\&=-2.515\sin\left[-1.88625\right]\\&=-2.390899\end{aligned}$$

Hence, the tide is falling at a speed of 2.39 meters per hour at 3:00 AM.

(b)

The speed ness of the tide is,

Substitute the time $t=6$ into the equation,

$$\begin{aligned}\frac{dD}{dt}&=-2.515\sin\left[0.503(t-6.75)\right]\\&=-2.515\sin\left[0.503(6-6.75)\right]\\&=-2.515\sin\left[-0.37725\right]\\&=-0.9264\end{aligned}$$

Hence, the tide is falling at a speed of 0.9264 meters per hour at 6:00 AM.

(c)

The speed ness of the tide is,

Substitute the time $t=9$ into the equation,

$$\begin{aligned}\frac{dD}{dt}&=-2.515\sin\left[0.503(t-6.75)\right]\\&=-2.515\sin\left[0.503(9-6.75)\right]\\&=-2.515\sin\left[1.13175\right]\\&=-2.2765\end{aligned}$$

Hence, the tide is falling at a speed of 2.2765 meters per hour at 9:00 AM.

(d)

The speed ness of the tide is,

Substitute the time $t=12$ into the equation,

$$\begin{aligned}\frac{dD}{dt}&=-2.515\sin\left[0.503(t-6.75)\right]\\&=-2.515\sin\left[0.503(12-6.75)\right]\\&=-2.515\sin\left[2.64075\right]\\&=-1.2077\end{aligned}$$

Hence, the tide is falling at a speed of 1.2077 meters per hour at 12:00 PM.

Consider the Boyle's Law,

$$PV = C$$

Where P is pressure and V is the volume of gas and C is a constant.

(a)

Need to find the rate of change of volume with respect to pressure.

The rate of change of volume with respect to pressure is equal to the derivative of V with respect to P.

Consider the Boyle's Law,

$$PV = C$$

It can be written as,

$$PV = C$$

$$V = \frac{C}{P} \qquad \text{Divide by } P \text{ on both sides}$$

Differentiate with respect to P:

$$\frac{dV}{dP} = \frac{d}{dp}\left(\frac{C}{P}\right)$$

$$= \frac{d}{dP}\left(CP^{-1}\right)$$

$$= C\frac{d}{dP}\left(P^{-1}\right)$$

$$= C(-1)P^{-1-1} \qquad \text{Use } \frac{d}{dr}r^n = nr^{n-1}$$

$$= -CP^{-2}$$

$$= -\frac{C}{P^2}$$

Hence, the rate of change of volume with respect to pressure is $\boxed{\frac{dV}{dP} = -\frac{C}{P^2}}$

(b)

Suppose, a sample of gas compressed for 10 minutes.

From part (a)

$$V = \frac{C}{P}$$

From the equation $V = \frac{C}{P}$, $V \alpha \frac{1}{P}$ this means, the volume decreases when the pressure increases.

At the end of the 10 minutes the presser of the gas is increases, because the volume of the gas decreases.

So, the pressure increases then, $\frac{1}{P^2}$ decreases.

Hence, the rate of change of volume with respect to pressure $-\frac{C}{P^2}$ is decreases.

(c)
Consider isothermal compressibility,

$$\beta = -\frac{1}{V}\frac{dV}{dP} \qquad \text{......} (1)$$

Need to prove that $\beta = \frac{1}{P}$

From part (a),

$$\frac{dV}{dP} = -\frac{C}{P^2}$$

The equation (1) can be written as,

$$\beta = -\frac{1}{V}\frac{dV}{dP}$$

$$= -\frac{1}{V}\left(-\frac{C}{P^2}\right) \qquad \text{Substitute } \frac{dV}{dP} = -\frac{C}{P^2}$$

$$= \frac{1}{V}\cdot\frac{C}{P^2}$$

$$= \frac{1}{V}\cdot\frac{PV}{P^2} \qquad \text{Use Boyles's law } PV = C$$

$$= \frac{1}{P} \qquad \text{Cancel out common factors}$$

Hence, the isothermal compressibility $\boxed{\beta = \frac{1}{P}}$

Chapter 2 Derivatives Exercise 2.7 24E

One molecule of the product C is formed from one molecule of the reactant A and one molecule of the reactant B, and the initial concentrations of A and B have an common value then the equation is $[\mathrm{C}] = \frac{a^2kt}{(akt+1)}$ where k is a constant.

(a)

To find the rate of reaction of a time t, differentiate the equation.

The rate of reaction at a time t is,

$$[\mathrm{C}] = \frac{a^2kt}{(akt+1)}$$

$$\frac{d[\mathrm{C}]}{dt} = \frac{d}{dt}\left[\frac{a^2kt}{(akt+1)}\right]$$

$$= a^2k\left[\frac{(akt+1) - t(ak)}{(akt+1)^2}\right]$$

$$= \frac{a^2k}{(akt+1)^2}$$

Hence, the rate of reaction at a time t is $\boxed{\frac{d[\mathrm{C}]}{dt} = \frac{a^2k}{(akt+1)^2}}$.

(b)

Substitute the concentration value in the above equation,

$$\frac{dx}{dt}=\frac{a^2k}{(akt+1)^2} \quad (\text{since } x=[C])$$

The claim is, $\frac{dx}{dt}=k(a-x)^2$ this means that we need to prove that $\frac{a^2k}{(akt+1)^2}=k(a-x)^2$.

So,

$$\begin{aligned}
k(a-x)^2 &= a^2k-2akx+x^2k \\
&= a^2k-2ak\left(\frac{a^2kt}{(akt+1)}\right)+k\left(\frac{a^2kt}{(akt+1)}\right)^2 \qquad \left(x=[\mathrm{C}]=\frac{a^2kt}{(akt+1)}\right) \\
&= a^2k-\left(\frac{2a^3k^2t}{(akt+1)}\right)+\frac{a^4k^3t^2}{(akt+1)^2} \\
&= \frac{a^2k(akt+1)^2-(2a^3k^2t)(akt+1)+a^4k^3t^2}{(akt+1)^2} \\
&= \frac{\cancel{a^4k^3t^2}+a^2k+\cancel{2a^3k^2t}-\cancel{2a^4k^3t^2}+\cancel{2a^3k^2t}+\cancel{a^4k^3t^2}}{(akt+1)^2} \\
&= \frac{a^2k}{(akt+1)^2} \\
&= \frac{d[C]}{dt} \\
&= \frac{dx}{dt}
\end{aligned}$$

Hence, we proved i.e. $\frac{dx}{dt}=k(a-x)^2$.

(c)

The concentrations are,

$$\frac{d[A]}{dt}=\frac{d[B]}{dt}$$

$$-\frac{dx}{dt}=-k(a-x)^2$$

$$\int_0^x \frac{1}{(a-x)^2}dx=k\int_0^t dt$$

$$\left[\frac{1}{(a-x)}\right]_0^x=k[t]_0^t$$

$$\frac{1}{(a-x)}-\frac{1}{a}=kt$$

$$\frac{x}{a(a-x)}=kt$$

As $t\to\infty$, the value $\frac{1}{t}$ goes to 0 and As $x=[C]=a$ then the concentrations $[A],[B]\to 0$

(d)

The rate of reaction is,

$$\begin{aligned}
r &= \frac{d}{dt}[C] \\
&= \frac{a^2k}{(akt+1)^2}
\end{aligned}$$

As $t\to\infty$, $r\to 0$, the reaction comes to stop after a long time.

(e)

From the result (c) means the end of the reaction A and B are used up to produce C.

From the reaction comes to a completion and stops after a long time.

Chapter 2 Derivatives Exercise 2.7 25E

(A)

We will get the slope of two secant lines from 1910 to 1920 and from 1920 to 1930 these slopes are called average rate of growth. After taking the average of these two secant line we can get the slope of tangent line which is called rate o population growth in 1920.

Average rate of population growth from 1910 to 1920 is

$$= \frac{P(1920)-P(1910)}{1920-1910} = \frac{1860-1750}{10}$$
$$= 11 \quad millions / year$$

Average rate of population growth from 1920 to 1930 is

$$= \frac{P(1930)-P(1920)}{1930-1920} = \frac{2070-1860}{10}$$
$$= 21 \quad millions / year$$

Then the rate of population growth in 1920 by taking the average of the avenges get above

$$= \frac{11+21}{2} = \boxed{16 \quad millions / year}$$

Now we will use the same process from 1980
Average rate of growth from 1970 to 1980 is

$$= \frac{P(1980)-P(1970)}{1980-1970} = \frac{4450-3710}{10} = 74 \quad millions / year$$

Average rate of growth from 1980 to 1990 is

$$= \frac{P(1990)-P(1980)}{1990-1980} = \frac{5280-4450}{10} = 83 \quad millions / year$$

Then the rate of population growth in 1980

$$= \frac{74+83}{2} = \boxed{78.5 \quad millions / year}$$

(B)

By the computer we can draw the graph and find the cubic function with help of this date. Such as

$$P(t) = 0.00129371t^3 - 7.061422t^2 + 12822.979t - 7743771$$

This is the form of $y = ax^3 + bx^2 + cx + d$

(C)

The rate of population growth is the derivative of $P(t)$ with respect to t is

$$\frac{d(P(t))}{dt} = \frac{d}{dt}\left[0.00129371t^3 - 7.061422t^2 + 12822.979t - 7743771\right]$$
$$= 3(0.00129371)t^2 - 2(7.061422)t + 12822.979$$

$$\boxed{\frac{d(P(t))}{dt} = 0.00388113t^2 - 14.122844t + 12822.979}$$

(D)

Rate of growth in 1920 is

$$P'(1920) = (0.00388113)(1920)^2 - 14.122844(1920) + 12822.979$$
$$= 14307.39763 - 27115.86048 + 12822.979$$

$$\boxed{P'(1920) = 14.5 \quad millons / year}$$

Rate of growth in 1980 is

$$P'(1980) = (0.00388113)(1980)^2 - 14.122844(1980) + 12822.979$$
$$= 15215.58205 - 27963.23112 + 12822.979$$

$$\boxed{P'(1980) = 75.33 \quad millons / year}$$

(E)

Rate of growth in 1985 is

$P'(1985) = (0.00388113)(1985)^2 - 14.122844(1985) + 12822.979$

$= 15292.52545 - 28033.8434 + 12822.979$

$\boxed{P'(1985) = 81.66 \quad millons/year}$

Chapter 2 Derivatives Exercise 2.7 26E

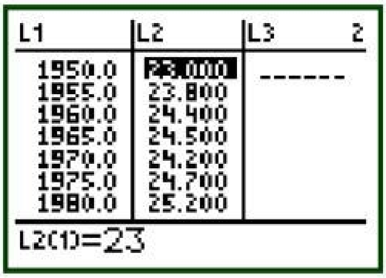
Let $A(t)$ be the average age of first marriage of woman.

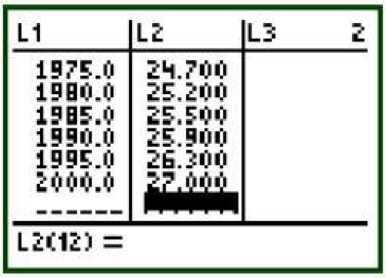
(a)

To determine the fourth – degree polynomial model, use the TI – 8 plus graphing calculator.

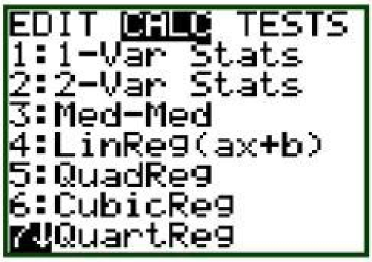
Press the STAT button as shown below.

Move to Edit, press enter and enter the values into it as shown below.

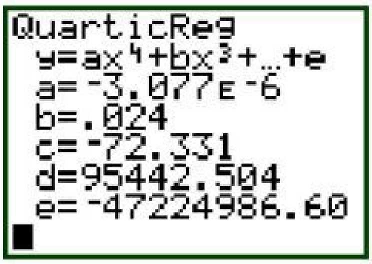




Press Stat, move to Calc and press 7. QuartReg as shown below.



Press enter button to get the final answer.



Hence, the fourth degree polynomial is

$\boxed{A(t) = -3.077\times10^{-6}t^4 + 0.024t^3 - 72.331t^2 + 95\times10^3 t - 47\times10^6}$.

(b)

To find the derivative of $A(t)$, we need to apply derivative formula.

$$A(t) = -3.077\times10^{-6}t^4 + 0.024t^3 - 72.331t^2 + 95\times10^3 t - 47\times10^6$$
$$A'(t) = \left(-3.077\times10^{-6}\right)(4)t^3 + (0.024)(3)t^2 - (72.331)(2)t + \left(95\times10^3\right)$$
$$= -12.3\times10^{-6}t^3 + 0.072t^2 - 144.7t + 95000$$

Hence, the model of $A'(t)$ is $\boxed{A'(t) = -12.3\times10^{-6}t^3 + 0.072t^2 - 144.7t + 95000}$.

(c)

To find the rate of change of marriage age for women in 1990, substitute the value 1990 into t.

The rate of change of marriage age for women in 1990 is,

$$A'(t) = -12.3\times10^{-6}t^3 + 0.072t^2 - 144.7t + 95000(2)t + \left(95\times10^3\right)$$
$$A'(1990) = -12.3\times10^{-6}(1990)^3 + 0.072(1990)^2 - 144.7(1990) + 95000$$
$$= -96924 + 285120 - 287953 + 95000$$
$$= -4757.17$$

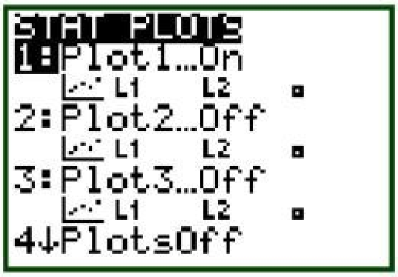
Hence, the rate of change of marriage age for women in 1990 is $\boxed{A'(1990) = -4757.17}$.

(d)

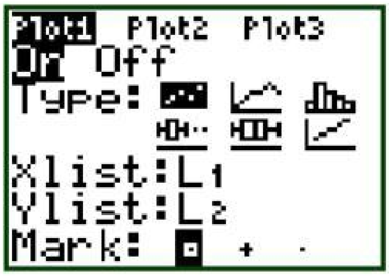
To plot the data points and models, use the TI-84 Plus graphing calculator.

First press 2nd + Y= button and on the 1. Plot1..On.

The display is as shown below.

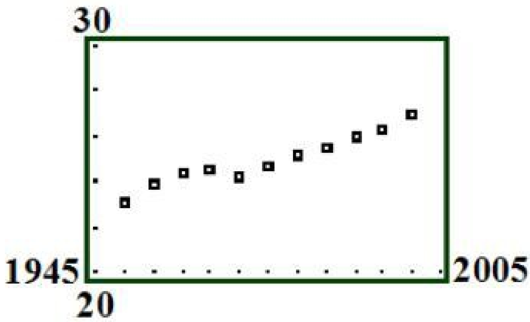


Press enter button then select options is as shown below.



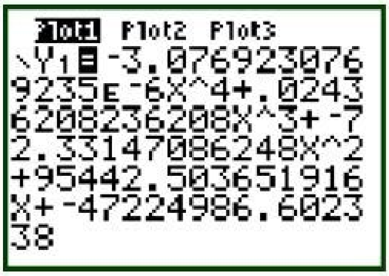
Use the window settings is as shown below.

Press graph button for final graph.

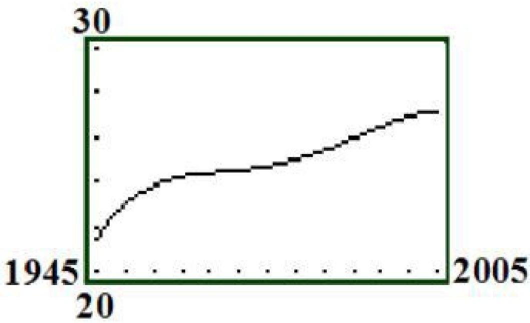


Press Y = button and enter $A(t) = -3.077\times10^{-6}t^4 + 0.024t^3 - 72.331t^2 + 95\times10^3 t - 47\times10^6$ model.

The display is as shown below.

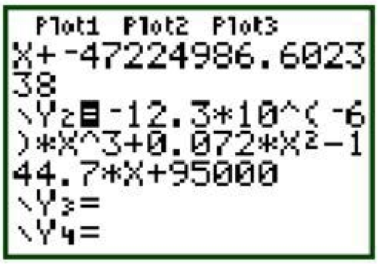


Press Graph button to get the final graph.

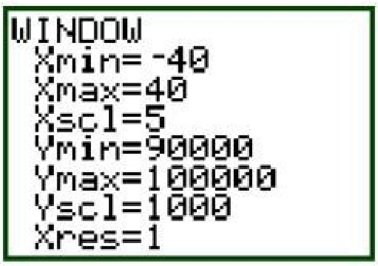


Press Y = button and enter $A'(t) = -12.3\times10^{-6}t^3 + 0.072t^2 - 144.7t + 95000$ model.

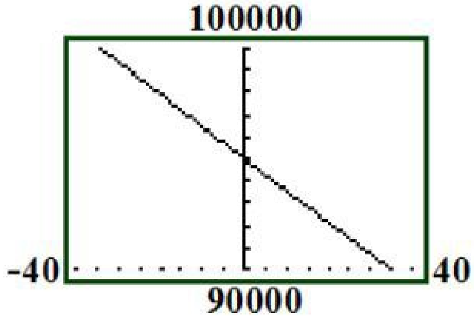
The display is as shown below.



Adjust the window settings is as shown below.



Press Graph button to get the final graph.



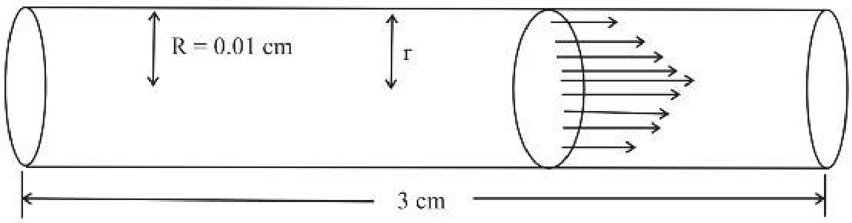
Chapter 2 Derivatives Exercise 2.7 27E

The relation ship between v and r is given by the law of Laminar flow such as

$$v = \frac{P}{4\eta l}\left(R^2 - r^2\right)$$

Where η is the viscosity of the blood and P is the pressure difference between the ends of the tube

Here P is given $= 3000$ $dynes/cm^2 v$ and R = 0.01 cm, $l = 3$cm



(A)

When r = 0, the velocity of the blood is

$$v = \frac{P}{4\eta l}\left(R^2 - 0\right) = \frac{PR^2}{4\eta l}$$

$$= \frac{3000 \times (0.01)^2}{4 \times (0.027)(3)} = \frac{3000 \times .001}{12 \times 0.027}$$

$$= \frac{0.3}{0.324}$$

$$\boxed{v = 0.9259\, cm/s}$$

When r = 0.005cm then velocity of the blood is

$$v = \frac{3000\left[(0.01)^2 - (0.005)^2\right]}{4 \times (0.027) \times 3} = \frac{3000 \times \left[7.5 \times 10^{-5}\right]}{12 \times 0.027}$$

$$= \frac{0.225}{0.324}$$

$$\boxed{v = 0.694\ cm/s}$$

When r = R then velocity of the blood is

$$v = \frac{3000\left((0.01)^2 - (0.01)^2\right)}{4 \times 0.027 \times 3}$$

$$\Rightarrow \boxed{v = 0\ cm/s}$$

(B) The velocity gradient $= \dfrac{dv}{dr}$

$$\frac{dv}{dr} = \frac{d}{dr}\left[\frac{P\left(R^2 - r^2\right)}{4\eta l}\right]$$

Here $P.\eta.R and\ l$ are constant so

$$\frac{dv}{dr} = \frac{P}{4\eta l}\frac{d}{dr}\left(R^2 - r^2\right)$$

$$= \frac{P}{4\eta l}\frac{d}{dr}[0-2r]$$

$$\boxed{\frac{dv}{dr} = -\frac{\Pr}{2\eta l}}$$

Velocity gradient at r = 0 cm

$$\frac{dv}{dr} = \frac{-P.0}{2\eta l} = \frac{0}{2\eta l}$$

$$\Rightarrow \left.\frac{dv}{dr}\right|_{r=0.005} = \frac{-P.(0.005)}{2\eta l}$$

$$= \frac{-3000\times(0.005)}{2\times 0.027\times 3}$$

$$= \frac{-15}{0.162}$$

$$\boxed{\Rightarrow \left.\frac{dv}{dr}\right|_{r=0.005} = -92.59\,(cm/s)/cm}$$

Velocity gradient at r = 0.01cm

$$\frac{dv}{dt} = \frac{-3000\times(0.01)}{2\times 0.027\times 3} = \frac{-30}{0.162}$$

$$\boxed{\Rightarrow \left.\frac{dv}{dr}\right|_{r=0.005} = -185.185\,(cm/s)/cm}$$

(C)

From part (A) we can see that when r = 0 the velocity v = 0.9259 and when r = R the velocity v = 0 hence we can conclude that velocity of blood is greatest along the central axis of the tube.

We see that when r = R the velocity gradient is lowest this means at the wall of the tube the velocity of blood is changing most.

Chapter 2 Derivatives Exercise 2.7 28E

The frequency of vibrations is given as

$$f = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

Where L is the length of string, T is tension and ρ is linear density

(A) (1) The rate of change with respect to the length when T and ρ are constants is.

$$\frac{df}{dL} = \frac{d}{dL}\left(\frac{1}{2L}\left(\sqrt{\frac{T}{\rho}}\right)\right)$$

$$= \frac{1}{2}\sqrt{\frac{T}{\rho}}\frac{d}{dL}\frac{1}{L}$$

$$= \frac{1}{2}\sqrt{\frac{T}{\rho}}\left(-\frac{1}{L^2}\right)$$

$$\frac{df}{dL} = -\frac{1}{2L^2}\sqrt{\frac{T}{\rho}}$$

(2) The rate of change with respect to the tension when L and ρ are constant

$$\frac{df}{dT} = \frac{d}{dT}\left(\frac{1}{2L}\sqrt{\frac{T}{\rho}}\right)$$

$$= \frac{1}{2L}\frac{d}{dT}\left(\sqrt{\frac{T}{\rho}}\right) = \frac{1}{2L}\frac{d}{dT}\frac{\sqrt{T}}{\sqrt{\rho}}$$

$$= \frac{1}{2L}\frac{1}{\sqrt{\rho}}\frac{d}{dT}\sqrt{T}$$

$$= \frac{1}{2L\sqrt{\rho}} \frac{d}{dT}(T)^{1/2} = \frac{1}{2L\sqrt{\rho}}\left(\frac{1}{2} \cdot \frac{1}{\sqrt{T}}\right)$$

$$\boxed{\frac{df}{dT} = \frac{1}{4L\sqrt{\rho T}}}$$

(3) The rate of change with respect to the linear density ρ, when L and T are constants

$$\frac{df}{d\rho} = \frac{d}{d\rho}\left(\frac{1}{2L}\sqrt{\frac{T}{\rho}}\right)$$

$$= \frac{1}{2L} \frac{d}{d\rho}\left(\sqrt{\frac{T}{\rho}}\right)$$

$$= \frac{\sqrt{T}}{2L} \frac{d}{d\rho}\left(\frac{1}{\sqrt{\rho}}\right) = \frac{\sqrt{T}}{2L} \frac{d}{d\rho}\left(\rho^{-1/2}\right)$$

$$= \frac{\sqrt{T}}{2L}\left(-\frac{1}{2}\rho^{-3/2}\right)$$

$$= -\frac{1}{4} \frac{\sqrt{T}}{L\rho\sqrt{\rho}}$$

$$\frac{df}{d\pi} = -\frac{1}{4} \frac{\sqrt{T}}{L\rho\sqrt{\rho}} = \boxed{-\frac{1}{4L\rho}\sqrt{\frac{T}{\rho}}}$$

(B)

(1) When L is decreasing so the frequency is increasing because the rate of change of frequency or derivative of frequency with respect to L is negative. Thus pitch is increasing.

(2) The rate of change of f or derivative of f with respect to T is positive so when T is increasing, f is also increasing thus pitch is increasing.

(3) The rate of change of f or derivative of f with respect to ρ is negative so when linear density is increasing, f is decreasing thus pitch is decreasing.

Chapter 2 Derivatives Exercise 2.7 29E

(b)

Substitute $x = 200$ in marginal cost function $C'(x) = 12 - 0.2x + 0.0015x^2$ dollars per yard.

$$C'(200) = 12 - 0.2(200) + 0.0015(200)^2 \text{ dollars per yard}$$

$$= 12 - 40 + 60$$

$$= \boxed{32 \text{ dollars/yard}}$$

This is the cost of producing 201st yard of fabric because it is how C is changing when x is already 200.

(c)

Substitute $x = 201$ in to the marginal function

$C'(x) = 12 - 0.2x + 0.0015x^2$ dollars per yard, then the value $C'(201)$ is,

$$C'(201) = 12 - 0.2(201) + 0.0015(201)^2$$

$$= 12 - 40.2 + 60.6015$$

$$= 32.4015$$

$$\approx 32 \text{ dollars/yard}$$

Observe the result in part (b) and part (c), it confirms that both results are the same.

Chapter 2 Derivatives Exercise 2.7 30E

Consider the cost function,

$$C(x)=339+25x-0.09x^2+0.0004x^3 \quad \cdots\cdots(1)$$

(a)

The objective is to find and interpret $C'(100)$.

Differentiate equation (1) with respect to *t* to obtain that,

$$\begin{aligned} C'(x) &= \frac{d}{dx}\left(339+25x-0.09x^2+0.0004x^3\right) \\ &= 25-0.18x+0.0012x^2 \end{aligned}$$

Replace *x* by 100 to obtain that,

$$\begin{aligned} C'(100) &= 25-0.18(100)+0.0012(100)^2 \\ &= 25-18+12 \\ &= 19 \end{aligned}$$

This is the rate of change of cost relative to production, so this estimates how much the cost will increase per unit increase in production. It will cost $\boxed{\$19}$ to produce each extra unit.

(b)

The objective is to compare $C'(100)$ with the cost of producing the 101st item.

The value of the function $C(101)$ gives the cost of producing 101st items. To get the cost of producing the 101st item, take that total cost and subtract the cost for producing the first 100 items.

$$\begin{aligned} C(101)-C(100) &= \left[339+25(101)-0.09(101)^2+0.0004(101)^3\right] \\ &\quad -\left[339+25(100)-0.09(100)^2+0.0004(100)^3\right] \\ &= [339+2525-918.09+412.1204]-[339+2500-900+400] \\ &= \boxed{\$19.0304} \end{aligned}$$

Thus, the actual cost of producing the 101st item is about $\boxed{3.04 \text{ cents}}$.

Chapter 2 Derivatives Exercise 2.7 31E

Average productivity of the work force at the plant is

$$A(x)=\frac{P(x)}{x}$$

(A)

$$\begin{aligned} A'(x) &= \frac{d}{dx}\left(A(x)\right) \\ &= \frac{d}{dx}\left[\frac{P(x)}{x}\right] \end{aligned}$$

By using Quotient law

$$= \frac{x.P'(x)-P(x)}{x^2}$$

$$\boxed{A'(x)=\frac{\left[xP'(x)-P(x)\right]}{x^2}}$$

$$A'(x)>0$$

$$\Rightarrow \frac{\left[xP'(x)-P(x)\right]}{x^2}>0$$

$$\Rightarrow xP'(x)-P(x)>0$$

$$\Rightarrow xP'(x)>P(x)$$

So productivity is directly proportional to the number of workers so if workers are increased, the productivity will also increase that is why the company wants to hire more workers for increasing the productivity

(B) If $A'(x) > 0$

$$\Rightarrow \frac{[xP'(x) - P(x)]}{x^2} > 0$$

$$\Rightarrow xP'(x) - P(x) > 0$$

$$\Rightarrow xP'(x) > P(x)$$

$$\Rightarrow P'(x) > P(x)/x$$

$$\Rightarrow \boxed{P'(x) > A(x)}$$ Thus $P'(x)$ is greater than $A(x)$

Chapter 2 Derivatives Exercise 2.7 32E

The area R of the pupil is defined as

$$R = \frac{40 + 24x^{0.4}}{1 + 4x^{0.4}}$$

(A) Sensitivity S = the rate of change of the reaction with respect to s

$$= \frac{dR}{dx}$$

Hence $\frac{dR}{dx} = \frac{d}{dx}\left(\frac{40 + 24x^{0.4}}{1 + 4x^{0.4}}\right)$

Using the Quotient rule

$$\frac{dR}{dx} = \frac{(1 + 4x^{0.4})(24.(0.4).x^{0.4-1}) - (40 + 24x^{0.4})(4.(0.4)\,x^{0.4-1})}{(1 + 4x^{0.4})^2}$$

$$= \frac{(1 + 4x^{0.4})(9.6x^{-0.6}) - (40 + 24x^{0.4})(1.6x^{-0.6})}{(1 + 4x^{0.4})^2}$$

$$= (1.6x^{-0.6})\frac{(6(1 + 4x^{0.4}) - (40 + 24x^{0.4}))}{(1 + 4x^{0.4})^2}$$

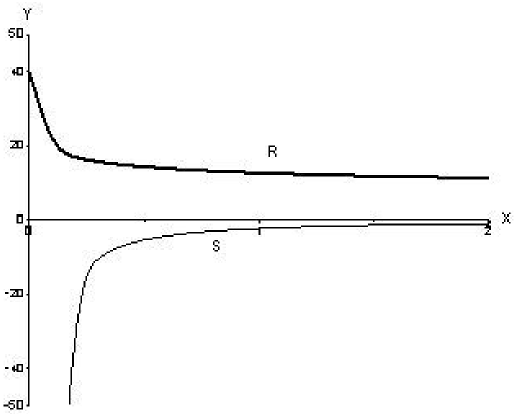
$$= \frac{1.6[6 + 24x^{0.4} - 40 - 24x^{0.4}]}{x^{0.6}(1 + 4x^{0.4})^2}$$

$$\boxed{S = \frac{dR}{dx} = \frac{-54.4}{x^{0.6}(1 + 4x^{0.4})^2}}$$ Square millimeters/ unit brightness

(B) For instant we take the values of x = 1, 2, 3, 4, 5, 6

x	R(x)	S(x)
1	12.8	-2.18
2	11.41	-0.91
3	10.71	-0.54
4	10.27	-0.37
5	9.95	-0.28
6	9.70	-0.22

With help of this data we can draw a graph of R(x) and S(x) in figure 1



At low level of brightness R is very high. It means the area of pupil R is higher. But the sensitivity S is decreasing fast at the law level of brightness. It means sensitivity will be lower at low level of brightness.
We expect that brightness should be optimum (not too much high) for good sensitivity

Chapter 2 Derivatives Exercise 2.7 33E

The gas law for a ideal gas at absolute temperature is

$$PV = nRT \quad \text{--- (1)}$$

Where T (in Kelvin), Pressure P (at m) and volume V (Liters) ,n and R are constants
Differentiate the equation (1) with respect to t (time)

$$\frac{d}{dt}(PV) = nR\frac{dT}{dt}$$

$$\Rightarrow \frac{dT}{dt} = \frac{1}{nR}\frac{d}{dt}(PV)$$

By using product rule

$$\Rightarrow \frac{dT}{dt} = \frac{1}{nR}\left[P.\frac{dV}{dt} + V.\frac{dP}{dt}\right] \quad \text{--- (2)}$$

Now we have rate of volume with respect to time = -0.15 L/min
So

$$\frac{dV}{dt} = -0.15 \quad L/\min$$

And rate of increase of pressure with respect to time t is

$$\frac{dP}{dt} = 0.10 \quad atm/\min$$

And R = 0.0821, n = 10 mol (given)
And P = 8.0 atm, V = 10L

Thus the rate of change of T with respect to time t is

$$\frac{dT}{dt} = \frac{1}{nR}\left[P\frac{dV}{dt} + V\frac{dP}{dt}\right]\backslash$$

$$= \frac{1}{10\times 0.0821}\left[8\times(-0.15) + 10\times(0.10)\right]$$

$$= \frac{[1-1.2]}{.0821}$$

$$= \frac{-0.2}{.0821}$$

$$\boxed{\frac{dT}{dt} \approx -0.2436 \quad Kelvin/\min}$$

Chapter 2 Derivatives Exercise 2.7 34E

(A)

The rate of population change of the fish is given as

$$\frac{dP}{dt} = r_o\left(1 - \frac{P(t)}{P_c}\right)P(t) - \beta P(t)$$

Where r_o is the birth rate of the fish, P_c is the carrying capacity and $\beta =$ harvesting rate.

$$\boxed{\frac{dP}{dt} = 0} \text{ Corresponds to a stable population}$$

(B)

$P_c = 10,000$, $r_o = 5\% = 0.05$ and $\beta = 4\% = 0.04$

We get the stable population level when $\frac{dP}{dt} = 0$

$$\Rightarrow r_o\left(1 - \frac{P(t)}{P_c}\right)P(t) - \beta P(t) = 0$$

$$\Rightarrow r_o\left(1 - \frac{P(t)}{P_c}\right) = \beta$$

$$\Rightarrow 0.05\left(1-\frac{P(t)}{10000}\right)=0.04$$

$$\Rightarrow 1-\frac{P(t)}{10000}=\frac{4}{5}$$

$$\Rightarrow \frac{P(t)}{10000}=1-\frac{4}{5}=\frac{1}{5}$$

$$\Rightarrow P(t)=\frac{10000}{5}\Rightarrow \boxed{P(t)=2000}$$ This is stable Population level

(C)

When $\beta = 5\% = 0.05$ so $P(t)=0$ thus stable population level will be 0

Thus there is no stable population.

Chapter 2 Derivatives Exercise 2.7 35E

The interaction has been modeled by the equations

$$\frac{dc}{dt}=ac-bcw,\ \frac{dw}{dt}=-cw+dcw$$

(A)

For the stable populations

$$\boxed{\frac{dc}{dt}=0} \text{ and } \boxed{\frac{dw}{dt}=0}$$

(B)

The caribou go extinct, means c = 0

(C)

Here, a = 0.05, b = 0.001, c = 0.05 and d = 0.0001 then

First pair (c, w) that lead to stable populations = (0, 0) another pairs we can get by putting the values of a, b, c and d in both the equation for stable population.

$$ac-bcw=0 \quad \text{and} \quad -cw+dcw=0$$

$$\Rightarrow a=bw \quad \text{and} \quad c=dc$$

$$\Rightarrow w=\frac{a}{b} \quad \text{and} \quad c=\frac{c}{d}$$

$$\Rightarrow w=\frac{0.05}{0.005} \quad \text{and} \quad c=\frac{0.5}{0.0001}$$

$$\Rightarrow w=50 \quad \text{and} \quad c=500$$

Thus we have two pair (0, 0) and (500, 50). Yes it is possible for the two species to live in balance.