Fractions and Decimals

Classification of Fractions and Their Conversion

A **common fraction** is written in the form $\frac{a}{b}$, where *a* and *b* both are integers and $b \neq 0$. Here, *a* is numerator and *b* is denominator. Common fractions are also known as **vulgar or simple fractions**.

To understand the concept of fractions better, look at the following video.

These numbers like $\frac{1}{2}$ and $\frac{1}{4}$ etc. are known as **fractions**. Both these fractions have 1 as the numerator. Their denominators are 2 and 4 respectively. Fractions are used to represent a part of a whole.

The fractions can be categorized into 3 types. They are

1. Proper Fraction

2. Improper Fraction

3. Mixed Fraction

To understand each type of fraction, look at the following video.

Important points to note

- If both the numerator and denominator of a fraction are equal, then the value of the fraction is 1. For example, $\frac{9}{9} = 1$.

- We know that an improper fraction is always greater than 1 and a proper fraction is always less than 1. Therefore, we can say that any improper fraction is always greater than any proper fraction. For example, the improper fraction $\frac{11}{9}$ is greater than the proper fraction $\frac{5}{7}$.

A mixed fraction can be converted into an improper fraction and vice-versa. Let us start with the conversion of improper fraction into mixed fraction.

It can be easily understood with the help of an example. Let us convert the improper fraction $\frac{34}{15}$ into mixed fraction. For this, we have to follow these steps.

Step 1: Divide the numerator by the denominator of the improper fraction to obtain the quotient and remainder.

The numerator and denominator of the fraction 34/15 are 34 and 15 respectively. Now, the division process can be done as

$$\begin{array}{r} 2 \\ 15 \overline{)\ 34} \\ -30 \\ \hline 4 \\ \hline \end{array}$$

In the above division process, the quotient = 2 and the remainder = 4.

Step 2: The improper fraction can then be expressed as a mixed fraction by writing it in the form of

$$\text{Quotient}\frac{\text{Remainder}}{\text{Denominator}}$$

For the improper fraction $\frac{34}{15}$, we have the denominator as 15, the remainder as 4, and the quotient as 2.

Therefore, we can write the improper fraction $\frac{34}{15}$ as a mixed fraction as $2\frac{4}{15}$.

Let us now try and convert a mixed fraction into an improper fraction.

We know that a mixed fraction is a combination of a whole number and a fraction. Let us consider a mixed fraction of the form of $\left(\text{Whole number}\frac{\text{Numerator}}{\text{Denominator}}\right)$.

The improper fraction of this mixed fraction is written in the form of

$$\frac{(\text{Whole Number} \times \text{Denominator}) + \text{Numerator}}{\text{Denominator}}$$

It can be easily understood with the help of an example.

Previously, we had converted the improper fraction $\frac{34}{15}$ into the mixed fraction $2\frac{4}{15}$. Let us now try to convert the mixed fraction $2\frac{4}{15}$ into its improper fraction form $\frac{34}{15}$.

Now, in the mixed fraction, the whole number is 2, the numerator is 4, and the denominator is 15. Thus, we can convert the mixed fraction as

$$\frac{(\text{Whole Number} \times \text{Denominator}) + \text{Numerator}}{\text{Denominator}} = \frac{2 \times 15 + 4}{15} = \frac{30 + 4}{15} = \frac{34}{15}$$

As we can see, we converted the mixed fraction $2\frac{4}{15}$ back into its improper fraction form.

It is, however, very important to remember that a proper fraction *cannot* be expressed as an improper or a mixed fraction. *Neither* can an improper or a mixed fraction be expressed as a proper fraction.

Example 1:

Identify the proper, improper and mixed fractions from the following fractions.

$$\frac{1}{3}, \frac{2}{5}, \frac{3}{2}, \frac{7}{11}, 1\frac{5}{6}, 3\frac{2}{5}, \frac{22}{21}, \frac{45}{55}, 12\frac{2}{7}$$

Solution:

The numerators of the fractions $\frac{1}{3}, \frac{2}{5}, \frac{7}{11}$, and $\frac{45}{55}$ are less than their corresponding denominators. Therefore, these fractions are proper fractions.

The numerators of the fractions $\frac{3}{2}$ and $\frac{22}{21}$ are greater than their corresponding denominators. Therefore, these fractions are improper fractions.

The fractions $1\frac{5}{6}$, $3\frac{2}{5}$, and $12\frac{2}{7}$ are formed by combining a whole number and a proper fraction. Therefore, these fractions are mixed fractions.

Example 2:

Express the mixed fraction $9\frac{3}{5}$ as an improper fraction. Then convert the improper fraction so obtained back into its mixed fraction equivalent to verify your answer.

Solution:

The mixed fraction $9\frac{3}{5}$ has 9 as the whole number, 3 as the numerator and 5 as the denominator. This fraction can be expressed as an improper fraction as

$$\frac{(\text{Whole Number} \times \text{Denominator}) + \text{Numerator}}{\text{Denominator}} = \frac{(9 \times 5) + 3}{5} = \frac{45 + 3}{5} = \frac{48}{5}$$

The improper fraction, thus obtained, is $\frac{48}{5}$. This fraction has 48 as its numerator and 5 as its denominator. We first need to divide the numerator (48) by the denominator (5), which can be done as:

$$\begin{array}{r} 9 \\ 5\overline{)\ 48} \\ -45 \\ \hline 3 \\ \hline \end{array}$$

We, thus, obtain 9 as the quotient and 3 as the remainder. We know that we can express an improper fraction as $\text{Quotient}\frac{\text{Remainder}}{\text{Denominator}}$. Thus, we can express the improper fraction $\frac{48}{5}$ as $9\frac{3}{5}$. We again obtained the same mixed fraction that we started off with. This verifies our calculations.

Simplest Form of Fractions

Consider the fraction $\frac{105}{45}$. We know that equivalent fractions of this fraction can be obtained by dividing the numerator and the denominator by the same number.

Thus, an equivalent fraction of $\frac{105}{45}$ is

$$\frac{105 \div 5}{45 \div 5} = \frac{21}{9}$$

Now, can we further divide the fraction $\frac{21}{9}$ to find its equivalent fraction?

Let us now try to find an equivalent fraction of this fraction.

The fraction $\frac{21}{9}$ can be written as $\frac{21 \div 3}{9 \div 3} = \frac{7}{3}$.

Now, can we find the equivalent fraction of $\frac{7}{3}$ by division?

In the fraction $\frac{7}{3}$, the numerator and the denominator have no common factor other than 1.

The fraction $\frac{7}{3}$ is the simplest form of the given fraction $\frac{105}{45}$.

"**In the simplest form of a fraction, its numerator and denominator have 1 as the only common factor".**

Observe that **if we divide the numerator and the denominator of a fraction by their highest common factor, then the fraction obtained is the simplest form of the given fraction.**

For example, let us again consider the fraction $\frac{105}{45}$.

The highest common factor of 105 and 45 is 15. Therefore, when we divide the numerator and denominator of the fraction $\frac{105}{45}$ by 15, we obtain the simplest form of this fraction.

Thus, the simplest form of the fraction $\frac{105}{45}$ is $\frac{105 \div 15}{45 \div 15} = \frac{7}{3}$.

Let us now look at some more examples to understand this concept better.

Example 1:

Reduce $\frac{48}{64}$ into its simplest form.

Solution:

The common factors of 48 and 64 are 1, 2, 4, 8, and 16. The highest common factor (HCF) is 16.

Now, $\frac{48}{64} = \frac{48 \div 16}{64 \div 16} = \frac{3}{4}$

3 and 4 have no common factor except 1.

Thus, the simplest form of $\frac{48}{64}$ is $\frac{3}{4}$.

Example 2:

Obtain the simplest form of the fraction $\frac{275}{125}$.

Solution:

The highest common factor of 275 and 125 is 25.

On dividing the numerator and the denominator by 25, we obtain

$$\frac{275 \div 25}{125 \div 25} = \frac{11}{5}$$

Thus, $\frac{11}{5}$ is the simplest form of the given fraction since the numerator and the denominator have no common factor other than 1.

Equivalent Fractions

Look at the fractions $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$. They seem to represent different fractions. However, do they represent the same or different quantities? Let us find out with help of the following video.

But the question that arises here is that if we are given a fraction, then **how will we find equivalent fractions of that fraction?**

We can find equivalent fraction by the following way.

An equivalent fraction of the given fraction can be found out by multiplying both the numerator and the denominator of the given fraction by the same number.

An equivalent fraction of a given number can also be found out by dividing both the numerator and the denominator of the given fraction by the same number.

For example, let us find an equivalent fraction of $\frac{2}{5}$ and $\frac{16}{32}$.

We can find an equivalent fraction of $\frac{2}{5}$ by multiplying the numerator and the denominator by the same number.

Therefore, an equivalent fraction of $\frac{2}{5}$ is $\frac{2\times 2}{5\times 2} = \frac{4}{10}$.

A fraction can have many equivalent fractions. For example, $\frac{6}{15}, \frac{8}{20}, \frac{10}{25}$ are also equivalent fractions of $\frac{2}{5}$.

An equivalent fraction of $\frac{16}{32}$ is $\frac{16 \div 16}{32 \div 16} = \frac{1}{2}$.

Observe that in order to find an equivalent fraction of the fraction 16/32, we preferred division of both the numerator and the denominator by the same number rather than multiplication. This is because if we multiply it by the same number, then the calculations will be more complex. That is why we preferred division in this case.

Another way to check whether the fractions are equivalent or not

If the cross products of the terms of two fractions are equal, then the two fractions are equivalent.

Consider the fractions $\frac{2}{5}$ and $\frac{6}{15}$.

$2 \times 15 = 5 \times 6$ (By cross multiplication)

$30 = 30$

$\therefore \frac{2}{5} = \frac{6}{15}$

Hence, the fractions $\frac{2}{5}$ and $\frac{6}{15}$ are equivalent.

Let us now look at some more examples to understand this concept better.

Example 1:

Find three equivalent fractions of the following fractions.

1. $\frac{3}{5}$
2. $\frac{36}{42}$

Solution:

1. By multiplying both the numerator and the denominator of a given fraction by the same number, an equivalent fraction of the given number can be found out. Therefore,

$$\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10}$$
$$\frac{3}{5} = \frac{3 \times 3}{5 \times 3} = \frac{9}{15}$$
$$\frac{3}{5} = \frac{3 \times 4}{5 \times 4} = \frac{12}{20}$$

Thus, three equivalent fractions of $\frac{3}{5}$ are $\frac{6}{10}, \frac{9}{15}$, and $\frac{12}{20}$.

2. We can find equivalent fractions of the given fractions by dividing the numerator and the denominator by the same number.

$$\frac{36}{42} = \frac{36 \div 2}{42 \div 2} = \frac{18}{21}$$
$$\frac{36}{42} = \frac{36 \div 3}{42 \div 3} = \frac{12}{14}$$
$$\frac{36}{42} = \frac{36 \div 6}{42 \div 6} = \frac{6}{7}$$

Thus, three equivalent fractions of $\frac{36}{42}$ are $\frac{18}{21}, \frac{12}{14}$, and $\frac{6}{7}$.

Example 2:

Check whether $\frac{5}{7}$ and $\frac{40}{56}$ are equivalent fractions.

Solution:

The given fractions are $\frac{5}{7}$ and $\frac{40}{56}$.

We know that 7 × 8 = 56. Now, we multiply both the numerator and the denominator of the fraction $\frac{5}{7}$ by 8.

$$\frac{5\times 8}{7\times 8}=\frac{40}{56}$$

Therefore, $\frac{5}{7}$ and $\frac{40}{56}$ are equivalent fractions.

Example 3:

Check whether $\frac{2}{5}$ and $\frac{36}{60}$ are equivalent.

Solution:

The given fractions are $\frac{2}{5}$ and $\frac{36}{60}$.

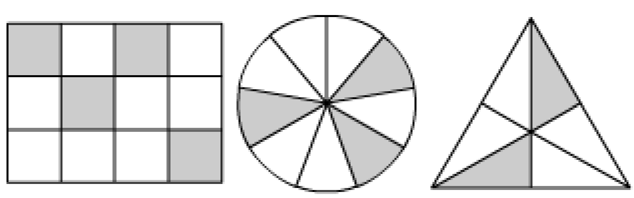
We know that 60 ÷ 12 = 5. Now, we divide both the numerator and the denominator of the fraction $\frac{36}{60}$ by 12.

$$\frac{36\div 12}{60\div 12}=\frac{3}{5}\neq\frac{2}{5}$$

Therefore, $\frac{2}{5}$ and $\frac{36}{60}$ are not equivalent.

Example 4:

Are the following fractions equivalent?



Solution:

The fraction represented by the first figure is $\frac{4}{12}$.

The fraction represented by the second figure is $\frac{3}{9}$.

The fraction represented by the third figure is $\frac{2}{6}$.

Now, $\frac{4}{12}=\frac{4\div 4}{12\div 4}=\frac{1}{3}$

$\frac{3}{9}=\frac{3\div 3}{9\div 3}=\frac{1}{3}$

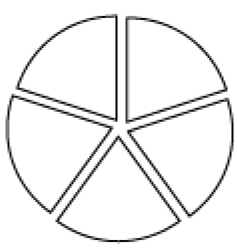
$\frac{2}{6}=\frac{2\div 2}{6\div 2}=\frac{1}{3}$

$\therefore \frac{4}{12}=\frac{3}{9}=\frac{2}{6}$

Thus, the above figures represent equivalent fractions.

Comparing and Ordering Like and Unlike Fractions

Suppose you invite two of your friends over to your house. You buy a pizza and cut it into five equal parts to divide it among yourselves. The pizza will look as follows:



Now, let us suppose you eat one slice of pizza and your friends eat two slices each. Now, how will you determine the exact quantity of pizza eaten by you?

There were five equal slices of pizza in the beginning. You ate one slice of it. You can say that you ate one out of five slices of pizza. This is represented by $\frac{1}{5}$. Similarly, each of your friends ate two slices, i.e., $\frac{2}{5}$ of the pizza. These types of numbers are known as **fractions.** Here, we will learn about **like** and **unlike fractions,** and how to compare them.

Let us first learn about like and unlike fractions.

Like Fraction

The fractions whose denominators are same are called like fractions.

For example: $\frac{2}{13}, \frac{5}{13}, \frac{7}{13}, \frac{8}{13}$ etc.

Unlike Fraction

Unlike fractions are the fractions whose denominators are different.

For example: $\frac{7}{9}, \frac{4}{7}, \frac{15}{20}, \frac{16}{21}$ etc.

Let us consider some examples based on like and unlike fractions.

Example 1:

Separate like and unlike fractions from the following fractions.

$$\mathbf{\frac{5}{19}, \frac{5}{25}, \frac{17}{27}, \frac{10}{38}, \frac{10}{13}, \frac{5}{26}, \frac{8}{19}, \frac{19}{13}, \frac{11}{19}, \frac{10}{18}, \frac{7}{14}, \frac{18}{19}, \frac{2}{13}}$$

Solution:

Like fractions: $\frac{5}{19}, \frac{8}{19}, \frac{11}{19}, \frac{18}{19}$

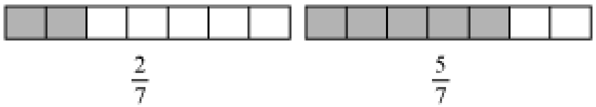
Like fractions: $\frac{10}{13}, \frac{2}{13}, \frac{19}{13}$

Unlike fractions: $\frac{5}{25}, \frac{17}{27}, \frac{10}{38}, \frac{5}{26}, \frac{10}{18}, \frac{7}{14}$

Now, that we know what like and unlike fractions are, let us now learn how to compare and order them.

Comparing and ordering of like fractions:

Let us compare two like fractions $\frac{2}{7}$ and $\frac{5}{7}$. We can represent them as:



In both the fractions, the whole is divided into 7 equal parts. To represent 2/7, we shade 2 parts out of the 7 equal parts; and to represent 5/7, we shade 5 parts out of the 7 equal parts. Clearly, out of the 7 equal parts in the two figures, more portions have been shaded in the figure corresponding to 5 parts than that in the figure corresponding to 2 parts.

$\therefore \frac{5}{7} > \frac{2}{7}$

We can generalise this as: "**Out of two like fractions, the fraction with the greater numerator is greater.**"

For example, among the like fractions $\frac{6}{8}, \frac{9}{8}$ and $\frac{4}{8}$:

$\frac{9}{8} > \frac{6}{8} > \frac{4}{8}$ as 9 > 6 > 4.

Let us discuss some examples keeping the above fact in mind.

Example 2:

Arrange the following fractions in ascending order.

$\mathbf{\frac{5}{18}, \frac{11}{18}, \frac{3}{18}, \frac{17}{18}, \frac{9}{18}, \frac{16}{18}}$ **and** $\mathbf{\frac{14}{18}}$

Solution:

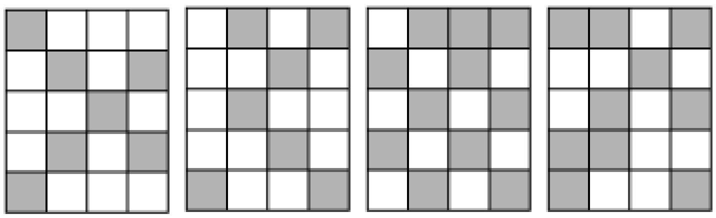
The fractions $\frac{5}{18}, \frac{11}{18}, \frac{3}{18}, \frac{17}{18}, \frac{9}{18}, \frac{16}{18}$ and $\frac{14}{18}$ are like fractions as the fractions have the same denominator.

If we arrange the numerator in ascending order we have 3, 5, 9, 11, 14, 16, and 17.

Hence, the arrangement of the given fractions in ascending order is $\frac{3}{18}, \frac{5}{18}, \frac{9}{18}, \frac{11}{18}, \frac{14}{18}, \frac{16}{18}, \frac{17}{18}$.

Example 3:

Write the fractions corresponding to each figure drawn below. Compare the fractions using the correct sign '<', '=', '>' between the fractions.



Solution:

The fractions represented by the given figures are respectively $\frac{7}{20}, \frac{7}{20}, \frac{11}{20}$ and $\frac{10}{20}$.

We can now arrange the fractions in ascending order as $\frac{7}{20} = \frac{7}{20} < \frac{10}{20} < \frac{11}{20}$.

Example 4:

Math Olympiad was conducted in Delhi, Mumbai and Hyderabad. 200, 300, and 450 candidates were supposed to appear for the test in Delhi, Mumbai and Hyderabad respectively. But due to certain reasons only 180, 90, and 315 candidates appeared for the test in the above cities respectively. Which city has the

(a) greatest fraction of candidates who appeared for the test?

(b) least fraction of candidates who appeared for the test?

Solution:

In **Delhi,** 180 candidates appeared for the test out of 200 candidates.

Hence, fraction of candidates who appeared for the test in Delhi = $\frac{180}{200} = \frac{180 \div 20}{200 \div 20} = \frac{9}{10}$

In **Mumbai,** 90 candidates appeared for the test out of 300 candidates.

Hence, fraction of candidates who appeared for the test in Mumbai =
$\frac{90}{300} = \frac{90 \div 30}{300 \div 30} = \frac{3}{10}$

In **Hyderabad,** 315 candidates appeared for the test out of 450 candidates.

Hence, fraction of candidates who appeared for the test in Hyderabad =
$\frac{315}{450} = \frac{315 \div 45}{450 \div 45} = \frac{7}{10}$

The fraction of candidates who appeared for the test in Delhi, Mumbai and Hyderabad are $\frac{9}{10}$, $\frac{3}{10}$, and $\frac{7}{10}$ respectively.

These fractions are like fractions (same denominator).

Comparing the numerators, we will obtain 9 > 7 > 3.

Hence, $\frac{9}{10} > \frac{7}{10} > \frac{3}{10}$.

1. Since the fraction $\frac{9}{10}$ is the greatest among the other fractions, Delhi is the city where the greatest fraction of candidates appeared for the test.

2. Since the fraction $\frac{3}{10}$ is the smallest among all, Mumbai is the city where least fraction of candidates appeared for the test.

So far we have learnt comparing and ordering of like fractions. Let us know how we can compare and order unlike fractions.

Comparing and ordering of unlike fractions:

Unlike fractions are of two types:

1. Unlike fractions with same numerator; and
2. Unlike fractions with different numerators

Let us try and understand how to compare and order unlike fractions of both types with the help of the following video.

Let us discuss some more examples based on comparison and ordering of unlike fractions.

Example 5:

Arrange the following unlike fractions in descending order.

$\frac{17}{5}, \frac{17}{19}, \frac{17}{1}, \frac{17}{45}, \frac{17}{18}$ **and** $\frac{17}{22}$

Solution:

The fractions $\frac{17}{5}, \frac{17}{19}, \frac{17}{1}, \frac{17}{45}, \frac{17}{18}$ and $\frac{17}{22}$ are unlike fractions with the same numerator.

We can arrange the denominators of these fractions in ascending order as:

1 < 5 < 18 < 19 < 22 < 45

Hence, the given fractions in descending order are $\frac{17}{1}, \frac{17}{5}, \frac{17}{18}, \frac{17}{19}, \frac{17}{22}, \frac{17}{45}$.

Example 6:

Arrange the following fractions in descending order.

$\frac{2}{5}, \frac{4}{7}, \frac{1}{2}$ **and** $\frac{5}{4}$

Solution:

The fractions $\frac{2}{5}, \frac{4}{7}, \frac{1}{2}$ and $\frac{5}{4}$ are unlike fractions with different denominators.

Now, we can find the LCM of 5, 7, 2, and 4 as:

$$
\begin{array}{l|l}
2 & 5,\ 7,\ 2,\ 4 \\ \hline
2 & 5,\ 7,\ 1,\ 2 \\ \hline
5 & 5,\ 7,\ 1,\ 1 \\ \hline
7 & 1,\ 7,\ 1,\ 1 \\ \hline
 & 1,\ 1,\ 1,\ 1
\end{array}
$$

Hence, LCM = 2 × 2 × 5 × 7 = 140

Now, we can convert the denominator of each fraction to 140 as:

$$\frac{2}{5} = \frac{2 \times 28}{5 \times 28} = \frac{56}{140}$$

$$\frac{4}{7} = \frac{4 \times 20}{7 \times 20} = \frac{80}{140}$$

$$\frac{1}{2} = \frac{1 \times 70}{2 \times 70} = \frac{70}{140}$$

$$\frac{5}{4} = \frac{5 \times 35}{4 \times 35} = \frac{175}{140}$$

If we arrange the numerator in descending order, we obtain 175 > 80 > 70 > 56.

So, $\frac{175}{140} > \frac{80}{140} > \frac{70}{140} > \frac{56}{140}$.

Hence, the given fractions in decreasing order are $\frac{5}{4} > \frac{4}{7} > \frac{1}{2} > \frac{2}{5}$.

Example 7:

In class A of 92 students, 69 got scholarship; in class B of 80 students, 64 got scholarship and in another class C of 99 students, 66 got scholarship. In which class did a greater fraction of students get scholarship?

Solution:

In class A of 92 students, 69 got scholarship.

So, fraction of the students who got scholarship in class A = $\frac{69}{92} = \frac{69 \div 23}{92 \div 23} = \frac{3}{4}$

(HCF of 69 and 92 is 23)

In class B of 80 students, 64 got scholarship.

So, fraction of the students who got scholarship in class B = $\frac{64}{80} = \frac{64 \div 16}{80 \div 16} = \frac{4}{5}$

(HCF of 64 and 80 is 16)

In class C of 99 students, 66 got scholarship.

So, fraction of the students who got scholarship in class C = $\frac{66}{99} = \frac{66 \div 33}{99 \div 33} = \frac{2}{3}$

(HCF of 66 and 99 is 33)

We have 3 fractions $\frac{2}{3}, \frac{3}{4}$ and $\frac{4}{5}$.

$$\begin{array}{r|l} 3 & 3, 4, 5 \\ \hline 4 & 1, 4, 5 \\ \hline 5 & 1, 1, 5 \\ \hline & 1, 1, 1 \end{array}$$

LCM = 3 × 4 × 5 = 60

$$\frac{2}{3} = \frac{2 \times 20}{3 \times 20} = \frac{40}{60}$$

$$\frac{3}{4} = \frac{3 \times 15}{4 \times 15} = \frac{45}{60}$$

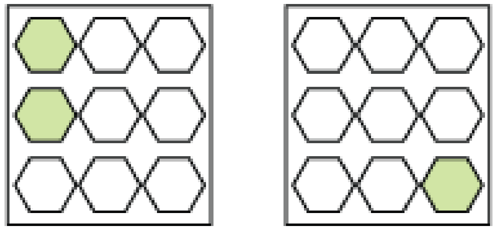
$$\frac{4}{5} = \frac{4 \times 12}{5 \times 12} = \frac{48}{60}$$

Clearly 4/5 is the greatest among all.

Hence, the students of class B got a greater fraction of scholarship.

Addition and Subtraction of Like and Unlike Fractions

Consider the fractions represented by the following figures.



The fractions represented by these figures are $\frac{2}{9}$ and $\frac{1}{9}$.

What will happen if we combine these two fractions?

Combining these two fractions means adding the two fractions.

To understand the addition and subtraction of like fractions, look at the given video.

To understand the addition and subtraction of unlike fractions, look at the given video.

Let us now see how to add and subtract mixed fractions.

In case of mixed fractions, we can add or subtract the whole part and the fraction part separately.

For example, let us add $7\frac{5}{9}$ to $2\frac{3}{4}$.

$$2\frac{3}{4}+7\frac{5}{9}$$

$$=2+\frac{3}{4}+7+\frac{5}{9}$$

$$=(2+7)+\left(\frac{3}{4}+\frac{5}{9}\right)$$

$$=9+\left(\frac{3}{4}+\frac{5}{9}\right)$$

$$= 9 + \left(\frac{3\times 9}{4\times 9} + \frac{5\times 4}{9\times 4}\right) \qquad (\because \text{The L.C.M. of 4 and 9 is 36})$$
$$= 9 + \left(\frac{27}{36} + \frac{20}{36}\right)$$
$$= 9 + \left(\frac{47}{36}\right)$$
$$= 9 + \left(\frac{36+11}{36}\right)$$
$$= 9 + \left(1 + \frac{11}{36}\right) = 10 + \frac{11}{36} = 10\frac{11}{36}$$

Another way of adding or subtracting mixed fractions is to convert the mixed fractions into improper fractions. Now, these improper fractions can be added in the usual manner.

Let us find the value of $3\frac{4}{5} - 2\frac{1}{6}$.

$$3\frac{4}{5} - 2\frac{1}{6}$$
$$= \frac{(3\times 5)+4}{5} - \frac{(2\times 6)+1}{6}$$
$$= \frac{19}{5} - \frac{13}{6}$$
$$= \frac{19\times 6}{5\times 6} - \frac{13\times 5}{6\times 5} \qquad (\text{The L.C.M. of 5 and 6 is 30})$$
$$= \frac{114}{30} - \frac{65}{30}$$
$$= \frac{114-65}{30}$$
$$= \frac{49}{30} = \frac{30+19}{30} = 1 + \frac{19}{30} = 1\frac{19}{30}$$

Let us now look at some more examples to understand the concept better.

Example 1:

Solve the following:

1. $\frac{3}{4}+\frac{5}{6}$
2. $\frac{31}{6}+3\frac{9}{16}-\frac{5}{18}$

Solution:

1. $\frac{3}{4}+\frac{5}{6}$

2	4	6
2	2	3
3	1	3
	1	1

The LCM of 4 and 6 = 2 ×2 ×3 = 12

$$\frac{3}{4}+\frac{5}{6}=\frac{3\times3}{4\times3}+\frac{5\times2}{6\times2}$$
$$=\frac{9}{12}+\frac{10}{12}=\frac{9+10}{12}=\frac{19}{12}$$

2. $\frac{31}{6}+3\frac{9}{16}-\frac{5}{18}$

$$=\frac{31}{6}+\frac{(3\times16)+9}{16}-\frac{5}{18}$$
$$=\frac{31}{6}+\frac{57}{16}-\frac{5}{18}$$

2	6	16	18
2	3	8	9
2	3	4	9
2	3	2	9
3	3	1	9
3	1	1	3
	1	1	1

The LCM of 6, 16, and 18 = 2 × 2 × 2 × 2 × 3 × 3 = 144

$\frac{31}{6}+\frac{57}{16}-\frac{5}{18}$

$=\frac{31\times 24}{6\times 24}+\frac{57\times 9}{16\times 9}-\frac{5\times 8}{18\times 8}$

$=\frac{744}{144}+\frac{513}{144}-\frac{40}{144}$

$=\frac{744+513-40}{144}$

$=\frac{1217}{144}$

$=\frac{(8\times 144)+65}{144}$

$=8+\frac{65}{144}$

$=8\frac{65}{144}$

Example 2:

Sonu walked 3/7 of a kilometre. After that, he ran for 2/7 of a kilometre. How much distance did he cover?

Solution:

Total distance travelled = $\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}$ of a kilometre

$=\frac{5}{7}$ of a kilometre

Example 3:

Isha bought $\frac{1}{2}$ kg of mangoes. If she ate $\frac{1}{6}$ kg of it, then how much is left with her?

Solution:

Weight of mangoes left = $\frac{1}{2}-\frac{1}{6}$

$$= \frac{1\times 3}{2\times 3} - \frac{1\times 1}{6\times 1} \qquad (\because \text{L.C.M. of 2 and 6 is 6})$$

$$= \frac{3}{6} - \frac{1}{6} = \frac{3-1}{6} = \frac{2}{6} = \frac{1}{3}\ \text{kg}$$

Example 4:

Sumit bought $3\frac{3}{4}$ litres of milk. Later he went out and bought $2\frac{1}{2}$ litres more. How much milk did he buy?

Solution:

Milk bought by Sumit = $3\frac{3}{4} + 2\frac{1}{2}$

$$= \frac{(3\times 4)+3}{4} + \frac{(2\times 2)+1}{2}$$

$$= \frac{15}{4} + \frac{5}{2}$$

$$= \frac{15\times 1}{4\times 1} + \frac{5\times 2}{2\times 2} \qquad (\text{The L.C.M. of 4 and 2 is 4})$$

$$= \frac{15}{4} + \frac{10}{4}$$

$$= \frac{15+10}{4}$$

$$= \frac{25}{4}$$

$$= \frac{(6\times 4)+1}{4}$$

$$= 6 + \frac{1}{4}$$

$$= 6\frac{1}{4}$$

Thus, Sumit bought $6\frac{1}{4}$ litres of milk.

Example 5:

Seema bought 18 kg of oranges. She gave $4\frac{1}{2}$ kg, $3\frac{1}{4}$ kg, $3\frac{3}{4}$ kg and 3 kg to her four friends. Find the weight of the oranges left with Seema.

Solution:
Total weight of total oranges = 18 kg

Weight of oranges given to friends

$$= 4\frac{1}{2} + 3\frac{1}{4} + 3\frac{3}{4} + 3$$
$$= \frac{9}{2} + \frac{13}{4} + \frac{15}{4} + 3$$
$$= \frac{18 + 13 + 15 + 12}{4}$$
$$= \frac{58}{4} \text{ kg}$$

Weight of remaining oranges

$$= 18 - \frac{58}{4}$$
$$= \frac{72 - 58}{4}$$
$$= \frac{14}{4} = \frac{7}{2}$$
$$= 3\frac{1}{2} \text{ kg}$$

Thus, $3\frac{1}{2}$ kg of oranges is left with Seema.

Multiplication of Fractions with Fractions

We have learnt how to multiply integers or decimals. But we have another form of numbers whose multiplication we have to learn. Those types of numbers are fractions.

Let us go through the given video to understand how to multiply two fractions.

There are several more properties in multiplication of fractions which we require to keep in mind while carrying out the multiplication.

These points are summed up below.

Points to remember

(i) ***The value of the product of two proper fractions is always less than the value of each of the fraction that is being multiplied.***

For example,

$$\frac{2}{3}\times\frac{4}{7}=\frac{2\times4}{3\times7}=\frac{8}{21}$$

We know that $\frac{2}{3}=\frac{2\times7}{3\times7}=\frac{14}{21}$

Since $\frac{8}{21}<\frac{14}{21}$

Therefore, $\frac{8}{21}<\frac{2}{3}$

Again, $\frac{4}{7}=\frac{4\times3}{7\times3}=\frac{12}{21}$

Since $\frac{8}{21}<\frac{12}{21}$

Therefore, $\frac{8}{21}<\frac{4}{7}$

(ii) ***The value of the product of two improper fractions is always greater than the value of each of the fraction that is being multiplied.***

For example,

$$\frac{7}{4}\times\frac{6}{5}=\frac{7\times6}{4\times5}=\frac{42}{20}$$

We know that $\frac{7}{4}=\frac{7\times5}{4\times5}=\frac{35}{20}$

Since $\frac{42}{20}>\frac{35}{20}$

Therefore, $\frac{42}{20} > \frac{7}{4}$

Again, $\frac{6}{5} = \frac{6\times4}{5\times4} = \frac{24}{20}$

Since $\frac{42}{20} > \frac{24}{20}$

Therefore, $\frac{42}{20} > \frac{6}{5}$

Let us now solve some examples.

Example 1:

Find the value of the following expressions.

(i) $\frac{2}{3}\times\frac{6}{5}$ **(ii)** $\frac{7}{4} \times 2\frac{4}{5}$

(iii) $3\frac{5}{9}\times7\frac{1}{2}$ **(iv)** $5\frac{3}{8}\times\frac{2}{5}$

Solution:

(i) $\frac{2}{3}\times\frac{6}{5} = \frac{2\times6}{3\times5}$

$= \frac{12}{15}$

$= \frac{4}{5}$

(ii) $\frac{7}{4}\times2\frac{4}{5} = \frac{7}{4}\times\frac{14}{5}$

$= \frac{7 \times 14}{4 \times 5}$

$= \frac{98}{20}$

$= \frac{49}{10}$

(iii) $3\frac{5}{9} \times 7\frac{1}{2} = \frac{32}{9} \times \frac{15}{2}$

$= \frac{32 \times 15}{9 \times 2}$

$= \frac{480}{18}$

$= \frac{80}{3}$

(iv) $5\frac{3}{8} \times \frac{2}{5} = \frac{43}{8} \times \frac{2}{5}$

$= \frac{43 \times 2}{8 \times 5}$

$= \frac{86}{40}$

$= \frac{43}{20}$

Example 2:

Simon can eat $\frac{3}{4}$ of an apple in 1 minute. How many apples can she eat in $2\frac{2}{3}$ minutes?

Solution:

Apples that Simon can eat in 1 minute = $\frac{3}{4}$ apple

Apples that she can eat in $2\frac{2}{3}$ minutes $= \frac{3}{4} \times 2\frac{2}{3}$

$= \frac{3}{4} \times \frac{8}{3}$

$= \frac{3 \times 8}{4 \times 3}$

$= \frac{24}{12}$

$= 2 \text{ apples}$

Thus, Simon can eat $2 \text{ apples in } 2\frac{2}{3} \text{ minutes.}$

Example 3:

The length of a rectangular park is $24\frac{3}{5}$ m. Find the width of the park, if its width is $\frac{1}{3}$ of the length.

Solution:

It is given that length of the park = $24\frac{3}{5}\text{ m}$

$= \frac{123}{5}\text{ m}$

Therefore, width of the park $= \frac{1}{3} \times \frac{123}{5}$

$$= \frac{1 \times 123}{3 \times 5}$$

$$= \frac{123}{15}$$

$$= \frac{41}{5}$$

$$= 8 + \frac{1}{5}$$

$$= 8\frac{1}{5} \text{ m}$$

Thus, the width of the park is $8\frac{1}{5}$ m.

Example 4:

Deepa bought $8\frac{3}{4}$ m long ribbon. She had given $\frac{2}{5}$ of the ribbon to her sister Tanu. Find the length of the remaining ribbon.

Solution:

Original length of the ribbon = $8\frac{3}{4} \text{ m} = \frac{35}{4} \text{ m}$

$\frac{2}{5}$ of the ribbon was given to her sister Tanu.

∴ Length of the ribbon given to Tanu = $\frac{2}{5} \times \frac{35}{4} = \frac{7}{2} \text{ m}$

Thus, length of the remaining ribbon = $\frac{35}{4} - \frac{7}{2} = \frac{35-14}{4} = \frac{21}{4} = 5\frac{1}{4} \text{ m}$

Multiplication of Fractions with Whole Numbers

Sometimes, we are required to add a fraction repeatedly. Let us consider one such situation. Suppose Mohit's school is located at a distance of $\frac{3}{4}$ km from his house. Now, how do we find the total distance that Mohit travels from his home to school and back?

Here, we know that Mohit's school is located at a distance of $\frac{3}{4}$ km from his house. He travels the same distance while going to school and while returning home every day.

Thus, total distance that Mohit travels every day = $\left(\frac{3}{4}+\frac{3}{4}\right)$ km

We could have also written this as $\left(2\times\frac{3}{4}\right)$ km.

Here, instead of multiplying the fraction with 2, we could have easily added it twice. However, what would have happened if we were to find the total distance traveled by Mohit to his school and back in 20 days? Then it would not be possible to add the fraction so many times. It would be easier if we multiplied the fraction with 40.

To multiply a fraction with a whole number, the whole number is multiplied with the numerator of the fraction, keeping the denominator same.

Thus, if *a* is a whole number and $\frac{b}{c}$ is a fraction, then $a\times\frac{b}{c}=\frac{a\times b}{c}$.

For example,

$$2\times\frac{1}{4}=\frac{2\times1}{4}=\frac{2}{4}=\frac{1}{2}$$
$$3\times\frac{7}{4}=\frac{3\times7}{4}=\frac{21}{4}$$
$$\frac{3}{5}\times8=\frac{3\times8}{5}=\frac{24}{5}\text{ etc.}$$

In the example that we were discussing in the beginning, we can multiply the fraction $\frac{3}{4}$ with 40 as

$$\frac{3}{4}\times40=\frac{3\times40}{4}=3\times10=30$$

Thus, Mohit would travel 30 km to his school and back in 20 days.

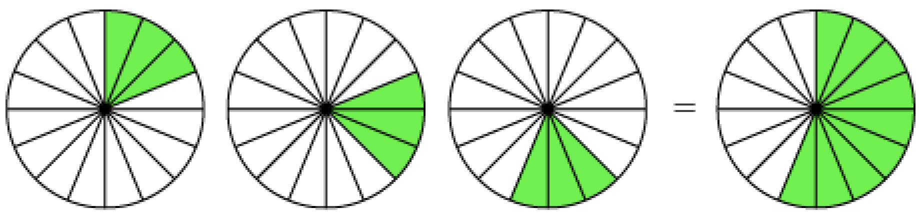
Now, suppose we want to **multiply a whole number with a mixed fraction**, say for example, we want to find the value of the expression $3 \times 2\frac{4}{5}$.

To multiply a mixed fraction with a whole number, convert the mixed fraction into an improper fraction and then carry out the multiplication operation.

$$\therefore 3 \times 2\frac{4}{5} = 3 \times \frac{14}{5} = \frac{3 \times 14}{5} = \frac{42}{5}$$

We can also represent the **multiplication of fractions with whole numbers with the help of figures.** Let us see how.

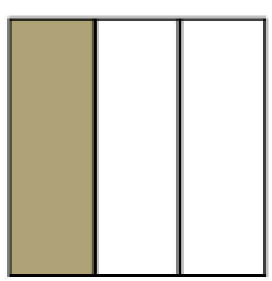
Which expression does the following set of figures represent?



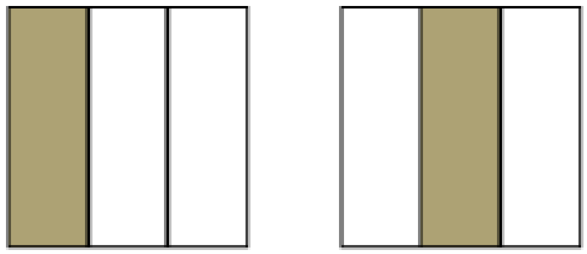
The figures represent the equation $\frac{3}{16} + \frac{3}{16} + \frac{3}{16} = \frac{9}{16}$. We can also say that it represents the equation $3 \times \frac{3}{16} = \frac{9}{16}$.

Note that we can also use the operator "of" instead of the multiplication sign "×".

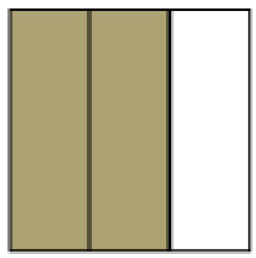
The shaded portion of the following figure represents $\frac{1}{3}$ of the whole, i.e., $\frac{1}{3}$ of 1.



Similarly, in the following figure, the shaded portions represent $\frac{1}{3}$ of 2 wholes, i.e., $\frac{1}{3}$ of 2.



We can combine these two figures to obtain the following figure:



This figure represents the fraction $\frac{2}{3}$.

$\therefore \frac{1}{3}$ of $2 = \frac{2}{3}$

However, when we combined the two figures to obtain the final figure, the expression was $\frac{1}{3} \times 2 = \frac{2}{3}$.

Thus, the expressions $\frac{1}{3} \times 2$ and $\frac{1}{3}$ of 2 are the same.

Let us now find the value of the expression $\frac{1}{5}$ of 6.

$\frac{1}{5}$ of $6 = \frac{1}{5} \times 6 = \frac{6}{5} = 1\frac{1}{5}$

This can also be represented as

Now, in order to understand this concept in greater detail, let us take a look at the following video.

Let us now see some more examples to understand the concept more clearly.

Example 1:

Find the value of the following expressions:

(i) $\frac{9}{7} \times 15$

(ii) $2\frac{2}{3} \times 5$

(iii) $\frac{7}{2}$ of 12

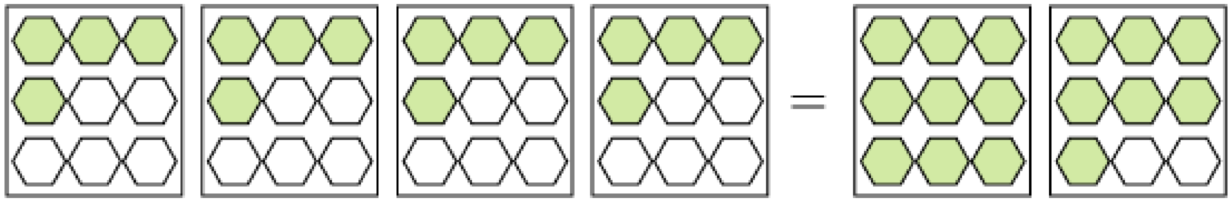
Solution:

(i) $\frac{9}{7} \times 15 = \frac{9 \times 15}{7} = \frac{135}{7}$

(ii) $2\frac{2}{3} \times 5 = \frac{8}{3} \times 5 = \frac{8 \times 5}{3} = \frac{40}{3}$

(iii) $\frac{7}{2} \text{ of } 12 = \frac{7}{2} \times 12 = \frac{7 \times 12}{2} = \frac{84}{2} = 42$

Example 2:



Which of the following expressions is represented by the figure drawn above?

(i) $3\times\frac{4}{9}= 1\frac{1}{3}$

(ii) $4\times\frac{5}{9}=1\frac{8}{9}$

(iii) $4\times\frac{4}{9}=1\frac{7}{9}$

(iv) $5\times\frac{5}{9}=1\frac{7}{9}$

Solution:

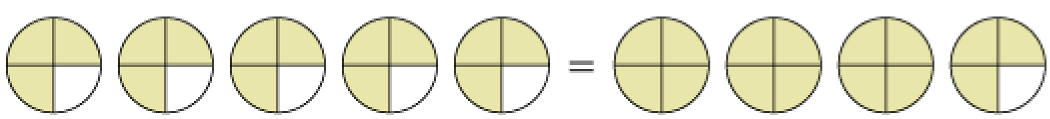
Expression (iii), i.e., $4\times\frac{4}{9}=1\frac{7}{9}$, is represented by the given figure.

Example 3:

Represent $5\times\frac{3}{4}=3\frac{3}{4}$ **pictorially.**

Solution:

The expression $5\times\frac{3}{4}=3\frac{3}{4}$ can be represented pictorially as follows:



Example 4:

Isha scored 450 marks in 5 subjects. In mathematics, she scored $\frac{1}{5}$ of the total marks scored. Find the sum of the marks scored by her in the other subjects.

Solution:

We are given that Isha scored $\frac{1}{5}$ of the total marks scored in mathematics.

∴ Her marks in other subjects = $\left(1-\frac{1}{5}\right)$ of the total marks scored

= $\frac{4}{5}$ of the total marks scored

$=\frac{4}{5}\times 450$

$=\frac{4\times 450}{5}$

= 4 × 90

= 360

Example 5:

Soma bought $3\frac{1}{4}$ kg of mangoes, Bala bought $4\frac{1}{2}$ kg of mangoes, Sunila bought $2\frac{3}{4}$ kg of mangoes. If the cost of 1 kg of mangoes is Rs 60 then find the total cost of mangoes bought by them.

Solution:

Total quantities of mangoes bought by three persons

$$= 3\frac{1}{4} + 4\frac{1}{2} + 2\frac{3}{4}$$
$$= \frac{13}{4} + \frac{9}{2} + \frac{11}{4}$$
$$= \frac{13 + 18 + 11}{4}$$
$$= \frac{42}{4}$$
$$= \frac{21}{2}\text{ kg}$$

Cost of 1 kg of mangoes = Rs 60

∴ Total cost of mangoes bought by all three persons

$$= \frac{21}{2} \times \text{Rs } 60$$
$$= 21 \times \text{Rs } 30$$
$$= \text{Rs } 630$$

Division of Whole Numbers by Fractions

We know how to divide fractions by whole numbers and fractions by fractions.

But what if we have to divide a whole number by a fraction? How will we perform such a division?

The division of a whole number by a fraction can also be performed in a similar way as the division of a fraction by a whole number or as the division of a fraction by another fraction.

In this case also, we convert the division operation into multiplication operation using the following rule.

"**To divide a whole number by a fraction, we replace the divisor by the reciprocal of the divisor and the division operation by multiplication operation".**

For example,

$$2 \div \frac{1}{2}$$

Here, first we replace the fraction by its reciprocal which is 2 and the division sign by multiplication.

Hence, the expression becomes

$= 2 \times 2$

Now, we can simply perform multiplication to obtain the required result

$= 4$

Thus, $2 \div \frac{1}{2}$ = 4

In this way, we can divide a given whole number by a fraction. Let us now solve some more examples.

Example 1:

Find the value of the following expressions.

(i) $5 \div \frac{4}{7}$ **(ii)** $19 \div 2\frac{6}{15}$

Solution:

(i) 5 ÷ 475 ÷ 47

$= 5 \times \frac{7}{4}$

$= \frac{5 \times 7}{4}$

$= \frac{35}{4}$

(ii) $19 \div 2\frac{6}{15}$

$$=19\div\frac{(2\times15)+6}{15}$$

$$=19\div\frac{36}{15}$$

$$=19\times\frac{15}{36}$$

$$=\frac{19\times15}{36}$$

$$=\frac{285}{36}$$

$$=\frac{285\div3}{36\div3}$$

$$=\frac{95}{12}$$

$$=\frac{(7\times12)+11}{12}$$

$$=7\frac{11}{12}$$

Example 2:

Fill in the blanks in the following expressions.

(1) $9\div\frac{19}{30}=9\times\square$

(2) $50\div\square=50\times2\frac{25}{16}$

Solution:

(1) $9\div\frac{19}{30}=9\times\boxed{\frac{30}{19}}$

(2) $2\frac{25}{16}=\frac{(16\times2)+25}{16}=\frac{57}{16}$

And the reciprocal of $\frac{57}{16}$ is $\frac{16}{57}$.

Thus, $50 \div \boxed{\frac{16}{57}} = 50 \times 2\frac{25}{16}$

Division of Fractions by Whole Numbers or Fractions

To divide a fraction by a whole number or by a fraction, we use the concept of reciprocal. Let us go through the video to understand the concept of reciprocal and relate it to the division of fractions by whole numbers or by fractions.

Let us see some more examples.

$$\frac{3}{5} \div \frac{1}{4} = \frac{3}{5} \times \frac{4}{1} = \frac{3 \times 4}{5 \times 1} = \frac{12}{5}$$
$$\frac{9}{7} \div \frac{3}{2} = \frac{9}{7} \times \frac{2}{3} = \frac{9 \times 2}{7 \times 3} = \frac{18}{21} = \frac{6}{7}$$

Let us solve some more examples to understand this concept better.

Example 1:

Evaluate the following.

$$\textbf{(i)}\ 3\frac{4}{7} \div 9 \qquad \textbf{(ii)}\ 4\frac{7}{9} \div 3\frac{1}{2}$$

Solution:

(i) $3\frac{4}{7} \div 9 = \frac{25}{7} \div 9$

$= \frac{25}{7} \times \frac{1}{9}$

$= \frac{25 \times 1}{7 \times 9}$

$= \frac{25}{63}$

(ii) $4\frac{7}{9} \div 3\frac{1}{2} = \frac{43}{9} \div \frac{7}{2}$

$= \frac{43}{9} \times \frac{2}{7}$

$= \frac{43 \times 2}{9 \times 7}$

$= \frac{86}{63}$

$= \frac{(1 \times 63) + 23}{63}$

$= 1 + \frac{23}{63}$

$= 1\frac{23}{63}$

Example 2:

Fill the boxes in the following expressions.

$\mathbf{(i)} \frac{2}{5} \times \frac{3}{7} = \frac{2}{5} \div \square$

$\mathbf{(ii)} 2\frac{3}{5} \times 1\frac{2}{3} = 2\frac{3}{5} \div \square$

Solution:

$(\mathbf{i}) \frac{2}{5} \times \frac{3}{7} = \frac{2}{5} \div \boxed{\frac{7}{3}}$

$(\mathbf{ii}) 1\frac{2}{3} = \frac{(1 \times 3) + 2}{3} = \frac{5}{3}$

Its reciprocal is $\frac{3}{5}$.

Therefore, $2\frac{3}{5} \times 1\frac{2}{3} = 2\frac{3}{5} \div \boxed{\frac{3}{5}}$

Example 3:

Mohit has $3\frac{1}{5}$ pizzas with him. How many people are required to eat this amount of pizza, if each person eats a portion equal to $\frac{2}{5}$ of the pizza?

Solution:

Amount of pizza which Mohit has = $3\frac{1}{5}$

Portion of pizza which each person will eat = $\frac{2}{5}$

Therefore, number of people required = $3\frac{1}{5} \div \frac{2}{5}$

$$= \frac{16}{5} \div \frac{2}{5}$$
$$= \frac{16}{5} \times \frac{5}{2}$$
$$= \frac{16 \times 5}{5 \times 2}$$
$$= \frac{80}{10}$$
$$= 8$$

Thus, the number of people required is 8.

Example 4:

A car travels $244\frac{1}{2}$ km in 6 hours. Find the speed of the car.

Solution:

Distance travelled by the car = $244\frac{1}{2}$ km

$= \frac{489}{2}$ km

Time taken to travel the distance = 6 h

Therefore, speed of the car $= \text{Distance} \div \text{time}$

$= \left(\frac{489}{2} \div 6\right)$ km/h

$= \frac{489}{2} \times \frac{1}{6}$

$= \frac{489 \times 1}{2 \times 6}$

$= \frac{489}{12}$

$= \frac{163}{4}$

$= \frac{(40 \times 4) + 3}{4}$

$= 40 + \frac{3}{4}$

$= 40\frac{3}{4}$ km/h

Thus, the speed of the car is $40\frac{3}{4}$ km/h.

Example 5:

A bag contains $8\frac{1}{2}$ kg of salt. In how many packets can it be filled if each packet can hold $\frac{1}{4}$ kg?

Answer:

Quantity of salt contained in the bag = $8\frac{1}{2} = \frac{17}{2}$ kg

Quantity of salt a packet can hold = $\frac{1}{4}$ kg

So, number of packets that can be filled by salt = $\frac{17}{2} \div \frac{1}{4} = \frac{17}{2} \times 4 = 17 \times 2 = 34$

Expressing Decimals in Different Forms and Vice-versa

Sohan has a ribbon of length 25 cm and 4 mm. Can you express the length of the ribbon only in terms of centimeters?

We know that 10 mm = 1 cm

1 mm = $\frac{1}{10}$ cm = one-tenth cm = 0.1 cm

Hence, length of Sohan's Ribbon = 25 cm and 4 mm

= 25cm and $\frac{4}{10}$ cm (four-tenth of a cm)

= $25\frac{4}{10}$ cm (read as twenty five and four-tenth)

= 25.4 cm (read as twenty five point four)

Such numbers are known as decimal numbers.

In this example, what we observe is that $25\frac{4}{10}$ is equivalent to 25.4.

Let us discuss more about a decimal number.

Place value of a decimal number

In a decimal number, different digits have different place values. In order to know about the place value, look at the following video.

Representation of a decimal using concrete material

We can also represent decimal numbers with the help of figures.

To understand the representation of decimal numbers using concrete material, look at the following video.

Let us consider some examples based on what we have learnt in this section.

Example 1:

Read these decimal numbers and write them in the place value table.

1. **890.253**
2. **12. 20**
3. **3.9**

Solution:

If we place the given decimal numbers in a place value table, then we will have the following table.

Decimal Number	Hundreds (100)	Tens (10)	Ones (1)	Tenths $\left(\frac{1}{10}\right)$	Hundredths $\left(\frac{1}{100}\right)$	Thousandths $\left(\frac{1}{1000}\right)$
890.253	8	9	0	2	5	3
12.20	0	1	2	2	0	0
3.9	0	0	3	9	0	0

1. 890.253 is read as eight hundred ninety point two five three (It is incorrect to read the digits after the decimal point as two hundred fifty three).
2. 12.20 is read as twelve point two zero.
3. 3.9 is read as three point nine.

Example 2:

Express each of the following as decimal numbers.

1. **Six tens five ones two tenths nine hundredths**
2. **Eight hundreds five ones three hundredths seven thousandths**

Solution:

(a) Six tens five ones two tenths nine hundredths

$= 60 + 5 + \frac{2}{10} + \frac{9}{100}$

= 65 + $2 \times \frac{1}{10} + 9 \times \frac{1}{100}$

= 65.29

(b) Eight hundreds five ones three hundredths seven thousandths

= 805 + $\frac{3}{100}$ + $\frac{7}{1000}$

= 805 + $0 \times \frac{1}{10} + 3 \times \frac{1}{100} + 7 \times \frac{1}{1000}$

= 805.037

Example 3:

Write each of the following in decimal form.

1. **900 + 50 + 6 +** $\frac{2}{10}$ **+** $\frac{7}{100}$ **+** $\frac{8}{1000}$
2. **300 + 5 +** $\frac{2}{100}$
3. **2 +** $\frac{6}{1000}$
4. $\frac{7}{100} + \frac{9}{1000}$

Solution:

(a) 900 + 50 + 6 + $\frac{2}{10}$ + $\frac{7}{100}$ + $\frac{8}{1000}$

= 956 + $2 \times \frac{1}{10} + 7 \times \frac{1}{100} + 8 \times \frac{1}{1000}$

= 956.278

(b) 300 + 5 + $\frac{2}{100}$

$= 305 + 0 \times \frac{1}{10} + 2 \times \frac{1}{100}$

= 305.02

(c) $2 + \frac{6}{1000}$

$= 2 + 0 \times \frac{1}{10} + 0 \times \frac{1}{100} + 6 \times \frac{1}{1000}$

= 2.006

(d) $\frac{7}{100} + \frac{9}{1000}$

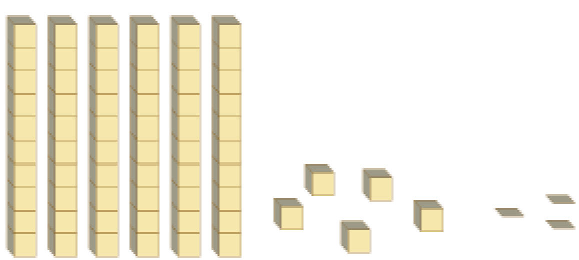
$= 0 + 0 \times \frac{1}{10} + 7 \times \frac{1}{100} + 9 \times \frac{1}{1000}$

= 0.079

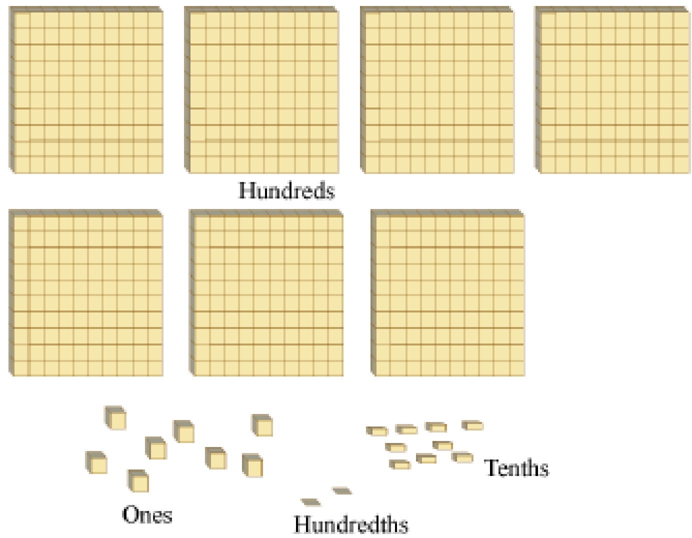
Example 4:

Write the following as numbers in the given table.

(a)



(b)



Hundreds (100)	**Tens (10)**	**Ones (1)**	**Tenths** $\left(\frac{1}{10}\right)$	**Hundredths** $\left(\frac{1}{100}\right)$	**Decimal Number**

Solution:

From the given figures, we can write the numbers in the given table as

Hundreds (100)	**Tens (10)**	**Ones (1)**	**Tenths** $\left(\frac{1}{10}\right)$	**Hundredths** $\left(\frac{1}{100}\right)$	**Decimal Number**
0	6	5	3	0	65.3 or 65.30
7	0	8	9	2	708.92

Conversion of Fractions into Terminating Decimals and Vice-versa

Raman took a piece of paper and cut it into 100 equal parts. Now, one such part is equal to $\frac{1}{100}$ of the whole piece of paper. Similarly, five such equal parts will be equal to $\frac{5}{100}$ of the whole piece of paper.

Can we write this fraction in decimal form?

Yes, we can. Let us see how we represent it in decimal form.

Let us discuss some more examples to understand this concept better.

Example:
Write the following fractions in decimal form.

(i) $\frac{1}{4}$ (ii) $\frac{3}{5}$ (iii) $\frac{23}{8}$ (iv) $\frac{4}{15}$

Solution:
(i)

$$\begin{array}{r} 0.25 \\ 4\overline{)10} \\ \underline{8} \\ 20 \\ \underline{20} \\ \underline{0} \end{array}$$

Thus, we get

$$\frac{1}{4} = 0.25$$

(ii)

$$\begin{array}{r} 0.6 \\ 5\overline{)30} \\ \underline{30} \\ \underline{0} \end{array}$$

Thus, we get

$\frac{3}{5} = 0.6$

(iii)

$$
\begin{array}{l}
\;\;2.875 \\
8\overline{)23} \\
\;\;\underline{16} \\
\;\;\;\;70 \\
\;\;\;\;\underline{64} \\
\;\;\;\;\;\;60 \\
\;\;\;\;\;\;\underline{56} \\
\;\;\;\;\;\;\;\;40 \\
\;\;\;\;\;\;\;\;\underline{40} \\
\;\;\;\;\;\;\;\;\;\underline{0}
\end{array}
$$

Thus, we get

$\frac{23}{8} = 2.875$

(iv)

$$
\begin{array}{l}
\;0.266 \\
15\overline{)40} \\
\;\;\;\;\underline{30} \\
\;\;\;\;100 \\
\;\;\;\;\;\;\underline{90} \\
\;\;\;\;\;\;100 \\
\;\;\;\;\;\;\;\;\underline{90} \\
\;\;\;\;\;\;\;\;\;10
\end{array}
$$

Thus, we get

$\frac{4}{15} = 0.266$

Example 2:

Convert the following fractions into decimal form.

(i) $36\frac{3}{5}$ **(ii)** $6\frac{26}{1000}$ **(iii)** $\frac{47}{20}$ **(iv)** $\frac{68}{125}$ **(v)** $200+10+\frac{3}{100}$

Solution:

(i) $36\frac{3}{5} = 36 + \frac{3}{5}$

$$= 36 + \frac{3\times 2}{5\times 2} = 36 + \frac{6}{10} = 36 + 0.6 = 36.6$$

(ii) $6\frac{26}{1000} = 6 + \frac{26}{1000}$

$= 6 + 0.026 = 6.026$

(iii) $\frac{47}{20} = \frac{47\times 5}{20\times 5}$

$= \frac{235}{100} = 2.35$

(iv) $\frac{68}{125} = \frac{68\times 8}{125\times 8}$

$= \frac{544}{1000} = 0.544$

(v) $200+10+\frac{3}{100}$

$= 210 + 0.03 = 210.03$

Example 3:

Convert the following decimals into fractions.

(i) 35.25

(ii) 40.069

(iii) 5.2

Solution:

(i) 35.25

$= 35 + 0.25$

$= 35 + \frac{25}{100}$

$= 35 + \frac{1}{4}$

$= 35\frac{1}{4}$

(ii) 40.069

$= 40 + 0.069$

$= 40 + \frac{69}{1000}$

$= 40\frac{69}{1000}$

(iii) 5.2 = $\frac{52}{10}$

= $\frac{26}{5}$ or $5\frac{1}{5}$

Example 4:

Carry out the following conversions.

(i) 4 rupees 30 paise to paise

(ii) 7 rupees 65 paise to rupees

(iii) 5 metres 8 centimetres to centimetres

(iv) 57 centimetres 42 millimetres to metres

(v) 2.68 kilograms to grams

(vi) 11 kilograms 55 grams to grams

Solution:

(i) 4 rupees 30 paise = 400 paise + 30 paise = 430 paise

(ii) 7 rupees 65 paise = Rs 7 + Re $\frac{65}{100}$ = Rs 7 + Re 0.65 = Rs 7.65

(iii) 5 metres 8 centimetres = 500 cm + 8 cm = 508 cm

(iv) 57 centimetres 42 millimetres = 570 mm+ 42 mm

= 612 mm

= $\frac{612}{1000}$ m

= 0.612 m

(v) 2.68 kilograms = (2.68 × 1000) g = 2680 g

(vi) 11 kilograms 55 grams = (11 × 1000) g + 55 g

= 11000 g + 55 g

= 11055 g

Example 5:

The height of a room is 273 cm. What is the height of the room in metres?

Solution:

273 cm = $\frac{273}{100}$ m

= 2.73 m

∴ Height of the room in metres = 2.73 m

Example 6:

Sanjeev bought 5 kg 250 g potatoes and Rajeev bought 5.25 kg potatoes. Who bought more potatoes?

Solution:

Weight of potatoes bought by Sanjeev

= 5 kg 250 g

= 5 kg + 250 g

= 5 kg + $\frac{250}{1000}$ kg

= 5 kg + 0.25 kg

= 5.25 kg

Weight of potatoes bought by Rajeev = 5.25 kg

Therefore, both Sanjeev and Rajeev bought an equal amount of potatoes.

Addition and Subtraction of Decimals

If we are asked to add two numbers 72 and 56 and then subtract 38 from their sum, then we can easily carry out the calculations because we have already studied about various mathematical operations such as addition and subtraction in our earlier classes.

However, what would you do if the numbers given are decimal numbers such as 5.12 and 7.43 instead of whole numbers?

In this case, the mathematical operations such as addition and subtraction are applied in the same way, but with a slight difference. Let us learn about the applications of these mathematical operations on decimal numbers in detail.

Let us discuss some more examples based on addition and subtraction of decimals.

Example 1:

Find the value of the following.

i) 25.928 + 187.53

ii) 24 + 87.39 + 0.009

iii) Addition of 7.35 to 398.8

iv) 438.23 – 261.473

v) Subtraction of 54.17 from 56.423

Solution:

i) 25.928 + 187.53

$$\begin{array}{r} 25.928 \\ +\ 187.530 \\ \hline 213.458 \end{array}$$

∴ 25.928 + 187.53 = 213.458

ii) 24 + 87.39 + 0.009

$$\begin{array}{r} 24.000 \\ +\ 87.390 \\ +\ 0.009 \\ \hline 111.399 \end{array}$$

∴ 24 + 87.39 + 0.009 = 111.399

iii) Addition of 7.35 to 398.8

$$\begin{array}{r} 398.80 \\ +\ 7.35 \\ \hline 406.15 \end{array}$$

Thus, the result of the addition of 7.35 to 398.8 is 406.15.

iv) 438.23 – 261.473

$$\begin{array}{r} 438.230 \\ -\ 261.473 \\ \hline 176.757 \end{array}$$

∴ 438.23 – 261.473 = 176.757

v) Subtraction of 54.17 from 56.423

$$\begin{array}{r} 56.423 \\ -\ 54.170 \\ \hline 2.253 \end{array}$$

Thus, the result of the subtraction of 54.17 from 56.423 is 2.253.

Example 2:

Out of 3 kg 25 g apples and 4.75 kg mangoes, 1.5 kg apples and 2 kg 85 g mangoes were eaten. Find the total weight of the fruits left uneaten.

Solution:

Original weight of apples = 3 kg 25 g

= 3 kg + 25 g

= 3.025 kg ($\because 25\text{ g} = \frac{25}{1000}\text{ kg} = 0.025\text{ kg}$)

Weight of apples eaten = 1.5 kg

∴ Weight of apples not eaten = (3.025 – 1.5) kg

= 1.525 kg

Original weight of mangoes = 4.75 kg

Weight of mangoes eaten = 2 kg 85 g

= 2 kg + 85 g

= 2.085 kg

Weight of mangoes not eaten = (4.75 – 2.085) kg

= 2.665 kg

Therefore, total weight of fruits not eaten

= Weight of apples not eaten + Weight of mangoes not eaten

= 1.525 kg + 2.665 kg

= (1.525 + 2.665) kg

= 4.190 kg

Example 3:

Deepika bought vegetables weighing 12 kg. Out of this, 4.5 kg is potatoes, 3 kg 75 g is onions, 2.25 kg is tomatoes, and the rest is cauliflowers. What is the weight of the cauliflowers?

Solution:

Total weight of vegetables = 12 kg = 12.000 kg

Weight of potatoes = 4.5 kg = 4.500 kg

Weight of onion = 3 kg 75 g

= 3 kg + 75 g

= 3 kg + 0.075 g $\left\{\because 75\text{ g} = \frac{75}{1000}\text{ kg} = 0.075\text{ kg}\right\}$

= 3.075 kg

Weight of tomatoes = 2.25 kg = 2.250 kg

Total weight of vegetables without cauliflower = (4.500 + 3.075 + 2.250) kg

= 9.825 kg

∴ Weight of cauliflower

= Total weight of vegetables – Weight of vegetables without cauliflower

= (12.000 – 9.825) kg

= 2.175 kg

Thus, the weight of cauliflowers is 2.175 kg.

Example 4:
The weights of four members in a family are 58.325 kg, 62.125 kg, 60.002 kg and 52.958 kg. Find the total weight of the members in the family.

Solution:
Weight of the 1^{st} member = 58.325 kg
Weight of the 2^{nd} member = 62.125 kg

Weight of the 3^{rd} member = 60.002 kg
Weight of the 4^{th} member = 52.958 kg

Total weight of all members = (58.325 + 62.125 + 60.002 + 52.958) kg = 233.410 kg

Example 5:
There are 300 music files occupying a space of 1750.25 MB stored in a memory card having total space of 4096 MB. Find the free space on the memory card.

Solution:
Total space on the memory card = 4096.00 MB
Space occupied by the music files = 1750.25 MB

Free space on the memory card = (4096.00 – 1750.25) MB = 2345.75 MB

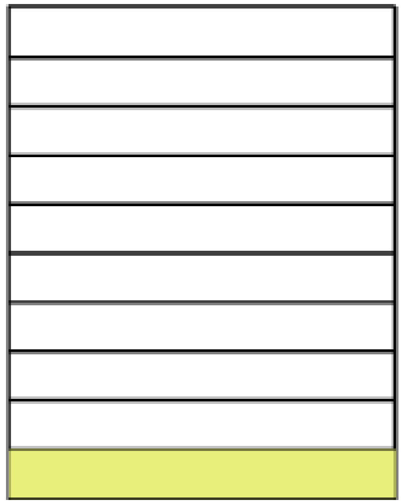
Multiplication of Decimals

Multiplication of decimal numbers is closely related to multiplication of fractions. Therefore, let us take a real-life situation and learn about multiplying two decimal numbers through this example.

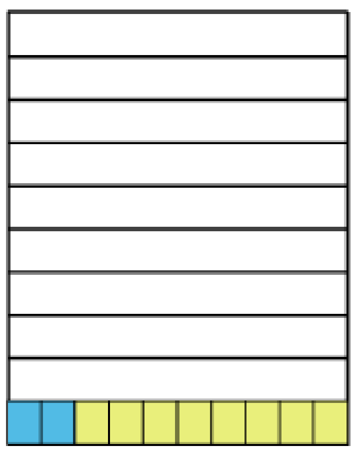
We can represent this multiplication pictorially as well. Let us see how this can be done.

The decimal 0.1 or 1/10 represents 1 part of 10 equal parts.

This can be shown as in the following figure.



Now, $\frac{2}{10} \times \frac{1}{10}$ means $\frac{2}{10}$ of $\frac{1}{10}$ i.e., divide the shaded portion of the above figure into 10 equal parts and take 2 parts out of these 10 parts. These two parts represent $\frac{2}{10}$ of $\frac{1}{10}$.



In the above figure, the blue shaded part represents $\frac{2}{10}$ of $\frac{1}{10}$ i.e., $\frac{2}{10} \times \frac{1}{10}$ or 0.2 × 0.1.

Let us now look at some more examples.

Let us find the value of

(a) 0.4 × 0.6

(b) 2 × 0.14

(c) 0.43 × 0.23

Now, $0.4\times 0.6=\frac{4}{10}\times\frac{6}{10}=\frac{4\times 6}{10\times 10}=\frac{24}{100}=0.24$

$$2\times 0.14=2\times\frac{14}{100}=\frac{2\times 14}{100}=\frac{28}{100}=0.28$$

$$0.43\times 0.23=\frac{43}{100}\times\frac{23}{100}=\frac{43\times 23}{100\times 100}=\frac{989}{10000}=0.0989$$

In the above examples, we can see that

> **“The number of digits to the right of the decimal point of the product is equal to the sum of the number of digits on the right of the decimal point on each of the numbers being multiplied”.**

Using this fact, we can multiply the decimals without converting them into fractions.

Let us look at the first example.

To perform the multiplication 0.4 × 0.6, we will first find the product of 4 and 6 and then put the decimal at the required position.

Thus, 4 × 6 = 24

Now we have to insert the decimal.

In this case, the number of digits on the right of decimal point in 0.4 and 0.6 are 1 and 1 respectively.

Now, 1 + 1 = 2, therefore, we insert the decimal before two digits starting from left.

Therefore, we obtain

0.6 × 0.4 = 0.24

Let us now look at some more examples.

Example 1:

Evaluate the following expressions.

1. **4 × 2.05 × 0.05**
2. **256.15 × 0.1**

Solution:

(i)

$$\begin{aligned}4\times2.05\times0.05&=4\times\frac{205}{100}\times\frac{5}{100}\\&=\frac{4\times205\times5}{100\times100}\\&=\frac{4100}{10000}\\&=\frac{41}{100}\\&=0.41\end{aligned}$$

(ii)

$$\begin{aligned}256.15\times0.1&=\frac{25615}{100}\times\frac{1}{10}\\&=\frac{25615\times1}{100\times10}\\&=\frac{25615}{1000}\\&=25.615\end{aligned}$$

Example 2:

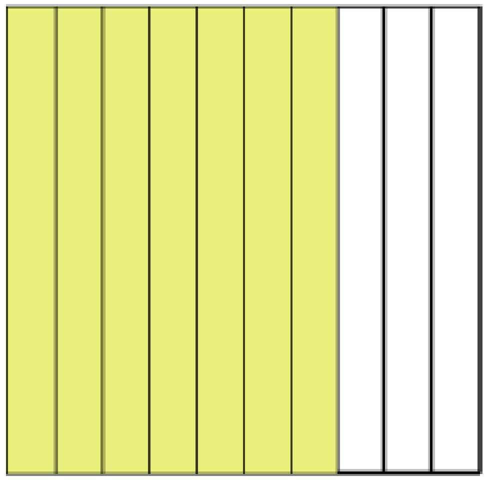
Evaluate 0.4 × 0.7 and represent the multiplication pictorially.

Solution:

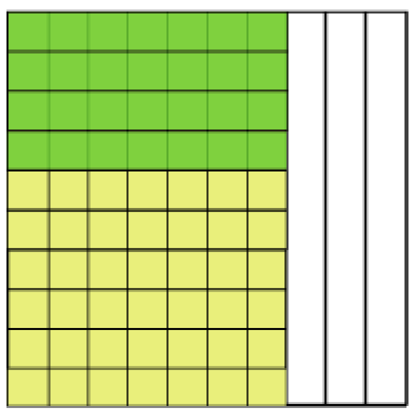
$$\begin{aligned}0.4\times0.7&=\frac{4}{10}\times\frac{7}{10}\\&=\frac{4\times7}{10\times10}\\&=\frac{28}{100}\\&=0.28\end{aligned}$$

Pictorial representation -

$0.7 = \frac{7}{10}$ means 7 parts of 10 equal parts as shown below.



$0.4 \times 0.7 = \frac{4}{10} \times \frac{7}{10}$ means $\frac{4}{10}$ of $\frac{7}{10}$. To represent $\frac{4}{10}$ of $\frac{7}{10}$, we have to take 4 parts out of 10 equal parts of $\frac{7}{10}$.



The green shaded part of the above figure represents $\frac{4}{10}$ of $\frac{7}{10}$ i.e., $\frac{4}{10} \times \frac{7}{10}$ or 0.4 × 0.7.

Example 3:
A metro train takes 2.25 minutes to travel from one station to the next station and it stops for half a minute at every station. If the total number of stations in the whole journey of the train is 32 then find the time taken by the train to reach its destination station.

Solution:
Time taken by the train to reach second station = 2.25 minutes
Stoppage time of the train on second station = 0.5 minute

∴ Total time taken by the train to pass through second station = (2.25 + 0.5) minutes = 2.75 minutes

In the same manner, the train will pass through 31 stations.

∴ Total time taken by the train to pass through 31^{st} station = (2.75 × 31**)** minutes = 85.25 minutes

Time taken by the train from 31^{st} station to 32^{nd} station = 2.25 minutes

∴ Total time taken by the train to reach its destination station i.e., 32^{nd} station = (85.25 + 2.25) minutes = 87.50 minutes

Multiplication of Decimals with 10, 100 and 1000

Let us start with an example.

Suppose Honey went to a shop to buy 10 toffees. The cost of one toffee is Rs 0.50. So, she started calculating the money which she would require to give to the shopkeeper.

She did the calculation as follows.

Cost of 1 toffee = Rs 0.50 = Rs $\frac{50}{100}$

∴ Cost of 10 toffees = $\text{Rs } 0.5\times10 =$ Rs $\left(\frac{50}{100}\times10\right)$ = Rs $\frac{500}{100}$

= Rs 5

Thus, she requires Rs 5 to buy 10 toffees.

When she returned home, she saw that her friend Reema was waiting for her. She told her about the toffees she had bought. Then Reema told her that she can calculate the cost of 100 toffees within seconds.

She then told her that the money required for 100 toffees is Rs 50.

Honey calculated it and was amazed that this was correct and asked Reema how she could do this.

Then Reema told her that whenever we have to multiply a number by any multiple of 10, then in the answer, the digits remain the same and the decimal is shifted to right by as many places as there are zeros after one in the multiple of 10.

The multiplication of a decimal number with a multiple of 10 can be summarised as follows.

"**When a decimal number is multiplied by 10, 100, or any other multiple of 10, then the digits in the product are the same as in the decimal number but the decimal point in the product is shifted to the right by as many places as there are zeroes after one".**

For example,

17.8562 × 10 = 178.562

17.8562 × 100 = 1785.62

17.8562 × 1000 = 17856.2

The given video will help you understand this better.

Now, let us solve some more examples.

Example 1:

Evaluate the following.

1. **0.007 × 10000**

2. **785.5478 × 100**

3. **0.8996 × 1000**

Solution:

1) Here, the decimal is shifted 4 places to the right as there are 4 zeroes after 1

$0.007 \times 10000 = 70$

2) $785.5478 \times 100 = 78554.78$

3) $0.8996 \times 1000 = 899.6$

Example 2:

The cost of 1 kg of apples is Rs 26.5. What will be the cost of 10 kg of apples?

Solution:

Cost of 1 kg of apples = Rs 26.5

Cost of 10 kg apples = Rs (26.5 × 10)

= Rs 265

Division of Decimals by Decimals

To divide a decimal number by another decimal number, we should follow the rule "First, both the divisor and dividend are changed into fractional forms and then the dividend is multiplied with the reciprocal of the divisor".

To understand this rule better, let's look at the given video.

We can follow the long division method also to solve such division problems.

First, we make divisor a whole number and then proceed in the same way as division of decimal by whole number.

Let us divide 650.112 by 1.6.

It can be seen that:

$$\frac{650.112}{1.6} = \frac{650.112 \times 10}{1.6 \times 10} = \frac{6501.12}{16}$$

Since the value of 650.112 ÷ 1.6 is equal to the value of 6501.12 ÷ 16, let us solve the second expression.

$$
\begin{array}{r}
406.32 \\
16\overline{)6501.12} \\
-64 \\
\hline
101 \\
-96 \\
\hline
51 \\
-48 \\
\hline
32 \\
-32 \\
\hline
0 \\
\hline
\end{array}
$$

The remainder is 0, so the division finishes here.

∴ 6501.12 ÷ 16 = 406.32

∴ 650.112 ÷ 1.6 = 406.32

Let us now go through some more examples to understand the method better.

Example 1:

Evaluate the following.

(1) 28.819 ÷ 0.23

(2) 299.91 ÷ 2.6

Solution:

(1)

It can be seen that:

$$\frac{28.819}{0.23} = \frac{28.819 \times 100}{0.23 \times 100} = \frac{2881.9}{23}$$

Since the value of 28.819 ÷ 0.23 is equal to the value of 2881.9 ÷ 23, let us solve the second expression.

$$
\begin{array}{r}
125.3 \\
23\overline{)2881.9} \\
-23 \\
\hline
58 \\
-46 \\
\hline
121 \\
-115 \\
\hline
69 \\
-69 \\
\hline
0 \\
\hline
\end{array}
$$

The remainder is 0, so the division finishes here.

∴ 2881.9 ÷ 23 = 125.3

∴ 28.819 ÷ 0.23 = 125.3

(2)

It can be seen that:

$$\frac{299.91}{2.6} = \frac{299.91 \times 10}{2.6 \times 10} = \frac{2999.1}{26}$$

Since the value of 299.91 ÷ 2.6 is equal to the value of 2999.1 ÷ 26, let us solve the second expression.

$$
\begin{array}{r}
115.35 \\
26\overline{)2999.10} \\
-26 \\
\hline
39 \\
-26 \\
\hline
139 \\
-130 \\
\hline
91 \\
-78 \\
\hline
130 \\
-130 \\
\hline
0 \\
\hline
\end{array}
$$

The remainder is 0, so the division finishes here.

∴ 2999.1 ÷ 26 = 115.35

∴ 299.91 ÷ 2.6 = 115.35

Example 2:

Evaluate the following.

(1) 1.25 ÷ 0.05

(2) 0.032 ÷ 0.8

Solution:

$$\textbf{(1)}\ 1.25 \div 0.05 = \frac{125}{100} \div \frac{5}{100}$$
$$= \frac{125}{100} \times \frac{100}{5}$$
$$= \frac{125}{5}$$
$$= 25$$

$$\textbf{(2)}\ 0.032 \div 0.8 = \frac{32}{1000} \div \frac{8}{10}$$
$$= \frac{32}{1000} \times \frac{10}{8}$$
$$= \frac{32}{100} \times \frac{1}{8}$$
$$= \frac{32 \times 1}{100 \times 8}$$
$$= \frac{4}{100}$$
$$= 0.04$$

Example 3:

Rohit has a pencil of length 5.4 cm and Mohit has a pencil of length 10.8 cm. How many times is Mohit’s pencil longer than Rohit’s pencil?

Solution:

It is given that, length of Rohit’s pencil = 5.4 cm

Length of Mohit's pencil = 10.8 cm

Therefore, Mohit's pencil is longer than Rohit's pencil by $\frac{10.8}{5.4}$ times.

$$\frac{10.8}{5.4} = 10.8 \div 5.4 = \frac{108}{10} \div \frac{54}{10} = \frac{108}{10} \times \frac{10}{54} = \frac{108}{54} = 2$$

Thus, Mohit's pencil is 2 times longer than Rohit's pencil.

Example 4:
Raju earns Rs 587.25 by selling mangoes at the rate of Rs 40.50. Find the total weight of the mangoes sold by Raju.

Solution:
Total amount earned by Raju = Rs 587.25
Rate = Rs 40.50
The weight of mangoes

$$= \frac{587.25}{40.50}$$
$$= 14.50 \text{ kg}$$

Division of Decimals by 10, 100 and 1000

Let us start with an example.

Rahul fills the tank of his bike with 10 L of petrol. He covered a distance of 125.3 km with this petrol. Can you find the distance covered by Rahul in 1 L of petrol?

We can find the distance covered by Rahul by unitary method.

Distance covered in 10 L of petrol = 125.3 km

Distance covered in 1 L of petrol = $\frac{125.3}{10}$ km

To find the distance covered in 1 L of petrol, we have to divide 125.3 by 10. We can divide it by long division method as follows.

$$
\begin{array}{r|l}
 & 12.53 \\
10 & 125.3 \\
 & -10 \\
\hline
 & 25 \\
 & -20 \\
\hline
 & 53 \\
 & -50 \\
\hline
 & 30 \\
 & -30 \\
\hline
 & 0 \\
\hline
\end{array}
$$

The long division method is very lengthy and time consuming. We can divide a number by the multiples of 10 within seconds. Now, let us know the method to divide a number by any multiple of 10 without carrying out the long division method.

“**When a decimal number is divided by 10, 100, 1000 etc., the digits remain the same, and only the decimal is shifted to the left by as many places as there are zeroes in the divisor”.**

In the above example, the number of zeroes in the divisor is 1. Therefore, we shift the decimal point one place to the left. Thus, we obtain

$\therefore \frac{125.3}{10} = 12.53$

Using this rule, we obtained the same answer as we obtained from the long division method.

Now, look at some more examples.

$2468.3 \div 10 = 246.83$

2468.3 ÷ 100 = 24.683

2468.3 ÷ 1000 = 2.4683

It can be clearly noted that the number of places by which the decimal is shifted to the left is equal to the number of zeroes in the divisor.

$2468.3 \div 10 = 2468.3 = 246.83$
$2468.3 \div 100 = 2468.3 = 24.683$
$2468.3 \div 1000 = 2468.3 = 2.4683$

Look at the given video to understand the concept explained above.

Let us see some more examples to understand the concept better.

Example 1:

Evaluate

(i) 6407 ÷ 1000

(ii) 25.32 ÷ 10000

Solution:

(i) 6407 ÷ 1000 = $\frac{6407}{1000}$ = 6.407

(ii) 25.32 ÷ 10000 = $\frac{25.32}{10000}$ = 0.002532

Example 2:

Fill in the blanks to make the following statements correct.

(i) $36.7 \div \boxed{} = 3.67$

(ii) $19.95 \div 1000 = \boxed{}$

Solution:

(i) $36.7 \div \boxed{10} = 3.67$

Because, 36.7 ÷ 10 $= \frac{36.7}{10}$

= 3.67

(ii) $19.95 \div 1000 = \boxed{0.01995}$

Because, 19.95 ÷ 1000 $= \frac{19.95}{1000}$

= 0.01995

Example 3:

Sumit bought 10 chocolates for Rs 127.5. What was the cost of one chocolate?

Solution:

Cost of 10 chocolates = Rs 127.5

∴ Cost of 1 chocolate = Rs $\left(\frac{127.5}{10}\right)$

= Rs 12.75

Terminating and Non-terminating Repeating Decimal Expansions of Rational Numbers

We can find the decimal expansion of rational numbers using long division method.

However, it is possible to check whether the decimal expansion is terminating or non-terminating without actually carrying out long division also.

Let us start by taking a few rational numbers in the decimal form.

(a)

$0.5632 = \frac{5632}{10000}$

On prime factorising the numerator and the denominator, we obtain

$\frac{5632}{10000} = \frac{2^9 \times 11}{2^4 \times 5^4} = \frac{2^5 \times 11}{5^4}$

(b)

$0.275 = \frac{275}{1000}$

On prime factorizing the numerator and the denominator, we obtain

$$\frac{275}{1000} = \frac{5^2 \times 11}{2^3 \times 5^3} = \frac{11}{2^3 \times 5}$$

Can you see a pattern in the two examples?

We notice that the given examples are rational numbers with terminating decimal expansions. When they are written in the $\frac{p}{q}$ form, where *p* and *q* are co-prime

(the HCF of *p* and *q* is 1), the denominator, when written in the form of prime factors, has 2 or 5 or both.

The above observation brings us to the given theorem.

If *x* is a rational number with terminating decimal expansion, then it can be expressed in the $\frac{p}{q}$ form, where *p* and *q* are co-prime (the HCF of *p* and *q* is 1) and the prime factorisation of *q* is of the form 2^n5^m, where *n* and *m* are non-negative integers.

Contrary to this, if the prime factorisation of *q* is not of the form 2^n5^m, where *n* and *m* are non-negative integers, then the decimal expansion is a

non-terminating one.

Let us see a few examples that will help verify this theorem.

(a) $\frac{7}{12} = \frac{7}{2^2 \times 3} = 0.58333...$

(b) $\frac{15}{16} = \frac{3 \times 5}{2^4} = \frac{3 \times 5 \times 5^4}{2^4 \times 5^4} = \frac{3 \times 5^5}{(10)^4} = 0.9375$

(c) $\frac{1}{14}=\frac{1}{7\times 2}=0.0714285714...$

(d) $\frac{125}{16}=\frac{5^3}{2^4}=\frac{5^3\times 5^4}{2^4\times 5^4}=\frac{5^7}{(10)^4}=\frac{78125}{10^4}=7.8125$

Note that in examples **(b)** and **(d)**, each of the denominators is composed only of the prime factors 2 and 5, because of which, the decimal expansion is terminating.

However, in examples **(a)** and **(c)**, each of the denominators has at least one prime factor other than 2 and 5 in their prime factorisation, because of which, the decimal expansion is non-terminating and repetitive.

To summarize the above results, we can say that:

Let *x* = $\frac{p}{q}$ be any rational number.

If the prime factorization of *q* is of the form 2^m5^n, where *m* and *n* are non-negative integers, then *x* has a terminating decimal expansion.

If the prime factorisation of *q* is not of the form 2^m5^n, where *m* and *n* are non-negative integers, then *x* has a non-terminating and repetitive decimal expansion.

Let us solve a few examples to understand this concept better.

Example 1:

Without carrying out the actual division, find if the following rational numbers have a terminating or a non-terminating decimal expansion.

(a) $\frac{17}{1600}$

(b) $\frac{723}{392}$

Solution:

(a) $\frac{17}{1600} = \frac{17}{2^6 \times 5^2}$

As the denominator can be written in the form $2^n 5^m$, where n = 6and m = 2 are non-negative integers, the given rational number has a terminating decimal expansion.

(b) $\frac{723}{392} = \frac{3 \times 241}{2^3 \times 7^2}$

As denominator cannot be written in the form $2^n 5^m$, where n and m are non-negative integers, the given rational number has a non-terminating decimal expansion.

Example 2:

Without carrying out the actual division, find if the expression $\frac{715}{128}$ has a terminating or a non-terminating decimal expansion.

Solution:

$\frac{715}{128} = \frac{715}{2^7}$

As the denominator can be written in the form $2^n 5^m$, where n = 7and m = 0 are non-negative integers, the given rational number has a terminating decimal expansion.

$$\frac{715}{128} = \frac{715}{2^7} = \frac{715 \times 5^7}{2^7 \times 5^7}$$
$$= \frac{715 \times 5^7}{10^7}$$

$$= \frac{715 \times 78125}{10^7} = \frac{55859375}{10^7} = 5.5859375$$

Hence, 5.5859375 is the decimal expansion of the given rational number.