CHAPTER 07

Levelling and Contouring

7.1 Introduction

- It is the method of determining the difference of elevations or levels of different points on the earth's surface.
- Distance measurement is always done from a reference point of known parameters (its elevation, height above MSL etc.). In surveying, we define this aspect of vertical distance measurement by the term **levelling.**

7.2 Terminologies in Levelling

(a) **Elevation:**

- It is the vertical distance above or below a reference surface. The reference surface is called as **datum** and is having **zero elevation.**
- This elevation is taken as **positive** when the point under consideration is above the datum and **negative,** when the point under consideration is below the datum. Usually **Mean Sea Level (MSL)** is taken as the **standard datum** but at times, any arbitrary surface can be taken as datum.

NOTE: Mean Sea Level (MSL) as a datum is obtained by taking average of hourly high and low tides elevations over a long period of time of about **19 years.**

(b) **Altitude :** It is the vertical distance of a point above MSL i.e., when datum is MSL, the elevation is termed as altitude.

(c) **Grade :** The term elevation when used in reference to construction activities, then it is called as **grade.**

(d) **Level :** The vertical distance of a point above or below a datum on earth's surface is called as **level** or **reduced level** and the process of determining the difference in reduced levels of the points is called as **levelling.**

- The method of levelling can either be **direct** like simple levelling, differential levelling, cross-sectioning, profile levelling etc. or may be **indirect** like barometric levelling, trigonometric levelling, hypsometry, aerial surveys etc.

(e) **Level surface :** A level surface is a curved surface in which all the points are at the same distance from the earth's surface.

- It represents a surface which is parallel to the mean spheroidal surface of earth. Level surface is normal to the plumb line at all the points.

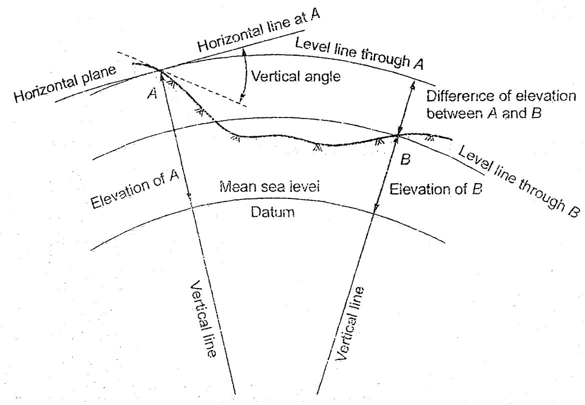


Fig. 7.1 Different terms in levelling

(f) **Level line:** Any line lying on the level surface is called as level line. Obviously level line is perpendicular to the plumb line.

(g) **Vertical line:** It is a line from any point on the earth's surface to the center of the earth. e.g. Plumb line.

(h) **Horizontal plane:** It is a plane tangential to the level surface at any point which is under consideration.

(i) **Horizontal line:** Any line lying in the horizontal plane is referred to as a horizontal line.

(j) **Axis of telescope:** It is a line joining the center of objective lens to the center of eyepiece of the telescope.

(k) **Line of sight:** It is the line joining the optical center of objective lens to the point of intersection of cross hairs. While sighting a point, the line of sight should remain horizontal, and thus the line of sight is called as **line of collimation.**

(l) **Axis of level/bubble tube:** It is the line which is tangential to the bubble of level tube when the bubble is at its mid-point.

(m) **Height of instrument:** When the instrument at a station is levelled the elevation of plane of collimation is called as the height of instrument.

(n) **Back sight:** It is the staff reading taken on a point of known elevation. e.g. if elevation of station A is known then the staff reading on station A is called as back sight (BS). Sometimes, back sight is also referred to as **plus sight.**

Thus, H.I. = Elevation of BM + BS

(o) **Fore sight:** It is the staff reading taken on a point whose elevation is yet to be determined i.e. elevation is not known at this point of time. e.g. If elevation of station B is to be determined, then staff reading taken on station B is called as fore sight (FS). Sometimes, fore sight is also referred to as **minus sight.**

Thus, Elevation of point = HI – FS

(p) **Intermediate sight:** It is the staff reading taken on the point of unknown elevation between the back sight and the fore sight.

(q) **Change point/turning point:** It is the point which denotes the shifting of the instrument/level. Both BS and FS are taken on the staff at the change point. This is required while levelling a long line like a highway or a railway.

(r) **Station:** It is the point whose elevation is to be determined and where the staff is kept to take staff readings.

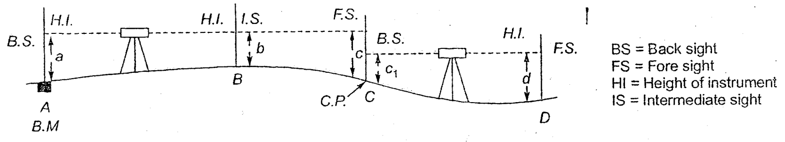


Fig. 7.2 *Sight measurements with level*

(s) **Bench mark:** It is the fixed reference point of known elevation. Depending on the permanency and the precision with which the bench mark is located, bench marks are classified as:

(i) **Great Trigonometric Survey (GTS) bench mark:** These bench marks are established by the **Survey of India** at an interval of about 100 km all over the country with the datum being taken as **mean sea level at Bombay Port.** The elevations of the bench marks are shown in the GTS Maps.

(ii) **Permanent bench mark:** In between the GTS benchmarks, permanent bench marks are established by the Government departments on a clearly defined and permanent point like the top of a parapet wall, a bridge or a culvert, kilometer stone, railway platform etc.

(iii) **Arbitrary bench mark:** These are the reference points whose elevations are arbitrarily assumed generally for a small levelling purpose.

- Their elevations do not refer to any fixed datum.

(iv) **Temporary bench mark:** These are the reference points on which a day's work is closed and from where levelling is continued the next day.

- Examples of such types of bench marks are the kilometer stones, parapets etc.

(t) **Balancing of Sights:** In order to minimize the error due to instrument or other reasons, the distance of the point where the back sight is taken and the distance of the point where the fore sight is taken should approximately be equal.

7.3 Basic Principle of Levelling

- When the levelling instrument has been properly levelled then the bubble tube and the line of sight are truly horizontal and the vertical axis of the instrument is truly vertical.
- From this setting of the levelling instrument, when the telescope of the instrument is rotated, the line of sight remains in the horizontal plane only.
- Because distances involved in levelling are very small compared to the radius of earth, there is not much difference between the level line and the horizontal line.
- In field, the levelling instrument is set up at a certain suitable location and staff at a location of known elevation.
- Reading is taken on the staff so as to determine the level of line of sight. This staff reading is known as back sight (BS).
- Now the staff is shifted to a location of unknown elevation and staff reading is again taken. This staff reading is known as fore sight (FS).
- Thus the elevation of the point is determined from the level of line of sight and the foresight reading at the location of unknown elevation.

7.4 Theory of Simple Levelling

- Simple levelling is used in situations where difference of levels of two points are required to be determined which are relatively close to each other and only one setting of instrument is enough.
- The levelling instrument is set at a convenient location roughly mid-way between the two points.

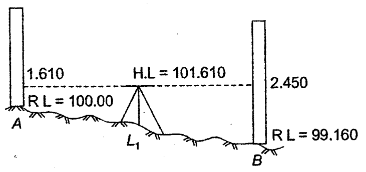


Fig. 7.3 *Simple levelling*

- In Fig. 7.3, instrument is set up at L_1. Elevation of point *A* is 100 (i.e. *RL* of *A* = 100 m). It is required to determine the elevation of point *B*. After doing the necessary adjustments, back sight is taken on the staff held at station *A*.

Thus,

$$\text{Height of instrument } (HI) = RL \text{ of } A + \text{Back Sight}$$

$$HI = 100 + \text{Back Sight}$$

After taking the back sight reading, turn the telescope towards the staff held at station *B* and take the reading on staff (fore sight). By turning the telescope in the horizontal plane, the level line also remains in the same horizontal plane.

Thus,

$$\text{Height of instrument} = RL \text{ of } B + \text{Fore Sight}$$

Here height of instrument is known from the staff reading at *A* and thus *RL* of *B* can be computed as,

$$RL \text{ of } B = \text{Height of instrument} - \text{Fore Sight}$$

- Here the *RL* of instrument station (i.e., L_1) does not come into picture.

7.5 Reciprocal Levelling

- Reciprocal leveling is used to determine the correct difference of elevations of the two points which are quite far apart and when it is not possible to set up the instrument mid-way between the two points to balance the fore sight and back sight.
- This reciprocal leveling is usually used to determine the difference of elevations of two points on the opposite banks of a river or a deep gorge.
- Reciprocal levelling eliminates the error due to curvature and refraction.

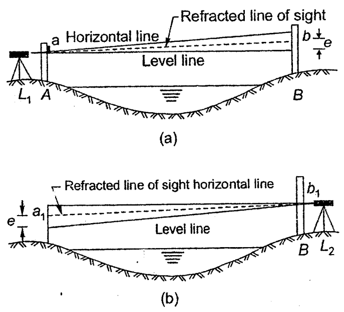


Fig. 7.4 *Reciprocal levelling*

In Fig. 7.4, let *A* and *B* be the two points on opposite banks of a river. The following procedure is used:

(a) Drive pegs at *A* and *B*. Set up the instrument at L_1 near peg *A*. Take readings '*a*' and '*b*' on staffs held at *A* and *B* respectively.

(b) Shift instrument to L_2 near peg B. Take readings 'a_1' and 'b_1' on staffs held at A and B respectively.

(c) Find the correct difference in elevations between A and B as per the procedure given below:

Let B is at higher elevation than A and e be the total error.

When instrument is at L_1

Apparent difference of levels, $h' = a - b$

Since A is very near to L_1, the error in a is not significant.

Thus, correct staff reading at $A = a$

Correct staff reading at $B = b - e$

Correct difference of levels, $h = a - (b - e)$ ...(7.1)

When instrument is at L_2

Apparent difference of levels, $h'' = a_1 - b_1$

Correct staff reading at $B = b_1$

Thus, correct staff reading at $A = a_1 - e$

Correct difference of levels, $h = (a_1 - e) - b_1$ ...(7.2)

Adding Eqs. (7.1) and (7.2),

$$2h = a - (b - e) + (a_1 - e) - b_1$$

$$h = [(a_1 - b_1) + (a - b)]/2$$

$$\Rightarrow \quad h = (h' + h'')/2 \quad ...(7.3)$$

7.6 Balancing of Foresights and Backsights

- It is quite beneficial that distance of the staff on which back sight is taken and the staff on which fore sight is taken as measured from the instrument station should approximately be equal. This is called as balancing of fore sight and back sight.
- If the sights are balanced then difference of elevations of the two points can be determined accurately even if the instrument is not in permanent adjustment and the line of sight is inclined.
- Apart from that, balancing of sight also neutralizes the effect of refraction and curvature.

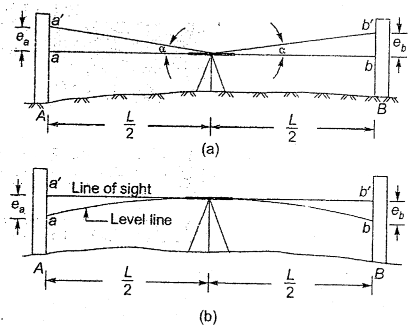


Fig. 7.5 *Balancing of FS and BS*

As shown in the figure 7.5 the error in the back sight of A will be equal to that in the fore sight of B if the instrument is set up exactly mid-way between the two stations and the line of sight is inclined upwards.

Thus, e_a for $BS = e_b$ for FS

$(e_l + e_c - e_r)$ for $BS = (e_l + e_c - e_r)$ for FS

Thus the difference of elevations can be computed directly from the observed readings a' and b' without applying the corrections for inclined sight, curvature and refraction.

correct staff readings, $a = a' - e_a$

$b = b' - e_b$

Correct difference of levels, $h = a - b = a' - b'$

7.7 Level Field Book

- It is the book used for entering the staff readings and reduced levels of various points.
- For computation of reduced levels of various points, two methods are used :
  - (a) Height of instrument / height of collimation method.
  - (b) Rise and fall method

7.7.1 Height of Instrument Method (Height of Collimation Method)

Station (1)	BS (2)	IS (3)	FS (4)	HI (5)	RL (6)	Remarks (7)
A	1.350			101.350	100	BM
B			2.150	101.350	99.2	

Fig. 7.6 *HI Method/ Height of collimation method*

- The first reading is always back sight.
- The last reading is always fore sight.
- Station at which both BS and FS readings are taken implies that it a change point i.e. instrument is shifted after taking the BS.
- Arithmetic check : After computation of all the levels, the following arithmetic check is applied :

$$\Sigma BS - \Sigma FS = \text{Last RL} - \text{First RL}$$

When $\Sigma BS > \Sigma FS \Rightarrow$ Last point is higher than the first point

When $\Sigma BS < \Sigma FS \Rightarrow$ Last point is lower than the first point

7.7.2 Rise and Fall Method

Station (1)	BS (2)	IS (3)	FS (4)	Rise (5)	Fall (6)	RL (7)	Remarks (8)
A	1.350					100	BM
B			2.150		0.8	99.2	

Fig. 7.7 *Rise and Fall Method*

- The first four and last two columns are exactly same as HI method.
- Here instead of column for HI, two columns of rise and fall are there.
- Here rise or fall of each station is determined w.r.t. previous station.
- Arithmetic check : After computation of all the levels, the following arithmetic check is applied :

$$\Sigma BS - \Sigma FS = \text{Last RL} - \text{First RL} = \Sigma\text{Rise} - \Sigma\text{Fall}$$

7.8 Errors in Levelling

7.8.1 Instrumental Errors

(a) Error incurred due to imperfect adjustment. In case the instrument is not perfect in adjustment, the resulting line of sight will be inclined when the bubble is at the center.

- This error can be eliminated by balancing the foresights and back sights.
- However balancing of back sights and intermediate sights is not possible when there are a large number of intermediate sights.
- This is compensating type of error but is cumulative if back sights are consistently greater or shorter than the foresights.

(b) Error incurred due to sluggish bubble

- Sluggish bubble comes to rest very quickly.
- Also it may shift position while the sight is taken.
- In order to avoid this error, it must be ensured that bubble is central before taking each reading.
- This is a compensating error.

(c) Error incurred due to defective staff

- In case the graduations of the staff are not accurate then obviously there will be an error.
- For too long staff, the correction is positive and for too short staff, the correction is negative.
- Also if the bottom of staff has worn out then it will give wrong height of instrument but the error in foresight is balanced by the back sight and thus the calculated elevation is correct.

(d) Error incurred due to defective tripod

- If tripod is not strong and stable then there will be an error.
- Also a significant amount of time gets wasted in levelling and taking the observations.

(e) Error due to faulty focusing tube

- If the focusing tube of the external focusing telescope is faulty then the objective moves in an inclined direction when tube is moved for focusing.
- This error can be eliminated by taking out the defective tube and aligning it properly.

7.8.2 Personnel Errors

(a) Error incurred due to careless setting up and leveling of the instrument.

(b) Error incurred due to bubble out of center

- There will be an error if the bubble is not at the center when the sights are taken.
- In case the bubble is not at the center then bring it to the center by turning the leveling screw.

(c) Error incurred due to imperfect focusing

- In case the eye piece and objective are not properly focused, there will be a parallax and staff readings will be erroneous.
- To avoid this error, check if any movement of the eye causes an apparent change in the staff reading. If so, then focus the eye piece and objective.

(d) Error incurred due to non-verticality of the staff

- If the staff is not held vertical, the staff reading obtained will be higher than the actual reading. Thus a positive systematic error occurs and thus correction will be negative.
- A small circular bubble tube or a plumb bob is attached to the back of the staff which is helpful in checking the verticality of the staff.

(e) Error incurred due to telescope staff not extended fully

- If the sections of the telescopic staff is not extended fully then the staff readings will be erroneous.
- This error can be avoided by ensuring that the sections are fully extended and the graduations are continuous from one section to the other.

(f) Error incurred due to sighting

- This error occurs when the cross hairs do not coincide with the staff graduations.
- This error is large when the sights are too long or when the cross hairs are too thick.
- This error can be minimized by keeping the distances small and taking care while sighting.

7.8.3 Error Due to Natural Causes

(a) Error due to curvature of earth and refraction

- In ordinary leveling, this error due to curvature and refraction is small and can be neglected without introducing any serious errors.
- When the distances are large, this error is quite significant and must be taken care of.

(b) Error due to temperature variation

- High atmospheric temperature causes the liquid of the level tube to expand and the bubble to shorten. The adjustments of the instruments are thus get disturbed by unequal heating of different parts leading to distortions and warping.
- If one end of the tube is warmer than the other end then bubble moves towards the warmer end and thus error occurs.
- Error may also occur due to change in the length of the staff due to temperature variations.
- The heating effect can be minimized by shielding the instrument from the direct rays of sun.
- This error is usually small and can also be neglected.

(c) Error due to sun and wind

- When bright sunshine occurs on the objective then staff reading cannot be taken accurately. In this case the ray shade provided on the telescope should be pulled out. If there is no shade provided with the objective then shades the objective with hand etc.
- Very high wind causes shaking of the tripod and the instrument and it becomes difficult to keep the bubble centered. It is better to stop the work in high winds.

(d) Error due to shimmering effect

- In very hot weather, the air very close to the ground gets heat up and the resulting refraction error is quite significant. It produces shimmering effect near the ground surfaces.
- This makes the staff to appear wavy and it becomes difficult to read the staff near the bottom.
- To avoid this error, the line of sight should be kept high above the ground.
- The reading on the lowermost 0.5 m of the staff should be avoided.

(e) Error due to tripod settlement

- If the tripod is not set on a firm ground then there is a possibility that it may settle.
- If tripod settles between a back sight and fore sight then observed fore sight will be smaller than the actual reading and computed RL will be higher than the actual value.
- To avoid this error, the tripod should be set up on a firm ground.

7.9 Mistakes in Levelling

(a) Mistakes while taking the readings

- Accidently reading the staff in wrong direction. It should be read in the direction where the readings get increased.
- Taking the readings of stadia hairs instead of central hair on the staff.
- Mistakes in reading the figures of meters and decimeters.

(b) Mistakes while making the recordings

- Recording the readings in the wrong column of level book.
- Recording the readings with interchanged digits. e.g. Recording 1.025 as 1.205.
- Accidently skipping the recording of an entry.

(c) Mistakes while making the computations

- The mistakes in computations can be easily detected by performing arithmetic check on the readings.

7.10 Contour

It is the line joining points of equal elevation on the surface of earth.

7.11 Characteristics of Contours

(a) Contour lines always form the closed loops; however they may close either on the map itself or outside the map.

(b) Contours are always perpendicular to the direction of steepest slope.

(c) The spacing of contour lines depends on the slope of the ground. For steep ground slopes, small spacing is desired and for relatively mild slopes, large spacing can be conveniently adopted.

(d) The horizontal distance between the consecutive contours is called as **horizontal equivalent.** For a given contour interval, scale of the map and the nature of terrain decide the horizontal equivalent. As the slope becomes more and more steeper, the horizontal equivalent decreases.

(e) Uniform slope is indicated by equally spaced contour lines.

(f) Straight, parallel and equally spaced contour lines represent a plane surface.

(g) Irregular contours represent rough terrain.

(h) For depressions, contours increase in elevation from inside to outside and in case of a hill, contours increase from outside to inside.

(i) Contour lines cross the ridge at right angles.

(j) Contour lines cross the valley at right angles.

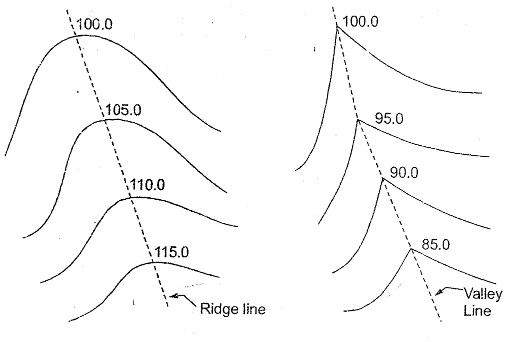


Fig. 7.8 *Ridge line and valley line*

(k) Two contour lines of altogether different elevations cannot cross each other as this will mean two elevations of the same point which is not possible. However in the special case of an overhanging cliff or a cave penetrating the hill inside, the contours appear to cross each other.

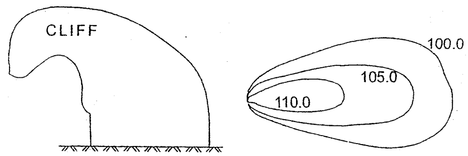


Fig. 7.9 *Overhangging cliff*

7.12 Various Methods of Locating the Contours

- By the term locating the contours, it implies locating the point on the ground whose position in the horizontal plane and its elevation are known.
- Thus location of a point involves the both horizontal and vertical control. The method of locating the contours depends on the instruments available for the purpose.
- Points of equal elevation can be determined with a levelling instrument.
- Once all such points have been determined, their horizontal location can be established by using any suitable horizontal control system.
- The horizontal control depends on the type and area to be covered and the accuracy being required. For areas of small extent, approximate horizontal control can be established through chain surveying and points may be located from offsets from the survey lines. Plane tabling is also suitable for small areas wherein the measurement and plotting are done simultaneously. For works of still higher level of accuracy, compass surveying can be used. For still higher precision work, a theodolite can be used.

All the methods of locating the contours can be classified as:

(a) **Direct method of contouring :** Here the contour to be plotted is actually traced on the ground by locating the points of that elevation. The horizontal position of the points so located is then determined and plotted on the plan. The pegs of different contours are coded so that one set cannot be mistaken for the other.

(b) **Indirect method of contouring :** Here the spot levels are taken at some selected points called as guide points and the levels of the points are determined. The horizontal positions of these points are then determined and points are plotted on the plan. The contours are then drawn from the interpolation from the levels of the guide points. The indirect method is more convenient than the direct method and is suitable for contouring large areas.

7.13 Direct Method of Contouring

In the direct method of contouring, the points on a contour are located directly by identifying the points of that elevation.

For example: If *HI* = 202.550 and contour of *RL* = 198.00 is to be located then all those points in that area which give staff reading of 4.550 (=202.550 – 198.00 = 4.550) will be on the contour line of 198.00. Similarly other points are located on the same contour and these points when joined will give a contour of 198.00.

Advantages: This is more accurate method than the indirect methods as the points are located directly and is generally used for small areas.

Disadvantages: This method being slow and cumbersome is not suitable for contouring large areas.

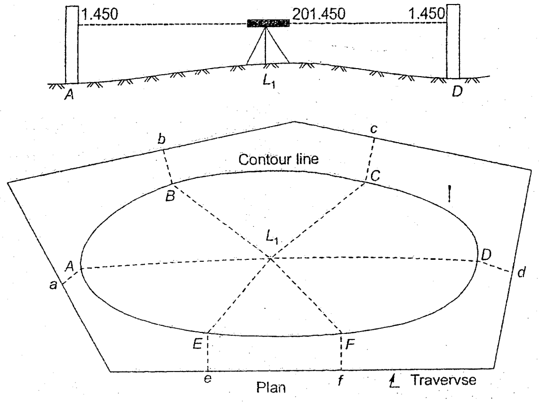


Fig. 7.10 *Direct method of contouring*

7.14 Indirect Methods of Contouring

(a) Grid method :

- This method is used when the area to be surveyed is not very large and also the ground is not much undulating.
- The area to be surveyed is divided into a number of squares, the size of the square depends on the nature of ground and the accuracy desired. Usually it varies from 5 m to 20 m.
- A theodolite is used to layout the grid lines at right angles to each other.
- Grid points are marked on the intersections of the grid lines. In case theodolite is not available then this process is carried out through a tape and a cross staff.
- The level is set up in the middle of the area to obtain the elevations of grid points. The height of the instrument is determined by taking back sight on the bench mark. Intermediate sights are then taken on each of the grid points. When required, the instrument is shifted and a new height of instrument is determined and the process is repeated. In this way, elevations of all the grid points are determined.

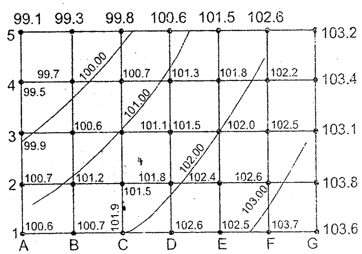


Fig. 7.11 *Grid method of contouring*

- From the elevations of the grid points, the points on the various contours are located by interpolation.

(b) Cross section method :

- This method is usually used for determination of contours along a fixed route such as a canal, railway, highway etc.
- Cross sections are located on the ground at right angles to the fixed line or center line of the route.
- Depending on the nature of ground and the contour spacing, the spacing of cross sections is decided.

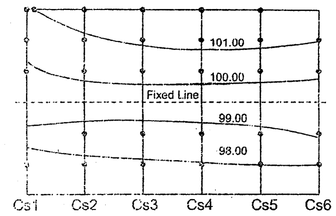


Fig. 7.12 *Cross-section method of contouring*

(c) Radial lines method :

- This method is particularly useful for contouring small hilly areas where radial lines are drawn from a peak to cover the entire area.
- Grid points are taken on the radial lines and their elevations are determined.
- The contour lines are drawn through interpolation.

(d) Controlling point method :

- Here the elevations are determined for a few selected key or controlling points. The contour lines are then drawn through interpolation.
- This however is an approximate but quick method.
- For fairly uniform sloping ground, this method gives quite good results.

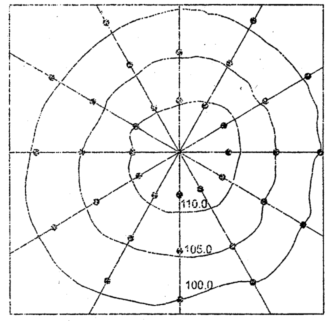


Fig. 7.13 *Radial line method of contouring*

7.15 Uses of Contour Maps

(a) Assessment of the terrain character
(b) Selection of suitable site for a project
(c) Determination of sections
(d) Inter-visibility between the two points
(e) Location of a route
(f) Catchment area assessment
(g) Estimation of reservoir capacity

7.16 Effect of Earth's Curvature and Refraction

- The effect of curvature of earth and refraction needs to be considered when line of sight is long.
- This effect occurs because the difference between the level line and horizontal line increases as the distance of the staff from the instrument station increases.
- Now since the horizontal line is tangential to the level surface at one point while the level is normal to the direction of gravity at different points.
- The line of sight of the instrument is horizontal while the level line is curved. Thus the levels obtained from the leveling instruments have to be corrected for the curvature of earth.

7.16.1 Effect of Curvature

- In Fig. 7.14 line *AD* is the horizontal line while line *AC* is the level line. While looking through the telescope, the staff reading *BD* is obtained while the level line has a reading of *BC*. Thus there occurs a positive error of CD due to curvature of earth and the resulting correction is negative i.e. the correct reading is equal to observed reading less the correction CD.

- Let AC be the level line through A and AD be the horizontal line. It is assumed that earth is a perfect sphere of radius R and thus level line AC will be an arc of the circle.

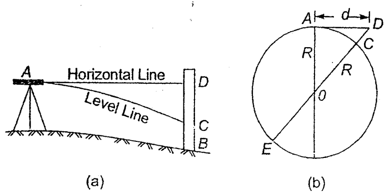


Fig. 7.14 *Curvature effect of earth*

Thus from geometry,

$$DC \times DE = AD^2 \quad \text{[From theorem of circle, } PB \times PC = PT^2\text{]}$$

$$DC(DC + CE) = AD^2$$

$$DC^2 + DC \times CE = AD^2$$

Now DC is very small as compared to diameter of earth CE (=2R) and thus the term DC^2 can be ignored without introducing any significant errors.

Thus,

$$DC = AD^2/CE = d^2/2R$$

where d = Horizontal distance between the two points

Now radius of earth R = 6367 km

So,

$$DC = d^2/2 \times 6367 = d^2/12734$$

where both d and R are in km.

If DC is required to be expressed in meter while keeping d in km then,

$$DC = (d^2/12734) \times 1000 = 0.0785d^2$$

Thus correction due to curvature,

$$C_c = -0.0785d^2$$

where C_c is in meters and d is in km. Curvature makes the staff point to appear lower and thus a larger reading is obtained than the actual value and thus the correction is negative.

7.16.2 Effect of Refraction

- With increase in altitude, the density of air decreases.
- Since air is denser near the earth, the ray of light from staff to the instrument travels from a lighter medium to a denser medium and thus it gets bent towards the normal. The line of sight does not remain horizontal but it bends towards the center of earth.
- **Fig. 7.15** shows the refracted ray AD. The actual line of sight touches the staff at D' and not at D.
- The intercept DD' is the error in reading due to refraction.
- Refraction causes the object to appear higher than the actual position.
- The resulting staff reading obtained is smaller than the actual since point D' will appear on the line of sight instead of point D. The correction to the staff reading is thus positive.

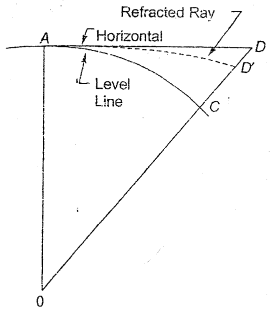


Fig. 7.15 *Effect of earth's atmospheric refraction*

Because of highly varying density of air, the refraction curve is highly irregular. For average atmospheric conditions, the curved path of the refracted ray may be assumed to be an arc of circle with radius 7R where R is the radius of earth.

Thus,

$$DD' = (1/7) \times 0.0785d^2 = 0.0112d^2$$

Thus correction due to refraction, C_r is,

$$C_r = +0.0112d^2$$

7.16.3 Combined Curvature and Refraction Correction

The combined correction due to curvature and refraction can arrived at as follows:

$$C = CD' = -CD + DD'$$

$$= C_c + C_r = -0.0785d^2 + 0.0112d^2 = -0.0673d^2$$

where correction C is in meters and distance d is in km.

7.17 Distance of Visible Horizon

Let there is a point C at an elevation h from the mean sea level. A is the point on the horizon as observed from C. The tangent from C meets the mean sea level at A. The distance AC is the distance of visible horizon.

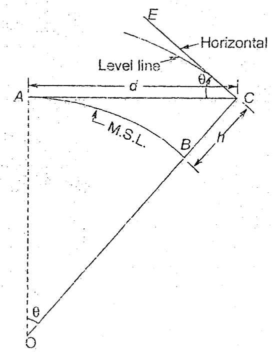


Fig. 7.16 *Distance of visible horizon*

$$h = 0.0673d^2$$

$$d = \frac{\sqrt{h}}{0.0673}$$

where d = Distance AC (in km) and h is in meters.

If height h is not negligible in comparison to radius of earth then,

$$d^2 = h(2R + h)$$

However the effect due to refraction is taken as 1/7 of that due to curvature of earth and is of opposite sign.

ILLUSTRATIVE EXAMPLES

Example 7.1 What is the combined correction for curvature and refraction for a distance of 5000 m?

Solution:

The combined correction for curvature and refraction is given by,

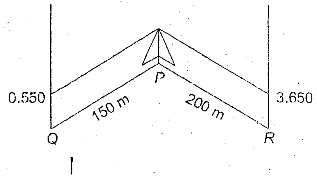
$$C = -0.0673\, d^2 \quad \text{where d is in km}$$

$$= -0.0673\,(5)^2$$

$$= -1.6825 \text{ m}$$

Example 7.2 A level was set up at P at a distance of 150 m from Q and 200 m from R. The staff readings at Q and R are respectively 0.550 and 3.650. What is the time elevation difference of points Q and R?

Solution:



Combined correction for $Q = -0.0673\,(0.15)^2$

$= -0.00151425$ m

Thus correct staff reading at $Q = 0.550 - 0.00151425$

$= 0.549$ m

Combined correction for $R = -0.0673\,(0.2)^2$

$= -0.002692$ m

Thus correct staff reading at $R = 3.650 - 0.002692 = 3.647$ m

∴ Elevation difference = 3.647 − 0.549 = 3.098 m

Example 7.3 From the horizon of a ship, a light house was just visible. If height of the light house is 265 m, find the distance between the light house and the ship.

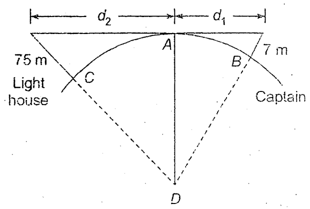
Solution:

Distance of visible horizon is given by,

$$d\,(\text{km}) = \sqrt{\frac{h(m)}{0.0673}} = \sqrt{\frac{265}{0.0673}} = 62.75 \text{ km}$$

Example 7.4 The caption of a ship standing on the deck, just sees a lighthouse of height 75 m above MSL. If height of captain's eye above MSL is 7 m then what is the distance of light house from the captain?

Solution:



From the figure, it is evident that,

$$d_1 = \sqrt{\frac{7}{0.0673}} = 10.2 \text{ km}$$

$$d_2 = \sqrt{\frac{75}{0.0673}} = 33.38 \text{ km}$$

∴Distance of captain from light house = $d_1 + d_2$

= 10.2 + 33.38 = 43.58 km

Example 7.5 What will be the level difference between the points *A* and *B* is curvature and refraction effects are also taken into account for the case as given below:

Level is set up at *A* and staff at *B*:

R.L. of *A* = 160 m

HI at *A* = 1.2 m

Reading of staff at *B* = 1.950 m

Distance between *A* and *B* = 425 m

Solution:

Combined correction due to curvature and refraction,

$C = -0.0673\, d^2$ where d is in km

$= -0.0673\,(0.425)^2 = -0.012156$ m

∴ Correct staff reading at B = 1.950 − 0.012156 = 1.938 m

∴ Level difference between A and B is given by,

RL of *A* + *HI* = Staff reading at *B* + *RL* of *B*

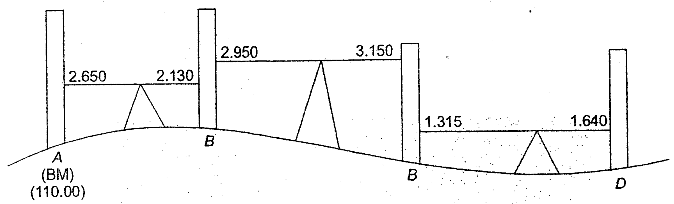
⇒ *RL* of *A* − *RL* of *B* = 1.938 − 1.2 = 0.738 m

∴ *B* is lower than *A*

∴ *RL* of *B* = *RL* of *A* − 0.738

= 160 − 0.738 = 159.262 m

Example 7.6 The figure given below shows various observations made of stations *A, B, C* and *D*. Enter the values in level field book and determine the *RLs* of different points by both the methods.



Solution:

HI Method

Station (1)	BS (2)	IS (3)	F S (4)	HI (m) (5)	RL (m) (6)	Remarks (7)
A	2.650		—	112.65	110	BM
B	2.950		2.130	113.47	110.52	TP1
C	1.315		3.150	111.635	110.32	TP2
D	—		1.640		109.995	
	ΣBS=6.915		ΣFS=6.920			

Check

ΣBS − ΣFS = Last RL − First RL

⇒ 6.915 − 6.92 = 109.995 − 110.00

⇒ −0.005 = −0.005 (O.K.)

Rise and Fall method

Station (1)	BS (2)	IS (3)	F S (4)	Rise(m) (5)	Fall(m) (6)	RL(m) (7)	Remarks (8)
A	2.650		—			110	BM
B	2.950		2.130	0.52		110.52	TP1
C	1.315		3.150		0.2	110.32	TP2
D	—		1.640		0.325	109.995	
	ΣBS = 6.915		ΣFS = 6.920	ΣRise = 0.52	ΣFall = 0.525		

Check :

ΣBS − ΣFS = Last RL − First RL = ΣRise − ΣFall

⇒ 6.915 − 6.92 = 109.995 − 110.00 = 0.52 − 0.525

⇒ −0.005 = −0.005 = −0.005 (O.K.)

Example 7.7 During testing of a dumpy level, the following observations were taken :

Instrument at	Staff reading at	
	A	B
A	1.035	2.115
B	1.115	2.050

Check whether the instrument is in proper adjustment. What should be the revised reading of line of collimation. When the instrument is at B? If A and B are 95 m apart then what is the angle of inclination of line of collimation?

Solution:

when the instrument is at A,

apparent difference in levels = 2.115 − 1.035 = 1.08 m

when the instrument is at B,

apparent difference in levels = 2.050 − 1.115 = 0.935 m

since the two difference in levels are not equal and thus line of collimation is not in proper adjustment.

$$\text{correct difference of levels} = \frac{1.08 + 0.935}{2} = 1.0075 \text{ m} \simeq 1.008 \text{ m}$$

collimation error when the instrument is at B

correct reading on staff at B = 2.050 m

∴ correct reading on staff at A should be = 2.050 − 1.008 = 1.042 m ≃ 1.040 m

Observed reading on A (1.115 m) > correct reading at A (1.040 m)

∴ Line of collimation is inclined upwards

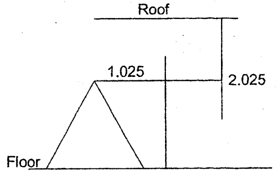
Upward inclination = 1.115 − 1.040 = 0.075 m

$$\therefore \text{ Angle of inclination of line of collimation} = \tan^{-1}\left(\frac{0.075}{95}\right) = 0.0452° = 00°02'43''.$$

Example 7.8 The RL of a floor level is 45.00 m and staff reading on the floor is 1.025 m. The staff reading when held vertical touching the roof slab comes out to be 2.025 m. What is the floor height?

Solution:

Floor height = 1.025 + 2.025 = 3.05 m



Example 7.9 Refer to the following notes for reciprocal levelling taken with simple level.

Instrument Station	Staff reading on		Remarks
	A	B	
A	1.305	2.725	Dist. AB = 905 m
B	1.355	2.050	R.L of A = 100 m collimation error - 0.003 / 150

Determine

(a) the true R.L. of B

(b) Collimation correction

(c) Refraction correction

(d) Combined correction

Solution:

(a) True RL of B

when instrument is at A, apparent level difference

= 2.725 − 1.305 = 1.42 m

when instrument is at B, apparent level difference

= 2.050 − 1.355 = 0.695 m

$$\therefore \quad \text{True level difference} = \frac{1.42 + 0.695}{2} = 1.0575 \text{ m} \simeq 1.058 \text{ m}$$

∴ True RL of B = RL of A − True level difference

= 100 − 1.058 = 98.942 m

(b) $$\text{Total collimation error} = \frac{0.003}{150} \times 905 = 0.0181 \text{ m}$$

∴ Total collimation correction = − 0.0181 m

(c) Refraction correction

Error due to curvature = 0.0785 d^2 = 0.0785 $(0.905)^2$ = 0.0643 m

∴ Curvature correction = − 0.0643 m

∴ combined correction (c) = − 0.0643 + C_R

where C_R = Refraction correction

Total correction for collimation, curvature and refraction

= − 0.0181 − 0.0643 + C_R

= − 0.0824 + C_R

This must be equal to true level difference between A and B

∴ $-0.0824 + C_R = 1.058 \Rightarrow C_R = 1.1404$ m

(d) Combined correction = − 0.0643 + 1.1404

= 1.0761 m

Example 7.10 The following consecutive readings were taken with a level on a 4 m staff on a continuously sloping ground at an interval of 20 :

0.650, 1.535, 1.850, 2.435, 2.995, 3.350, 1.150, 1.850, 2.045, 3.635, 0.905, 1.055, 1.750, 2.550. The RL of first point A is 110.780 m. Rule out a page of level field book and record the above readings. Calculate the reduced levels of points. Calculate also the gradient of line joining first and last points.

Solution :

The first reading (at station A) must be B.S. Now readings are taken on a continuously sloping ground with 4m staff, the largest reading that can be taken is 4 m so position of level must be changed after the largest reading of given series.

Level last reading (2.550) is FS.

Thus readings 3.350, 3.635 and 2.550 are fore sights of these, each of the fore sight readings 3.350? 3.635 are followed by back sight readings.

Thus 1.150 and 0.905 are BS. All remaining readings are IS.

Height of instrument / Height of Collimation Method

Station	Distance (m)	BS	IS	FS	HI(m)	RL(m)	Remarks
A	0	0.650			111.43	110.780	BM
	20		1.535			109.895	
	40		1.850			109.580	
	60		2.435			108.995	
	80		2.995			108.435	
	100	1.150		3.350	109.230	108.080	Change point (*CP*–1)
	120		1.850			107.380	
	140		2.045			107.185	
	160	0.905		3.635	106.50	105.595	Change point (*CP*–2)
	180		1.055			105.445	
	200		1.750			104.750	
B	220			2.550		103.950	

$\Sigma BS = 2.705$ $\Sigma FS = 9.535$ Last RL – First RL

$\Sigma BS - \Sigma FS = -6.83$ $= 103.950 - 110.780$

$= -6.83$ (O.K.)

Rise and fall Method

Station	Distance (m)	BS	IS	FS	Rise (m)	Fall (m)	RL(m)	Remarks
A	0	0.650					110.780	BM
	20		1.535			0.885	109.895	
	40		1.850			0.315	109.580	
	60		2.435			0.585	108.995	
	80		2.995			0.560	108.435	
	100	1.150		3.350		0.355	108.080	Change point (*CP*–1)
	120		1.850			0.700	107.380	
	140		2.045			0.195	107.185	
	160	0.905		3.635		1.59	105.595	Change point (*CP*–2)
	180		1.055			0.150	105.445	
	200		1.750			0.695	104.750	
B	220			2.550		0.800	103.950	

$\Sigma BS = 2.705$ $\Sigma FS = 9.535$ Σ Rise = 0 Σ Fall = 6.83 Last RL – First RL = 103.95 – 110.78 = –6.83

$$\therefore \quad \Sigma BS - \Sigma FS = \text{Last RL} - \text{First RL} = \Sigma\text{Rise} - \Sigma\text{fall}$$

$$\Rightarrow \quad 2.705 - 9.535 = 103.95 - 110.78 = 0 - 6.83$$

$$\Rightarrow \quad -6.83 = -6.83 = -6.83 \quad \text{(O.K.)}$$

$$\therefore \quad \text{Gradient of line } AB = \frac{\text{Total fall}}{\text{Total distance from } A \text{ to } B} = \frac{6.83}{220}$$

$$= 0.03105 \text{ or } 1 \text{ in } 32.211$$

Objective Brain Teasers

Q.1 The sensitivity of bubble tube can be enhanced by:
(a) Increasing the length of divisions
(b) Increasing the tube length
(c) Increasing the tube diameter
(d) Increasing the radius of curvature of tube

Q.2 While doing the levelling process on a steep slope, the instrument should preferably be set up successively along a:
(a) Zig-zag path
(b) Curved path
(c) Straight path
(d) All of the above

Q.3 The levelling work can be best checked by:
(a) Open loop method
(b) Rise and fall method
(c) Closed loop method
(d) HI method

Q.4 The datum adopted in India is the MSL at:
(a) Karachi (b) Mumbai
(c) Chennai (d) Kolkata

Q.5 The sensitivity of a bubble tube is 20." A staff is located at a distance of 200 m. Find the error in reading if the bubble is out by one division.
(a) 0.0194 m (b) 0.0805 m
(c) 0.8050 m (d) 0.1945 m

Q.6 What is the curvature correction for a distance of 1000 m?
(a) 0.0785 m (b) 78.50 m
(c) 67.03 m (d) 0.0673 m

Q.7 Reciprocal levelling eliminate the effect of:
(i) Mistakes in staff reading
(ii) Errors due to collimation line
(iii) Errors due to atmospheric refraction
(iv) Errors due to earth's curvature
Choose the correct option among the following:
(a) (i) and (ii) (b) (i), (ii) and (iii)
(c) (ii), (iii) and (iv) (d) (iii) and (iv)

Q.8 Choose the correct statement
(a) Hypsometry is NOT a type of levelling
(b) Most common method of spirit levelling is the differential levelling
(c) In spirit levelling, horizontal as well as vertical angles are also measured
(d) All of the above

Q.9 The liquid of the level tube must be:
(a) Non-freezing (b) Quick acting
(c) Both (a) and (b) (d) Viscous

Q.10 The contours at ridge line intersect at ______
(a) 60° (b) 30°
(c) 0° (d) 90°

Q.11 Identify the incorrect statement
(a) Intersection of level surface with surface of earth is called as contour
(b) Gentle slope is represented by closely spaced contours
(c) In a contour map the direction of steepest slope is normal to the contour line.
(d) All of the above

Q.12 Which of the following eliminates the error due to curvature and refraction?
(a) Equating the BS and FS
(b) Fly levelling
(c) Profile levelling
(d) Check levelling

Q.13 Which of following can be eliminated by using reciprocal levelling?

(i) Error due to earth's curvature
(ii) Error due to earth's atmospheric refraction.
(iii) Mistakes increased while taking staff readings
(iv) Error due to line of collimation

(a) (i) and (iv) (b) (ii), (iii) and (iv)
(c) (i), (ii) and (iv) (d) (i), (iii) and (iv)

Q.14 Lines joining points of equal elevation on the surface of earth are known as

(a) Isogonic lines (b) Isobars
(c) Agonic lines (d) Contours

Q.15 Interpolation of contours can be done by

(i) Arithmetic methods
(ii) Trigonometric methods
(iii) Graphical methods
(iv) Numerical methods

Of these, the correct ones are

(a) (i) and (iii) (b) (i) and (iv)
(c) (ii) and (iii) (d) (i) and (ii)

Q.16 The fluid of level tube should be

(a) Anti-freezing
(b) Temperature insusceptible
(c) Quick acting
(d) All of the above

Q.17 Levelling deals with measurements made in

(a) Vertical plane
(b) Horizontal plane
(c) Inclined plane
(d) Both horizontal and vertical plane

Q.18 The back sight on a BM of 102.50 m is 2.25 m. The inverted staff reading to the bottom of roof is 1.515 m. The R.L. of bottom of roof is

(a) 103.235 m (b) 101.765 m
(c) 106.265 m (d) 98.735 m

Q.19 In India, the reference datum is taken as MSL of

(a) Kolkata (b) Mumbai
(c) Panji (d) Chennai

Answers

1. (d)	2. (a)	3. (c)	4. (b)	5. (a)
6. (a)	7. (c)	8. (b)	9. (c)	10. (d)
11. (b)	12. (a)	13. (c)	14. (d)	15. (a)
16. (d)	17. (a)	18. (c)	19. (b)	

Student's Assignments

Ex.1 From the table given below, fill in the missing entries. Apply all checks.

Station	BS	CS	FS	Rise (m)	Fall (m)	RL (m)	Remarks
1.	3.125					—	BM
2.	—		—	1.325		125.505	TP –1
3.		2.320			0.055	—	
4.		—		—		125.850	
5.	—		2.655		—	—	TP –2
6.	1.620		3.205		2.165	—	TP –3
7.		3.625			—	—	
8.			—			123.090	

Ex.2 The following consecutive readings were taken with a level and a 4 m long levelling staff on a continuously sloping ground at 30 m interval.
0.680, 1.455, 1.855, 2.330, 2.885, 3.380, 1.055, 1.860
2.265, 3.540, 0.835, 0.945, 1.530 and 2.250
With the above readings, complete the page of the level field book if RL of starting point is 80.750 m. Also determine the gradient of the line joining first and last point.

Ans. Gradient = 1 in 50 falling

Ex.3 Determine the corrections for curvature of earth and refraction for a distance of 1.6 km.

Ans. –0.2 m, +0.0287 m, –0.172 m