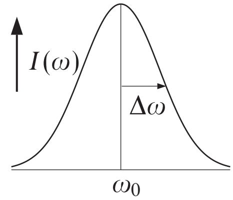
8.8 Line radiation

Spectral line broadening

Natural broadening[a]	$I(\omega)=\dfrac{(2\pi\tau)^{-1}}{(2\tau)^{-2}+(\omega-\omega_0)^2}$	(8.112)
Natural half-width	$\Delta\omega=\dfrac{1}{2\tau}$	(8.113)
Collision broadening	$I(\omega)=\dfrac{(\pi\tau_c)^{-1}}{(\tau_c)^{-2}+(\omega-\omega_0)^2}$	(8.114)
Collision and pressure half-width[c]	$\Delta\omega=\dfrac{1}{\tau_c}=p\pi d^2\left(\dfrac{\pi mkT}{16}\right)^{-1/2}$	(8.115)
Doppler broadening	$I(\omega)=\left(\dfrac{mc^2}{2kT\omega_0^2\pi}\right)^{1/2}\exp\left[-\dfrac{mc^2}{2kT}\dfrac{(\omega-\omega_0)^2}{\omega_0^2}\right]$	(8.116)
Doppler half-width	$\Delta\omega=\omega_0\left(\dfrac{2kT\ln 2}{mc^2}\right)^{1/2}$	(8.117)

$I(\omega)$	normalised intensity[b]
τ	lifetime of excited state
ω	angular frequency ($=2\pi\nu$)
$\Delta\omega$	half-width at half-power
ω_0	centre frequency
τ_c	mean time between collisions
p	pressure
d	effective atomic diameter
m	gas particle mass
k	Boltzmann constant
T	temperature
c	speed of light



[a]The transition probability per unit time for the state is $=1/\tau$. In the classical limit of a damped oscillator, the e-folding time of the electric field is 2τ. Both the natural and collision profiles described here are Lorentzian.
[b]The intensity spectra are normalised so that $\int I(\omega)\,\mathrm{d}\omega=1$, assuming $\Delta\omega/\omega_0\ll 1$.
[c]The pressure-broadening relation combines Equations (5.78), (5.86) and (5.89) and assumes an otherwise perfect gas of finite-sized atoms. More accurate expressions are considerably more complicated.

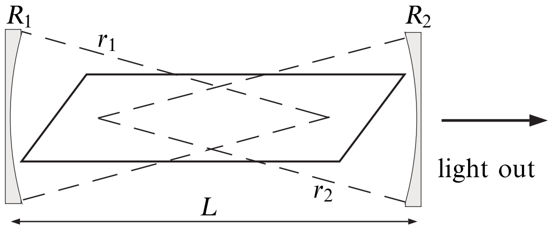
Einstein coefficients[a]

Absorption	$R_{12}=B_{12}I_\nu n_1$	(8.118)
Spontaneous emission	$R_{21}=A_{21}n_2$	(8.119)
Stimulated emission	$R'_{21}=B_{21}I_\nu n_2$	(8.120)
Coefficient ratios	$\dfrac{A_{21}}{B_{12}}=\dfrac{2h\nu^3}{c^2}\dfrac{g_1}{g_2}$	(8.121)
	$\dfrac{B_{21}}{B_{12}}=\dfrac{g_1}{g_2}$	(8.122)

R_{ij}	transition rate, level $i\rightarrow j$ ($\mathrm{m^{-3}\,s^{-1}}$)
B_{ij}	Einstein B coefficients
I_ν	specific intensity of radiation field
A_{21}	Einstein A coefficient
n_i	number density of atoms in quantum level i ($\mathrm{m^{-3}}$)
h	Planck constant
ν	frequency
c	speed of light
g_i	degeneracy of ith level

[a]Note that the coefficients can also be defined in terms of spectral energy density, $u_\nu=4\pi I_\nu/c$ rather than I_ν. In this case $\frac{A_{21}}{B_{12}}=\frac{8\pi h\nu^3}{c^3}\frac{g_1}{g_2}$. See also *Population densities* on page 116.

Lasers[a]



Cavity stability condition	$0 \leq \left(1-\frac{L}{r_1}\right)\left(1-\frac{L}{r_2}\right) \leq 1$	(8.123)	$r_{1,2}$ radii of curvature of end-mirrors L distance between mirror centres
Longitudinal cavity modes[b]	$\nu_n = \frac{c}{2L}n$	(8.124)	ν_n mode frequency n integer c speed of light
Cavity Q	$Q = \frac{2\pi L(R_1R_2)^{1/4}}{\lambda[1-(R_1R_2)^{1/2}]}$	(8.125)	Q quality factor $R_{1,2}$ mirror (power) reflectances λ wavelength
	$\simeq \frac{4\pi L}{\lambda(1-R_1R_2)}$	(8.126)	
Cavity line width	$\Delta\nu_c = \frac{\nu_n}{Q} = 1/(2\pi\tau_c)$	(8.127)	$\Delta\nu_c$ cavity line width (FWHP) τ_c cavity photon lifetime
Schawlow–Townes line width	$\frac{\Delta\nu}{\nu_n} = \frac{2\pi h(\Delta\nu_c)^2}{P}\left(\frac{g_l N_u}{g_l N_u - g_u N_l}\right)$	(8.128)	$\Delta\nu$ line width (FWHP) P laser power $g_{u,l}$ degeneracy of upper/lower levels $N_{u,l}$ number density of upper/lower levels
Threshold lasing condition	$R_1R_2\exp[2(\alpha-\beta)L] > 1$	(8.129)	α gain per unit length of medium β loss per unit length of medium

[a]Also see the *Fabry-Perot etalon* on page 163. Note that "cavity" refers to the empty cavity, with no lasing medium present.
[b]The mode spacing equals the cavity free spectral range.