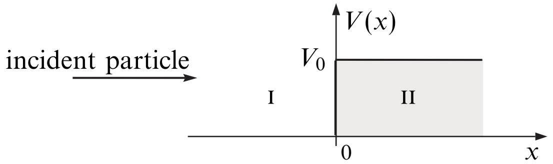
4.3 Wave mechanics

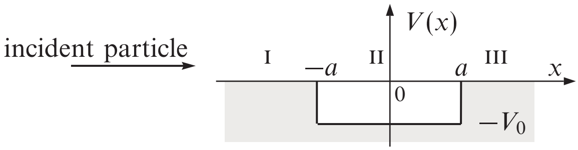
Potential step[a]



Potential function	$V(x)=\begin{cases}0 & (x<0)\\ V_0 & (x\geq 0)\end{cases}$	(4.38)	V V_0 $\hbar$	particle potential energy step height (Planck constant)/(2π)
Wavenumbers	$\hbar^2k^2=2mE \quad (x<0)$ $\hbar^2q^2=2m(E-V_0) \quad (x>0)$	(4.39) (4.40)	k,q m E	particle wavenumbers particle mass total particle energy
Amplitude reflection coefficient	$r=\dfrac{k-q}{k+q}$	(4.41)	r	amplitude reflection coefficient
Amplitude transmission coefficient	$t=\dfrac{2k}{k+q}$	(4.42)	t	amplitude transmission coefficient
Probability currents[b]	$j_{\rm I}=\dfrac{\hbar k}{m}(1-\vert r\vert^2)$ $j_{\rm II}=\dfrac{\hbar q}{m}\vert t\vert^2$	(4.43) (4.44)	$j_{\rm I}$ $j_{\rm II}$	particle flux in zone I particle flux in zone II

[a]One-dimensional interaction with an incident particle of total energy $E=\mathrm{KE}+V$. If $E<V_0$ then q is imaginary and $|r|^2=1$. $1/|q|$ is then a measure of the tunnelling depth.
[b]Particle flux with the sign of increasing x.

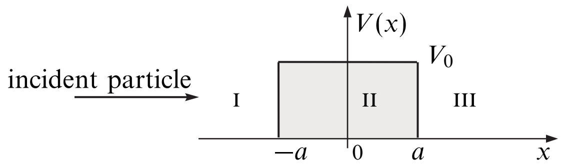
Potential well[a]



Potential function	$V(x)=\begin{cases}0 & (\lvert x\rvert>a)\\ -V_0 & (\lvert x\rvert\le a)\end{cases}$	(4.45)	V V_0 $\hbar$ $2a$	particle potential energy well depth (Planck constant)/(2π) well width
Wavenumbers	$\hbar^2k^2=2mE \quad (\lvert x\rvert>a)$ $\hbar^2q^2=2m(E+V_0) \quad (\lvert x\rvert<a)$	(4.46) (4.47)	k,q m E	particle wavenumbers particle mass total particle energy
Amplitude reflection coefficient	$r=\dfrac{\mathbf{i}\mathrm{e}^{-2\mathbf{i}ka}(q^2-k^2)\sin 2qa}{2kq\cos 2qa-\mathbf{i}(q^2+k^2)\sin 2qa}$	(4.48)	r	amplitude reflection coefficient
Amplitude transmission coefficient	$t=\dfrac{2kq\mathrm{e}^{-2\mathbf{i}ka}}{2kq\cos 2qa-\mathbf{i}(q^2+k^2)\sin 2qa}$	(4.49)	t	amplitude transmission coefficient
Probability currents[b]	$j_{\mathrm{I}}=\dfrac{\hbar k}{m}(1-\lvert r\rvert^2)$ $j_{\mathrm{III}}=\dfrac{\hbar k}{m}\lvert t\rvert^2$	(4.50) (4.51)	j_{I} j_{III}	particle flux in zone I particle flux in zone III
Ramsauer effect[c]	$E_n=-V_0+\dfrac{n^2\hbar^2\pi^2}{8ma^2}$	(4.52)	n E_n	integer >0 Ramsauer energy
Bound states $(V_0<E<0)$[d]	$\tan qa=\begin{cases}\lvert k\rvert/q & \text{even parity}\\ -q/\lvert k\rvert & \text{odd parity}\end{cases}$ $q^2-\lvert k\rvert^2=2mV_0/\hbar^2$	(4.53) (4.54)		

[a]One-dimensional interaction with an incident particle of total energy $E=\mathrm{KE}+V>0$.
[b]Particle flux in the sense of increasing x.
[c]Incident energy for which $2qa=n\pi$, $|r|=0$, and $|t|=1$.
[d]When $E<0$, k is purely imaginary. $|k|$ and q are obtained by solving these implicit equations.

Barrier tunnelling[a]

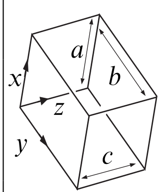


Potential function	$V(x)=\begin{cases}0 & (\lvert x\rvert>a)\\ V_0 & (\lvert x\rvert\le a)\end{cases}$ (4.55)	V particle potential energy V_0 well depth $\hbar$ (Planck constant)/(2π) $2a$ barrier width
Wavenumber and tunnelling constant	$\hbar^2k^2=2mE \quad (\lvert x\rvert>a)$ (4.56) $\hbar^2\kappa^2=2m(V_0-E) \quad (\lvert x\rvert<a)$ (4.57)	k incident wavenumber κ tunnelling constant m particle mass E total energy ($<V_0$)
Amplitude reflection coefficient	$r=\dfrac{-\mathrm{i}\mathrm{e}^{-2\mathrm{i}ka}(k^2+\kappa^2)\sinh 2\kappa a}{2k\kappa\cosh 2\kappa a-\mathrm{i}(k^2-\kappa^2)\sinh 2\kappa a}$ (4.58)	r amplitude reflection coefficient
Amplitude transmission coefficient	$t=\dfrac{2k\kappa\mathrm{e}^{-2\mathrm{i}ka}}{2k\kappa\cosh 2\kappa a-\mathrm{i}(k^2-\kappa^2)\sinh 2\kappa a}$ (4.59)	t amplitude transmission coefficient
Tunnelling probability	$\lvert t\rvert^2=\dfrac{4k^2\kappa^2}{(k^2+\kappa^2)^2\sinh^2 2\kappa a+4k^2\kappa^2}$ (4.60) $\simeq\dfrac{16k^2\kappa^2}{(k^2+\kappa^2)^2}\exp(-4\kappa a) \quad (\lvert t\rvert^2\ll 1)$ (4.61)	$\lvert t\rvert^2$ tunnelling probability
Probability currents[b]	$j_{\mathrm{I}}=\dfrac{\hbar k}{m}(1-\lvert r\rvert^2)$ (4.62) $j_{\mathrm{III}}=\dfrac{\hbar k}{m}\lvert t\rvert^2$ (4.63)	j_{I} particle flux in zone I j_{III} particle flux in zone III

[a]By a particle of total energy $E=\mathrm{KE}+V$, through a one-dimensional rectangular potential barrier height $V_0>E$.
[b]Particle flux in the sense of increasing x.

Particle in a rectangular box[a]

Eigen-functions	$\Psi_{lmn}=\left(\dfrac{8}{abc}\right)^{1/2}\sin\dfrac{l\pi x}{a}\sin\dfrac{m\pi y}{b}\sin\dfrac{n\pi z}{c}$ (4.64)	Ψ_{lmn} eigenfunctions a,b,c box dimensions l,m,n integers ≥ 1
Energy levels	$E_{lmn}=\dfrac{h^2}{8M}\left(\dfrac{l^2}{a^2}+\dfrac{m^2}{b^2}+\dfrac{n^2}{c^2}\right)$ (4.65)	E_{lmn} energy h Planck constant
Density of states	$\rho(E)\,\mathrm{d}E=\dfrac{4\pi}{h^3}(2M^3E)^{1/2}\,\mathrm{d}E$ (4.66)	M particle mass $\rho(E)$ density of states (per unit volume)



[a]Spinless particle in a rectangular box bounded by the planes $x=0$, $y=0$, $z=0$, $x=a$, $y=b$, and $z=c$. The potential is zero inside and infinite outside the box.

Harmonic oscillator

Schrödinger equation	$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_n}{\partial x^2}+\frac{1}{2}m\omega^2x^2\psi_n=E_n\psi_n \quad (4.67)$$	$\hbar$ (Planck constant)/(2π) m mass ψ_n nth eigenfunction x displacement
Energy levels[a]	$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega \quad (4.68)$$	n integer ≥ 0 ω angular frequency E_n total energy in nth state
Eigen-functions	$$\psi_n=\frac{H_n(x/a)\exp[-x^2/(2a^2)]}{(n!2^na\pi^{1/2})^{1/2}} \quad (4.69)$$ where $a=\left(\frac{\hbar}{m\omega}\right)^{1/2}$	H_n Hermite polynomials
Hermite polynomials	$H_0(y)=1, \quad H_1(y)=2y, \quad H_2(y)=4y^2-2$ $$H_{n+1}(y)=2yH_n(y)-2nH_{n-1}(y) \quad (4.70)$$	y dummy variable

[a] E_0 is the zero-point energy of the oscillator.