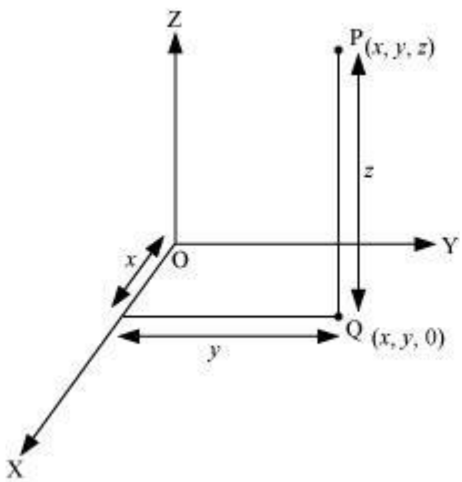
Introduction to Three Dimensional Geometry

- **Three-dimensions coordinate planes**
  - The coordinate axes of a rectangular Cartesian coordinate system are three mutually perpendicular lines. The axes are called x, y, and z-axes.
  - The three planes determined by the pair of axes are the coordinate planes, called XY, YZ and ZX-planes.
  - The three coordinate planes divide the space into eight parts known as octants.
  - In three-dimensional geometry, the coordinates of a point P are always written in the form of triplets i.e., (x, y, z). Here, x, y, and z are the distances from the YZ, ZX and XY-planes. Also, the coordinates of the origin are (0, 0, 0).



- The sign of the coordinates of a point determine the octant in which the point lies. The following table shows the signs of the coordinates in the eight octants.

Octants → / Coordinates ↓	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

Example: The point (−5, 6, −7) lies in the VI octant.

- In Coordinates of points lying on different axes:
  - Any point on the x-axis is of the form $(x, 0, 0)$
  - Any point on the y-axis is of the form $(0, y, 0)$

  - Any point on the z-axis is of the form (0, 0, z)
- Coordinates of points lying in different planes:
  - Coordinates of a point in the YZ-plane are of the form (0, y, z)
  - Coordinates of a point in the XY-plane are of the form (x, y, 0)
  - Coordinates of a point in the ZX-plane are of the form (x, 0, z)

  Example: The points (–5, 6, 0), (0, –5, 6), (–5, 0, 6) lies in the XY-plane, YZ-plane and ZX-plane respectively.

- **distance formula** Distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is given by

  $$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

  Example: Find the point(s), lying on the z-axis, whose distance from point (2, –1, 3) is 3 units.

  Solution: Let the required point be (0, 0, z). We know that the distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1) + (z_2 - z_1)^2}$.

  Therefore,

  $$\sqrt{(2-0)^2 + (-1-0)^2 + (3-z)^2} = 3$$

  On squaring both the sides, we get

  $$4 + 1 + 9 + z^2 - 6z = 9$$

  $$\Rightarrow z^2 - 6z + 5 = 0$$

  $$\Rightarrow z^2 - 5z - z + 5 = 0$$

  $$\Rightarrow z(z-5) - 1(z-5) = 0$$

  $$\Rightarrow z = 1, 5$$

  Thus, the required points on the z-axis are (0, 0, 1) and (0, 0, 5).