Mensuration

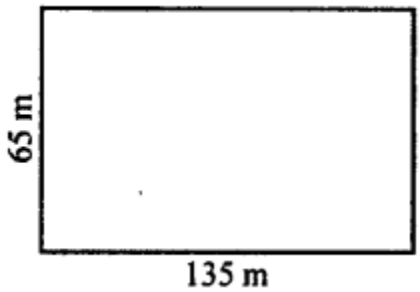
EXERCISE 20 (A)

Question 1.

The length and the breadth of a rectangular plot are 135 m and 65 m. Find, its perimeter and the cost of fencing it at the rate of ₹60 per m.

Solution:

Given :
Length (l) = 135 m
Breadth (b) = 65 m



Perimeter = 2 (l + b)
= 2(135 + 65)
= 2(200) = 400 m
∴Perimeter of rectangular plot is = 400 m
Cost of fencing per m = ₹60
∴Cost of fencing 400 m = ₹60 x 400 m = ₹24000

Question 2.

The length and breadth of a rectangular field are in the ratio 7 : 4. If its perimeter is 440 m, find its length and breadth. Also, find the cost of fencing it @ ₹150 per m.

Solution:

Given : Perimeter = 440 m
Let the length of rectangular field = lx and breadth = 4x
2(l + b) = Perimeter
2(7x + 4x) = 440 m
2(11x) = 440 m
22x = 440 m
$x = \frac{440}{22}$
x = 11 m
∴Length = 7x = 7 x 11 = 77 m
Breadth = Ax = 4 x 11 = 44 m
Cost of fencing per m = ₹150
Cost of fencing 440 m = ₹150 x 440 = ₹66,000

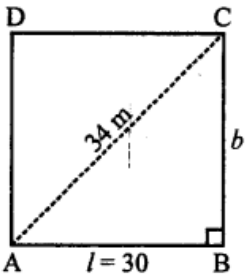
Question 3.

The length of a rectangular field is 30 m and its diagonal is 34 m. Find the breadth of the field and its perimeter.

Solution:

Length = 30 m

Diagonals = 34 m



Let the breadth of the rectangle = b m

Applying Pythagoras Theorem in triangle ABC,

We get,

$AC^2 = AB^2 + BC^2$

$(34)^2 = (30)^2 + b^2$

$1156 = 900 + b^2$

$1156 - 900 = b^2$

$256 = b^2$

$\Rightarrow b = \sqrt{256} = 16$ m

Perimeter = $2(l + b)$

$= 2(30 + 16) = 2 \times 46 = 92$ m

Question 4.

The diagonal of a square is $12\sqrt{2}$ cm. Find its perimeter.

Solution:

Diagonal of square = Its side x $\sqrt{2}$

Side $\sqrt{2}$ = $\sqrt{2}\sqrt{2}$

i.e. side = 12 cm

Perimeter of a square = 4 x Side

= 4 x 12 = 48 cm

Question 5.

Find the perimeter of a rectangle whose length = 22.5 m and breadth = 16 dm.

Solution:

Length = 22.5 m

Breadth = 16 dm = 1.6 m

Perimeter of rectangle = 2(l + b)

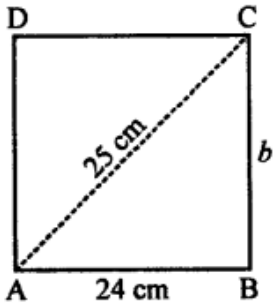
– 2(22.5 + 1.6)
– 2(24.1) = 48.2 m

Question 6.

Find the perimeter of a rectangle with length = 24 cm and diagonal = 25 cm

Solution:

Length of a rectangle (l) = 24 cm Diagonal = 25 cm



Let breadth of the rectangle = b m
Applying Pythagoras Theorem in triangle ABC,
We get, $(AC)^2 = (AB)^2 + (BC)^2$
$(25)^2 = (24)^2 + (b)^2$
$625 = 576 + (b)^2$
$625 – 576 = b^2$
49 = A2
$\sqrt{7x7}$ =b
∴b = 7 cm
Now, perimeter of the rectangle
= 2(1 + b)
= 2(24 + 7)
= 2(31)
= 62 cm

Question 7.

The length and breadth of rectangular piece of land are in the ratio of 5 : 3. If the total cost of fencing it at the rate of ₹48 per metre is ₹19,200, find its length and breadth.

Solution:

Ratio in length and breadth of a rectangular piece of land = 5:3
Cost of fencing =₹ 19,200
and rate = ₹48 per m
∴Perimeter = $\frac{19200}{48}$ = 400 m 48
Let length = 5x.
Then breadth = 3x
∴Perimeter = 2(l + b)
400 = 2(5x + 3x)
400 = 2 x 8x= 16x

∴16x = 400
⇒ x = $\frac{400}{16}$ = 25
∴Length of the land = 5x= 5 x 25 = 125 m and breadth = 3x = 3 x 25 = 75 m

Question 8.
A wire is in the shape of square of side 20 cm. If the wire is bent into a rectangle of length 24 cm, find its breadth.
Solution:
Side of square = 20 cm
Perimeter of square = 4 x 20 = 80 cm
Or perimeter of rectangle = 80 cm
Length of a rectangle = 24 cm
∴ Perimeter of a rectangle = 2(l + b)
b = $\frac{80}{2}$ – 24
b = 40 – 24 = 16 m

Question 9.
If P = perimeter of a rectangle, l= its length and b = its breadth find :
(i) P, if l = 38 cm and b = 27 cm
(ii) b, if P = 88 cm and l = 24 cm
(iii) l, if P = 96 m and b = 28 m
Solution:

(*i*) Length (l) = 38 cm
Breadth (b) = 27 cm
Perimeter of a rectangle = $2(l + b)$
$= 2(38 + 27)$
$= 2(65) = 130$ cm

(*ii*) Perimeter of a rectangle = 88 cm
Length (l) = 24 cm
Let breadth = b
$P = 2(l + b)$

$$b = \frac{P}{2} - l$$

$$b = \frac{88}{2} - 24 \text{ cm} = 44 \text{ cm} - 24 \text{ cm}$$

∴ Breadth of a rectangle = 20 cm

(*iii*) Perimeter of a rectangle = 96 m
Breadth (b) = 28 m
Let length = l
$P = 2(l + b)$

$$l = \frac{P}{2} - b$$

$$= \frac{96}{2} - 28 = 48 - 42 = 20 \text{ m}$$

∴ Length of a rectangle = 20 m

Question 10.

The cost of fencing a square field at the rate of
Cost of fencing 440 m = ₹150 x 440 = ₹75 per meter is
Cost of fencing 440 m = ₹150 x 440 = ₹67,500. Find the perimeter and the side of the square field.

Solution:

Length of the fence × its rate = ₹67,500

$\Rightarrow$ Length of the fence = $₹\frac{67500}{75} = 900$ m

$\therefore$ Perimeter of a square field = length of its fence = 900 m

Since, perimeter of a square = 4 × Length of its side

$\Rightarrow$ Length of the side of the square

$= \frac{\text{Perimeter}}{4} = \frac{900}{4} = 225$ m

Question 11.

The length and the breadth of a rectangle are 36 cm and 28 cm. If its perimeter is equal to the perimeter of a square, find the side of the square.

Solution:

Length of rectangle = 36 cm

Breadth of rectangle = 28 cm

Perimeter of the rectangle = $2(l + b)$

$= 2(36 + 28)$

$= 2(64) = 128$ cm

Given, perimeter of the square = perimeter of rectangle = 128 cm

$\therefore$ Side of the square = $\frac{\text{Perimeter}}{4}$

$= \frac{128}{4} = 32$ cm

Question 12.

The radius of a circle is 21 cm. Find the circumference (Take π = $3\frac{1}{7}$).

Solution:

Given, radius (r) = 21 cm and $\pi = \frac{22}{7}$

Circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 21$ cm

$= 2 \times 22 \times 3$ cm = 132 cm

Question 13.

The circumference of a circle is 440 cm. Find its radius and diameter. (Take π = $\frac{22}{7}$

Solution:

(*i*) Circumference of circle = 440 cm

$\text{Radius} = \frac{C}{2\pi} = \frac{440 \times 7}{2 \times 22}$ cm

$= \frac{3088}{44} = 70$ cm

(*ii*) Diameter = 2 × radius

= 2 × 70 = 140 cm

Question 14.

The diameter of a circular field is 56 m. Find its circumference and cost of fencing it at the rate of ₹80 per m. (Take n = $\frac{22}{7}$)

Solution:

Given, Diameter of a circular field = 56 m

$\therefore$ Radius = $\frac{56}{2} = 28$ m

Circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 28$ m

= 2 × 22 × 4 m = 176 m

Cost of fencing of 176 m is

= 176 m × ₹80 per m = ₹1,40,780

Question 15.

The radii of two circles are 20 cm and 13 cm. Find the difference between their circumferences. (Take π = $\frac{22}{7}$)

Solution:

Radius of 1st circle = 20 cm

Circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 20$

$= \frac{880}{7} = 122.8$ cm

Radius of 2nd circle = 13 cm

Circumference of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 13 = \frac{572}{7} = 81.7$

$\therefore$ Difference of circumference of two circles

$= 122.8 - 81.7$ cm $= 41.1$ cm

Question 16.

The diameter of a circle is 42 cm, find its perimeter. If the perimeter of the circle is doubled, what will be the radius of the new circle. (Take π = $\frac{22}{7}$)

Solution:

Given, Diameter of a circle = 42 cm

$\therefore$ Radius of circle $= \frac{42}{2} = 21$ cm

Perimeter of the circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 21 = 132$ cm

If the perimeter of the circle doubled

$= 2 \times 132 = 264$ cm

$$\text{Radius} = \frac{C}{2\pi} = \frac{264}{2 \times \frac{22}{7}}$$

$$= \frac{264 \times 7}{2 \times 22} = 42 \text{ cm}$$

Question 17.

The perimeter of a square and the circumference of a circle are equal. If the length of each side of the square is 22 cm, find:
(i) perimeter of the square.
(ii) circumference of the circle.
(iii) radius of the circle.

Solution:

(*i*) Side of square = 22 cm

Perimeter of square = 4 × Side

= 4 × 22 = 88 cm

(*ii*) Circumference of circle

Given, Perimeter of square = Circumferene of circle

= 88 cm

(*iii*) Circumference of circle = 88 cm

$$\therefore \text{Radius} = \frac{C}{2\pi} = \frac{88 \times 7}{2 \times 22} = \frac{616}{44} = 14 \text{ cm}$$

Question 18.

Find the radius of the circle whose circumference is equal to the sum of the circumferences of the circles having radii 15 cm and 8 cm.

Solution:

For circle with radius = 15 cm

Circumference of circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times 15 \text{ cm} = \frac{660}{7} \text{cm}$$

For circle with radius = 8 cm

Circumference of circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times 8 \text{ cm} = \frac{352}{7} \text{cm}$$

Sum of the circumferences of these two circles

$$= \frac{660}{7} \text{cm} + \frac{352}{7} \text{cm} = \frac{1012}{7} \text{cm}$$

If the required radius = R cm

Its circumference = $2\pi R$

$$= 2 \times \frac{22}{7} \times R \text{ cm} = \frac{44}{7} R \text{ cm}$$

Given, $\frac{44}{7} R = \frac{1012}{7}$

$$\Rightarrow R = \frac{7}{44} \times \frac{1012}{7} \text{cm} = 23 \text{ cm}$$

$\therefore$ Required radius = 23 cm

Question 19.

Find the diameter of a circle whose circumference is equal to the sum of circumference of circles with radii 10 cm, 12 cm and 18 cm.

Solution:

Let the radius of the circle = R cm

$\therefore$ $2\pi R = 2\pi \times 10 + 2\pi \times 12 + 2\pi \times 18$

On dividing each terms by 2π, we get :

$R = 10 + 12 + 18 = 40$ cm

$\therefore$ Radius of the circle obtained = 40 cm

And, its diameter = 2 × Radius

= 2 × 40 cm = 80 cm

Question 20.

The circumference of a circle is eigth time the circumference of the circle with radius 12 cm. Find its diameter.

Solution:

Radius of the given circle = 12 cm

Circumference of the given circle = $2\pi r$

$= 2 \times \frac{22}{7} \times 12 = \frac{528}{7}$ cm

Circumference of the required circle = 5 ×

$\frac{528}{7} = \frac{2640}{7}$ cm

If the radius of the required circle = R cm

Its circumference = $2\pi R$

$= 2 \times \frac{22}{7} \times R = \frac{44}{7} \times R$ cm

Given : $\frac{44}{7} \times R = \frac{2640}{7}$

$\Rightarrow R = \frac{2640}{7} \times \frac{7}{44} = 60$ cm

$\therefore$ Required radius = 60 cm

Question 21.

The radii of two circles are in the ratio 3 : 5, find the ratio between their circumferences.

Solution:

The ratio of the radii of the circles = 3 : 5

Let radius of the first circle = $3x$

and radius of second circle = $5x$

$\therefore$ Circumference of first circle = $2\pi r$

$= 2\pi \times 3x = 6\pi x$

and circumference of second circle = $2\pi r$

$= 2\pi \times 5x = 10\pi x$

$\therefore$ Ratio between their circumference

$= 6\pi x : 10\pi x$

$= 16 : 10$

$= 3 : 5$

Question 22.

The circumferences of two circles are in the ratio 5 : 7, find the ratio between their radii.

Solution:

The ratio of the circumference of the circle

$= 5 : 7$

Let circumference of first ratio = $5x$

$\therefore\ 2\pi r = 5x \quad \Rightarrow r = \dfrac{5x}{2\pi}$

and the circumference of second ratio = $7x$

$\therefore\ 2\pi r = 7x \quad \Rightarrow r = \dfrac{7x}{2\pi}$

Ratio between their radii = $\dfrac{5x}{2\pi} : \dfrac{7x}{2\pi}$

$= 5 : 7$

Question 23.

The perimeters of two squares are in the ratio 8:15, find the ratio between the lengths of their sides.

Solution:

Let the perimeter of first square = $8x$

$\therefore$ Side of the first square = $\frac{\text{Perimeter}}{4} = \frac{8x}{4}$

and the perimeter of second square = $7x$

$\therefore$ Side of the second square = $\frac{\text{Perimeter}}{4}$

$= \frac{15x}{4}$

Now, the ratio between the sides of the square = $\frac{8x}{4} : \frac{15x}{4}$

= 8 : 15

Question 24.

The lengths of the sides of two squares are in the ratio 8:15, find the ratio between their perimeters.

Solution:

Let the side of first square = 8x

∴Perimeter of first square = 4 x Side = 4 x 8x = 32 x

and the side of second squares = 15x

∴Perimeter of second square = 4 x Side = 4 x 15s = 60s

Now, the ratio between their perimeter = 32x: 60x= 8: 15

Question 25.

Each side of a square is 44 cm. Find its perimeter. If this perimeter is equal to the circumference of a circle, find the radius of the circle.

Solution:

The side of a square = 44 cm

$\therefore$ Its perimeter = 4 × Side

= 4 × 44 = 176 cm

Since, It is given that, Circumference of a circle = Perimeter of a square

$\therefore$ Circumference of a circle = 176 cm

Let, the radius of the circle = r

$\Rightarrow 2\pi r = 176$ cm

$r = \frac{176 \times 7}{2 \times 22} = 28$ cm

$\therefore$ The radius of the circle = 28 cm

EXERCISE 20 (B)

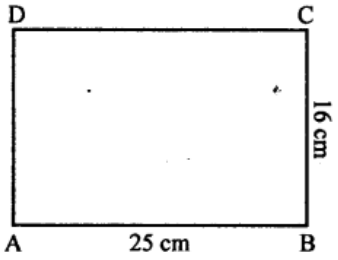
Question 1.

Find the area of a rectangle whose length and breadth are 25 cm and 16 cm.

Solution:

Length of rectangle = 25 cm

Breadth of rectangle = 16 cm



Area of rectangle = $l \times b$ or AB × BC

= 25 × 16 cm^2 = 400 cm^2

∴ Area of rectangle = 400 cm^2

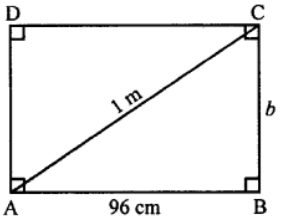
Question 2.

The diagonal of a rectangular board is 1 m and its length is 96 cm. Find the area of the board.

Solution:

Length of diagonal (AB) = 96 cm

Diagonal (AC) = 1 m = 100 cm



In right-angled triangle ABC,

By applying Pythagoras Theorem,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (100)^2 = (96)^2 + BC^2$

$10000 = 9216 + BC^2$

$10000 - 9216 = BC^2$

$\sqrt{784} = BC$

∴ BC = 28 cm

Area of rectangular board

= $l \times b$ or AB × BC

= 96 × 28 = 2688 cm^2

Question 3.

The sides of a rectangular park are in the ratio 4 : 3. If its area is 1728 m^2, find
(i) its perimeter
(ii) cost of fencing it at the rate of ₹40 per meter.

Solution:

Ratio in the sides of a rectangle = 4 : 3

Area = 1728 m^2

Let length = $4x$, and breadth = $3x$

$\therefore$ Area = $l \times b$

$1728 = 4x \times 3x$

$\Rightarrow 12x^2 = 1728$

$$\Rightarrow x^2 = \frac{1728}{12}$$

$\Rightarrow x^2 = 144 = (12)^2$

$\therefore x = 12$

$\therefore$ Length = $4x = 4 \times 12 = 48$ m

Breadth = $3x = 3 \times 12 = 36$ m

(*i*) Now perimeter = $2(l + b)$

$= 2(48 + 36)$ m

$= 2 \times 84 = 168$ m

(*ii*) Rate of fencing = ₹40 per metre

Total cost = 168×40 = ₹6720

Question 4.

A floor is 40 m long and 15 m broad. It is covered with tiles, each measuring 60 cm by 50 cm. Find the number of tiles required to cover the floor.

Solution:

Length of floor (l) = 40 m

Breadth of floor (b) = 15 m

$\therefore$ Area of floor = $l \times b = 40 \times 15 = 600$ m^2

Length of one tile = 60 cm = $\frac{6}{10}$ m

and breadth = 50 cm = $\frac{5}{10}$ m

$\therefore$ Area of one tile = $\frac{6}{10} \times \frac{5}{10}$

$= \frac{30}{100} = \frac{3}{10}$ m^2

$\therefore$ Number of tiles = $\frac{\text{Total area of floor}}{\text{Area of one tile}}$

$= \frac{600}{\frac{3}{10}} = \frac{600 \times 10}{3} = 2000$

Question 5.

The length and breadth of a rectangular piece of land are in the ratio 5 : 3. If the total cost of fencing it at the rate of ₹24 per meter is ₹9600, find its :

(i) length and breadth

(ii) area

(iii) cost of levelling at the rate of ₹60 per m^2.

Solution:

Ratio in length and breadth of a rectangular piece of land = 5 : 3

Cost of fencing = ₹9600

and rate = ₹24 per m

$$\text{Perimeter} = \frac{\text{Total cost of fencing}}{\text{Rate per m}} = \frac{9600}{24} = 400 \text{ m}$$

Let length = $5x$

Then breadth = $3x$

$\therefore$ Perimeter = $2(l + b)$

$400 = 2 \times (5x + 3x)$

$\therefore$ $16x = 400$

$$x = \frac{400}{16} = 25$$

(*i*) $\therefore$ Length of land = $5x = 5 \times 25 = 125$ m

and breadth = $3x = 3 \times 25 = 75$ m

(*ii*) Area = $l \times b$

$= 125 \times 75 = 9375 \text{ m}^2$

(*iii*) Cost of levelling at rate ₹60 per m^2

$= ₹60 \times 9375 \text{ m}^2 = ₹5,62,500$

Question 6.

Find the area of the square whose perimeter is 56 cm.

Solution:

Perimeter of square = 56 cm

$\Rightarrow$ $4 \times$ side = 56 cm

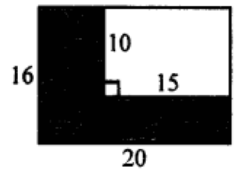
$$\Rightarrow \text{Side} = \frac{56}{4} \text{ cm}$$

$\Rightarrow$ Side = 14 cm

$\therefore$ Area of square = $(\text{Side})^2 = (14)^2$

$= 14 \times 14 \text{ cm}^2 = 196 \text{ cm}^2$

(i)



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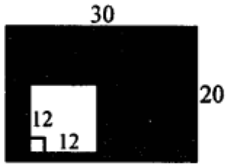

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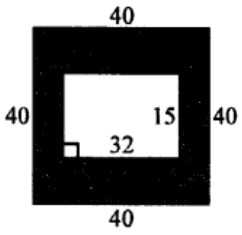

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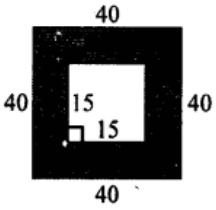

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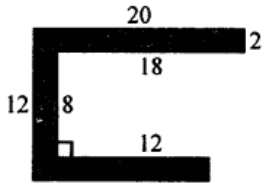


Question 7.

A square lawn is surrounded by a path 2.5 m wide. If the area of the path is 165 m^2 find the area of the lawn.

Solution:

Area of path = 165 m^2

Width of path = 2.5 m

Let side of square lawn = x m

$\therefore$ Outer side = $x + 2 \times 2.5$

$= (x + 5)$ m

$\therefore$ Area of path = $(x + 5)^2 - x^2$

$\Rightarrow x^2 + 10x + 25 - x^2 = 165$

$\Rightarrow 10x = 165 - 25 = 140$

$\Rightarrow x = \frac{140}{10} = 14$ m

$\therefore$ Side of lawn = 14 m

and area of lawn = $(14)^2$ m^2 = 196 m^2

Question 8.

For each figure, given below, find the area of shaded region : (All measurements are in cm)

Solution:

(*i*) Outer length = 20 cm

and breadth = 16 cm

$\therefore$ Outer area = $l \times b$

$= 20 \times 16\ cm^2 = 320\ cm^2$

Inner length = 15 cm

and Inner breadth = 10 cm

$\therefore$ Inner area = $15 \times 10 = 150\ cm^2$

$\therefore$ Area of shaded region = Area of whole region – Area of unshaded region

$= 320 - 150\ cm^2 = 170\ cm^2$

(*ii*) Outer length = 30 cm

and Outer breadth = 20 cm

$\therefore$ Outer area = $l \times b$

$= 30 \times 20 = 600\ cm^2$

Inner length = 12 cm and inner breadth = 12 cm

Inner area = $l \times b = 12 \times 12 = 144\ cm^2$

Area of shaded portion = Area of outer figure – Area a of inner figure

$= 600 - 144 = 456\ cm^2$

(*iii*) Area of shaded portion = Area of outer region – Area of unshaded region

$= 40 \times 40 - 32 \times 15$

$= 1600 - 480\ cm^2 = 1120\ cm^2$

(*iv*) Area of shaded region = Area of outer region – Area of inner region

$= 40 \times 40 - 15 \times 15$

$= 1600 - 225 = 1375\ cm^2$

(*v*) Area of shaded portion

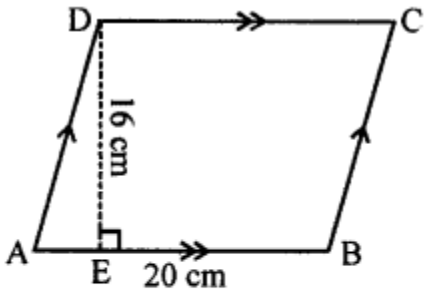
$= 2 \times 20 + 2 \times 8 + 2 \times (12 + 2)$

$= 40 + 16 + 28\ cm^2 = 84\ cm^2$

Question 9.

One side of a parallelogram is 20 cm and its distance from the opposite side is 16 cm. Find the area of the parallelogram.

Solution:



Area of parallelogram = Base × Height

= AB × DE

= 20 × 16 cm = 320 cm^2

∴ Area of parallelogram = 320 cm^2

Question 10.

The base of a parallelogram is thrice it height. If its area is 768 cm^2, find the base and the height of the parallelogram.

Solution:

Area of the parallelogram = 768 cm^2

Let the height of the parallelogram = x

Then base = $3x$

∴ Area = Base × Height

$\Rightarrow 768 = 3x \times x$

$\Rightarrow 768 = 3x^2$

$\Rightarrow x^2 = \frac{768}{3} = 256$ cm

$\therefore x = \sqrt{16 \times 16} = 16$ cm

∴ Base = 3 × 16 = 48 cm

and height = x = 16 cm

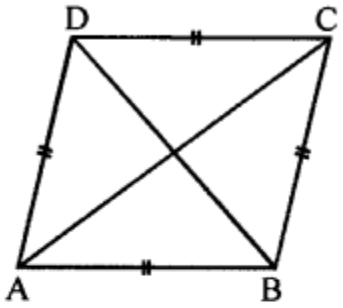
Question 11.

Find the area of the rhombus, if its diagonals are 30 cm and 24 cm.

Solution:

Given, diagonal $(d_1) = 30$ cm

Other diagonal $(d_2) = 24$ cm



If AC and BD are the diagonals of a rhombus its

$$\text{Area} = \frac{1}{2} \times \text{Product of it diagonals}$$

$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 30 \times 24 \text{ cm}^2$$

$$= 15 \times 24 = 360 \text{ cm}^2$$

$\therefore$ Area of rhombus = 360 cm^2

Question 12.

If the area of a rhombus is 112 cm^2 and one of its diagonals is 14 cm, find its other diagonal.

Solution:

Area of rhombus = 112 cm^2

One diagonal = 14 cm

Let second diagonal = x

Then, area = $\frac{\text{Product of diagonal}}{2}$

$\Rightarrow 112 = \frac{14 \times x}{2}$

$\Rightarrow x = \frac{112 \times 2}{14} = \frac{224}{14}$

$\Rightarrow x = 16$

$\therefore$ Second diagonal = 16 cm

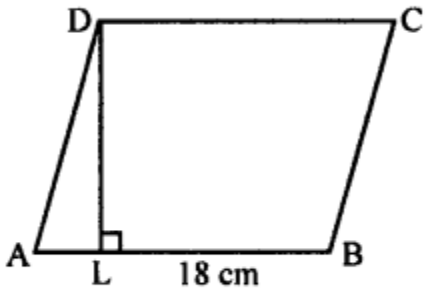
Question 13.

One side of a parallelogram is 18 cm and its area is 153 cm^2. Find the distance of the given side from its opposite side.

Solution:

Area of parallelogram ABCD = 153 cm^2

Side (Base) AB = 18 cm



$\therefore$ Distance DL between AB and DC (altitude)

$= \frac{\text{Area}}{\text{Base}} = \frac{153}{18} = \frac{17}{2}\text{cm} = 8.5 \text{ cm}$

Question 14.

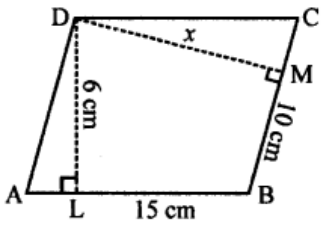
The adjacent sides of a parallelogram are 15 cm and 10 cm. If the distance between the longer sides is 6 cm, find the distance between the shorter sides.

Solution:

In parallelogram ABCD

AB = DC = 15 cm

BC = AD = 10 cm



Distance between longer sides AB and DC is 6 cm

i.e., perpendicular DL = 6 cm

DM ⊥ BC

Area of parallelogram = Base × Altitude

= AB × DL = 15 × 6 = 90 cm^2

Again let DM = x cm

$\therefore$ Area of parallelogram ABCD = BC × DM

$= 10 \times x = 10x \text{ cm}^2$

$\therefore 10x \text{ cm}^2 = 90 \text{ cm}^2$

$$\Rightarrow x = \frac{90}{10} = 9 \text{ cm}$$

Question 15.

The area of a rhombus is 84 cm2 and its perimeter is 56 cm. Find its height.

Solution:

$\therefore$ Area of rhombus = 84 cm^2

Perimeter = 56 cm

$$\therefore \text{Its side} = \frac{56}{4} = 14 \text{ cm}$$

$$\therefore \text{Height} = \frac{\text{Area of rhombus}}{\text{Base}} = \frac{84}{14} = 6 \text{ cm}$$

Question 16.

Find the area of a triangle whose base is 30 cm and height is 18 cm.

Solution:

Base of triangle = 30 cm

Height of triangle = 18 cm

$$\therefore \text{Area} = \frac{1}{2} \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 30 \times 18 = 270 \text{ cm}^2$$

Question 17.

Find the height of a triangle whose base is 18 cm and area is 270 cm^2.

Solution:

Base of triangle = 18 cm

Area of triangle = 270 cm^2

$$\therefore \text{Height} = \frac{\text{Area} \times 2}{\text{Base}}$$

$$= \frac{270 \times 2}{18} = \frac{540}{18} = 30 \text{ cm}$$

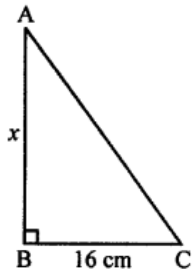
Question 18.

The area of a right-angled triangle is 160 cm^2. If its one leg is 16 cm long, find the length of the other leg.

Solution:

Area of the right angled triangle = 160 cm^2

Let base (one side) = 16 cm



$\therefore$ Altitude (second side)

$$= \frac{\text{Area} \times 2}{\text{Base}} = \frac{160 \times 2}{16} = \frac{320}{16} = 20 \text{ cm}$$

Question 19.

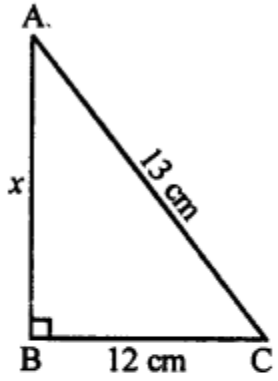
Find the area of a right-angled triangle whose hypotenuse is 13 cm long and one of its legs is 12 cm long.

Solution:

In right angled ΔABC,

Base BC = 12 cm

and hypotenuse AC = 13 cm



Applying Pythagoras Theorem,

$(AC)^2 = (AB)^2 + (BC)^2$

$(13)^2 = (AB)^2 + (12)^2$

$169 = (AB)^2 + 144$

$(AB)^2 = 169 - 144$

$(AB)^2 = 25$

$\therefore AB = \sqrt{25}$

$= \sqrt{5 \times 5} = 5$ cm

Now, area of $\Delta ABC = \frac{1}{2}$ base $\times$ altitude

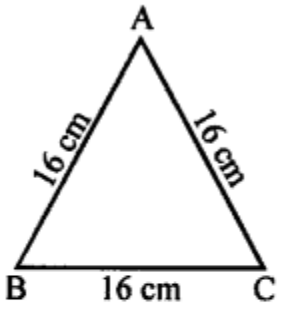
$= \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$

Question 20.

Find the area of an equilateral triangle whose each side is 16 cm. (Take $\sqrt{3}$= 1.73)

Solution:

Side of the equilateral triangle = 16 cm



$$\therefore \text{ Area} = \frac{\sqrt{3}}{4}(a)^2$$

$$= \frac{\sqrt{3}}{4} \times 16 \times 16$$

$$= 1.73 \times 4 \times 16 = 110.72 \text{ cm}^2$$

Question 21.

The sides of a triangle are 21 cm, 17 cm and 10 cm. Find its area.

Solution:

Let a = 21 cm, b = 17 cm and c = 10 cm

$\therefore$ $a + b + c$

= 21 cm + 17 cm + 10 cm = 48 cm

$$s = \frac{a+b+c}{2} = \frac{48}{2} = 24 \text{ cm}$$

Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{24(24-20)(24-17)(24-10)}$$

$$= \sqrt{24 \times 3 \times 7 \times 14}$$

$$= \sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 2 \times 7}$$

$$= 2 \times 2 \times 3 \times 7 = \text{cm}^2 = 84 \text{ cm}^2$$

Question 22.

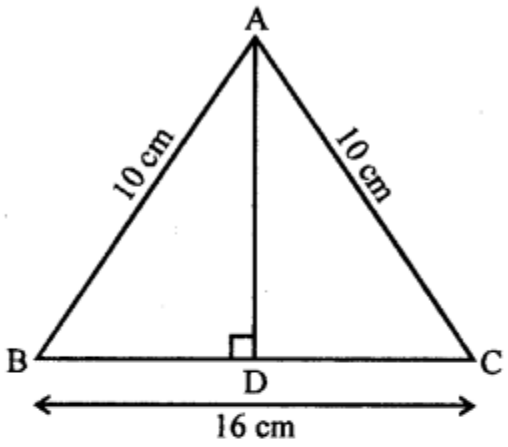
Find the area of an isosceles triangle whose base is 16 cm and length of each of the equal sides is 10 cm.

Solution:

In isosceles ΔABC

Base $BC = 16$ cm

and $AB = AC = 10$ cm



Let $AD \perp BC$ and $BD = \frac{1}{2}BC = \frac{16}{2}$

$\therefore$ $BD = 8$ cm

In right ΔABD

$AB^2 = AD^2 + BD^2$ (Pythagoras Theorem)

$(10)^2 = AD^2 + (8)^2$

$100 = AD^2 + 64$

$100 - 64 = AD^2$

$36 = AD^2$

$AD = \sqrt{36} = \sqrt{6 \times 6}$

$\therefore$ $AD = 6$ cm

Now, area of triangle $= \frac{\text{Base} \times \text{Altitude}}{2}$

$= \frac{16 \times 6}{2} = 48 \text{ cm}^2$

Question 23.

Find the base of a triangle whose area is 360 cm^2 and height is 24 cm.

Solution:

Area of triangle = 360 cm^2

and height (h) = 24 cm

$$\therefore \text{Base} = \frac{\text{Area} \times 2}{\text{Height}}$$

$$= \frac{360 \times 2}{24} = \frac{720}{24} = 30 \text{ cm}$$

Question 24.

The legs of a right-angled triangle are in the ratio 4 :3 and its area is 4056 cm^2. Find the length of its legs.

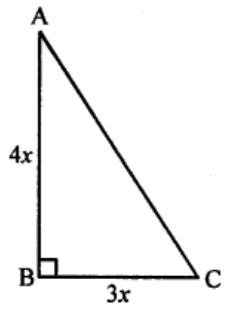
Solution:

Area of right angled triangle = 4056 cm^2

Legs of a right angled triangle are in the ratio *i.e.* 4 : 3

Let one leg (base) = $3x$

Then second leg (altitude) = $4x$



$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Altitude}$$

$$= \frac{1}{2} \times 3x \times 4x = 6x^2$$

$$\therefore 6x^2 = 4056$$

$$x^2 = \frac{4056}{6} = 676$$

$$x = \sqrt{676} = \sqrt{26 \times 26}$$

$$\therefore x = 26 \text{ cm}$$

$$\therefore \text{One leg (base)} = 3x = 3 \times 26 = 78 \text{ cm}$$

and second leg (altitude) $4x = 4 \times 26$

$= 104$ cm

Question 25.

The area of an equilateral triangle is (64 x $\sqrt{3}$) cm^2– Find the length of each side of the triangle.

Solution:

Area of equilateral triangle = $64\sqrt{3}$ cm^2

Let each side = a

Then, $\frac{\sqrt{3}}{4}a^2 = 64\sqrt{3}$

$a^2 = \frac{64\sqrt{3}\times 4}{\sqrt{3}} = 256$

$a = (16)^2$

$\therefore$ a = 16 cm

$\therefore$ Each side = 16 cm

Question 26.

The sides of a triangle are in the ratio 15 : 13 : 14 and its perimeter is 168 cm. Find the area of the triangle.

Solution:

Perimeter of the triangle = 168 cm

Sum of ratios of sides = 15 + 13 + 14 = 42

Let the first side = $\frac{168\times 15}{42}$ = 60 cm

Second side = $\frac{168\times 13}{42}$ = 52 cm

Third side = $\frac{168\times 14}{42}$ = 56 cm

Now, $s = \frac{a+b+c}{2}$

$= \frac{60+52+56}{2} = \frac{168}{2} = 84$

$\therefore$ Area = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{84(84-60)(84-52)(84-56)}$

$= \sqrt{84\times 24\times 32\times 28}$

$= \sqrt{\begin{matrix}2\times 2\times 3\times 7\times 2\times 2\times 2\times 3\times 2\\ \times 2\times 2\times 2\times 2\times 2\times 7\end{matrix}}$

$= 2\times 2\times 2\times 2\times 2\times 2\times 3\times 7 = 1344$ cm^2

Question 27.

The diameter of a circle is 20 cm. Taking π = 3.14, find the circumference and its area.

Solution:

Diameter of circle $(d) = 20$ cm

$\therefore$ Circumference $= d\pi = 20 \times 3.14 = 62.8$ cm

$$\text{Radius } (r) = \frac{d}{2} = 10 \text{ cm}$$

$\therefore$ Area of a circle $= \pi r^2$

$= 3.14 \times 10 \times 10 = 314 \text{ cm}^2$

Question 28.

The circumference of a circle exceeds its diameter by 18 cm. Find the radius of the circle.

Solution:

Let c be the circumference and d be the diameter of the circle.

$\therefore c = d + 18$

$\Rightarrow d\pi = d + 18 \Rightarrow d\pi - d = 18$

$d(\pi - 1) = 18$

$$\Rightarrow d\left(\frac{22}{7} - 1\right) = 18$$

$$\Rightarrow d\left(\frac{15}{7}\right) = 18$$

$$\Rightarrow d = \frac{18 \times 7}{15} = \frac{126}{15} = 8.4 \text{ cm}$$

$$\therefore \text{Radius} = \frac{d}{2} = \frac{8.4}{2} = 4.2 \text{ cm}$$

Question 29.

The ratio between the radii of two circles is 5 : 7. Find the ratio between their :

(i) circumference

(ii) areas

Solution:

(*i*) The ratio of the radii of the circles

$= 5 : 7$

Let radius of first circle $= 5x$

and radius of second circle $= 7x$

$\therefore$ Circumference of first circle $= 2\pi r$

$= 2\pi \times 5x = 10\pi x$

and circumference of second circle

$= 2\pi \times 7x = 14\pi x$

$\therefore$ Ratio between their circumference

$= 10\pi x : 14\pi x$

$= 10 : 14 = 5 : 7$

(*ii*) Area of first circle $= \pi r^2$

$$= \frac{22}{7} \times 5x \times 5x = \frac{550}{7}x^2$$

and area of second circle $= \pi r_2^2$

$$= \frac{22}{7} \times 7x \times 7x = \frac{1078}{7}x^2$$

Ratio between their areas

$$= \frac{550}{7}x^2 : \frac{1078}{7}x^2$$

$= 550 : 1078$ (Dividing by 22)

$= 25 : 49$

Question 30.

The ratio between the areas of two circles is 16 : 9. Find the ratio between their :

(i) radii

(ii) diameters

(iii) circumference

Solution:

(*i*) Let the radius of first circle = r_1
and radius of second circle = r_2
Given that ratio of the areas of circles
= 16 : 9

$$\Rightarrow \frac{\pi r_1^2}{\pi r_2^2} = \frac{16}{9}$$

$$\Rightarrow \frac{\pi r_1^2}{\pi r_2^2} = \frac{4^2}{3^2}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$$

(*ii*) Let the diameter of first circle = d_1
and diameter of second circle = d_2
Since, we know that diameter = 2 × radius
$\therefore$ $d_1 = 2 \times r_1 = 2 \times 4x = 8x$
and $d_2 = 2 \times r_2 = 2 \times 3x = 6x$
Now, the ratio between the diameter of two circles = $d_1 : d_2$
= $8x : 6x = 4 : 3$

(*iii*) Now, consider the ratio of circumference of the circles

$$= \frac{2\pi r_1}{2\pi r_2} = \frac{r_1}{r_2} = \frac{4}{3}$$

$\therefore$ The ratio between the circumference of two circles = 4 : 3

Question 31.

A circular racing track has inner circumference 528 m and outer circumference 616 m. Find the width of the track.

Solution:

Outer circumference = 616 m

$$\text{Radius (R)} = \frac{C}{2\pi} = \frac{616 \times 7}{2 \times 22}\text{ m} = 98\text{ m}$$

Inner circumference = 528 m

$$\therefore \text{ Inner radius } (r) = \frac{528 \times 7}{2 \times 22}\text{ m} = 84\text{ m}$$

$\therefore$ Width of track = R – r

= 98 – 84 = 14 m

Question 32.

The inner circumference of a circular track is 264 m and the width of the track is 7 m. Find:

(i) the radius of the inner track.

(ii) the radius of the outer circumference.

(iii) the length of the outer circumference.

(iv) the cost of fencing the outer circumference at the rate of ₹50 per m.

Solution:

Inner circumference of the circular track

= 264 m

(*i*) $\therefore$ Inner radius $(r) = \frac{C}{2\pi}$

$$= \frac{264 \times 7}{2 \times 22} = \frac{1848}{44} = 42\text{ cm}$$

(*ii*) Width of the track = 7 m

$\therefore$ Outer radius (R) = 42 + 7 = 49 m

(*iii*) Outer circumference = 2πR

$$= 2 \times \frac{22}{7} \times 49 = 308\text{ m}$$

(*iv*) Rate of fencing = ₹50 per metre

$\therefore$ Total cost of fencing outer circumference

= ₹50 × 308 = ₹15,400

Question 33.

The diameter of every wheel of a car is 63 cm. How much distance will the car move during 2000 revolutions of its wheel.

Solution:

Diameter of car wheel $(d) = 63$ cm

$\therefore$ Circumference $= \pi d = \frac{22}{7} \times 63 = 198$ cm

Distance covered in 2000 revolutions

$= 2000 \times 198$ cm

$= \frac{2000 \times 198}{100} = 3960$ m $= 3.96$ km

Question 34.

The diameter of the wheel of a car is 70 cm. How many revolutions will it make to travel one kilometre?

Solution:

Diameter of car wheel $(d) = 70$ cm

$\therefore$ Circumference $= \pi d = \frac{22}{7} \times 70$ cm

$= 220$ cm $= \frac{220}{100}$ m

No. of revolutions in 1 km

$= 1$ km $\div \frac{220}{100}$ m

$= 1 \times 1000 \times \frac{100}{220}$ m $= \frac{5000}{11} = 454\frac{6}{11}$ km

Question 35.

A metal wire, when bent in the form of a square of largest area, encloses an area of 484 cm^2. Find the length of the wire. If the same wire is bent to a largest circle, find:

(i) radius of the circle formed.

(ii) area of the circle.

Solution:

Area of the square made wire $= 484$ cm^2

$\therefore$ Length (side) $= \sqrt{\text{Area}} = \sqrt{484} = 22$ cm

(i) Perimeter of wire $= 4 \times$ Side

$= 4 \times 22 = 88$ cm

$\therefore$ Circumference of circular wire = 88 cm

$\therefore$ Radius $(r) = \frac{C}{2\pi} = \frac{88 \times 7}{2 \times 22}$ cm = 14 cm

(*ii*) $\therefore$ Area of the circle = πr^2

$= \frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2$

Question 36.

A wire is along the boundary of a circle with radius 28 cm. If the same wire is bent in the form of a square, find the area of the square formed.

Solution:

Radius of circular wire = 28 cm

$\therefore$ Circumference = $2\pi r$

$= 2 \times \frac{22}{7} \times 28 \text{ cm} = 176 \text{ cm}$

$\therefore$ Perimeter of the square formed by this wire = 176 cm

$\therefore$ Side $(a) = \frac{176}{4} = 44$ cm

Area of square so formed = $a^2 = (44)^2 \text{ cm}^2$

$= 1936 \text{ cm}^2$

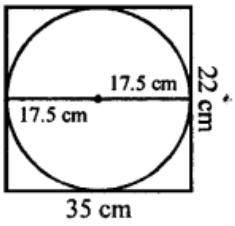
Question 37.

The length and the breadth of a rectangular paper are 35 cm and 22 cm. Find the area of the largest circle which can be cut out of this paper.

Solution:

Length of rectangular paper (l) = 35 cm

Breadth of rectangular paper (b) = 22 cm



$\therefore$ Area = $35 \times 22 = 770 \text{ cm}^2$

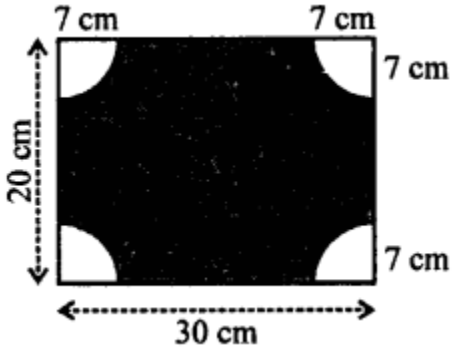
The largest circle which can be cut from the rectangular paper will have radius of 17.5 cm

$\therefore$ Area of a circle = πr^2

$$= \frac{22}{7} \times 17.5 \times 17.5 = 962.50 \text{ cm}^2$$

Question 38.

From each comer of a rectangular paper (30 cm x 20 cm) a quadrant of a circle of radius 7 cm is cut. Find the area of the remaining paper i.e., shaded portion.



Solution:

Length of paper (l) = 30 cm

and breadth (b) = 20 cm

$\therefore$ Area of rectangular paper = $l \times b$

$= 30 \times 20 = 600$ cm

Radius of each quadrant at the corner

= 7 cm

Area of 4 quadrants = $4 \times \frac{1}{4}\pi r^2$

$= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$

$\therefore$ Area of remaining paper

$= 600 - 154 = 446 \text{ cm}^2$