Mathematics

(Chapter – 2) (Inverse Trigonometric Functions) (Exercise 2.1)
(Class – XII)

Question 1:

Find the principal value of $sin^{-1}\left(-\frac{1}{2}\right)$

Answer 1:

Let $sin^{-1}\left(-\frac{1}{2}\right) = y$, then $sin\ y = -\frac{1}{2} = -sin\left(\frac{\pi}{6}\right) = sin\left(-\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$

Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}$.

Question 2:

Find the principal value of $cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Answer 2:

Let $cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$, then $\cos y = \frac{\sqrt{3}}{2} = cos\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Therefore, the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

Question 3:

Find the principal value of $\text{cosec}^{-1}(2)$.

Answer 3:

Let $\text{cosec}^{-1}(2) = y$. then, $\text{cosec}\ y = 2 = \text{cosec}\left(\frac{\pi}{6}\right)$

The range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ -{0} and $\text{cosec}\left(\frac{\pi}{6}\right) = 2$.

Therefore, the principal value of $\text{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

Question 4:

Find the principal value of $tan^{-1}(-\sqrt{3})$.

Answer 4:

Let $\tan^{-1}(-\sqrt{3}) = y$, then $\tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$

Therefore, the principal value of $\tan^{-1}(-\sqrt{3})$ is $-\frac{\pi}{3}$.

Question 5:

Find the principal value of $cos^{-1}\left(-\frac{1}{2}\right)$.

Answer 5:

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$, then $\cos y = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

Question 6:

Find the principal value of $\tan^{-1}(-1)$.

Answer 6:

Let $\tan^{-1}(-1) = y$.

Then, $\tan y = -1 = -\tan\left(\frac{\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right) = -1$

Therefore, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$.

Question 7:

Find the principal value of $sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$.

Answer 7:

Let $sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$, then $\sec y = \frac{2}{\sqrt{3}} = sec\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ and $sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$.

Therefore, the principal value of $sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

Question 8:

Find the principal value of $cot^{-1}\sqrt{3}$.

Answer 8:

Let $\cot^{-1}\sqrt{3} = y$, then $\cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$.

We know that the range of the principal value branch of $\cot^{-1}$ is $(0, \pi)$ and $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$.

Therefore, the principal value of $\cot^{-1}\sqrt{3}$ is $\frac{\pi}{6}$.

Question 9:

Find the principal value of $cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

Answer 9:

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$, then $\cos y = -\frac{1}{\sqrt{2}} = -\cos\left(\frac{\pi}{4}\right) = \cos\left(\pi - \frac{\pi}{4}\right) = \cos\left(\frac{3\pi}{4}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Question 10:

Find the principal value of $cosec^{-1}(-\sqrt{2})$.

Answer 10:

Let $cosec^{-1}(-\sqrt{2}) = y$, then $cosec\ y = -\sqrt{2} = -cosec\left(\frac{\pi}{4}\right) = cosec\left(-\frac{\pi}{4}\right)$

We know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$-{0} and $\text{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}.$

Therefore, the principal value of $\text{cosec}^{-1}(-\sqrt{2})$ is $-\frac{\pi}{4}$.

Question 11:

Find the value of $tan^{-1}(1) + cos^{-1}\left(-\frac{1}{2}\right) + sin^{-1}\left(-\frac{1}{2}\right)$.

Answer 11:

Let $tan^{-1}(1) = x$, then $\tan x = 1 = tan\frac{\pi}{4}$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

$\therefore tan^{-1}(1) = \frac{\pi}{4}$

Let $cos^{-1}\left(-\frac{1}{2}\right) = y$, then

$$\cos y = -\frac{1}{2} = -cos\frac{\pi}{3} = cos\left(\pi - \frac{\pi}{3}\right) = cos\left(\frac{2\pi}{3}\right)$$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

$\therefore cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let $sin^{-1}\left(-\frac{1}{2}\right) = z$, then

$$\sin z = -\frac{1}{2} = -sin\frac{\pi}{6} = sin\left(-\frac{\pi}{6}\right)$$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Now,

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

Question 12:

Find the value of $cos^{-1}\left(\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right)$

Answer 12:

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$, then $\cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = y$, then $\sin y = \frac{1}{2} = \sin\frac{\pi}{6}$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\therefore sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Now, $cos^{-1}\left(\frac{1}{2}\right) + 2sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2 \times \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$.

Question 13:

If $\sin^{-1} x = y$, then

(A) $0 \le y \le \pi$ (B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$ (C) $0 < y < \pi$ (D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Answer 13:

It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Hence, the option (B) is correct.

Question 14:

$tan^{-1}\sqrt{3} - sec^{-1}(-2)$ is equal to

(A) π (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Answer 14:

Let $tan^{-1}\sqrt{3} = x$, then $\tan x = \sqrt{3} = tan\frac{\pi}{3}$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

Let $\sec^{-1}(-2) = y$, then $\sec y = -2 = -\sec\frac{\pi}{3} = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\left(\frac{2\pi}{3}\right)$

We know that the range of the principal value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\therefore \sec^{-1}(-2) = \frac{2\pi}{3}$

Now, $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

Hence, the option (B) is correct.

Mathematics

(Chapter – 2) (Inverse Trigonometric Functions) (Exercise 2.2)
(Class – XII)

Question 1:

Prove that $3sin^{-1}x = sin^{-1}(3x - 4x^3),\ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

Answer 1:

Let $sin^{-1}x = \theta$, then $x = \sin\theta$. We have,

RHS $= sin^{-1}(3x - 4x^3) = sin^{-1}(3\sin\theta - 4sin^3\theta)$

$= sin^{-1}(\sin 3\theta) = 3\theta = 3sin^{-1}x =$ LHS

Question 2:

Prove that $3cos^{-1}x = cos^{-1}(4x^3 - 3x),\ x \in \left[\frac{1}{2}, 1\right]$.

Answer 2:

Let $cos^{-1}x = \theta$, then $x = \cos\theta$. We have,

RHS $= cos^{-1}(4x^3 - 3x) = cos^{-1}(4cos^3\theta - 3cos\theta)$

$= cos^{-1}(\cos 3\theta) = 3\theta = 3cos^{-1}x =$ LHS

Question 3:

Write the function $tan^{-1}\frac{\sqrt{1+x^2}-1}{x}, x \neq 0$, in the simplest form.

Answer 3:

Given function $tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

Let $x = \tan\theta$

$$\therefore tan^{-1}\frac{\sqrt{1+x^2}-1}{x} = tan^{-1}\frac{\sqrt{1+tan^2\theta}-1}{\tan\theta}$$

$$= tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= tan^{-1}\left(\frac{2sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}cos\frac{\theta}{2}}\right) = tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2} = \frac{1}{2}tan^{-1}x$$

Question 4:

Write the function $tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x < \pi,$ in the simplest form.

Answer 4:

The given function is $tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$, Now,

$$tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = tan^{-1}\left(\sqrt{\frac{2sin^2\frac{x}{2}}{2cos^2\frac{x}{2}}}\right)$$

$$= tan^{-1}\left(\sqrt{tan^2\frac{x}{2}}\right) == tan^{-1}\left(tan\frac{x}{2}\right) = \frac{x}{2}$$

Question 5:

Write the function $tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right), -\frac{\pi}{4} < x < \frac{3\pi}{4}$, in the simplest form.

Answer 5:

The given function is $tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right)$

Now,

$$tan^{-1}\left(\frac{\cos x-\sin x}{\cos x+\sin x}\right) = tan^{-1}\left(\frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\right) = tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$$

$$= tan^{-1}\left(\frac{1-\tan x}{1+1.\tan x}\right) = tan^{-1}\left(\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}.\tan x}\right)$$

$$= tan^{-1}\left[tan\left(\frac{\pi}{4}-x\right)\right] = \frac{\pi}{4}-x$$

Question 6:

Write the function $tan^{-1}\frac{x}{\sqrt{a^2-x^2}}, |x| < a$, in the simplest form.

Answer 6:

The given function is $tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$.

Let $x = \text{a}\sin\theta$

$$\therefore tan^{-1}\frac{x}{\sqrt{a^2-x^2}} = tan^{-1}\left(\frac{\text{a}\sin\theta}{\sqrt{a^2-a^2sin^2\theta}}\right) = tan^{-1}\left(\frac{\text{a}\sin\theta}{a\sqrt{1-sin^2\theta}}\right)$$

$$= tan^{-1}\left(\frac{\text{a}\sin\theta}{\text{a}\sin\theta}\right) = tan^{-1}(\tan\theta) = \theta = sin^{-1}\frac{x}{a}$$

Question 7:

Write the function in $tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right), a > 0; \frac{-a}{\sqrt{3}} \leq x\frac{a}{\sqrt{3}}$, the simplest form.

Answer 7:

The given function is $tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$

Let $x = a\tan\theta$

$$\therefore tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = tan^{-1}\left(\frac{3a^2.a\tan\theta - a^3tan^3\theta}{a^3 - 3a.a^2tan^2\theta}\right)$$

$$= tan^{-1}\left(\frac{3a^3tan\theta - a^3tan^3\theta}{a^3 - 3a^3tan^2\theta}\right)$$

$$= tan^{-1}\left(\frac{3tan\theta - tan^3\theta}{1 - 3tan^2\theta}\right)$$

$$= tan^{-1}(tan3\theta) = 3\theta$$

$$= 3tan^{-1}\frac{x}{a}$$

Question 8:

Find the value of $tan^{-1}\left[2cos\left(2sin^{-1}\frac{1}{2}\right)\right]$

Answer 8:

The given function is $tan^{-1}\left[2cos\left(2sin^{-1}\frac{1}{2}\right)\right]$

$\therefore tan^{-1}\left[2cos\left(2sin^{-1}\frac{1}{2}\right)\right] = tan^{-1}\left[2cos\left(2sin^{-1}\left(sin\frac{\pi}{6}\right)\right)\right]$

$= tan^{-1}\left[2cos\left(2\times\frac{\pi}{6}\right)\right] = tan^{-1}\left[2cos\left(\frac{\pi}{3}\right)\right] = tan^{-1}\left[2\times\frac{1}{2}\right] = tan^{-1}[1] = \frac{\pi}{4}$

Question 9:

Find the value of $tan\frac{1}{2}\left[sin^{-1}\frac{2x}{1+x^2} + cos^{-1}\frac{1-y^2}{1+y^2}\right], |x| < 1, y > 0$ and $xy < 1$.

Answer 9:

The given function is $tan\frac{1}{2}\left[sin^{-1}\frac{2x}{1+x^2} + cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$\therefore tan\frac{1}{2}\left[sin^{-1}\frac{2x}{1+x^2} + cos^{-1}\frac{1-y^2}{1+y^2}\right]$

$= tan\frac{1}{2}[2tan^{-1}x + 2tan^{-1}y]$ $\quad$ [as $2tan^{-1}x = sin^{-1}\frac{2x}{1+x^2} = cos^{-1}\frac{1-x^2}{1+x^2}$]

$= tan\frac{1}{2}[2(tan^{-1}x + tan^{-1}y)] = tan[tan^{-1}x + tan^{-1}y] = tan\left[tan^{-1}\frac{x+y}{1-xy}\right] = \frac{x+y}{1-xy}$

Question 10:

Find the values of $sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

Answer 10:

Given that $sin^{-1}\left(\sin\frac{2\pi}{3}\right)$.

We know that $\sin^{-1}(\sin x) = x$ if $x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$, which is the principal value branch of $\sin^{-1}x$.

$\therefore sin^{-1}\left(\sin\frac{2\pi}{3}\right) = sin^{-1}\left(\sin\left\{\pi - \frac{\pi}{3}\right\}\right) = sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3} \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Hence, $sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \frac{\pi}{3}$

Question 11:

Find the values of $tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.

Answer 11:

Given that $tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

We know that $\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

$\therefore tan^{-1}\left(\tan\frac{3\pi}{4}\right) = tan^{-1}\left(\tan\left\{\pi - \frac{\pi}{4}\right\}\right) = tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

$= tan^{-1}\left(\tan\left\{-\frac{\pi}{4}\right\}\right) = -\frac{\pi}{4} \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Hence, $tan^{-1}\left(\tan\frac{3\pi}{4}\right) = -\frac{\pi}{4}$

Question 12:

Find the values of $tan\left(sin^{-1}\frac{3}{5}+cot^{-1}\frac{3}{2}\right)$.

Answer 12:

Given that $tan\left(sin^{-1}\frac{3}{5}+cot^{-1}\frac{3}{2}\right)$

$$\therefore tan\left(sin^{-1}\frac{3}{5}+cot^{-1}\frac{3}{2}\right)=tan\left(tan^{-1}\frac{3}{\sqrt{5^2-3^2}}+tan^{-1}\frac{2}{3}\right)$$

$$\left[as\ sin^{-1}\frac{a}{b}=tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\ and\ cot^{-1}\frac{a}{b}=tan^{-1}\frac{b}{a}\right]$$

$$=tan\left(tan^{-1}\frac{3}{4}+tan^{-1}\frac{2}{3}\right)$$

$$=tan\left[tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}\right)\right]=tan\left[tan^{-1}\left(\frac{\frac{9+8}{4\times 3}}{\frac{4\times 3-3\times 2}{4\times 3}}\right)\right]=tan\left(tan^{-1}\frac{17}{6}\right)=\frac{17}{6}$$

Question 13:

$cos^{-1}\left(cos\frac{7\pi}{6}\right)$ is equal to

(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Answer 13:

Given that $cos^{-1}\left(cos\frac{7\pi}{6}\right)$

We know that $\cos^{-1}(\cos x)=x$, if $x\in[0,\pi]$, which is the principal value branch of $\cos^{-1}x$.

$$\therefore cos^{-1}\left(cos\frac{7\pi}{6}\right)$$

$$=cos^{-1}\left[cos\left(2\pi-\frac{5\pi}{6}\right)\right]$$

$$=cos^{-1}\left(cos\frac{5\pi}{6}\right)=\frac{5\pi}{6}\in[0,\pi]$$

Hence, $cos^{-1}\left(cos\frac{7\pi}{6}\right)=\frac{5\pi}{6}$

Hence, the option (B) is correct.

Question 14: $sin\left(\frac{\pi}{3}-sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

Answer 14:

Given that $sin\left(\frac{\pi}{3}-sin^{-1}\left(-\frac{1}{2}\right)\right)$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

$\therefore sin\left(\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right)$

$= sin\left[\frac{\pi}{3} - sin^{-1}\left(-sin\frac{\pi}{6}\right)\right]$

$= sin\left[\frac{\pi}{3} - sin^{-1}\left\{sin\left(-\frac{\pi}{6}\right)\right\}\right]$

$= sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = sin\left(\frac{3\pi}{6}\right)$

$= sin\frac{\pi}{2} = 1$

Hence, $sin\left(\frac{\pi}{3} - sin^{-1}\left(-\frac{1}{2}\right)\right) = 1$

Hence, the option (D) is correct.

Question 15:

$tan^{-1}\sqrt{3} - cot^{-1}(-\sqrt{3})$ is equal to

(A) π (B) $-\frac{\pi}{2}$ (C) 0 (D) $2\sqrt{3}$

Answer 15:

Given that $tan^{-1}\sqrt{3} - cot^{-1}(-\sqrt{3})$

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\cot^{-1}$ is $(0, \pi)$.

$\therefore tan^{-1}\sqrt{3} - cot^{-1}(-\sqrt{3})$

$= tan^{-1}\left(tan\frac{\pi}{3}\right) - cot^{-1}\left(-cot\frac{\pi}{6}\right)$

$= \frac{\pi}{3} - cot^{-1}\left[cot\left(\pi - \frac{\pi}{6}\right)\right]$

$= \frac{\pi}{3} - cot^{-1}\left(cot\frac{5\pi}{6}\right)$

$= \frac{\pi}{3} - \frac{5\pi}{6} = \frac{2\pi - 5\pi}{6} = -\frac{3\pi}{6} = -\frac{\pi}{2}$

Hence, $tan^{-1}\sqrt{3} - cot^{-1}(-\sqrt{3}) = -\frac{\pi}{2}$

Hence, the options (B) is correct.

Mathematics

(Chapter – 2) (Inverse Trigonometric Functions) (Miscellaneous Exercise)
(Class – XII)

Question 1:

Find the value of $cos^{-1}\left(cos\frac{13\pi}{6}\right)$.

Answer 1:

Given that $cos^{-1}\left(cos\frac{13\pi}{6}\right)$

We know that $\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$, which is the principal value branch of $\cos^{-1}x$.

$\therefore cos^{-1}\left(cos\frac{13\pi}{6}\right) = cos^{-1}\left[cos\left(2\pi + \frac{\pi}{6}\right)\right] = cos^{-1}\left(cos\frac{\pi}{6}\right) = \frac{\pi}{6} \in [0, \pi]$

Hence, $cos^{-1}\left(cos\frac{13\pi}{6}\right) = \frac{\pi}{6}$

Question 2:

Find the value of $tan^{-1}\left(tan\frac{7\pi}{6}\right)$.

Answer 2:

Given that $tan^{-1}\left(tan\frac{7\pi}{6}\right)$

We know that $\tan^{-1}(\tan x) = x$ if, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, which is the principal value branch of $\tan^{-1}x$.

$\therefore tan^{-1}\left(tan\frac{7\pi}{6}\right) = tan^{-1}\left[tan\left(\pi + \frac{\pi}{6}\right)\right] = tan^{-1}\left(tan\frac{\pi}{6}\right) = \frac{\pi}{6}$

Hence, $tan^{-1}\left(tan\frac{7\pi}{6}\right) = \frac{\pi}{6}$

Question 3:

Prove that $2sin^{-1}\frac{3}{5} = tan^{-1}\frac{24}{7}$.

Answer 3:

LHS$= 2sin^{-1}\frac{3}{5} = 2tan^{-1}\frac{3}{\sqrt{5^2-3^2}}$ $\left[as\ sin^{-1}\frac{a}{b} = tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right]$

$= 2tan^{-1}\frac{3}{4} = tan^{-1}\left[\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right]$ $\left[as\ 2tan^{-1}x = tan^{-1}\frac{2x}{1-x^2}\right]$

$= tan^{-1}\left[\frac{\frac{3}{2}}{\frac{16-9}{16}}\right] = tan^{-1}\left(\frac{3}{2}\times\frac{16}{7}\right) = tan^{-1}\frac{24}{7}$ = RHS

Question 4:

Prove that $sin^{-1}\frac{8}{17} + sin^{-1}\frac{3}{5} = tan^{-1}\frac{77}{36}$.

Answer 4:

LHS $= sin^{-1}\frac{8}{17} + sin^{-1}\frac{3}{5}$

$= tan^{-1}\frac{8}{\sqrt{17^2-8^2}} + tan^{-1}\frac{3}{\sqrt{5^2-3^2}}$ $\left[as\ sin^{-1}\frac{a}{b} = tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right]$

$= tan^{-1}\frac{8}{15} + tan^{-1}\frac{3}{4} = tan^{-1}\left[\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15}\times\frac{3}{4}}\right]$ $\left[as\ tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= tan^{-1}\left[\frac{\frac{32+45}{15\times4}}{\frac{15\times4-8\times3}{15\times4}}\right] = tan^{-1}\left[\frac{\frac{77}{60}}{\frac{36}{60}}\right] = tan^{-1}\frac{77}{36}$ = RHS

Question 5:

Prove that $cos^{-1}\frac{4}{5} + cos^{-1}\frac{12}{13} = cos^{-1}\frac{33}{65}$

Answer 5:

LHS $= cos^{-1}\frac{4}{5} + cos^{-1}\frac{12}{13} = tan^{-1}\frac{\sqrt{5^2-4^2}}{4} + tan^{-1}\frac{\sqrt{13^2-12^2}}{12}$ $\left[as\ cos^{-1}\frac{a}{b} = tan^{-1}\frac{\sqrt{b^2-a^2}}{a}\right]$

$= tan^{-1}\frac{3}{4} + tan^{-1}\frac{5}{12} = tan^{-1}\left[\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}\right]$ $\left[as\ tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= tan^{-1}\left[\frac{\frac{36+20}{4\times 12}}{\frac{4\times 12-3\times 5}{4\times 12}}\right] = tan^{-1}\frac{56}{33} = cos^{-1}\frac{33}{\sqrt{56^2+33^2}}$ $\left[as\ tan^{-1}\frac{a}{b} = cos^{-1}\frac{b}{\sqrt{a^2+b^2}}\right]$

$= cos^{-1}\frac{33}{\sqrt{4225}} = cos^{-1}\frac{33}{65} =$ RHS

Question 6:

Prove that $cos^{-1}\frac{12}{13} + sin^{-1}\frac{3}{5} = sin^{-1}\frac{56}{65}$

Answer 6:

LHS $= cos^{-1}\frac{12}{13} + sin^{-1}\frac{3}{5}$

$= tan^{-1}\frac{\sqrt{13^2-12^2}}{12} + tan^{-1}\frac{3}{\sqrt{5^2-3^2}}$ $\left[as\ cos^{-1}\frac{a}{b} = tan^{-1}\frac{\sqrt{b^2-a^2}}{a}\text{ and }sin^{-1}\frac{a}{b} = tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right]$

$= tan^{-1}\frac{5}{12} + tan^{-1}\frac{3}{4} = tan^{-1}\left[\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\times\frac{3}{4}}\right]$ $\left[as\ tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= tan^{-1}\left[\frac{\frac{20+36}{12\times 4}}{\frac{12\times 4-5\times 3}{12\times 4}}\right] = tan^{-1}\frac{56}{33}$

$= sin^{-1}\frac{56}{\sqrt{56^2+33^2}}$ $\left[as\ tan^{-1}\frac{a}{b} = sin^{-1}\frac{a}{\sqrt{a^2+b^2}}\right]$

$= sin^{-1}\frac{56}{\sqrt{4225}} = sin^{-1}\frac{56}{65} =$ RHS

Question 7:

Prove that $tan^{-1}\frac{63}{16} = sin^{-1}\frac{5}{13} + cos^{-1}\frac{3}{5}$

Answer 7:

RHS $= sin^{-1}\frac{5}{13} + cos^{-1}\frac{3}{5}$

$= tan^{-1}\frac{5}{\sqrt{13^2-5^2}} + tan^{-1}\frac{\sqrt{5^2-3^2}}{3}$ $\left[as\ cos^{-1}\frac{a}{b} = tan^{-1}\frac{\sqrt{b^2-a^2}}{a}\text{ and }sin^{-1}\frac{a}{b} = tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right]$

$= tan^{-1}\frac{5}{12} + tan^{-1}\frac{4}{3} = tan^{-1}\left[\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\times\frac{4}{3}}\right]$ $\left[as\ tan^{-1}x + tan^{-1}y = tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$

$= tan^{-1}\left[\frac{\frac{15+48}{12\times 3}}{\frac{12\times 3-5\times 4}{12\times 3}}\right] = tan^{-1}\frac{63}{16} =$ RHS

Question 8:

Prove that $tan^{-1}\sqrt{x} = \frac{1}{2}cos^{-1}\left(\frac{1-x}{1+x}\right), x \in [0, 1]$

Answer 8:

$$\text{LHS} = tan^{-1}\sqrt{x} = \frac{1}{2} \times 2tan^{-1}\sqrt{x} = \frac{1}{2} \times 2tan^{-1}\sqrt{x}$$

$$= \frac{1}{2}cos^{-1}\left[\frac{1-(\sqrt{x})^2}{1+(\sqrt{x})^2}\right] \qquad \left[as\ 2tan^{-1}x = cos^{-1}\left[\frac{1-x^2}{1+x^2}\right]\right]$$

$$= \frac{1}{2}cos^{-1}\left(\frac{1-x}{1+x}\right) = \text{RHS}$$

Question 9:

Prove that $cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

Answer 9:

$$\text{LHS} = cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = cot^{-1}\left(\frac{\sqrt{1+\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{1-\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{1+\cos\left(\frac{\pi}{2}-x\right)}-\sqrt{1-\cos\left(\frac{\pi}{2}-x\right)}}\right)$$

$$= cot^{-1}\left(\frac{\sqrt{1+\cos y}+\sqrt{1-\cos y}}{\sqrt{1+\cos y}-\sqrt{1-\cos y}}\right) \qquad \left[Let\ \frac{\pi}{2} - x = y\right]$$

$$= cot^{-1}\left(\frac{\sqrt{2cos^2\frac{y}{2}}+\sqrt{2sin^2\frac{y}{2}}}{\sqrt{2cos^2\frac{y}{2}}-\sqrt{2sin^2\frac{y}{2}}}\right) \qquad \left[as\ 1+\cos y = 2cos^2\frac{y}{2} and\ 1-cosy = 2sin^2\frac{y}{2}\right]$$

$$= cot^{-1}\left(\frac{\sqrt{2}cos\frac{y}{2}+\sqrt{2}sin\frac{y}{2}}{\sqrt{2}cos\frac{y}{2}-\sqrt{2}sin\frac{y}{2}}\right)$$

$$= cot^{-1}\left(\frac{1+tan\frac{y}{2}}{1-tan\frac{y}{2}}\right) \qquad \left[Dividing\ each\ term\ by\ \sqrt{2}cos\frac{y}{2}\right]$$

$$= cot^{-1}\left(\frac{tan\frac{\pi}{4}+tan\frac{y}{2}}{1-tan\frac{\pi}{4}.tan\frac{y}{2}}\right) = cot^{-1}\left[tan\left(\frac{\pi}{4}+\frac{y}{2}\right)\right]$$

$$= cot^{-1}\left[cot\left\{\frac{\pi}{2}-\left(\frac{\pi}{4}+\frac{y}{2}\right)\right\}\right] = \frac{\pi}{2}-\left(\frac{\pi}{4}+\frac{y}{2}\right) = \frac{\pi}{4}-\frac{y}{2}$$

$$= \frac{\pi}{4}-\frac{1}{2}\left(\frac{\pi}{2}-x\right) \qquad \left[as\ \frac{\pi}{2}-x = y\right]$$

$$= \frac{x}{2} = \text{RHS}$$

Question 10:

Prove that $tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}cos^{-1}x, -\frac{1}{\sqrt{2}} \le x \le 1.$

Answer 10:

$$\text{LHS} = tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$$

$$= tan^{-1}\left(\frac{\sqrt{1+\cos y}-\sqrt{1-\cos y}}{\sqrt{1+\cos y}+\sqrt{1-\cos y}}\right) \qquad [Let\ x = \cos y]$$

$$= tan^{-1}\left(\frac{\sqrt{2cos^2\frac{y}{2}}-\sqrt{2sin^2\frac{y}{2}}}{\sqrt{2cos^2\frac{y}{2}}+\sqrt{2sin^2\frac{y}{2}}}\right) \qquad \left[as\ 1+\cos y = 2cos^2\frac{y}{2} and\ 1-cosy = 2sin^2\frac{y}{2}\right]$$

$$= tan^{-1}\left(\frac{\sqrt{2}cos\frac{y}{2}-\sqrt{2}sin\frac{y}{2}}{\sqrt{2}cos\frac{y}{2}+\sqrt{2}sin\frac{y}{2}}\right)$$

$$= tan^{-1}\left(\frac{1-tan\frac{y}{2}}{1+tan\frac{y}{2}}\right) \qquad \left[Dividing\ each\ term\ by\ \sqrt{2}cos\frac{y}{2}\right]$$

$$= tan^{-1}\left(\frac{tan\frac{\pi}{4}-tan\frac{y}{2}}{1+tan\frac{\pi}{4}.tan\frac{y}{2}}\right) = tan^{-1}\left[tan\left(\frac{\pi}{4}-\frac{y}{2}\right)\right]$$

$$= \frac{\pi}{4}-\frac{y}{2} = \frac{\pi}{4}-\frac{1}{2}cos^{-1}x = \text{RHS}$$

Question 11:

Solve for x: $2tan^{-1}(\cos x) = tan^{-1}(2\text{cosec}\, x)$

Answer 11:

Given that $2tan^{-1}(\cos x) = tan^{-1}(2\text{cosec}\, x)$

$\Rightarrow tan^{-1}\left(\frac{2\cos x}{1-cos^2x}\right) = tan^{-1}(2\text{cosec}\, x) \quad \left[as\ 2tan^{-1}x = tan^{-1}\frac{2x}{1-x^2}\right]$

$\Rightarrow \frac{2\cos x}{1-cos^2x} = 2\text{cosec}\, x$

$\Rightarrow \frac{2\cos x}{sin^2x} = \frac{2}{\sin x} \quad \Rightarrow 2\sin x.\cos x = 2sin^2x$

$\Rightarrow 2\sin x.\cos x - 2sin^2x = 0$

$\Rightarrow 2\sin x(\cos x - \sin x) = 0$

$\Rightarrow 2\sin x = 0 \qquad or \qquad \cos x - \sin x = 0$

But $\sin x \neq 0$ as it does not satisfy the equation

$\therefore \cos x - \sin x = 0 \quad \Rightarrow \cos x = \sin x \quad \Rightarrow \tan x = 1$

$\therefore x = \frac{\pi}{4}$

Question 12:

Solve for x: $tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}tan^{-1}x,\ (x > 0)$

Answer 12:

Given that $tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}tan^{-1}x$

$\Rightarrow tan^{-1}1 - tan^{-1}x = \frac{1}{2}tan^{-1}x \qquad \left[\because\ tan^{-1}x - tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

$\Rightarrow \frac{\pi}{4} = \frac{3}{2}tan^{-1}x$

$\Rightarrow \frac{\pi}{6} = tan^{-1}x$

$\Rightarrow tan\left(\frac{\pi}{6}\right) = x$

$\therefore x = \frac{1}{\sqrt{3}}$

Question 13:

$sin(tan^{-1}x), |x| < 1$ is equal to

(A) $\frac{x}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$ (C) $\frac{1}{\sqrt{1+x^2}}$ (D) $\frac{x}{\sqrt{1+x^2}}$

Answer 13:

Given that: $sin(tan^{-1}x)$

$= sin\left(sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$ $\left[as\ tan^{-1}\frac{a}{b} = sin^{-1}\frac{a}{\sqrt{a^2+b^2}}\right]$

$= \frac{x}{\sqrt{1+x^2}}$

Hence, the option (D) is correct.

Question 14:

$sin^{-1}(1-x) - 2sin^{-1}x = \frac{\pi}{2}$, then x is equal to

(A) 0, $\frac{1}{2}$ (B) 1, $\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$

Answer 14:

Given that $sin^{-1}(1-x) - 2sin^{-1}x = \frac{\pi}{2}$

Let $x = sin\ y$

$\therefore sin^{-1}(1-\sin y) - 2y = \frac{\pi}{2}$

$\Rightarrow sin^{-1}(1-\sin y) = \frac{\pi}{2} + 2y$

$\Rightarrow 1 - \sin y = sin\left(\frac{\pi}{2} + 2y\right)$

$\Rightarrow 1 - \sin y = \cos 2y$

$\Rightarrow 1 - \sin y = 1 - 2sin^2y$ $[as\ \cos 2y = 1 - 2sin^2y]$

$\Rightarrow 2sin^2y - \sin y = 0$

$\Rightarrow 2x^2 - x = 0$ $[as\ x = sin\ y]$

$\Rightarrow x(2x-1) = 0$

$\Rightarrow x = 0 \quad OR \quad x = \frac{1}{2}$

But $x \neq \frac{1}{2}$, as it does not satisfy the given equation.

$\therefore x = 0$ is the solution of the given equation.

Hence, the option (C) is correct.