Mechanical Properties Of Solids

Elastic Behaviour of Solids and The Concept of Stress and Strain

Elasticity

It is the property of a body by virtue of which it tends to regain its original size and shape after the applied force is removed.

Examples of elastic materials − quartz fibre, phosphor bronze, etc.

Plasticity

It is the inability of a body in regaining its original status on the removal of the deforming forces.

Examples of plastic materials − bakelite, plastic

Stress

The restoring force or deforming force experienced by a unit area is called stress.

S.I unit = Nm^{-2}

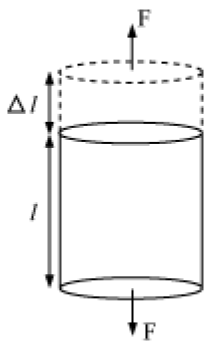
Types of Stress

- **Normal Stress**

When the elastic restoring force or deforming force acts perpendicular to the area, the stress is called normal stress. Normal stress can be sub-divided into the following categories:

(a) Tensile Stress

When there is an increase in the length or the extension of the body in the direction of the force applied, the stress set up is called tensile stress



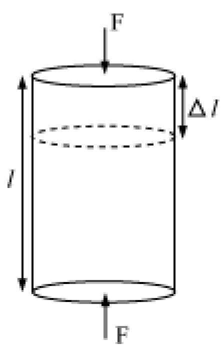
Here,

l = Original length

Δ*l* = Increase in length

(b) Compressive Stress

When there is a decrease in the length or the compression of the body due to the force applied, the stress set up is called compressive stress.



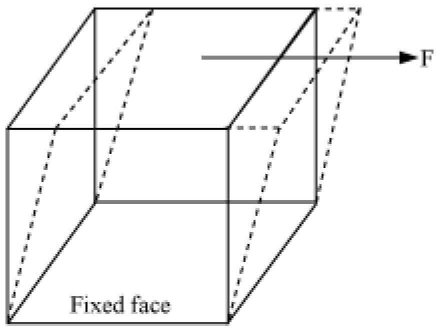
Here,

l = Original length

Δ*l* = Decrease in length

(c) Tangential or Shearing Stress

When the elastic restoring force or deforming force acts parallel to the surface area, the stress is called tangential stress.



Strain

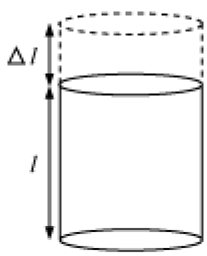
Ratio of change in configuration to the original configuration

Strain = $\dfrac{\text{Change in configuration}}{\text{Original configuration}}$

It is a dimensionless quantity.

Types of Strain

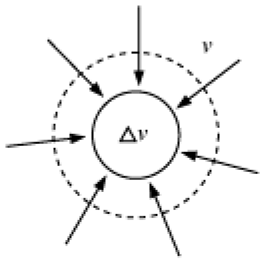
- **Longitudinal Strain**



Longitudinal Strain = $\dfrac{\text{Change in length}\,(\Delta l)}{\text{Original length}\,(l)}$

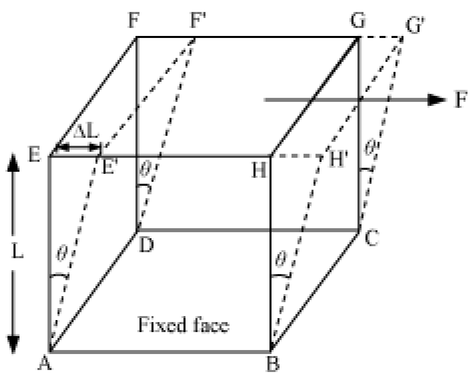
- **Volumetric Strain**

Volumetric Strain = $\dfrac{\text{Change in volume}\,(\Delta V)}{\text{Original volume}\,(V)}$



- **Shearing Strain**

  An angle (in radian) through which a plane perpendicular to the fixed surface of the cubical body gets turned under the effect of a tangential force.



Shearing Strain $= \theta = \dfrac{\Delta L}{L}$

Hooke's Law

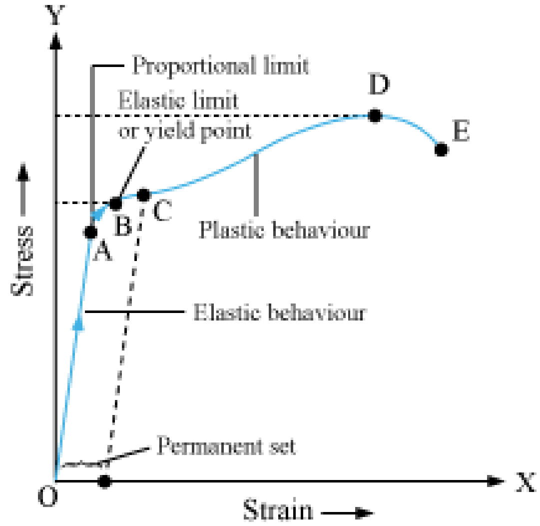
- For small deformations, stress and strain are proportional to each other

  Stress α strain

  Stress = *k* × strain

  Where, *k* is the proportionality constant, and is known as the modulus of elasticity

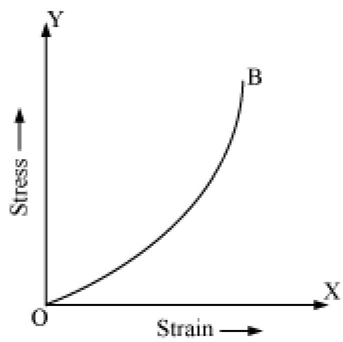
- Stress-strain curve for brittle materials:



Note:

When the material does not regain its original dimension, it is said to have a permanent set, and the deformation is said to be plastic deformation.

- Stress-strain curve for elastomers:



They do not obey Hooke's law, and always return to their original shape.

Elastic Moduli

Modulus of Elasticity:

According to Hooke's law, within the elastic limit, we have:

Stress ∝ Strain

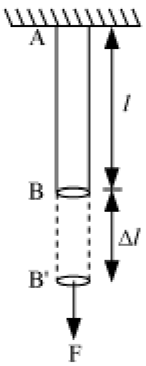
⇒ Stress = k × Strain

$\Rightarrow \frac{\text{Stress}}{\text{Strain}} = k = \text{constant}$

Here, *k* is known as the modulus of elasticity.

Types of Modulus of Elasticity

- **Young's modulus of elasticity (*Y*)**



$Y = \frac{\text{Normal stress}}{\text{Longitudinal strain}}$

$\Rightarrow Y = \frac{F / \pi r^2}{\Delta l / l}$

$\therefore Y = \frac{Fl}{\pi r^2 \Delta l}$

Here,

F = force applied

r = radius of the wire

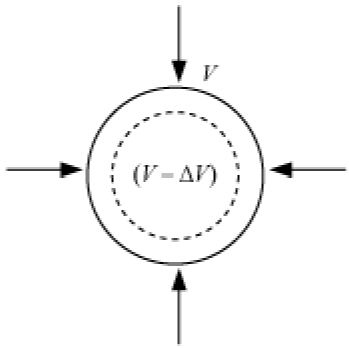
l = original length

Δl = change in length

Its unit is Nm^{-2} or Pascal (denoted by Pa).

- **Bulk modulus of elasticity (*B*)**

$B = \frac{\text{Normal stress}}{\text{Volumetric strain}}$



$\Rightarrow B = \frac{F/a}{-\Delta V/V} = \frac{-FV}{a\Delta V}$

If P is the increase in pressure applied on the spherical body, then

$P = F/a$

$\therefore B = \frac{-PV}{\Delta V}$

Here,
F = force applied

a = area of the object

V = original volume

ΔV = change in volume

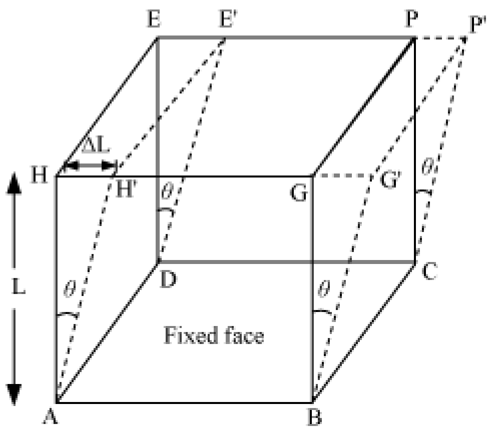
Its unit is Nm^{-2} or Pascal.

Compressibility (*k*): It is the reciprocal of bulk modulus of elasticity (B).

i.e., $k = 1/B$

- **Modulus of rigidity or shear modulus of elasticity (*G*)**

$G = \frac{\text{Tangential stress}}{\text{Shearing strain}}$



Here, $\angle HAH' = \theta = \angle GBG'$ and $HH' = \Delta L$

Shearing strain $= \theta = \frac{HH'}{AH} = \frac{\Delta L}{L}$

Tangential stress = F/a

$\therefore G = \frac{F/a}{\theta} = \frac{F}{a\theta} = \frac{FL}{a\Delta L}$

Here,

F = force applied

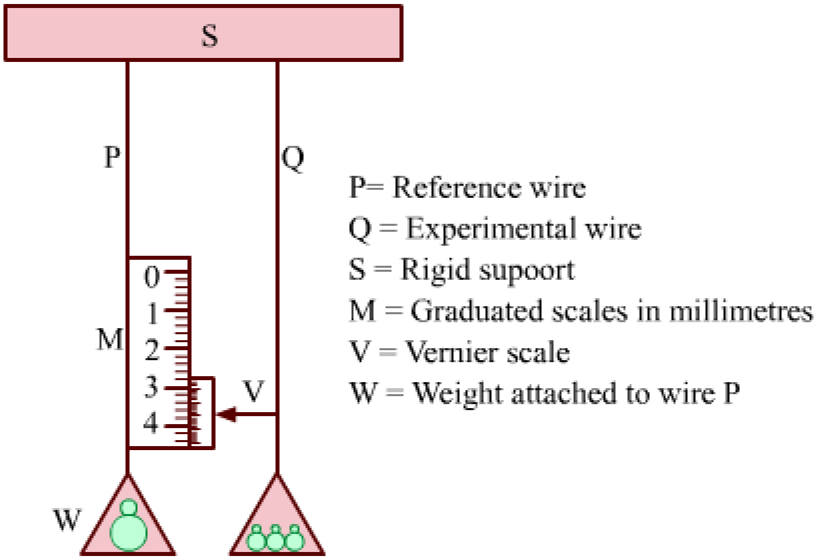
a = area

L = original length

ΔL = change in length

Its unit is Nm^{-2} or Pascal.

Determination of Young's Modulus of Material of Wire



The experimental apparatus for determining Young's modulus of the material of wire comprises two long straight wires of the same material, with the same length and equal radius suspended side by side from a fixed rigid support F.
Suppose that

r = initial radius of wire Q
L = initial length of wire Q
ΔL = elongation produced in the wire Q
M = mass producing elongation
πr^2= area of cross-section of wire Q

The use of the reference wire is to compensate for any change in the length that may occur when there is change in the room temperature. An initial small load is applied to both the wires (the reference wire and the experimental wire) to keep them straight and the corresponding vernier reading is recorded.

The experimental wire is loaded gradually with more weights to bring it under a tensile stress. The vernier reading is noted again. The difference between the two vernier readings gives the elongation produced in the wire.

If M is the mass that produced an elongation ΔL in the wire, then the applied force is equal to Mg, where g is the acceleration due to gravity.

By definition, Young's modulus of a material is the ratio of the longitudinal stress to the longitudinal strain. Young's modulus of the experimental wire is given by

$$Y = \frac{Mg/\pi r^2}{l/L}$$
$$\Rightarrow Y = \frac{MgL}{\pi r^2 l}$$

Poisson's ratio(σ):

On applying a force to a body along any direction, it is observed that there are changes in:

- the size along the direction of the force and
- the size in the perpendicular direction.

It is concluded that a deforming force when applied in one direction can produce strains in the other directions. If a wire is under a longitudinal strain, the lateral dimensions (radius of cross section) will undergo a small change.

Within the elastic limits,

Lateral strain∝∝Longitudinal strain
⇒ Lateral strain =σ××Longitudinal strain
The constant of proportionality σ is called the **Poisson's ratio**.

Poisson's ratio(σ) = lateral strain/ longitudinal strain

- Poisson's ratio (σ) is a unitless and dimensionless quantity.
- The value of Poisson's ratio for isotropic materials lies in the range [−1,0.5].
- For practical purposes, the value of Poisson's ratio is always positive and lies in range [0.2,0.4].

Relation Between Young's Modulus, Coefficient of Linear Expansion and Thermal Stress

Suppose that
L_0 = initial length of a metallic rod at t^o C
Δt^o C = increase in temperature
$L = L_0(1+\alpha\Delta t)$ = final length of the rod
Here, α is the coefficient of linear expansion of the material of the rod.

Linear strain in the wire = $\frac{L-L_0}{L_0} = \alpha\,\Delta\,t$
Stress = Young's modulus $\times$ strain
$\Rightarrow$ Stress = $Y\alpha\Delta t$

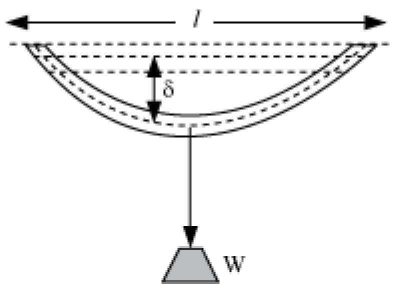
∴ Force exerted by the rod due to heating = thermal stress ×× Area = $Y\alpha\Delta t$××A

Applications of Elastic Behaviour of Materials

- The metallic parts in machinery are never subjected to a stress beyond the elastic limit; else, they may get permanently deformed.

- The thickness of the metallic rope used in a crane, for lifting a given load, depends on the elastic limit of the material of the rope and the factor of safety.

- Bridges are designed in such a way that they do not bend much or break under the load of heavy traffic, force of strong wind or their own weights.



From the given figure, the depression *δ* produced at the middle point of the bar is given by

$$\delta = \frac{Wl^3}{4Ybd^3}$$

Here,

Y = Young's modulus

W = load attached at its middle point

l = length of the bar

b = breadth of the bar

d = depth supported horizontally

In order to have a smaller depression (δ), for a given load, *l* should be small, while *Y*, *b* and *d* should be large.

Elastic energy

When a wire is stretched by a certain force, the work done by the force is stored as potential energy.

Consider a wire of length *l* and area of cross section *A* that is suspended from a rigid support. By applying a force *F* at the free end, it is stretched by a length *x.*

Longitudinal stress on the wire = $\frac{F}{A}$

Longitudinal strain on the wire = $\frac{x}{L}$

Young's modulus, $Y = \frac{F/A}{x/L} = \frac{FL}{Ax}$

$\Rightarrow F = \frac{YAx}{L}$

To further elongate the wire by *dx*, *dW* amount of work is done on it.

Now, work done = force $\times$ displacement

$\Rightarrow dW = \left(\frac{YAx}{L}\right)dx$

The total work done in stretching the wire from $x = 0$ to $x = l$ is obtained by integrating d*W*.

$$W = \int_0^l dW$$

$$\Rightarrow W = \int_0^l \left(\frac{YAx}{L}\right)dx$$

$$\Rightarrow W = \frac{YA}{L}\int_0^l x\,dx$$

$$\Rightarrow W = \frac{YA}{L} \times \left|\frac{x^2}{2}\right|_0^l$$

$$\Rightarrow W = \frac{YA}{L} \times \left[\frac{l^2}{2} - 0\right] = \frac{YAl^2}{2L}$$

$$\Rightarrow W = \frac{1}{2}\left(\frac{YAl}{L}\right)(l) = \frac{1}{2} \times F \times l$$

i.e., work done = $\frac{1}{2}\times$(load)$\times$(extension)

Thus, elastic energy stored in the wire on elongating it by a length $l = \frac{1}{2}\times$(load)$\times$(extension)