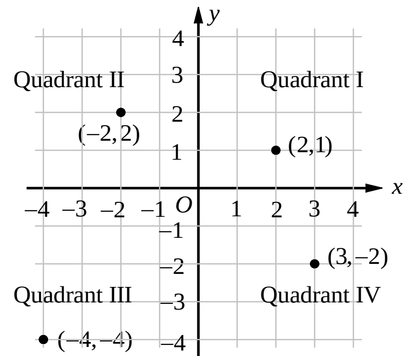
CHAPTER 3
Functions and Linear Equations

3-1. Relations and Functions

A **coordinate plane** is formed by the horizontal line called the **x- axis** and the vertical line called the **y- axis**, which meet at the **origin** $(0,0)$. The axes divide the plane into four parts called **quadrants**.

An **ordered pair** gives the coordinates and location of a point. The ordered pairs $(2,1)$, $(-2,2)$, $(-4,-4)$, and $(3,-2)$ are located in Quadrant I, Quadrant II, Quadrant III, and Quadrant IV respectively.

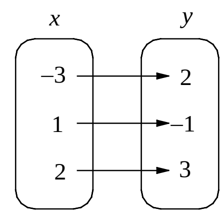


A **relation** is a set of ordered pairs. A relation can be represented by a graph, a table, or a **mapping**.
The **domain** of a relation is the set of all x- coordinates and the **range** of a relation is the set of all y- coordinates from the ordered pairs.
A **function** is a special type of relation in which each element of the domain is paired with exactly one element of the range.

Table

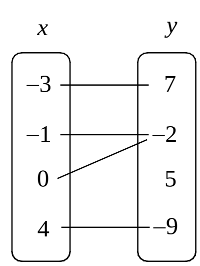
x	y
–3	2
1	–1
2	3

Mapping

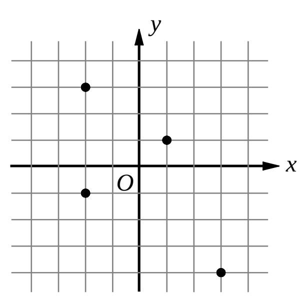


Example 1 □ Express each relation below as a set of ordered pairs and determine whether it is a function.

a.



b.



c.

x	y
1	–2
2	3
3	5
4	–2

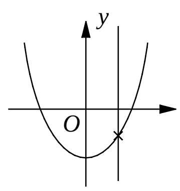
Solution □ a. $\{(-3,7), (-1,-2), (0,-2), (4,-9)\}$
The mapping represents a function.

b. $\{(1,1), (-2,3), (-2,-1), (3,-4)\}$
The element -2 in the domain is paired with both 3 and -1 in the range. This relation does not represent a function.

c. $\{(1,-2), (2,3), (3,5), (4,-2)\}$
The table represents a function.

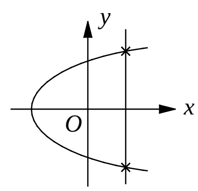
You can use the **vertical line test** to see if a graph represents a function. A relation is a function if and only if no vertical line intersects its graph more than once.

Function



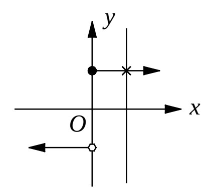
No vertical line intersects the graph more than once.

Not a Function



A vertical line intersects the graph at two points.

Function



No vertical line intersects the graph more than once.

Function Values

Equations that are functions can be written in a form called **function notation**.
In function notation, the equation $y = 2x + 3$ is written as $f(x) = 2x + 3$.
The **function value** of f at $x = c$ is denoted as $f(c)$. For instance, if $f(x) = 5x - 3$, $f(2)$ is the value of f at $x = 2$ and $f(2) = 5(2) - 3 = 7$.

Example 2 □ If $f(x) = 3x + 2$, find each value.

a. $f(-2)$ b. $f(c-2)$

c. $-2[f(-1)] + f(-2)$ d. $f(-\frac{1}{2}x + 1)$

Solution □ a. $f(-2) = 3(-2) + 2$ Substitute -2 for x.
$= -6 + 2 = -4$ Multiply and simplify.

b. $f(c-2) = 3(c-2) + 2$ Substitute $c-2$ for x.
$= 3c - 6 + 2$ Multiply.
$= 3c - 4$ Simplify.

c. $-2[f(-1)] + f(-2)$
$= -2[3(-1) + 2] + [3(-2) + 2]$ Substitute -1 for x and -2 for x.
$= -2[-3 + 2] + [-6 + 2]$ Multiply.
$= -2[-1] + [-4]$ Simplify.
$= 2 - 4$ Simplify.
$= -2$ Simplify.

d. $f(-\frac{1}{2}x + 1) = 3(-\frac{1}{2}x + 1) + 2$ Substitute $-\frac{1}{2}x + 1$ for x.
$= -\frac{3}{2}x + 3 + 2$ Multiply.
$= -\frac{3}{2}x + 5$ Simplify.

Exercises - Relations and Functions

1

What is the domain of the function that contains points at $(-5,2)$, $(-2,1)$, $(0,2)$, and $(4,-3)$?

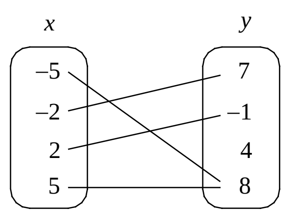
A) $\{-3,\ 1,\ 2\}$

B) $\{-2,\ 1,\ 0\}$

C) $\{-5,\ -2,\ 1,\ 2\}$

D) $\{-5,\ -2,\ 0,\ 4\}$

2



Which of the following relation is a correct representation of the mapping shown above?

A) $\{(-5,7),\ (-2,-1),\ (2,4),\ (5,8)\}$

B) $\{(-5,8),\ (-2,7),\ (2,-1),\ (5,8)\}$

C) $\{(7,-5),\ (-1,-2),\ (4,2),\ (8,5)\}$

D) $\{(8,-5),\ (7,-2),\ (-1,2),\ (8,5)\}$

3

If point $(7,b)$ is in Quadrant I and point $(a,-3)$ is in Quadrant III, in which Quadrant is the point (a,b) ?

A) Quadrant I

B) Quadrant II

C) Quadrant III

D) Quadrant IV

4

If $f(x) = -2x+7$, what is $f(\frac{1}{2}x+3)$ equal to?

A) $-x+1$

B) $-x+3$

C) $-x+5$

D) $-x+10$

5

$$g(x) = kx^3 + 3$$

For the function g defined above, k is a constant and $g(-1) = 5$. What is the value of $g(1)$?

A) -3

B) -1

C) 1

D) 3

6

If $f(x+1) = -\frac{1}{2}x + 6$, what is the value of $f(-3)$?

7

$$f(x) = x^2 - b$$

In the function above, b is a constant.
If $f(-2) = 7$, what is the value of $f(b)$?

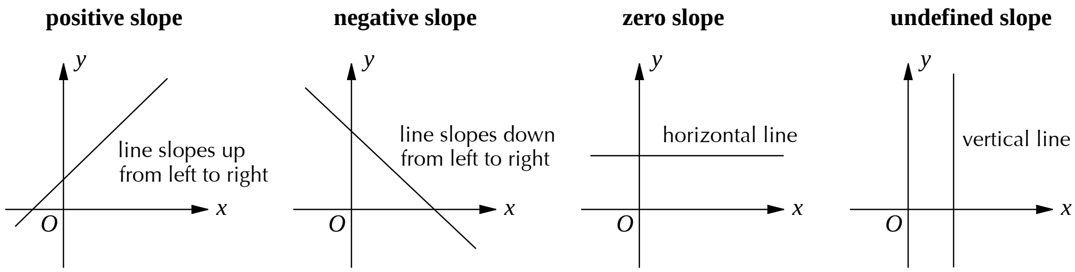
3-2. Rate of Change and Slope

The **average rate of change** is a ratio that describes, on average, change in one quantity with respect to change in another quantity. If x is the independent variable and y is the dependent variable, then

the average rate of change $= \dfrac{\text{change in } y}{\text{change in } x}$.

Geometrically, the rate of change is the slope of the line through the points (x_1, y_1) and (x_2, y_2).

The **slope** m of a line through (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{\text{rise}}{\text{run}}$.



The **standard form** of a linear equation of a line is $\boldsymbol{Ax + By = C}$, in which A, B, and C are integers.

The x-coordinate of the point at which the graph of an equation crosses the x-axis is called the ***x*- intercept**. To find the x-intercept, let $y = 0$ and solve the equation for x.

The y-coordinate of the point at which the graph of an equation crosses the y-axis is called the ***y*- intercept**. To find the y-intercept, let $x = 0$ and solve the equation for y.

The intercepts of a line provide a quick way to sketch the line.

Values of x for which $f(x) = 0$ are called **zeros** of the function f. A function's zero is its x-intercept.

Example 1 □ The table at the right shows Evan's height from age 12 to 18. Find the average rate of change in Evan's height from age 12 to 18.

Age (years)	12	14	16	18
Height (inches)	61	64	68	70

Solution □ Average rate of change $= \dfrac{\text{change in height}}{\text{change in years}}$

$$= \frac{70 - 61}{18 - 12} = \frac{9}{6} = 1.5 \text{ inches per year}$$

Example 2 □ Find the slope of the line that passes through $(3, -2)$ and $(-5, 4)$.

Solution □ $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - (-2)}{-5 - 3} = \dfrac{6}{-8} = -\dfrac{3}{4}$

Example 3□ Find the x-intercept and y-intercept of $2x + 3y = 6$.

Solution □ To find the x-intercept, let $y = 0$.

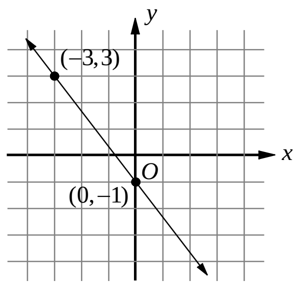
$2x + 3(0) = 6 \Rightarrow x = 3$ The x-intercept is 3.

To find the y-intercept, let $x = 0$.

$2(0) + 3y = 6 \Rightarrow y = 2$ The y-intercept is 2.

Exercises - Rate of Change and Slope

1



What is the rate of change shown in the graph of the line above?

A) $-\frac{4}{3}$

B) $-\frac{3}{4}$

C) $\frac{3}{4}$

D) $\frac{4}{3}$

2

x	-3	0	3	6
y	-1	1	3	5

What is the average rate of change for the relation shown in the table above?

A) $\frac{1}{3}$

B) $\frac{1}{2}$

C) $\frac{2}{3}$

D) $\frac{5}{6}$

3

The graph of the linear function f passes through the points $(a,1)$ and $(1,b)$ in the xy- plane. If the slope of the graph of f is 1, which of the following is true?

A) $a-b=1$

B) $a+b=1$

C) $a-b=2$

D) $a+b=2$

4

What is the slope of the line that passes through $(3,2)$ and $(-1,-8)$?

5

What is the value of r if the line that passes through $(4,3)$ and $(-5,r)$ has a slope of -1?

6

What is the value of a if the line that passes through $(a,7)$ and $(1,a)$ has a slope of $-\frac{5}{9}$?

7

$$-x+4y=6$$

What is the slope of the line in the equation above?
.

3-3. Slope-Intercept Form and Point-Slope Form

The **slope-intercept form** of the equation of a line is $\boldsymbol{y = mx + b}$, in which m is the slope and b is the y-intercept.

The **point-slope form** of the equation of a line is $\boldsymbol{y - y_1 = m(x - x_1)}$, in which (x_1, y_1) are the coordinates of a point on the line and m is the slope of the line.

Example 1 □ The graph of a linear equation is shown on the diagram at the right.

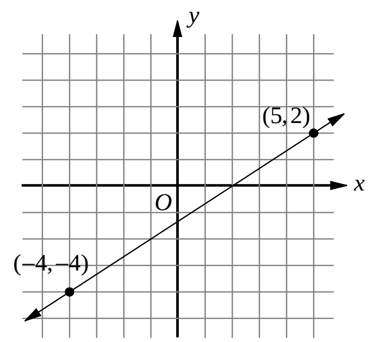
a. Find the slope of the line on the graph.

b. Write the equation of the line in point-slope form.

c. Write the equation of the line in slope-intercept form and find the y-intercept.

d. Write the equation of the line in standard form.

e. Find the x-intercept on the graph.



Solution □ a. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{2-(-4)}{5-(-4)}$ $\quad$ $(x_1, y_1) = (-4,-4)$, $(x_2, y_2) = (5,2)$

$= \dfrac{6}{9} = \dfrac{2}{3}$ $\quad$ Simplify.

b. $y - y_1 = m(x - x_1)$ $\quad$ Point-slope form

$y - 2 = \dfrac{2}{3}(x-5)$ $\quad$ $m = \dfrac{2}{3}$. Choose either point for (x_1, y_1).

c. $y - 2 = \dfrac{2}{3}x - \dfrac{10}{3}$ $\quad$ Simplify point-slope form.

$y = \dfrac{2}{3}x - \dfrac{4}{3}$ $\quad$ Add 2 to each side.

The y-intercept is $-\dfrac{4}{3}$.

d. $y = \dfrac{2}{3}x - \dfrac{4}{3}$ $\quad$ Slope-intercept form

$3y = 3(\dfrac{2}{3}x - \dfrac{4}{3})$ $\quad$ Multiply each side by 3.

$3y = 2x - 4$ $\quad$ Simplify.

$-2x + 3y = -4$ or $2x - 3y = 4$ $\quad$ Subtract $2x$ from each side.

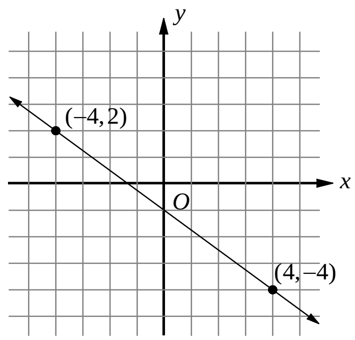
e. $-2x + 3(0) = -4$ $\quad$ To find the x-intercept, let $y = 0$.

$x = 2$

The x-intercept is 2.

Exercises - Slope-Intercept Form and Point-Slope Form

Questions 1-3 refer to the following information.



The graph of a linear equation is shown in the diagram above.

1

Which of the following is the equation of the line in point-slope form?

A) $y+4=-\frac{4}{3}(x-4)$

B) $y-4=-\frac{4}{3}(x+4)$

C) $y-2=-\frac{3}{4}(x+4)$

D) $y+2=-\frac{3}{4}(x-4)$

2

Which of the following is the equation of the line in slope-intercept form?

A) $y=-\frac{3}{4}x+1$

B) $y=-\frac{3}{4}x-1$

C) $y=-\frac{4}{3}x+1$

D) $y=-\frac{4}{3}x-1$

3

Which of the following is the equation of the line in standard form?

A) $4x-3y=-4$

B) $4x+3y=-4$

C) $3x-4y=-4$

D) $3x+4y=-4$

4

In 2005, 120 students at Lincoln High School had smart phones. By 2010, 345 students in the same school had smart phones. Which of the following best describes the annual rate of change in the number of smart phones students had from 2005 to 2010 at Lincoln High School?

A) The average increase in the number of smart phones per year is 40.

B) The average increase in the number of smart phones per year is 45.

C) The average increase in the number of smart phones per year is 50.

D) The average increase in the number of smart phones per year is 55.

5

Which of the following is the equation of the line that passes through point $(4,-1)$ and has slope -2?

A) $x+2y=2$

B) $x-2y=6$

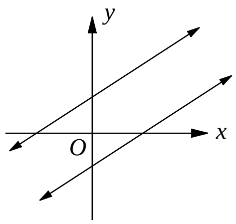
C) $2x-y=9$

D) $2x+y=7$

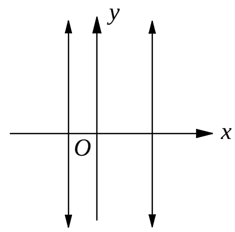
3-4. Parallel and Perpendicular Lines

Lines in the same plane that do not intersect are called **parallel lines**. Parallel lines have the same slope.
If two nonvertical lines have the same slope, then they are parallel.

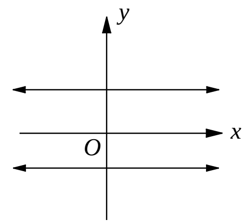
Parallel Lines in a Coordinate Plane



Parallel lines have the same slope.



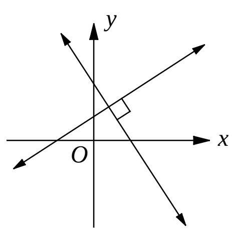
All vertical lines are parallel.



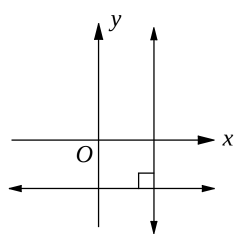
All horizontal lines are parallel.

Lines that intersect at right angles are called **perpendicular lines**.
If the product of the slopes of two nonvertical lines is -1, then the lines are perpendicular.

Perpendicular Lines in a Coordinate Plane



The slopes of perpendicular lines are negative reciprocals of each other.



Vertical lines and horizontal lines are perpendicular.

Example 1 □ Write the equation in point-slope form of the lines through point $(1,2)$ that are (a) parallel to (b) perpendicular to, $3x - y = -2$.

Solution □ a.

$3x - y = -2$	Original equation
$3x - y - 3x = -2 - 3x$	Subtract $3x$ from each side.
$-y = -3x - 2$	Simplify.
$(-1)(-y) = (-1)(-3x - 2)$	Multiply each side by -1.
$y = 3x + 2$	Simplify.

Parallel lines have the same slope. Replace m with 3 and (x_1, y_1) with $(1,2)$ in point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 2 = 3(x - 1)$	Substitution

b. The line perpendicular to $y = 3x + 2$ has a slope of $-\dfrac{1}{3}$, which is the negative reciprocal of 3. Replace m with $-\dfrac{1}{3}$ and (x_1, y_1) with $(1,2)$ in point-slope form.

$y - 2 = -\dfrac{1}{3}(x - 1)$	$m = -\dfrac{1}{3}$ and $(x_1, y_1) = (1,2)$

Exercises - Parallel and Perpendicular Lines

1

Which of the following equations represents a line that is parallel to the line with equation $y=-\frac{1}{2}x+5$ and contains the point $(-2,\frac{1}{2})$?

A) $x-2y=-3$

B) $x+2y=-1$

C) $2x-y=-5$

D) $2x+y=-3$

2

Which of the following equations represents a line that passes through $(7,6)$ and is parallel to the x- axis?

A) $x=6$

B) $y=7$

C) $y=7$

D) $y=6$

3

Which of the following equations represents a line that passes through $(-5,1)$ and is parallel to the y- axis?

A) $y=-5$

B) $y=1$

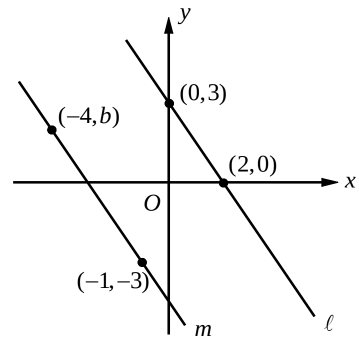
C) $x=-5$

D) $x=1$

4

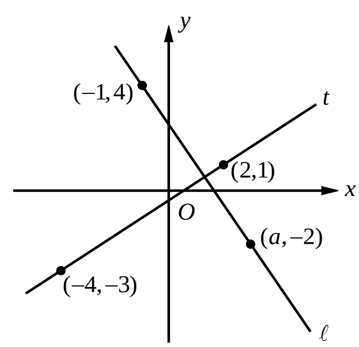
A line passes through the points $(-1,2)$ and $(5,b)$, and is parallel to the graph of the equation $4x-2y=13$. What is the value of b?

5



In the xy- plane above, line ℓ is parallel to line m. What is the value of b?

6



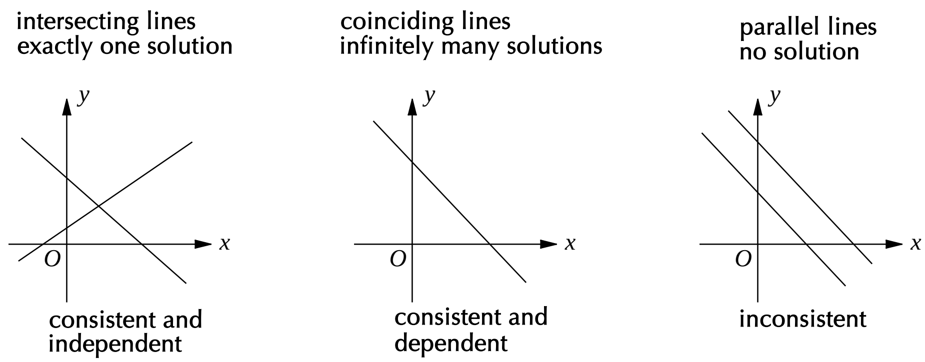
In the xy- plane above, if line ℓ is perpendicular to line t, what is the value of a?

3-5. Solving Systems of Linear Equations

A set of linear equations with the same two variables is called a **system of linear equations**.
A system of two linear equations can have no solution, one solution, or an infinite number of solutions.

Solving Linear Systems by Graphing

Three types of graphs of linear systems are illustrated below.



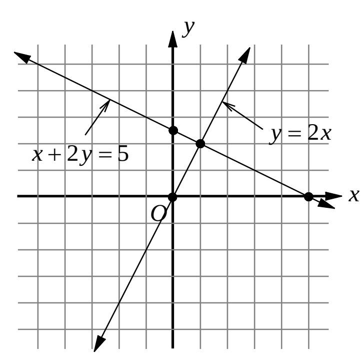
Example 1 □ Solve the system of equations by graphing.

a. $y = 2x$
$x + 2y = 5$

b. $-9x + 6y = 12$
$3x - 2y = -4$

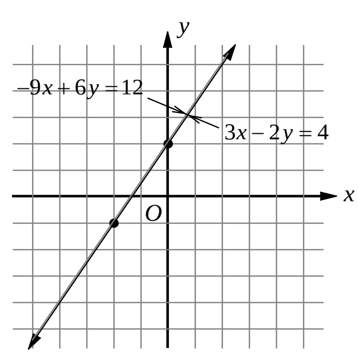
Solution □ a. $y = 2x$ has the y- intercept 0 and the slope 2. Graph $(0,0)$. From $(0,0)$, move right 1 unit and up 2 units. Draw a dot. Draw a line through the points.

To find the x- intercept of $x + 2y = 5$, let $y = 0$. $x + 2(0) = 5$ implies $x = 5$.
To find the y- intercept let $x = 0$.
$0 + 2y = 5$ implies $y = 2.5$.
Plot $(5,0)$ and $(0,2.5)$. Then draw a line through these two points.



The lines have different slopes and intersect at $(1,2)$.
The solution of the system is $(1,2)$.

b. The slope-intercept form of $-9x + 6y = 12$ is $y = \frac{3}{2}x + 2$.
The slope-intercept form of $3x - 2y = -4$ is $y = \frac{3}{2}x + 2$.
Since the equations are equivalent, their graphs show the same line. Any ordered pair representing a point on that line will satisfy both equations. So, there are infinitely many solutions to this system.



Solving Linear Systems by Substitution

The basic steps in the substitution method are as follows:

1. Solve one of the equations for one of its variables.
2. Substitute the resulting expression into the other equation and solve.
3. Substitute the resulting value into either of the original equations and solve for the other variable.

Example 2 □ Solve the linear system by substitution method.

$$y = x - 1$$
$$2x + y = 5$$

Solution □

$2x + y = 3$	Second equation
$2x + (x - 1) = 5$	Since the first equation is $y = x - 1$, substitute $x - 1$ for y in the second equation.
$3x - 1 = 5$	Combine like terms.
$3x = 6$	Add 1 to each side.
$x = 2$	Divide each side by 3 and simplify.
$y = (2) - 1$	Substitute 2 for x in the first equation.
$= 1$	Simplify.

The solution is $(2,1)$.

Solving Linear Systems by Elimination

The basic steps in the elimination method are as follows:

1. Arrange the equations with the like terms in columns.
2. Multiply one or both equations to obtain new coefficients for x (or y) that are opposites.
3. Add the equations and solve for the remaining variable.
4. Substitute this value into either of the original equations and solve for the other variable.

Example 3 □ Solve the linear system by elimination method.

$$2x - 3y = 13$$
$$-3x + 2y = -12$$

Solution □

$3(2x - 3y = 13)$	Multiply the first equation by 3.
$2(-3x + 2y = -12)$	Multiply the second equation by 2.

By multiplying the first equation by 3 and multiplying the second equation by 2, we obtain coefficients of x that are opposites.

$6x - 9y = 39$	First equation modified.
$\underline{-6x + 4y = -24}$	Second equation modified.
$-5y = 15$	Sum of equations
$y = -3$	Simplify.
$2x - 3(-3) = 13$	Substitute -3 for y in the first equation.
$2x + 9 = 13$	Simplify.
$2x = 4$	Subtract 9 from each side.
$x = 2$	Divide each side by 2 and simplify.

The solution is $(2,-3)$.

Systems of Equations with No Solution and Infinitely Many Solutions

1. A system of equations has no solution if the two equations have the same slope but different y- intercepts.
2. A system of equations has infinitely many solutions if the two equations are equivalent. Therefore the two equations have the same slope and same y- intercepts.

When you are asked if a system of equations has no solution or infinitely many solutions, you need to change the equations into *slope-intercept form* and check the slopes and y- intercepts.

Example 4 □ For what value of c will the system of equations below have no solution?

$$cx - 2y = 6$$
$$3x + 4y = 4$$

Solution □ Change the equations into slope-intercept form.

$y = \frac{c}{2}x - 3$ First equation in slope-intercept form

$y = -\frac{3}{4}x + 1$ Second equation in slope-intercept form

If two equations have the same slope and different y- intercepts, the system has no solution. So, let $\frac{c}{2} = -\frac{3}{4}$. Solving this equation for c gives $c = -\frac{3}{2}$.

Since the lines are parallel and the y- intercepts are -3 and 1, the two equations are not identical. Therefore, when $c = -\frac{3}{2}$, the system of equations has no solution.

Example 5 □ For what value of b will the system of equations below have infinitely many solutions?

$$-2x + y = 4$$
$$5x - by = -10$$

Solution □ $y = 2x + 4$ First equation in slope-intercept form

$y = \frac{5}{b}x + \frac{10}{b}$ Second equation in slope-intercept form

If two equations have infinitely many solutions, they are equivalent .

Therefore, $2 = \frac{5}{b}$ or $4 = \frac{10}{b}$. Solving these equations for b gives $b = \frac{5}{2}$.

Example 6 □ Solve the linear system.

$$2x - 3y = 5$$
$$y = \frac{2}{3}x + 2$$

Solution □ Substitute the expression for y from the second equation into the first equation.

$2x - 3y = 5$ First equation

$2x - 3(\frac{2}{3}x + 2) = 5$ Substitute $\frac{2}{3}x + 2$ for y in the first equation.

$2x - 2x - 6 = 5$ Simplify.

$-6 = 5$ Simplify.

Since $-6 = 5$ is false, the system has no solution.

Exercises - Solving Systems of Linear Equations

1

$$y = 2x + 4$$
$$x - y = -1$$

Which ordered pair (x, y) satisfies the system of equations shown above?

A) $(-2, -3)$

B) $(-3, -2)$

C) $(-1, 2)$

D) $(-2, 0)$

2

$$\frac{1}{2}x + y = 1$$
$$-2x - y = 5$$

If (x, y) is a solution to the system of equations above, what is the value of $x + y$?

A) -2

B) -1

C) 1

D) 2

3

$$2x - ky = 14$$
$$5x - 2y = 5$$

In the system of equations above, k is a constant and x and y are variables. For what values of k will the system of equations have no solution?

4

Which of the following systems of equations has infinitely many solutions?

A) $x + y = 1$

$x - y = 1$

B) $-2x + y = 1$

$-2x + y = 5$

C) $\frac{1}{2}x - \frac{1}{3}y = 1$

$3x - 2y = 6$

D) $2x + 3y = 1$

$3x - 2y = 1$

5

$$ax - y = 0$$
$$x - by = 1$$

In the system of equations above, a and b are constants and x and y are variables. If the system of equations above has no solution, what is the value of $a \cdot b$?

6

$$2x - \frac{1}{2}y = 15$$
$$ax - \frac{1}{3}y = 10$$

In the system of equations above, a is a constant and x and y are variables. For what values of a will the system of equations have infinitely many solution?

3-6. Absolute Value Equations

The **absolute value** of a number is the distance on a number line between the graph of the number and the origin.

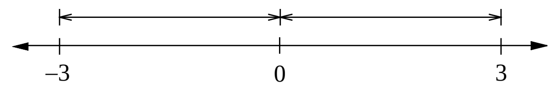
The distance between -3 and the origin is 3. Thus $|-3| = 3$.

The distance between 3 and the origin is 3. Thus $|3| = 3$.

Therefore, if $|x| = 3$, then $x = 3$ or $x = -3$.

$|-3| = 3$, because the distance from the origin to -3 is 3.

$|3| = 3$, because the distance from the origin to 3 is 3.



An **absolute value function** is a function written as $f(x) = |x|$, for all values of x.

An absolute function can be written using two or more expressions such as $f(x) = |x| = \begin{cases} -x \text{ if } x < 0 \\ x \text{ if } x \geq 0 \end{cases}$.

Example 1 □ Solve each absolute value equation.

a. $|3x-5| = 7$ b. $|x+3| = 0$ c. $|x-8| = -3$

Solution □ a. $|3x-5| = 7$

$3x-5=7$ or $3x-5=-7$ If $|x| = a$, then $x = a$ or $x = -a$.

$3x = 12$ or $3x = -2$ Add 5 to each side.

$x = 4$ or $x = \dfrac{-2}{3}$ Divide each side by 3.

b. $|x+3| = 0$

$x+3=0$ If $|x| = 0$, then $x = 0$.

$x = -3$ Subtract 3 from each side.

c. $|x-8| = -3$

$|x-8| = -3$ means that the distance between x and 8 is -3. Since distance cannot be negative, the equation has no solution.

To sketch the graph of $y = a|x+h| + k$, use the following steps.

1. Find the x-coordinate of the vertex by finding the value of x for which $x+h=0$.
2. Make a table of values using the x-coordinate of the vertex. Find two x-values to its left and two to its right.
3. Plot the points from inside the table. If $a > 0$, the vertex will be the minimum point and if $a < 0$, the vertex will be the maximum point.

The graph of $y = |x|$

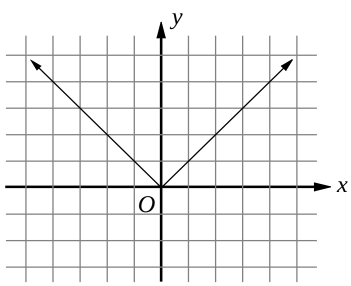


Table of values

x	$y = \|x\|$	
-2	2	
-1	1	
0	0	← Vertex
1	1	
2	2	

Exercises - Absolute Value Equations

1

Which of the following expressions is equal to -1 for some values of x?

A) $|1-x|+6$

B) $|1-x|+4$

C) $|1-x|+2$

D) $|1-x|-2$

2

If $|2x+7|=5$, which of the following could be the value of x?

A) -6

B) -4

C) -2

D) 0

3

For what value of x is $|x-1|-1$ equal to 1?

A) -1

B) 0

C) 1

D) 2

4

For what value of x is $|3x-5|=-1$?

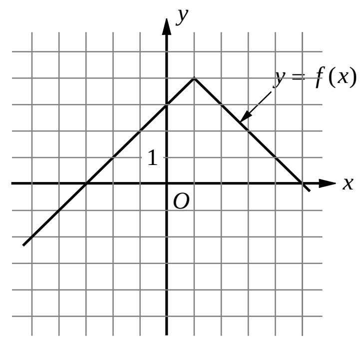
A) -2

B) -1

C) 0

D) There is no such value of x.

5



The graph of the function f is shown in the xy-plane above. For what value of x is the value of $f(x)$ at its maximum?

A) -3

B) -1

C) 1

D) 3

6

For what value of n is $3-|3-n|$ equal to 3?

Chapter 3 Practice Test

1

x	–4	0	6
$f(x)$	–4	–1	k

In the table above, if $f(x)$ is a linear function, what is the value of k?

A) 2.5

B) 3

C) 3.5

D) 4

2

The graph of a line in the xy- plane has slope $\frac{1}{3}$ and contains the point $(9,1)$. The graph of a second line passes through the points $(-2,4)$ and $(5,-3)$. If the two lines intersect at (a,b), what is the value of $a+b$?

A) –2

B) 2

C) 4

D) 6

3

Which of the following expressions is equal to 0 for some value of x?

A) $5+|x+5|$

B) $5+|x-5|$

C) $-5+|x+5|$

D) $-5-|x-5|$

4

Line ℓ in the xy- plane contains points from each of the Quadrants I, III, and IV, but no points from Quadrant II. Which of the following must be true?

A) The slope of line ℓ is zero.

B) The slope of line ℓ is undefined.

C) The slope of line ℓ is positive.

D) The slope of line ℓ is negative.

5

x	–3	–1	1	5
$f(x)$	9	5	1	–7

The table above shows some values of the linear function f. Which of the following defines f?

A) $f(x)=2x-3$

B) $f(x)=-2x+3$

C) $f(x)=2x-1$

D) $f(x)=-2x+1$

6

If $f(x)=-6x+1$, what is $f(\frac{1}{2}x-1)$ equal to?

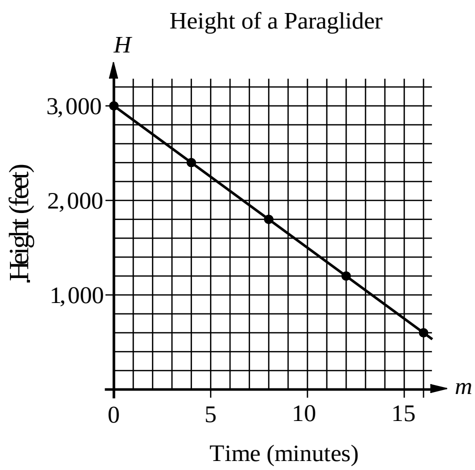
A) $-3x+7$

B) $-3x-5$

C) $-3x+1$

D) $-3x-1$

Questions 7 and 8 refer to the following information.



The graph above shows the relationship between the height of paraglider H, in feet, and time m, in minutes.

7

Which of the following represents the relationship between H and m?

A) $H = -100m + 3000$

B) $H = -150m + 3000$

C) $H = -175m + 3000$

D) $H = -225m + 3000$

8

If the height of the paraglider is 1,350 feet, which of the following best approximates the time the paraglider has been flying?

A) 10 minutes

B) 10 minutes and 30 seconds

C) 11 minutes

D) 11 minutes and 30 seconds

9

A line in the xy- plane passes through the point $(1,-2)$ and has a slope of $\frac{1}{3}$.Which of the following points lies on the line?

A) $(3,-2)$

B) $(2,-\frac{4}{3})$

C) $(0,-2)$

D) $(-1,-\frac{8}{3})$

10

$$f(x) = ax + 2$$

In the function above, a is a constant. If $f(-1) = 4$, what is the value of $f(-\frac{1}{2})$?

11

If the slope of the line in the xy- plane that passes through the points $(2,-4)$ and $(6,k)$ is $\frac{3}{2}$, what is the value of k?

12

$$\frac{1}{3}x - \frac{3}{4}y = -11$$
$$\frac{1}{2}x + \frac{1}{6}y = -1$$

If (x, y) is the solution to the system of equations above, what is the value of $x + y$?

Answer Key

Section 3-1

1. D	2. B	3. B	4. A	5. C
6. 8	7. 12			

Section 3-2

1. A	2. C	3. D	4. $\frac{5}{2}$ or 2.5
5. 12	6. $\frac{29}{2}$ or 14.5		7. $\frac{1}{4}$ or 0.25

Section 3-3

1. C	2. B	3. D	4. B	5. D

Section 3-4

1. B	2. D	3. C	4. 14	5. $\frac{3}{2}$ or 1.5
6. 3				

Section 3-5

1. B	2. B	3. $\frac{4}{5}$ or 0.8	4. C
5. 1	6. $\frac{4}{3}$ or 1.33		

Section 3-6

1. D	2. A	3. A	4. D	5. C
6. 3				

Chapter 3 Practice Test

1. C	2. B	3. C	4. C	5. B
6. A	7. B	8. C	9. D	10. 3
11. 2	12. 6			

Answers and Explanations

Section 3-1

1. D

The domain of a function is the set of all x-coordinates. Therefore, $\{-5, -2, 0, 4\}$ is the domain of the given function.

2. B

The ordered pairs $\{(-5,8), (-2,7), (2,-1), (5,8)\}$ is a correct representation of the mapping shown.

3. B

If point $(7,b)$ is in Quadrant I, b is positive.
If point $(a,-3)$ is in Quadrant III, a is negative.
Therefore, point (a,b) is in Quadrant II.

4. A

$f(x) = -2x+7$

To find $f(\frac{1}{2}x+3)$, substitute $\frac{1}{2}x+3$ for x, in the given function.

$f(\frac{1}{2}x+3) = -2(\frac{1}{2}x+3)+7$

$= -x-6+7 = -x+1$

5. C

$g(x) = kx^3+3$

$g(-1) = k(-1)^3+3 = 5$ $\quad g(-1)=5$

$-k+3=5$ $\quad$ Simplify.

$k=-2$ $\quad$ Solve for k.

Substitute -2 for k in the given function.

$g(x) = kx^3+3 = -2x^3+3$

$g(1) = -2(1)^3+3 = 1$

6. 8

$f(x+1) = -\frac{1}{2}x+6$

To find $f(-3)$, first solve $x+1=-3$.

$x+1=-3 \Rightarrow x=-4$.

Substitute -4 for x in the given function.

$f(-3) = -\frac{1}{2}(-4)+6 = 8$.

7. 12

$f(x) = x^2-b$

$f(-2) = 7 \Rightarrow (-2)^2-b = 7$

$\Rightarrow 4-b=7 \Rightarrow b=-3$

Therefore, $f(x) = x^2+3$.

$f(b) = f(-3) = (-3)^2+3 = 12$

Section 3-2

1. A

Rate of change $= \dfrac{\text{change in } y}{\text{change in } x} = \dfrac{-1-3}{0-(-3)} = \dfrac{-4}{3}$

2. C

Pick any two points from the table.
Let's pick $(-3,-1)$ and $(6,5)$.

Average rate of change $= \dfrac{\text{change in } y}{\text{change in } x}$

$= \dfrac{5-(-1)}{6-(-3)} = \dfrac{6}{9} = \dfrac{2}{3}$

3. D

slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{b-1}{1-a} = 1$

$\Rightarrow b-1 = 1-a \quad \Rightarrow a+b=2$

4. $\dfrac{5}{2}$ or 2.5

slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-8-2}{-1-3} = \dfrac{-10}{-4} = \dfrac{5}{2}$

5. 12

slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{r-3}{-5-4} = \dfrac{r-3}{-9} = -1$

$\Rightarrow r-3=9 \Rightarrow r=12$

6. $\dfrac{29}{2}$ or 14.5

slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{a-7}{1-a} = -\dfrac{5}{9}$

$\Rightarrow 9(a-7) = -5(1-a)$

$\Rightarrow 9a - 63 = -5 + 5a$

$\Rightarrow 4a = 58 \Rightarrow a = \dfrac{58}{4} = \dfrac{29}{2}$

7. $\dfrac{1}{4}$ or 0.25

$-x+4y=6$

Write the equation in slope-intercept form.

$-x+4y=6 \Rightarrow 4y = x+6 \Rightarrow y = \dfrac{x}{4} + \dfrac{6}{4}$

The slope of the line is $\dfrac{1}{4}$.

Section 3-3

1. C

. Since the points $(-4,2)$ and $(4,-4)$ lie on the line, the slope of the line is $\dfrac{2-(-4)}{-4-4} = \dfrac{6}{-8} = -\dfrac{3}{4}$.

If we use the point $(4,-4)$ and the slope $m = -\dfrac{3}{4}$, the point-slope form of the line is

$y-(-4) = -\dfrac{3}{4}(x-4)$ or $y+4 = -\dfrac{3}{4}(x-4)$.

If we use the point $(-4,2)$ and the slope $m = -\dfrac{3}{4}$, the point-slope form of the line is

$y-2 = -\dfrac{3}{4}(x-(-4))$ or $y-2 = -\dfrac{3}{4}(x+4)$.

Choice C is correct.

2. B

$y-2 = -\dfrac{3}{4}(x+4)$	Point-slope form of the line.
$y-2 = -\dfrac{3}{4}x - 3$	Distributive Property
$y = -\dfrac{3}{4}x - 1$	Add 2 to each side and simplify.

3. D

$y = -\dfrac{3}{4}x - 1$	Slope-intercept form
$4y = 4(-\dfrac{3}{4}x - 1)$	Multiply each side by 4.
$4y = -3x - 4$	Simplify.
$4y + 3x = -3x - 4 + 3x$	Add $3x$ to each side.
$3x + 4y = -4$	Simplify.

4. B

Average rate of change

$= \dfrac{\text{change in number of smart phones}}{\text{change in years}}$

$= \dfrac{345-120}{2010-2005} = \dfrac{225}{5} = 45$

The increase in the average number of smart phones is 45 each year.

5. D

Since the line passes through point $(4,-1)$ and has slope -2, the point-slope form of the line is $y-(-1)=-2(x-4)$.

$y+1=-2(x-4)$	Point-slope form simplified.
$y+1=-2x+8$	Distributive Property
$2x+y=7$	$2x-1$ is added to each side.

Section 3-4

1. B

Lines that are parallel have the same slope. So, we need to find the equation of a line with the slope $-\frac{1}{2}$ and the point $(-2,\frac{1}{2})$.

The point-slope form of this line is

$y-\frac{1}{2}=-\frac{1}{2}(x-(-2))$.

$y-\frac{1}{2}=-\frac{1}{2}x-1$	Simplified.
$2(y-\frac{1}{2})=2(-\frac{1}{2}x-1)$	Multiply each side by 2.
$2y-1=-x-2$	Simplify.
$x+2y=-1$	$x+1$ is added to each side.

2. D

A line parallel to the x-axis has slope 0.

$y-y_1=m(x-x_1)$	Point-slope form
$y-6=0(x-7)$	$m=0$, $x_1=7$, and $y_1=6$
$y-6=0$	Simplify.
$y=6$	

3. C

If a line is parallel to the y-axis, it is a vertical line and the equation is given in the form $x=a$, in which a is the x-coordinate of the point the line passes through. Therefore, the equation of the vertical line that passes through $(-5,1)$ is $x=-5$.

4. 14

$4x-2y=13$ can be rewritten as $y=2x-\frac{13}{2}$.

The line has slope 2. Lines that are parallel have the same slope. Therefore, $2=\frac{b-2}{5+1}$.

Solving the equation for b gives $b=14$.

5. $\frac{3}{2}$ or 1.5

Since lines ℓ and m are parallel, the two lines have the same slope. Therefore,

$\frac{0-3}{2-0}=\frac{-3-b}{-1-(-4)}$.

$\frac{-3}{2}=\frac{-3-b}{3}$	Simplified.
$-9=-6-2b$	Cross Multiplication
$-3=-2b$	Add 6 to each side.
$\frac{3}{2}=b$	Divide each side by -2.

6. 3

The slope of line t is $\frac{1-(-3)}{2-(-4)}$, or $\frac{2}{3}$. So, the slope of the line perpendicular to line t is the negative reciprocal of $\frac{2}{3}$, or $-\frac{3}{2}$. Therefore,

$-\frac{3}{2}=\frac{-2-4}{a+1} \Rightarrow -3(a+1)=2(-6)$

$\Rightarrow -3a-3=-12$

$\Rightarrow -3a=-9 \Rightarrow a=3$

Section 3-5

1. B

$y=2x+4$	First equation
$x-y=-1$	Second equation

Substituting $2x+4$ for y in the second equation gives $x-(2x+4)=-1$.

$x-(2x+4)=-1 \Rightarrow x-2x-4=-1$

$\Rightarrow -x-4=-1 \Rightarrow -x=3$ or $x=-3$

Substituting -3 for x in the first equation gives $y=2(-3)+4=-2$. Therefore, the solution (x,y) to the given system of equations is $(-3,-2)$.

2. B

$\frac{1}{2}x+y=1$	First equation
$\underline{-2x-y=5}$	Second equation
$-\frac{3}{2}x \quad =6$	Add the equations.
$-\frac{2}{3}(-\frac{3}{2}x)=-\frac{2}{3}(6)$	Multiply each side by $-\frac{2}{3}$

$x = -4$	Simplify.
$\frac{1}{2}(-4) + y = 1$	Substitute -4 for x in the first equation.
$-2 + y = 1$	Simplify.
$y = 3$	Add 2 to each side.

Therefore, $x + y = -4 + 3 = -1$

3. $\frac{4}{5}$

If a system of two linear equations has no solution, then the lines represented by the equations in the coordinate plane are parallel. So, the slopes of the line are equal.

$2x - ky = 14$	1st equation
$y = \frac{2}{k}x - \frac{14}{k}$	1st equation in slope-intercept form
$5x - 2y = 5$	2nd equation
$y = \frac{5}{2}x - \frac{5}{2}$	2nd equation in slope-intercept form

The system of equations will have no solution if $\frac{2}{k} = \frac{5}{2}$. Solving for k yields $k = \frac{4}{5}$.

If $k = \frac{4}{5}$, the y- intercept of the first equation is $-\frac{35}{2}$, and the y- intercept of the second equation is $-\frac{5}{2}$. Therefore, the lines are parallel, but not identical.

4. C

In order for a system of two linear equations to have infinitely many solutions, the two equations must be equivalent. The two equations in the answer choice A have different slopes. The two equations in the answer choice B have different y- intercepts. For answer choice C, multiply by 6 on each side of the first equation.

$6(\frac{1}{2}x - \frac{1}{3}y) = 6(1) \Rightarrow 3x - 2y = 6$.

The result is identical to the second equation. Therefore, the two equations are equivalent. The two equations in answer choice D have different slopes,

5. 1

Change the two equations into slope-intercept form.

$ax - y = 0 \Rightarrow y = ax$

$x - by = 1 \Rightarrow y = \frac{1}{b}x - \frac{1}{b}$

If $a = \frac{1}{b}$, the system of equations will have no solution. Therefore, $a \cdot b = 1$

6. $\frac{4}{3}$

In order for a system of two linear equations to have infinitely many solutions, the two equations must be equivalent. The equation $2x - \frac{1}{2}y = 15$ can be rewritten as $y = 4x - 30$ and the equation $ax - \frac{1}{3}y = 10$ can be rewritten as $y = 3ax - 30$. If two equations are equivalent, then $4x = 3ax$ or $a = \frac{4}{3}$.

Section 3-6

1. D

By definition, the absolute value of any expression is a nonnegative number. Therefore, $|1-x| + 6 > 0$, $|1-x| + 4 > 0$, and $|1-x| + 2 > 0$. Only $|1-x| - 2$ could be a negative number.

$|1-x| - 2 = -1 \Rightarrow |1-x| = 1 \Rightarrow x = 2$ or $x = 0$.

2. A

$|2x+7| = 5$

$2x + 7 = 5$ or $2x + 7 = -5$

$2x = -2$ or $2x = -12$

$x = -1$ or $x = -6$

3. A

$\lvert x-1\rvert - 1 = 1$	
$\lvert x-1\rvert = 2$	Add 1 to each side.
$x - 1 = 2$ or $x - 1 = -2$	The expression can be 2 or -2.
$x = 3$ or $x = -1$	Add 1 to each side.

4. D

The expression $|3x-5|$ is the absolute value of $3x - 5$, and the absolute value can never be a negative number. Thus $|3x-5| = -1$ has no solution

5. C

The maximum value of the function corresponds to the y- coordinate of the point on the graph, which is highest along the vertical axis. The highest point along the y- axis has coordinates $(1,4)$. Therefore, the value of x at the maximum of $f(x)$ is 1.

6. 3

$3-|3-n|=3$

$-|3-n|=0$ Subtract 3 from each side.

If $-|3-n|=0$ or $|3-n|=0$, then $3-n=0$,

Thus $n=3$.

Chapter 3 Practice Test

1. C

Use the slope formula to find the slope of the function. Since $f(x)$ is a linear function, the slope between $(-4,-4)$ and $(0,-1)$ equals the slope between $(0,-1)$ and $(6,k)$.

Therefore, $\frac{-1-(-4)}{0-(-4)}=\frac{k-(-1)}{6-0}$.

$\frac{3}{4}=\frac{k+1}{6}$ Simplify.

$4(k+1)=18$ Cross Multiplication

$4k+4=18$ Distributive Property

$4k=14$ Subtract 4 from each side.

$k=\frac{7}{2}$ or 3.5 Divide each side by 4.

2. B

The equation of the line with slope $\frac{1}{3}$ and point $(9,1)$ is $y-1=\frac{1}{3}(x-9)$ or $y=\frac{1}{3}x-2$.

The slope of the second line is $\frac{-3-4}{5-(-2)}$ or -1.

The equation of the second line is $y-4=-1(x+2)$ or $y=-x+2$. To find the point of intersection, substitute $\frac{1}{3}x-2$ for y in the second equation and solve for x.

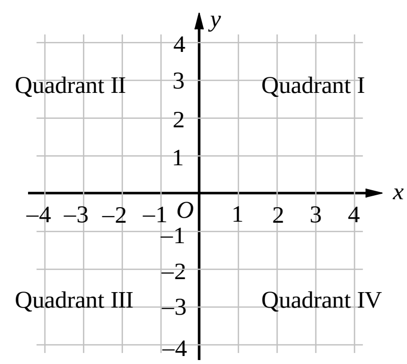
$\frac{1}{3}x-2=-x+2$

Solving for x yields $x=3$. Substituting 3 for x in the equation of the second line yields $y=-1$. Therefore, $(a,b)=(3,-1)$ and $a+b=3-1=2$.

3. C

The expressions $|x+5|$ or $|x-5|$ can never be a negative number. Thus $5+|x+5|$ or $5+|x-5|$ can not equal zero. The expression $-|x-5|$ can never be a positive number. Thus $-5-|x-5|$ can not equal zero. If $-5+|x+5|=0$, then $|x+5|=5$, when $x=0$.

4. C



If the slope of a line is positive, it is possible that the line contains no points from Quadrant II or from Quadrant IV. If the slope of a line is negative, it is possible that the line contains no points from Quadrant I or from Quadrant III. Since the line ℓ contains points from each of the Quadrants I, III, and IV, but no points from Quadrant II, the slope of line ℓ must be positive.

5. B

x	-3	-1	1	5
$f(x)$	9	5	1	-7

First, find the slope of the linear function f. We can choose any two points from the table. Let's use $(1,1)$ and $(-1,5)$ to find the slope m of f. $m=\frac{5-1}{-1-1}=\frac{4}{-2}=-2$. Thus the slope intercept form of f can be written as $f(x)=-2x+b$. From the table we know $f(1)=1$. $f(1)=-2(1)+b=1$ implies $b=3$. Thus f is defined as $f(x)=-2x+3$.

6. A

$f(x) = -6x + 1$

$f(\frac{1}{2}x - 1) = -6(\frac{1}{2}x - 1) + 1$	Substitute $\frac{1}{2}x - 1$ for x.
$= -3x + 6 + 1$	Distributive Property
$= -3x + 7$	Simplify.

7. B

Since the points $(0, 3000)$ and $(4, 2400)$ lie on the line, the slope of the line is $\frac{2400 - 3000}{4 - 0} = -150$.
The H-intercept of the line is 3,000. Therefore the relationship between H and m can be represented by $H = -150m + 3000$, the slope-intercept form of the line.

8. C

$H = -150m + 3000$	Equation of the line
$1350 = -150m + 3000$	Substitute 1350 for H.

Solving for m yields $m = 11$.

9. D

The point-slope form of the line that passes through the point $(1, -2)$ and has a slope of $\frac{1}{3}$ is $y + 2 = \frac{1}{3}(x - 1)$. The slope-intercept form of the line is $y = \frac{1}{3}x - \frac{7}{3}$. We can replace $f(x)$ for y to get the function form. Thus, $f(x) = \frac{1}{3}x - \frac{7}{3}$.

Now check each answer choices.

A) $(3, -2)$ $\quad f(3) = \frac{1}{3}(3) - \frac{7}{3} = -\frac{4}{3} \neq -2$

B) $(2, -\frac{4}{3})$ $\quad f(2) = \frac{1}{3}(2) - \frac{7}{3} = -\frac{5}{3} \neq -\frac{4}{3}$

C) $(0, -2)$ $\quad f(0) = \frac{1}{3}(0) - \frac{7}{3} = -\frac{7}{3} \neq -2$

D) $(-1, -\frac{8}{3})$ $\quad f(-1) = \frac{1}{3}(-1) - \frac{7}{3} = -\frac{8}{3}$

Choice D is correct.

10. 3

$f(x) = ax + 2$

If $f(-1) = 4$, then $f(-1) = a(-1) + 2 = 4$.

Solving for a yields $a = -2$.
Thus $f(x) = -2x + 2$ and

$f(-\frac{1}{2}) = -2(-\frac{1}{2}) + 2 = 3$.

11. 2

Use the slope formula.

Slope $= \frac{k - (-4)}{6 - 2} = \frac{3}{2}$.

$\frac{k + 4}{4} = \frac{3}{2}$	Simplify.
$2(k + 4) = 3 \cdot 4$	Cross Product
$2k + 8 = 12$	Distributive Property

Solving for k yields $k = 2$.

12. 6

$$\frac{1}{3}x - \frac{3}{4}y = -11 \quad \overset{\text{Multiply by 3}}{\Rightarrow} \quad x - \frac{9}{4}y = -33$$

$$\frac{1}{2}x + \frac{1}{6}y = -1 \quad \overset{\text{Multiply by } -2}{\Rightarrow} \quad -x - \frac{1}{3}y = 2$$

Add the equations and we get $-\frac{9}{4}y - \frac{1}{3}y = -31$.

$12(-\frac{9}{4}y - \frac{1}{3}y) = 12(-31)$	Multiply each side by 12.
$-27y - 4y = -372$	Distributive Property
$-31y = -372$	Simplify.
$\frac{-31y}{-31} = \frac{-372}{-31}$	Divide each side by -31.
$y = 12$	Simplify.
$\frac{1}{3}x - \frac{3}{4}y = -11$	First equation
$\frac{1}{3}x - \frac{3}{4}(12) = -11$	$y = 12$
$\frac{1}{3}x - 9 = -11$	Simplify.
$\frac{1}{3}x - 9 + 9 = -11 + 9$	Add 9 to each side.
$\frac{1}{3}x = -2$	Simplify.
$3(\frac{1}{3}x) = 3(-2)$	Multiply each side by -2.
$x = -6$	Simplify.

Therefore, $x + y = -6 + 12 = 6$.