Sample Paper 12

Class- X Exam - 2022-23

Mathematics - Basic

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section - A consists of 20 questions of 1 mark each.)

1. The standard form of the equation $y(2y + 15) = 3(y^2 + y + 8)$ is:

(a) $y^2 + 25y + 24 = 0$

(b) $y^2 - 27y - 23 = 0$

(c) $y^2 - 27y + 25 = 0$

(d) $y^2 - 12y + 24 = 0$ 1

2. The value of c for which pair of linear equations $cx - y = 2$ and $6x - 2y = 4$ will have infinitely many solutions is:

(a) 3 (b) 5

(c) –1 (d) 0 1

3. The distance between (7, 0) and (1, – 8) is:

(a) 7 units (b) 10 units

(c) 8 units (d) 9 units 1

4. School divides its students into 5 houses A,B, C, D and E. Class XA has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is chosen at random to become the monitor of the class. The probability that the chosen student is not from houses A, B and C is:

(a) $\frac{4}{23}$ (b) $\frac{6}{23}$

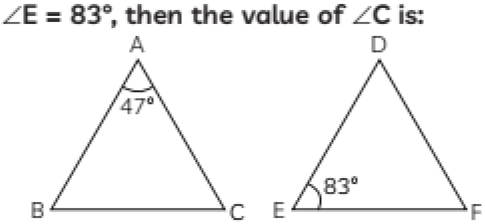
(c) $\frac{8}{23}$ (d) $\frac{17}{23}$ 1

5. A line of length 10 units has one end at the point (–3, 2). If the ordinate of the other end is 10, then the abscissa will be:

(a) 9 or 3 (b) 4 or –3

(c) –3 or 5 (d) –9 or 3 1

6. If $\Delta ABC \sim \Delta DEF$, such that $\angle A = 47°$ and $\angle E = 83°$, then the value of $\angle C$ is:



(a) 50° (b) 60°

(c) 45° (d) 90° 1

7. The zeros of $2x^2 - x - 45$ are respectively:

(a) $5, \frac{1}{2}$ (b) $2, -\frac{9}{2}$

(c) $5, -\frac{9}{2}$ (d) 5, –9 1

8. If $\cot A + \frac{1}{\cot A} = 2$, then the value of $\cot^2 A + \frac{1}{\cot^2 A}$ is:

(a) 2 (b) 4

(c) 1 (d) 5 1

9. The mean of twenty observations is 15. If the observations 3 and 14 are replaced by 8 and 5 respectively, then the new mean will be:

(a) 15.65 (b) 15.2

(c) 15 (d) 14.8 1

10. If $\sin A = \frac{1}{2}$, then the value of $(\cot A - \cos A)$ is:

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $\frac{3}{2}$ (d) $\frac{\sqrt{3}}{2}$ 1

11. From a group of 4 girls and 6 boys, a child is selected. The probability that the selected child is a girl is:

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$

(c) $\frac{4}{5}$ (d) $\frac{1}{5}$ 1

12. The perimeter of a quadrant of a circle of radius 'r' is:

(a) $\pi + 4$ (b) $\frac{r^2}{2}(\pi+2)$

(c) 0 (d) $\frac{r}{2}(\pi+4)$ 1

13. The total surface area of the hemisphere of radius 'r' is:

(a) $3\pi r^2$ (b) $5\pi r^2$

(c) $2\pi r^2$ (d) $4\pi r^2$ 1

14. What is the upper limit of the median class for the given below distribution?

Class interval	0-5	5-10	10-15	15-20	20-25
Frequency	13	10	15	8	11

(a) 14 (b) 10

(c) 15 (d) 20 1

15. Two coins are tossed simultaneously. The probability of getting at least one head is:

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$

(c) $\frac{2}{5}$ (d) $\frac{5}{4}$ 1

16. The common difference of the A.P., $\sqrt{3}, \sqrt{12}, \sqrt{27}$.................... is:

(a) $\sqrt{12}$ (b) $\sqrt{4}$

(c) $\sqrt{3}$ (d) $2\sqrt{3}$ 1

17. The nature of roots of the quadratic equation $ax^2 - 3bx - 4a = 0$ $(a \neq 0)$ is:

(a) equal

(b) real and distinct

(c) unreal and equal

(d) unreal 1

18. From a point Q, the length of the tangent to a circle is 12 cm and distance of d from the centre is 13 cm. The radius of the circle is:

(a) 4 cm (b) 6 cm

(c) 5 cm (d) 9 cm 1

Direction for questions 19 and 20: In question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct option:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. Assertion (A) : From a pack of 52 cards, the probability of drawing a red queen is $\frac{1}{20}$.

Reason (R) : Probability of occurring of an event P(A) = $\frac{\text{Favourable outcomes}}{\text{Total outcomes}}$ 1

20. Assertion (A) : The simplest form of $\frac{1095}{1168}$ is $\frac{15}{16}$.

Reason (R) : For finding the simplest form of a fraction the numerator and denominator are divided by their HCF. 1

SECTION - B **10 marks**

(Section - B consists of 5 questions of 2 marks each.)

21. Write the prime factorisation of 8190.

OR

Find the HCF of $(2^3 \times 3^2 \times 5^1)$, $(2^2 \times 3^3 \times 5^2)$ and $(2^4 \times 3^1 \times 5^2 \times 7)$. 2

22. Form a quadratic polynomial whose zeros are $3 + \sqrt{2}$ and $3 - \sqrt{2}$. 2

23. In a right angle triangle ABC, right-angled at B, if $\sin(A - C) = \frac{1}{2}$ find the measures of angles A and C.

OR

If $\sin\theta = \frac{2mn}{m^2 + n^2}$, find the value of $\frac{\sin\theta \cot\theta}{\cos\theta}$. 2

24. A path of width 7 m runs around outside a circular park whose radius is 18 m. Find the area of the path. 2

25. A die is thrown once. Find the probability of getting:

(A) a prime number greater than 3

(B) an even prime number greater than 3. 2

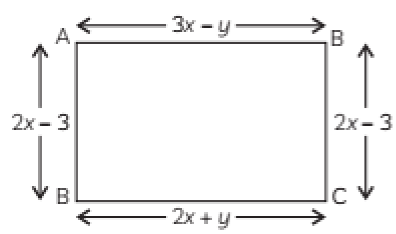
SECTION - C **18 marks**

(Section - C consists of 6 questions of 3 marks each.)

26. Prove that: $2\sqrt{3} - 4$ is an irrational number, using the fact that $\sqrt{3}$ is an irrational number.

OR

The figure shows a rectangle with its length and breadth as indicated.



Given that the perimeter of the rectangle is 120 cm, find:

(A) the values of x and y;

(B) the length and the breadth;

(C) the area of the rectangle. 3

27. If Q (0, 1) is equidistant from P (5, –3) and R (x, 6); find the values of x. Also, find the distances QR and PR.

3

28. Prove that the area of the semi-circle drawn on the hypotenuse of a right-angled triangle is equal to the sum of the area of the semi-circles drawn on the other two sides of the triangle. 3

29. If the median of the distribution given below is 28.5, find the values of x and y.

Class internal	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	20	15	y	5	60

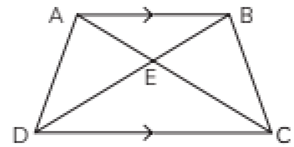
3

30. If $\sin A = m \sin B$ and $\tan A = n \tan B$ then show that $(n^2 - 1) \cos^2 A = m^2 - 1$.

OR

Find the mass of a 3.5 m long lead pipe if the external diameter of the pipe is 2.4 cm, thickness of the metal is 2 mm; and the mass of 1 cm^3 of lead is 12 grams. 3

31. ABCD is a trapezium with AB || DC. If $\triangle AED \sim \triangle BEC$, then prove that AD = BC.



3

SECTION - D

20 marks

(Section - D consists of 4 questions of 5 marks each.)

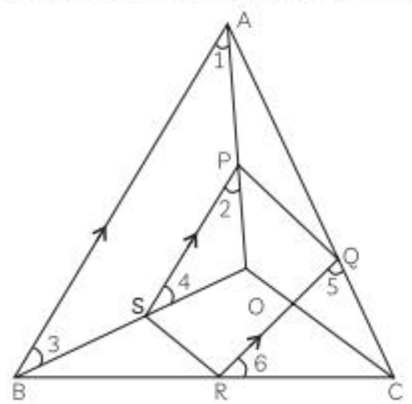
32. Solve the pair of equations graphically:
$4x - y = 5; x + y = 5$.

OR

Which term of the A.P. 3, 8, 13, 18, ... is 78? 5

33. Rahul got a total of 28 in math and science on a class quiz. The product of his scores would have been 180 if he had received 3 more in mathematics and 4 less in science. Find out what he scored in the two subjects. 5

34. A parallelogram, PQRS is inside a DABC in which AB || PS. Prove that OC || SR.



5

35. A Ferris wheel, often known as a big wheel in the UK, is a type of amusement ride that consists of a rotating, upright wheel with several passenger-carrying units or passenger cars linked to the rim, so that they remain upright as the wheel rotates. AB is a chord of the outer wheel which touches the inner wheel at P. The radius of the inner wheel = 8 m and radius of outer wheel = 10 m.

(A) Find the length of the chord AB of the outer circle.

(B) The chord AB of the inner wheel is extended to a point C. If BC = 9 m, then find distance of the point C from the centre of the wheel.

OR

The angle of elevation of the top of a tower from two points 8 m and 32 m from its base and in the same straight line with it, are complementary. Find the height of the tower. 5

SECTION - E

12 marks

(Case Study Based Questions)

(Section - E consists of 3 questions. All are compulsory.)

36. *Satellite TV manufacturing businesses tend to have what economists call "economies of scale." When economies of scale exist, bigness can be its own reward.*

The more TV's you manufacture in a single run, lower the costs per unit, which in turn increases your bottom-line margins.

Keeping that in mind, a T.V. manufacturing company increases its production uniformly by fixed number every year. The company produces 8000 sets in the 6^{th} year and 11,300 sets in the 9^{th} year.

On the basis of the above information, answer the following questions:

(A) Find the company's production of the first year.

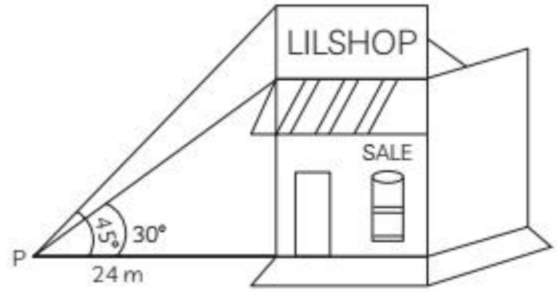
OR

In which year the company's production is 9100 sets ? 2

(B) Find the company's production of the 8^{th} year. 1

(C) Find the company's total production of the first 6 years. 1

37. *Eshan purchased a new building for her business. Being in the prime location, she decided to make some more money by putting up an advertisement sign for a rental ad income on the roof of the building.*



From a point P on the ground level, the angle of elevation of the roof of the building is 30° and the angle of elevation of the top of the sign board is 45°. The point P is at a distance of 24 m from the base of the building.

On the basis of the above information, answer the following questions:

(A) Find The height of the building (without the sign board).

OR

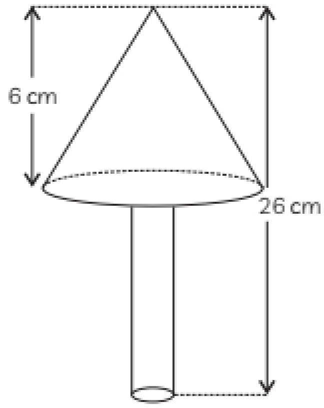
The height of the building (with the sign board). 2

(B) Find The height of the sign board. 1

(C) Find the distance of the point P from the top of the sign board. 1

38. *In a toys manufacturing company, wooden parts are assembled and painted to prepare a toy. One specific toy is in the shape of a cone mounted on a cylinder.*

For the wood processing activity center, the wood is taken out of storage to be sawed, after which it undergoes rough polishing, then is cut, drilled and has holes punched in it. It is then fine polished using sandpaper.



For the retail packaging and delivery activity center, the polished wood sub-parts are assembled together, then decorated using paint.

The total height of the toy is 26 cm and the height of its conical part is 6 cm. The diameters of the base of the conical part is 5 cm and that of the cylindrical part is 4 cm.

On the basis of the above information, answer the following questions:

(A) If its cylindrical part is to be painted yellow, then find the surface area need to be painted. 1

(B) If its conical part is to be painted green, then find the surface area need to be painted and also find the volume of the wood used in making this toy.

OR

If the cost of painting the toy is 3 paise per sq cm, then find the cost of painting the toy. (Use π = 3.14) 2

(C) The paint company gives a discount of 5% if the number of toys to be painted is 100 or above. then find the cost of painting 200 toys. 1

SOLUTION

SECTION - A

1. *(d)* $y^2 - 12y + 24 = 0$

Explanation:

$$y(2y + 15) = 3(y^2 + y + 8)$$

$$\Rightarrow \quad 2y^2 + 15y = 3y^2 + 3y + 24$$

$$\Rightarrow \quad y^2 - 12y + 24 = 0$$

2. *(a) 3*

Explanation: We have

$$cx - y = 2$$

and $6x - 2y = 4$

For infinitely many solutions

$$\frac{c}{6} = \frac{-1}{-2} = \frac{2}{4}$$

$$\frac{c}{6} = \frac{1}{2}$$

$$\Rightarrow \quad c = 3$$

⚠ **Caution**

➥ *While comparing the given equations with standard equation, we should also consider the signs of constants.*

3. *(b) 10 units*

Explanation: Distance between (7, 0) and (1, – 8)

[By distance formula]

$$= \sqrt{(1-7)^2 + (-8-0)^2}$$

$$= \sqrt{(-6)^2 + (-8)^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100} = 10 \text{ units}$$

⚠ **Caution**

➥ *The distance formula* $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. *It gives the same answer.*

4. *(b)* $\frac{6}{23}$

Explanation: Total students in class XA = 23

students from house A = 4

students from house B = 8

students from house C = 5

students from house D = 2

students from house E = 4

Total students in house D and E = 2 + 4

= 6

Required probability = $\frac{6}{23}$

5. *(d) –9 or 3*

Explanation: Let A and B denote the points (–3, 2) and (a, 10), where a is to be determined.

Here, AB = 10 units.

$$\text{So,} \quad \sqrt{(a+3)^2 + (10-2)^2} = 10$$

$$\Rightarrow \quad (a+3)^2 + 64 = 100$$

$$\text{or} \quad (a+3)^2 = 36$$

$$\Rightarrow \quad a + 3 = \pm 6$$

$$\Rightarrow \quad a = -9 \text{ or } 3.$$

6. *(a) 50°*

Explanation: Since ΔABC ~ ΔDEF.

∠A = ∠D, ∠B = ∠E and ∠C = ∠F

⇒ ∠A = 47° and ∠B = 83°

So, in ΔABC, ∠C = 180° – (∠A + ∠B)

= 180° – (47° + 83°) = 50°

7. *(c)* $5, -\frac{9}{2}$

Explanation:

$$2x^2 - x - 45 = 2x^2 - 10x + 9x - 45$$
$$= 2x(x-5) + 9(x-5)$$
$$= (2x+9)(x-5)$$

So, its zeros are 5 and $-\frac{9}{2}$.

8. *(a) 2*

Explanation:

Given, $\cot A + \frac{1}{\cot A} = 2$, we have

on squaring both sides, we get

$$\left(\cot A + \frac{1}{\cot A}\right)^2 = (2)^2$$

$$\Rightarrow \cot^2 A + \frac{1}{\cot^2 A} + 2\cot A \cdot \frac{1}{\cot A} = 4$$

$$\Rightarrow \quad \cot^2 A + \frac{1}{\cot^2 A} + 2 = 4$$

$$\Rightarrow \quad \cot^2 A + \frac{1}{\cot^2 A} = 2$$

9. *(d) 14.8*

Explanation: We know,

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Total number of observations}}$$

$$\Rightarrow \quad 15 = \frac{\text{Sum of observations}}{20}$$

⇒ Sum of observations

= 15 × 20 = 300

Since, 3 and 14 are replaced by 8 and 5

∴ New sum = 300 – (3 + 14) + (8 + 5)

= 296

∴ New mean = $\frac{296}{20}$ = 14.8

10. *(d)* $\frac{\sqrt{3}}{2}$

Explanation:

sin A = $\frac{1}{2}$ gives A = 30°

So, cot A – cos A = cot 30° – cos 30°

$$= \sqrt{3} - \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

11. *(a)* $\frac{2}{5}$

Explanation:

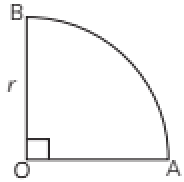
Total no. of children = 4 + 6 = 10

∴ P (a girl) = $\frac{4}{10}$ *i.e.* $\frac{2}{5}$

12. *(d)* $\frac{r}{2}(\neq + 4)$

Explanation: Perimeter of a quadrant

$$= \overline{OA} + \widehat{AB} + \overline{OB}$$



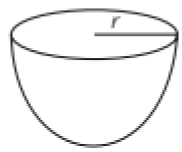
$= r + \frac{90°}{360°} \times 2\neq r + r$

$= 2r + \frac{\pi r}{2}$

$= \frac{r}{2}(\pi + 4)$

13. *(a)* $3\pi r^2$

Explanation: Total surface area of hemisphere



$= \frac{4\pi r^2}{2} + \pi r^2$

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

14. *(c) 15*

Explanation:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	13	10	15	8	11
Cumulative Frequency	13	23	38	46	57

Here, $N = 57$. So, $\frac{N}{2} = 28.5$

Cumulative frequency just greater than 28.5 is 38, which belongs to 10 – 15.

So, the median class is 10 – 15

Thus, its upper limit is 15.

15. *(a)* $\frac{3}{4}$

Explanation: Possible outcomes are {HH, HT, TH, TT}

Favourable outcomes are {HH, HT, TH}

So, required probability = $\frac{3}{4}$

16. *(c)* $\sqrt{3}$

Explanation: Here the terms of the A.P. are:

$\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$

So, the common difference $= (2\sqrt{3} - \sqrt{3})$,

$= \sqrt{3}$.

Caution

- *Common difference could be negative, positive or zero.*

17. *(b) real and distinct*

Explanation: Here, given quadratic equation is:

$ax^2 - 3bx - 4a = 0$

Discriminant D,

$b^2 - 4ac = (-3b)^2 - 4(a)(-4a)$

$= 9b^2 + 16a^2 > 0$

So, the roots are real and distinct.

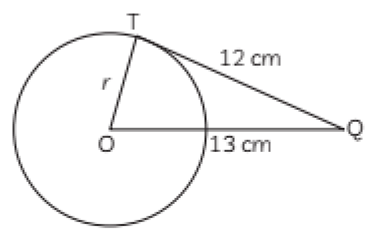
18. *(c) 5 cm*

Explanation: Let the radius of circle be '*r*'.

And, tangent is perpendicular to radius at the point of contact.

$\therefore \quad \angle OTQ = 90°$

In ΔOQT, by Pythagoras thereom,



$OQ^2 = OT^2 + QT^2$

$\Rightarrow \quad 13^2 = r^2 + 12^2$

$\Rightarrow \quad r^2 = 13^2 - 12^2$

$= (13 - 12)(13 + 12) = 25$

$\therefore \quad r = 5$ cm

19. *(d) Assertion (A) is false but reason (R) is true.*

Explanation: Red queens in pack of 52 cards = 2

Total number of cards = 52

$P(\text{red queen}) = \frac{2}{52} = \frac{1}{26}$

20. *(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)*

Explanation: Here, HCF of 1095 and 1168 is 73.

$\frac{1095 \div 73}{1168 \div 73} = \frac{15}{16}$, is the simplest form of fraction.

SECTION - B

21. The prime factorisation of 8190 is:

$$8190 = 2 \times 3 \times 3 \times 5 \times 7 \times 13.$$

2	8190
3	4095
3	1365
5	455
7	91
13	13
	1

⚠ **Caution**

➥ *While calculating prime factors, start with the lowest prime number.*

OR

The given factors are $(2^3 \times 3^2 \times 5^1)$, $(2^2 \times 3^3 \times 5^2)$ and $(2^4 \times 3^1 \times 5^2 \times 7)$

Now, HCF = Product of each prime factors with smallest power.

$$\text{HCF} = 2^2 \times 3^1 \times 5^1 \text{ i.e. } 60.$$

22. Sum of zeros $= (3 + \sqrt{2}) + (3 - \sqrt{2}) = 6$

Product of roots $= (3 + \sqrt{2})(3 - \sqrt{2})$

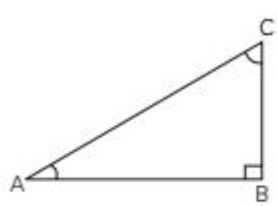
$$= 9 - 2 = 7$$

A quadratic polynomial with sum and product of zeros is given as,

x^2 – (sum of zeros) x + (Product of zeros)

i.e., $x^2 - 6x + 7$.

23. Since $\sin (A - C) = \frac{1}{2}$



$A - C = 30°$

But, $A + C = 90°$ (as, $A + B + C = 180°$)

So, $C = 30°$ and $A = 60°$

OR

Given, $\sin \theta = \frac{2mn}{m^2 + n^2}$, we have

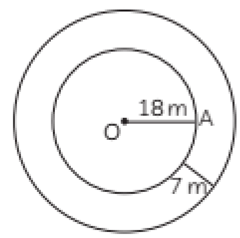
$\cos \theta = \frac{m^2 - n^2}{m^2 + n^2}$ and $\tan \theta = \frac{2mn}{m^2 - n^2}$

So, $$\frac{\sin\theta\cot\theta}{\cos\theta} = \frac{\frac{2mn}{m^2 + n^2} \times \frac{m^2 - n^2}{2mn}}{\frac{m^2 - n^2}{m^2 + n^2}}$$

$$= 1$$

24. Area of the path = Area of the outer circle – Area of the inner circle

$$= [\pi(18 + 7)^2 - \pi(18)^2] \text{ sq. m}$$



$$= (625\pi - 324\pi) \text{ sq. m}$$

$$= 301\pi \text{ sq. m, or } 946 \text{ sq. m.}$$

⚠ **Caution**

➥ *It is important to know that the area of the path = area of outer circle – area of the inner circle.*

25. (A) P (prime number greater than 3) $= \frac{1}{6}$

($\because$ Only 5 is the prime number greater than 3)

(B) P (even prime number greater than 3) $= \frac{0}{6}$, i.e., 0.

($\because$ Only even prime number is 2, which is not greater than 3)

SECTION - C

26. Let us assume on the contrary, that $2\sqrt{3} - 4$ be a rational number.

Then, $2\sqrt{3} - 4 = \frac{p}{q}$, where p and q are co-primes and $q \neq 0$.

$$\Rightarrow \sqrt{3} = \frac{1}{2}\left(\frac{p}{q} + 4\right)$$

Since p and q are integers, $\frac{1}{2}\left(\frac{p}{q} + 4\right)$ is rational and so $\sqrt{3}$ is rational. But, this contradicts the fact that $\sqrt{3}$ is irrational.

Hence, $2\sqrt{3} - 4$ is an irrational number.

OR

(A) Perimeter of the rectangle ABCD

$= AD + AB + BC + CD$

$= 3x - y + 2x - 3 + 2x + y + 2x - 3$

$= 9x - 6$

$\Rightarrow \quad 9x - 6 = 120$

$\Rightarrow \quad x = \frac{126}{9} = 14$

Also, opposite sides of a rectangle are equal. So,

$AD = BC$

$3x - y = 2x + y$

$\Rightarrow \quad x = 2y$

Thus, $\quad y = 7 \qquad$ (as $x = 14$)

(B) Length $= 3x - y = 2x + y = 35$ cm

Breadth $= 2x - 3 = 25$ cm

(C) Area of rectangle = (35 × 25) sq. cm, *i.e.* 875 sq. cm.

27. Since Q (0, 1) is equidistant from P (5, –3) and R (x, 6),

$$PQ = QR$$

$$\Rightarrow \quad PQ^2 = QR^2$$

i.e. $(5 - 0)^2 + (-3 - 1)^2 = (x - 0)^2 + (6 - 1)^2$

i.e. $\quad 25 + 16 = x^2 + 25$

i.e. $\quad x^2 = 16 \quad \text{or} \quad x = \pm 4$

Thus, R is R (4, 6) or R (–4, 6)

For R (4, 6),

$$QR = \sqrt{(4 - 0)^2 + (6 - 1)^2}$$

$$= \sqrt{16 + 25} = \sqrt{41}$$

and $\quad PR = \sqrt{(4 - 5)^2 + (6 + 3)^2}$

$$= \sqrt{1 + 81} = \sqrt{82}$$

For $\quad$ R (–4, 6),

$$QR = \sqrt{(-4 - 0)^2 + (6 - 1)^2}$$

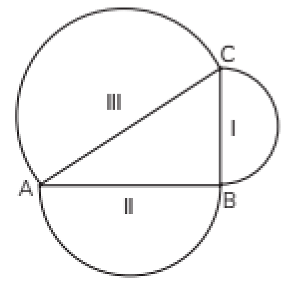
$$= \sqrt{16 + 25} = \sqrt{41}$$

$$PR = \sqrt{(-4 - 5)^2 + (6 + 3)^2}$$

$$= \sqrt{81 + 81} = \sqrt{162} = 9\sqrt{2}$$

28. We need to prove that

ar (semi-circle III) = ar (semi-circle I) + ar (semi-circle II)



Here, ΔABC is a right triangle, right-angled at B.

So, using Pythagoras Theorm,

$$AC^2 = AB^2 + BC^2 \qquad ...(1)$$

Now, ar (semi-circle I)$= \frac{\pi}{2}\left(\frac{BC}{2}\right)^2$.

$$= \frac{\neq}{8}BC^2 \qquad ...(2)$$

ar (semi-circle II)

$$= \frac{\pi}{2}\left(\frac{AB}{2}\right)^2 \cdot \text{ or } \frac{\neq}{8}AB^2 \qquad ...(3)$$

and ar (semi-circle III)

$$= \frac{\pi}{2}\left(\frac{AC}{2}\right)^2 \cdot \text{ or } \frac{\neq}{8}AC^2 \qquad ...(4)$$

Adding (2) and (3), and using (1), we have

ar (semi-circle I) + ar (semi-circle II)

$$= \frac{\pi}{8}(BC^2 + AB^2)$$

$$= \frac{\neq}{8}AC^2$$

= ar (semi-circle III)

Hence, proved.

29. With the given frequency distribution table, we first prepare a cumulative frequency distribution table as given below:

Class interval	Frequency	Cum. frequency
0-10	5	5
10-20	x	$5 + x$
20-30	20	$25 + x$
30-40	15	$40 + x$
40-50	y	$40 + x + y$
50-60	5	$45 + x + y = 60$

Since, median given is 28.5, the median class is 20-30.

For this class,

$l = 20, h = 10, \frac{n}{2} = 30, f = 20$ and $cf = 5 + x$.

According to the formula,

$$\Rightarrow \quad \text{median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$28.5 = 20 + \frac{30 - (5 + x)}{20} \times 10$$

$$\Rightarrow \quad \frac{25 - x}{2} = 8.5$$

$$\Rightarrow \quad x = 8$$

Since, $45 + x + y = 60$

$\Rightarrow \quad 45 + 8 + y = 60$

$\Rightarrow \quad y = 60 - 53 = 7$

Thus, required values of x and y are 8 and 7 respectively.

30. **Given:** $\tan A = n \tan B$

and $\sin A = m \sin B$

To Prove: $(n^2 - 1) \cos^2 A = m^2 - 1$

Proof: $\sin A = m \sin B$ (given) ...(i)

$\tan A = n \tan B$

$$\frac{\sin A}{\cos A} = n \frac{\sin B}{\cos B} \quad \text{...(ii)}$$

on substituting sin B from eq. (i)

We get

$$\Rightarrow \quad \cos B = \frac{n}{m} \cos A \quad \text{...(iii)}$$

and $\sin^2 A = m^2 \sin^2 B$

$\Rightarrow (1 - \cos^2 A) = m^2 (1 - \cos^2 B)$

substituting eq. (iii), we get

$$1 - \cos^2 A = m^2 \left(1 - \frac{n^2}{m^2} \cos^2 A\right)$$

$$\Rightarrow \quad 1 - \cos^2 A = m^2 \left(\frac{m^2 - n^2 \cos^2 A}{m^2}\right)$$

$$1 - \cos^2 A = m^2 - n^2 \cos^2 A$$

$$n^2 \cos^2 A - \cos^2 A = m^2 - 1$$

$$\cos^2 A (n^2 - 1) = m^2 - 1$$

Hence, Proved

OR

Length of pipe (h) = 3.5 m = 350 cm

External radius (R) = $\frac{2.4}{2}$ = 1.2 cm

Thickness (t) = 2 mm = 0.2 cm

Internal radius (r) = 1.2 cm – 0.2 cm

= 1.0 cm

Volume of lead in the pipe

$= \pi[(1.2)^2 - (1)^2] \times 350$ cu.cm

$= \frac{22}{7} \times (1.44 - 1) \times 350$ cu.cm

= 484 cu.cm

$\therefore$ total mass of lead = (484 × 12) grams

= 5808 grams

= 5.808 kg

31. For triangles AEB and CED, we have:

$\angle EAB = \angle ECD$ and $\angle EBA = \angle EDC$

[alternate angles as AB || DC]

$\therefore$ By AA similarly criterion, we have:

$\Delta AEB \sim \Delta CED$

$$\Rightarrow \quad \frac{AE}{CE} = \frac{AB}{CD} = \frac{EB}{ED}$$

$$\Rightarrow \quad \frac{AE}{EB} = \frac{CE}{ED} \quad \text{.....(i)}$$

It is also given that, $\Delta AED \sim \Delta BEC$

$$\text{So,} \quad \frac{AE}{BE} = \frac{ED}{EC} = \frac{AD}{BC} \quad \text{.....(ii)}$$

From (i) and (ii), we have:

$$\frac{CE}{ED} = \frac{ED}{EC}$$

$\Rightarrow \quad EC^2 = ED^2$ or $EC = ED$.

Substituting EC = ED in (ii), we have:

AD = BC

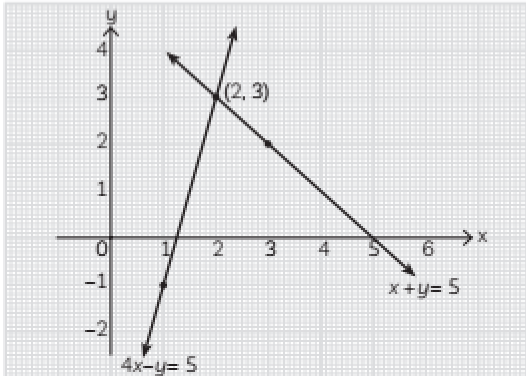
SECTION - D

32. $4x - y = 5$

x	1	2
y	–1	3

$x + y = 5$

x	3	2
y	2	3



The solution is $x = 2$, $y = 3$.

OR

Let n^{th} term of the A.P. be 78. Then,

$a_n = a + (n - 1)d = 78$

$\Rightarrow 3 + (n - 1)(5) = 78$ (Here, $a = 3$, $d = 5$)

$\Rightarrow \quad 5(n - 1) = 75$

$\Rightarrow \quad n - 1 = 15$

$\Rightarrow \quad n = 16$

So, 16^{th} term of the A.P. is 78.

33. Let Rahul has obtained 'x' marks in mathematics and 'y' marks in science.

Then by the problem,

$x + y = 28$ ...(i)

If Rahul would have got 3 marks more in mathematics, then Rahul would have got $(x + 3)$ in mathematics and if Rahul would have got marks less in science then Rahul would have got $(y - 4)$ marks in science.

A.T.Q.

$(x + 3)(y - 4) = 180$

$(x + 3)(28 - x - 4) = 180$

$\Rightarrow \quad (x + 3)(24 - x) = 180$

$\Rightarrow \quad 72 + 21x - x^2 = 180$

$\Rightarrow \quad x^2 - 21x + 108 = 0$

$\Rightarrow \quad (x - 12)(x - 9) = 0$

$\Rightarrow \quad x = 12, 9$

Then, $\quad y = 16, 19$

Hence, Rahul obtained 12 marks in mathematics and 16 marks in science.

Or Rahul obtained 9 marks in mathematics and 19 marks in science.

34. Given: PQRS is Parallelogram in which AB || PS

To Prove: OC || SR

Proof: In ΔOAB and ΔOPS

PS || AB (given ...(i)

$\therefore \quad \angle 1 = \angle 2$

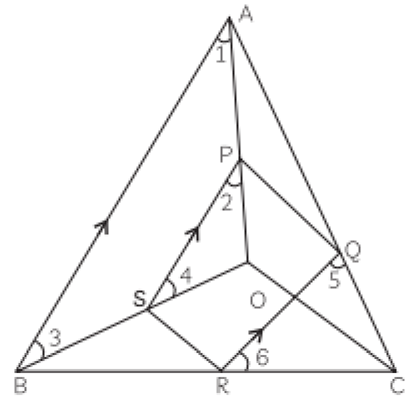
(Corresponding Pair of angles)

$\angle 3 = \angle 4$

(Corresponding Pair of angles)

∴ΔOPS ~ ΔOAB (by AA-similarity criterion)

$$\frac{OP}{OA} = \frac{OS}{OB} = \frac{PS}{AB} \quad \text{...(ii)}$$



PQRS is a parallelogram, so PS || QR ...(iii)

$\Rightarrow$ QR || AB (from (i) & (iii))

In ΔCQR and ΔCAB

$\angle 5 = \angle CAB$

(Coresponding angle)

$\angle 6 = \angle CBA$

(Coresponding angle)

∴ ΔCQR ~ ΔCAB (by AA similarity)

$$\frac{CQ}{CR} = \frac{CR}{CB} = \frac{QR}{AB}$$

(Corresponding sides of similar triangles)

∴ PS = QR

$$\therefore \quad \frac{PS}{AB} = \frac{CR}{CB} = \frac{CQ}{CA} \quad \text{....(v)}$$

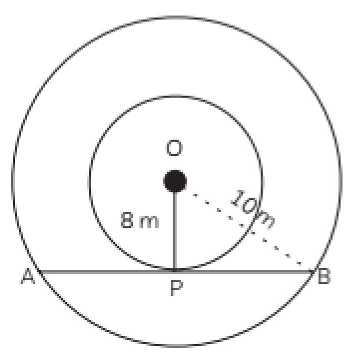
$$\Rightarrow \quad \frac{CR}{CB} = \frac{OS}{OB} \quad \text{[from (ii) and (v)]}$$

These are the rates of two sides of ΔBOC and are equal.

So, by converse of BPT, SR || OC.

Hence, Proved

35. (A) Triangle OPB is right-angled at P since OP ⊥ AB.



OP = 8 m (radius of inner wheel) and OB = 10

m (radius of outer wheel).

Therefore by Pythagoras theorem in ΔOPB,

$$OB^2 = OP^2 + BP^2$$

$$\Rightarrow \quad BP^2 = OB^2 - OP^2$$

$$= 10^2 - 8^2 = 36 = 6^2$$

$$\Rightarrow \quad BP = 6 \text{ m}$$

$$AP = BP = \frac{1}{2} AB$$

So, $AB = 2PB$

$= 2 \times 6$

$= 12$ m

(B) As BC = 9 m

$\therefore \quad PC = PB + BC$

$= 6 + 9 = 15$ m

Triangle OPC is right angled at P.

Therefore, applying Pythagoras theorem, we get

$$OC^2 = OP^2 + PC^2$$

$$= 8^2 + 15^2$$

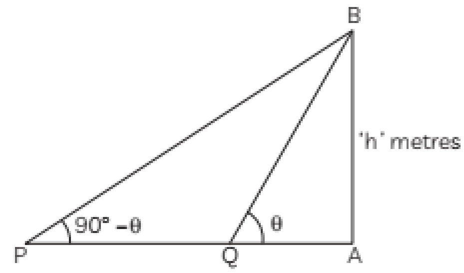
$$= 64 + 225 = 289$$

$$\Rightarrow \quad OC = 17 \text{ m.}$$

OR

Let AB represents the tower and points P and Q be two points at a distances of 32 m and 8 m from the base A of the tower, respectively.

$\therefore \quad AQ = 8$ and $AP = 32$ m



Let 'h' metres be the height of the tower.

Also, let $\angle AQB = \theta$

Then, $\angle APB = 90° - \theta$

[$\because$ $\angle P$ and $\angle Q$ are comptementary *i.e.*, their sum is 90°]

In ΔQAB,

$$\frac{AB}{AQ} = \tan\theta, \text{ or } AB = 8\tan\theta \quad ...(i)$$

In ΔPAB,

$$\frac{AB}{AP} = \tan(90° - \theta) = \cot\theta$$

or, $AB = 32 \cot\theta$ ...(ii)

From eqn. (i) and (ii) [$\because$ $\tan\theta . \cot\theta = 1$]

$$AB . AB = 8\tan\theta \times 32\cot\theta$$

$$\Rightarrow \quad (AB)^2 = 256$$

$$\Rightarrow \quad AB = 16$$

Thus, the height of the tower is 16 metres.

SECTION - E

36. (A) Given $a_6 = 8000$

$a_9 = 11{,}300$

Let, the first term be 'a' and common difference be 'd'.

Then, $a + (6 - 1)d = 8000$

$\Rightarrow \quad a + 5d = 8{,}000$ ...(i)

and $\quad a + 8d = 11{,}300$ ...(ii)

On solving (i) and (ii) we get

$d = 1{,}100$

$\Rightarrow \quad a = 8000 - 5 \times 1100$

$= 2500$

OR

Let, the year in which production is 9100 be 'n'

Then, $a_n = a + (n - 1)d$

$9100 = 2500 + (n - 1) \times 1100$

$\Rightarrow (n - 1) \times 100 = 6600$

$\Rightarrow \quad n - 1 = 6$

$\Rightarrow \quad n = 7$

(B) Since, $a = 2500$ and $d = 1100$

$\therefore \quad a_8 = a + (8 - 1)d$

$= 2500 + 7 \times 1100$

$= 2500 + 7700$

$= 10{,}200$

(C) Production in 6^{th} year = 8,000

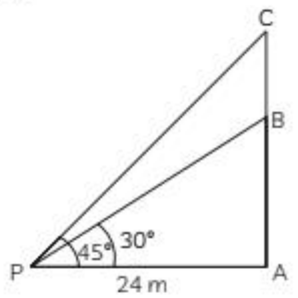
$$\therefore \quad S_n = \frac{n}{2}[a + l] = \frac{6}{2}[2500 + 8000]$$

$= 3 \times 10500$

$= 31{,}500$

37. (A) Without the sign board, the height of the shop is AB.

In ΔPAB,



$$\tan 30° = \frac{AB}{PA}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{24}$$

$$AB = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= 8\sqrt{3} \text{ m} = 13.85 \simeq 14 \text{ m}$$

Caution

- *In solving word problems, drawing of correct figure is very important, otherwise the answers obtained will be wrong.*

OR

Considering, the diagram in the above question, AC as the new height of the shop including the sign-board.

$\therefore$ In ΔAPC,

$$\tan 45° = \frac{AC}{AP}$$

$$1 = \frac{AC}{24}$$

$\Rightarrow \quad AC = 24$ m

(B) Length of sign board, BC = AC – AB

= 24 – 14

= 10 m

(C) In ΔAPC,

$$\cos 45° = \frac{AP}{AC} \Rightarrow \frac{1}{\sqrt{2}} = \frac{24}{PC}$$

$\Rightarrow \quad PC = 24\sqrt{2}$ m

38. (A) C.S.A. of cylinder $= 2\pi rH + \pi r^2$

$= \pi r(2H + r)$

$= 2\pi\ (2 \times 20 + 2)$

$[\because H = 26 - 6 = 20]$

$= 84\pi$

(B) C.S.A. of cone $= \pi rl + \pi(R^2 - r^2)$

$$= \pi\left[r\sqrt{h^2 + r^2} + (R^2 - r^2)\right]$$

$$= \pi\left[2.5\sqrt{2.5^2 + 6^2} + (2.5^2 - 2^2)\right]$$

$= \pi\ [2.5 \times 6.5 + 0.5 \times 4.5]$

$= \pi\ [16.25 + 2.23]$

$= 18.5\pi$ sq units

Volume of toy

= Volume of cone + Volume of cylinder

$$= \frac{1}{3}\pi r^2 h + \pi R^2 H$$

$$= \pi\left[\frac{1}{3} \times 2.5 \times 2.5 \times 6 + 2 \times 2 \times 20\right]$$

$= \pi\ [12.5 + 80]$

$= 92.5\pi\ cm^3$

OR

Surface area

= S.A. of cone + S. A. of cylinder

$= 84\pi + 18.5\pi$

$= 102.5\pi$

$\therefore$ cost of painting

$= 0.03 \times 102.5\pi$

= ₹ 9.65

(C) Cost of painting 200 toys

$= 9.65 \times 200$

= ₹ 1930

$\therefore$ Discount $= \frac{5}{100} \times 1930$

= 96.50

$\therefore$ cost = 1930 – 96.50

= ₹ 1833.5