28. Heat Transfer

Short Answer

1. Question

The heat current is written as $\frac{\Delta Q}{\Delta t}$. Why don't we written $\frac{dQ}{dt}$?

Answer

The term $\frac{\Delta Q}{\Delta t}$ represents the partial derivative of Q (or the amount of heat transfer) with respect to time, while $\frac{dQ}{dt}$ represents the absolute derivative of Q with respect to time. The the amount of heat transfer (Q) depends not only on time but also on the factors such as the area of cross-section, temperature difference, etc. So, we cannot write it as a complete derivative of time only. Hence the better way of representation of heat current is as a partial derivative or in the given form, which is, $\frac{\Delta Q}{\Delta t}$.

2. Question

Does a body at 20°C radiate in a room, where the room temperature is 30°C? If yes, why does its temperature not fall further?

Answer

Yes, the body will radiate at 20°C. Every entity with a temperature greater than 0K will radiate. In this case, as the room temperature(30°C) is greater than the body temperature, the temperature of the body would not fall further. Since the surrounding is warmer, the rate of absorption of radiation of the body from the surrounding will be comparatively greater than the thermal loss of the body by its own radiation. As a result, the temperature of the body will does not further fall.

3. Question

Why does blowing over a spoonful of hot tea cools it? Does evaporation play a role? Does radiation play a role?

Answer

The cooling down of tea is facilitated by Convection more than evaporation or radiation. Since the tea is a hot body, it does radiate and evaporate. But when we blow over the hot tea, convection plays the major role in cooling the tea down.

When we blow, the cool air of wind will replace the hot air from the upper part of the hot tea. This cool air will go down near to the tea because of the higher density compared to hot air. This will eventually make the tea cool down faster.

4. Question

On a hot summer day we want to cool our room by opening the refrigerator door and closing all the windows and doors. Will the process work?

Answer

No, This will rather cause an increase in room temperature than decreasing it.

The refrigerator is a device which takes the heat from the refrigerator cabin and expels it outside the fridge, with the help of external work. If we keep the refrigerator door opened while keeping the windows and doors closed, the refrigerator will take in the heat from the room and expel it into the room itself. Since no instrument can work with 100% efficiency, the loss in the electrical energy of the fridge will dissipate as heat, and this along with the heat after refrigeration, will be expelled into the room itself. Hence, in a room with an opened fridge door, it will expel more heat than it takes in from the room.

Since all the windows of the room are closed, opening the refrigerator door will result in an increase in the room temperature instead of decreasing it.

5. Question

On a cold winter night you are asked to sit on a chair. Would you like to choose a metal chair or a wooden chair? Both are kept in the same lawn and are at the same temperature.

Answer

It would be better to choose the wooden chair over the metal chair, on a winter night. This is because of the fact that wooden chair transfers less heat into the human body than the metal chair. These behaviors can be quantified by the difference in the conductivity between the two materials of the chairs. Metal has more thermal conductivity than wood. Hence metal chair will tend to transfer more heat or cold to one's body while in contact with it.

This can also be explained by the concept of specific heat capacity. The specific heat capacity of metal is lower than that of wood. Hence, it is easier to increase the temperature of the metal, by one degree, compared to that of wood. So, when the human body comes in contact with a metal and a wood, both having the same lower temperature than the human skin, the metal will absorb heat at a faster rate than wood. This will result in the cooling of the human body. Since the metal chair with a lower temperature has a faster rate of heat transfer, the wooden chair will be comfortable on a winter day.

So, on a winter day, it is preferable to sit in the wooden chair, which transfers comparatively less cold, hence helps to avoid the high cold of the surroundings.

6. Question

Two identical metal balls one at $T1 = 300$ K and the other at $T2 = 600$ K are kept at a distance of 1m in vacuum. Will the temperatures equalize by radiation? Will the rate of heat gained by the colder sphere be proportional to $T_2^4 - T_1^4$ as may be expected from the Stefan's law?

Answer

Yes, both the metal balls will radiate since their temperature is greater than 0 K. The radiation heat transfer is irrespective the surrounding medium.

The thermal radiation from a body depends on its temperature, according to Stefan–Boltzmann law. The law states that the radiation heat transfer from a black body is proportional to the fourth power of its absolute temperature (T).

Or,

$$Q = \sigma A T^4$$

Where,

$Q =$ Radiation heat transfer

$\sigma =$ Stefan–Boltzmann constant$=$ 5.67 x 10^{-8} W/m^2 k^4

$A =$Area of the black body

Hence, the rate of radiation heat loss from the body at 300K will be less compared to the rate of radiation heat loss from the body at 600K. Since the bodies are in vicinity of each other and the radiation is happening in all the directions, the interactions of thermal radiation can cause the equalization of the temperatures on the balls.

Also, the heat loss from the hotter ball to the cooler ball or the heat gained by the cooler ball will be proportional to $T_2^4 - T_1^4$ according to Stefan's law.

7. Question

An ordinary electric fan does not cool the air, still it gives comfort in summer. Explain.

Answer

An electric fan does not cool the air. It just creates a forced circulation of the air present inside the room. This circulation will facilitate the evaporation of sweat, that is present on the surface of one's skin. The heat for the evaporation is taken from the skin itself, and this will cause a sense of cooling.

Hence, on a summer day, even though the air is not getting cooled by the fan, the evaporation of the sweat will create comfort for the body.

8. Question

The temperature of the atmosphere at a high altitude is around 500°C. Yet an animal there would freeze to death and not boil. Explain.

Answer

At high altitude, even though the temperature is higher, the density of the air surrounding an animal is very low. This will cause a drastic decrease in the conduction mode of heat transfer, as the heat transfer occurs due to the close interaction of the molecules. So, there is very less amount of molecule in the atmosphere to transfer the heat to the animal's body. But, the animal skin contains water on the surface and the boiling point of this water will get decreased drastically due to the extreme drop of pressure in the higher altitude; and thus, water will take the heat from inside the body, instead of taking it from the atmosphere. This phenomenon will cause to drop the temperature inside the animal body.

Because of this lack of heat transfer from the surrounding air, the animal body will get freezed instead of getting heated.

9. Question

Standing in the sun is more pleasant on a cold winter day than standing in shade. Is the temperature of air in the sun considerably higher than that of the air in shade?

Answer

Yes. The temperature in an open place will be higher than that of a shady place due to the presence of more number of heated molecule in the former case.

Human body needs thermal comfort despite the surrounding condition. On a cold environment, human body needs to transfer less amount of heat compared to that in a normal environment.

The temperature of the surrounding is lower than the usual temperature on a winter day, and human body will transfer more heat to surrounding by radiation, convection and conduction*, causing a drop in the temperature of the outer skin. In an open space, where there is more radiation from sun, the temperature of the surrounding will be comparatively higher than a shady place. So, the heat transfer from human body will be lower in the open space since all the above modes of heat transfers are directly proportional to the Temperature difference between human body and the surroundings.

In the case of shady place, the temperature of the surroundings will be comparatively lower due to 2 reasons;

1. There is lesser availability of direct radiation from the sun

2. The phenomenon of transpiration** from the leaves of the trees will give a cooling effect.

So, there will be a higher amount of heat transfer from human skin to the colder surroundings.

To conclude, on a winter day, a hot environment will be convenient in order to reduce the heat transfer from the body.

*There are other types of heat transfer such as the heat transfer caused by Breathing, etc.

**Transpiration is the process of removal of water from leaves through the pores called Stomata. The evaporation of this will cause a cooling effect.

10. Question

Cloudy nights are warmer than the nights with clean sky. Explain.

Answer

The temperature of the environment is warm when the air gas molecules get warm. The accumulated heat from the sun, mainly by radiation, on the day times are emitted back from the earth at nights.

The earth's surface radiate energy in the form of Infrared radiation at night. This will attribute to the heating of air molecules in the environment. However, the heat from the radiation is usually gone into space; But if the clouds are present in the sky, they will trap the infrared radiations and prevent the heat from leaving the environment. As clouds are a form of water vapors, they have a tendency to absorb more heat compared to air molecules. This heat will be emitted back to the earth surface from the clouds, which causes the temperature to rise further. While in the case of clear sky, there is nothing to prevent radiation from escaping from the environment.

11. Question

Why is a white dress more comfortable than a dark dress in summer?

Answer

The comfort of wearing a white dress over a dark colored dress on a summer day is due to the difference in absorption of the sun's light by different colors.

The light is an electromagnetic wave, that contains mass and energy. Based on the absorption of certain wavelengths of light energy source like Sun, objects will exhibit its color. A darker object is the one which absorbs all the colors (the wavelengths in the visible spectrum) of the light and a white object will reflect all the colors of the light.

Hence, a dark-colored dress will absorb all the visible wavelengths from the sun and this energy will get converted into heat energy. While in the case of a white dress, all the visible wavelengths are reflected from the dress causing effectively no absorption (in the ideal cases).

So, in the summer, as the radiation from the sun is higher, wearing a white dress will help to not absorb the energy but to reflect it, while a darker dress will absorb most of the radiation. Hence it would be more comfortable in a white dress on a summer day.

Objective I

1. Question

The thermal conductivity of a rod depends on

A. length

B. massC. area of cross section

D. material of the rod.

Answer

The Thermal conductivity of rod depends on the material of the rod. Heat transfer is due to free electrons and conductors like metals have electrons in their outer shell so as to move freely along the road. If the rod is made up of non-conducting material then the transfer of heat would not pe possible as they don't have free electrons .Thermal conductivity does not depend on mass as the mass of the rod remains the same and it doesn't affect the number of free electrons in the rod.Thermal conductivity does not depend on length and area of cross section as thermal conductivity is the **ability** of a material to conduct heat and the dimensions of the material does not affect its ability.Thus, option (d) is the correct option.

2. Question

In a room containing air, heat can go from one place to another

A. by conduction only

B. by convection only

C. by radiation only

D. by all the three modes

Answer

Transfer of heat due to conduction is the molecular vibrations which does not involve mass movement. In the room, heat conduction is possible due the molecular collision of air molecules and other molecules present.Transfer of heat due to convection is due to actual movement of heated material or molecules. In the room, the movement of air molecules from one place to another can result in heat transfer.Transfer of heat due to Radiation does not need any medium or material. So, in a room containing air energy is radiated by every body or object, thus heat transfer is possible.Thus, option (d) is the correct option.

3. Question

A solid at temperature T_1 is kept in an evacuated chamber at temperature $T_2 > T_1$. The rate of increase of temperature of the body is proportional to

A. $T_2 – T_1$

B. $T_2^2 – T_1^2$C. $T_2^3 - T_1^3$

D. $T_2^4 - T_1^4$

Answer

Stefan- Boltzmann Law is used.According to the law, the energy of thermal radiation emitted per unit time by a body having surface area A is given as: $u = e\sigma AT^4$Here, e is the emissivity of the body and σ is the Stefan-Boltzmann constant and T is the Temperature.As $T_1 < T_2$, temperature of solid will increase.Now, for the solid the energy of thermal radiation is:$u_1 = e\sigma AT_1^4$Energy of the thermal radiation in evacuated chamber is:$u_2 = e\sigma AT_2^4$Now, the net difference in energy is $u_2 - u_1$ .Hence the net energy difference is proportional to: $u_2 - u_1 \; \alpha \; T_2^4 - T_1^4$Hence, The rate of increase of temperature of the body is proportional to $T_2^4 - T_1^4$ .For options (a), (b) and (c) the power od temperature difference is not 4. Hence, they are incorrect.Thus, option (d) is the correct option.

4. Question

The thermal radiation emitted by a body in proportional of T^n where T is its absolute temperature. The value of n is exactly 4 for

A. a blackbody

B. all bodies

C. bodies painted black only

D. polished bodies only

Answer

The thermal radiation emitted by a body in proportional of T^n where T is its absolute temperature. The value of n is exactly 4 for **all bodies.**According to Stefan-Boltzmann Law:The energy of thermal radiation emitted per unit time by a body having surface area A is given as:$u = e\sigma AT^4$Here, e (between 0 to 1) is the emissivity of the body and σ is the Stefan-Boltzmann constant and T is the Temperature.For black body, e=1.Thus, option (b) is the correct option.

5. Question

Two bodies A and B having equal surface areas are maintained at temperatures 10°C and 20°C. The thermal radiation emitted in a given time by A and B are in the ratio

A. 1: 1.15

B. 1: 2

C. 1: 4

D. 1: 16

Answer

The thermal radiation emitted in a given time by A and B are in the ratio **1: 1.15.**Stefan-Boltzmann Law is given as:The energy of thermal radiation emitted per unit time by a body having surface area A is given as: $\boldsymbol{u = e\sigma A T^4}$ Here, e (between 0 to 1) is the emissivity of the body and σ is the Stefan-Boltzmann constant and T is the Temperature.Now the temperature should be in Kelvin.For body A : $u_A = e\sigma A T_A^4$ For body B : $u_B = e\sigma A T_B^4$ Here T_A = 273 + 10 ° C = 283 K.T_B = 273 + 20° C= 293 KSubstituting we get, $\frac{u_A}{u_B} = \frac{e\sigma A(283)^4}{e\sigma A(293)^4} \therefore \frac{u_A}{u_B} = 0.8703$ Out of all the options only 1:1.15 gives answer close to 0.8703 hence, $\therefore$ $\boldsymbol{u_A : u_B = 1 : 1.15}$ Thus option (a) is the correct option.

6. Question

One end of a metal rod is kept in a furnace. In steady state, the temperature of the rod.

A. increases

B. decreases

C. remains constant

D. is nonuniform

Answer

In steady state, the temperature at any point of the material remains unchanged as time passes. Hence the end near the furnace will be extremely hot whereas the other end will have minimum temperature. Thus, showcasing nonuniformity in the temperature throughout the metal rod.In steady state there is no change in temperature hence first two options are not valid.The temperature us not constant throughout the rod, it's different at different points. Hence third option is incorrect too.Thus option (d) is the correct option.

7. Question

Newton's law of cooling is a special case of

A. Wien's displacement law

B. Kirchhoff's law

C. Stefan's law

D. Planck's law

Answer

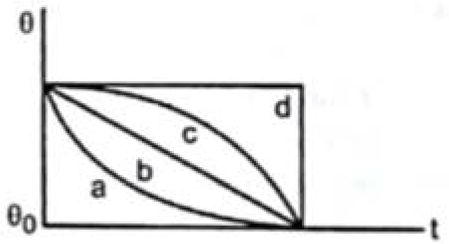
Newton's law of cooling is a special case of **Stefan's law**.Stefan's Law states that:The energy of thermal radiation emitted per unit time by a body having surface area A is given as: $\boldsymbol{u = e\sigma A T^4}$ Now, the energy absorbed by a body due to radiations emitted by the walls per unit time is given as: $u_0 = e\sigma A T_0^4$ Here T_0 is the temperature of the surrounding.Now the net rate of loss of energy of the body due to thermal radiation is: $u - u_0 = e\sigma A T^4 - e\sigma A T_0^4 \therefore \Delta u_1 = e\sigma A(T^4 - T_0^4)$ If the temperature difference is small, we can write $T = T_0 + \Delta T$

$\therefore T^4 - T_0^4 = (T_0 + \Delta T)^4 - T_0^4 \therefore\ T^4 - T_0^4 = T_0^4\left(1 + \left(\frac{\Delta T}{T_0}\right)\right)^4 - T_0^4$Using Binomial Expansion for the forth power we get and using equation relating heat and specific heat $Q = msdT$ we get,

$\therefore\ -\frac{dT}{dt} = bA(T - T_0)$For a small difference in temperature of the body and the surroundings, the rate of cooling is directly proportional to the Area exposed of the body and the temperature difference.It can also be written as,$\frac{dT}{dt} = -bA(T - T_0)$ This is Newton's Law of Cooling. b is a constant and depends on the nature of the surface and surrounding conditions.Negative sign shows that the temperature decrease with time.Hence. We started off with Stefan's Law and Ended with Newton's Law of cooling.Thus, option (c) is the correct option.

8. Question

A hot liquid is kept in a big room. Its temperature is plotted as a function of time. Which of the following curves may represent the plot?



Answer

The temperature of the hot liquid will decrease with time. The walls of the room will absorb the emitted radiation from the liquid. Since the room is big the temperature difference between emitted radiation and absorbed radiation is large.According to Stefan's Law, the energy of thermal radiation emitted per unit time is directly proportional to T^4. Thus, decrease in temperature with time will also be proportional to T^4. Thus the curve must be exponentially decreasing curve which is curve **a**.Thus, curve a represents the plot.

9. Question

A hot liquid is kept in a big room. The logarithm of the numerical value of the temperature difference between the liquid and the room is plotted against time. The plot will be very nearly.

A. a straight line

B. a circular arc

C. a parabola

D. an ellipse

Answer

When a hot liquid is kept in a big room, the temperature of the hot liquid will decrease with time. The room is big hence the difference between emitted radiation and absorbed radiation by the walls would be large. According to Stefan's Law, the energy of thermal radiation emitted per unit time is directly proportional to T^4. The Temperature would show an exponentially decreasing behavior against time. After a sometime, temperature of the liquid becomes equal to the temperature of the surrounding resulting in zero temperature difference. When logarithm of temperature difference is taken, the fourth power becomes coefficient as per the logarithmic properties,$\boldsymbol{log(T-T_0)^4 = 4log(T-T_0)}$Here T is the emitted radiation and T_0 is the absorbed radiation. Thus, the logarithmic plot would be a straight line as logarithmic graphs are nonlinear variables converted to linear graphs.Thus, option (a) is the correct option.

10. Question

A body cools down from 65°C to 60°C in 5 minutes. It will cool down from 60°C to 55°C in

A. 5 minutes

B. less than 5 minutes

C. more than 5 minutes

D. less than or more than 5 minutes depending on whether its mass is more than or less than 1 kg.

Answer

According to Newton's Law of Cooling:$\frac{dT}{dt} = -bA(T-T_0)$Here $\frac{dT}{dt}$ is the rate of fall of temperature is the constant depending on nature of surface and surrounding conditions, A is the area of cross-section of the body, T is the average Temperature of the body and T_0 is the surrounding temperature.It can also be written in Celsius scale as$\frac{d\theta}{dt} = -k(\theta-\theta_0)$Where θ and θ_0 are the temperature of the body and surrounding in Celsius scale respectively , k is a constant.First condition:Temperature falls from 65° C to 60° C.Which means 5°C falls in 5 minutes,∴ $\frac{d\theta}{dt} = \frac{5\ °C}{5\ min} = 1°C\ \ min^{-1}$ θ = $\frac{65+60}{2}$ = 62.5 ° CSubstituting we get,
$1°C\ \ min^{-1} = -k(\,62.5-\theta_0)°C \therefore -k = \frac{1}{62.5-\theta_0}\ \ min^{-1}$ Second Condition:Temperature further falls from 60 °C to 55 °C.∴ dθ= 5 °C.θ = $\frac{60+55}{2} = 57.5\ °C$ Substituting value of -k,dθ and θ in the equation we get,
$\therefore \frac{5\ °C}{t\ min} = \left(\frac{1}{62.5-\theta_0}\ \ min^{-1}\right) \times (57.5-\theta_0)°C \therefore t = \frac{5 \times (\,62.5-\theta_0)}{(57.5-\theta_0)}$ Here we can conclude that time required for the body to go from 60 ° C to 55° C will be, $\boldsymbol{\therefore t > 5\ min}$Thus, option (c) is the correct option.

Objective II

1. Question

One end of a metal rod is dipped in boiling water and the other is dipped in melting ice.

A. All parts of the rod are in thermal equilibrium with each other.

B. We can assign a temperature to the rod.

C. We can assign a temperature to the rod after steady state is reached.

D. The state of the rod does not change after steady state is reached.

Answer

When one end of the metal rod is dipped in boiling water, the temperature at that end would be maximum. When the other end of the rod is dipped in melting ice, the temperature of that end would be minimum. Due to conduction, the temperature of the rod starts to increase from maximum temperature end to minimum temperature end. After some time, equilibrium is established with the surrounding.When Steady state is reached, the temperature throughout the rod is nonuniform and constant. Temperature will be different at different points on the rod. Due to equilibrium the state of the rod does not change.Thus, option (d) is the correct option.

2. Question

A blackbody does not

A. emit radiation

B. absorb radiation

C. reflect radiation

D. refract radiation

Answer

A blackbody is a body which absorbs all incident radiation falling on it and the radiation emitted by it is called as Blackbody Radiation.A 100% Blackbody doesn't Reflect as it absorbs all the incident radiation neither does it refracts radiation as it does not have refractive properties.Lampblack is close to a black body, it reflects about 1% of the incident radiationThus, options (c) and (d) are correct options.

3. Question

In summer, a mild wind is often found on the shore of a calm river. This is caused due to

A. difference in thermal conductivity of water and soil

B. convection currents

C. conduction between air and the soil

D. radiation from the soil

Answer

In summer, the shore is warmer than the river. Hence air above the shore is at high temperature compared to the temperature of the air above of the river. The warmer air particles flow from river to the shore due to convection. Therefore, there is a mild wind at the shore of a calm river. Whereas during winters it's opposite. The temperature of the air above the river is warmer compared to that of air above land. Thus, convection current flows from land to river.Thus, option (b) is the correct option.

4. Question

A piece of charcoal and a piece of shining steel of the same surface area are kept for a long time in an open lawn in bright sun.

A. The steel will absorb more heat than the charcoal.

B. The temperature of the steel will be higher than that of the charcoal.

C. If both are picked up by bare hands, the steel will be felt hotter than the charcoal.

D. If the two are picked up from the lawn and kept in a cold chamber, the charcoal will lose heat at a faster rate than the steel.

Answer

Steel is a good conductor of heat whereas charcoal isn't. So, the sunlight falling on the steel surface will make the surface hot and since charcoal is an absorber the incident sunlight would be absorbed by the charcoal. Since charcoal is bad conductor of heat, it won't be too hot. Therefore, if both are picked up by bare hands, the steel will be felt hotter than the charcoal.Secondly, Blackbodies are good absorbers of radiation and hence good emitters of radiation. Charcoal is a black body and thus it is a good emitter compared to steel. Hence, If the two are picked up from the lawn and kept in a cold chamber, the charcoal will lose heat at a faster rate than the steel.Option (a) is incorrect as steel is a bad absorber of heat compared to charcoal as charcoal is a blackbody.Option (b) is incorrect as temperature is the intensity of heat present in the substance. Steel is hot at the surface but charcoal has more heat intensity absorbed within it.Thus, options (c) and (d) are the correct options.

5. Question

A heated body emits radiation which has maximum intensity near the frequency v_0. The emissivity of the material is 0.5. If the absolute temperature of the body is doubled.

A. the maximum intensity of radiation will be near the frequency $2v_0$.

B. the maximum intensity of radiation will be bear the frequency $v_0/2$

C. the total energy emitted will increase by a factor of 16.

D. the total energy emitted will increase by a factor of 8.

Answer

According to Wein's Displacement Law:The wavelength of the peak of the blackbody radiation is inversely proportion to the absolute temperature of the emitter.$\lambda_m T = b$Here, λ_m is the wavelength at maximum intensity, T is the absolute temperature in kelvins and b is the Wein constant.Also,$\lambda = \frac{c}{v}$Where c is speed of light and v is the frequency.Now when the absolute temperature of the body is doubled,T becomes 2T. Since LHS is equal to a constant, the RHS of the Wein's displacement law must be doubled too.This becomes,$c \times 2T = 2v_0 \times b$ Here v_0 is the frequency at maximum intensity. As we can see doubling absolute temperature doubles frequency too.Hence option (a) is correct.Now, According to Stefan's Law, the rate of emission of radiated energy is proportional to the fourth power of absolute Temperature,$u\ \alpha\ (2T)^4 \therefore \boldsymbol{u\ \alpha\ 16T^4}$where u is the radiated energy. Thus, the total energy emitted will increase by a factor of 16.Thus, options (a) and (c) are the correct options.

6. Question

A solid sphere and a hollow sphere of the same material and of equal radii are heated to the same temperature.

A. Both will emit equal amount of radiation per unit time in the beginning.

B. Both will absorb equal amount of radiation from the surrounding in the beginning.

C. The initial rate of cooling (dT/dt) will be the same for the two spheres.

D. The two spheres will have equal temperatures at any instant.

Answer

According to Stefan- Boltzmann Law:The energy of thermal radiation emitted per unit time by a body having surface area A is given as:$\boldsymbol{u = e\sigma AT^4}$Here, e (between 0 to 1) is the emissivity of the body and σ is the Stefan-Boltzmann constant and T is the Temperature.Since the spheres have equal radii and are of same material, the area of cross section would be same for both the spheres.Thus Stefan's Law will hold true for both the spheres in the exact same way. Hence, both will emit equal amount of radiation per unit time in the beginning.The energy of radiation absorbed by the spheres should be equal to the energy of radiation emitted by them. Applying same Stefan-BoltzmannLaw we get,$u_0 = eAT_0^4$Where u_0 is the energy of radiation absorbed and T_0 is the initial surrounding temperature.Again due to equal Area of cross section and same material, both will absorb equal amount of radiation from the surrounding in the beginning.Thus, options (a) and (b) are the correct options.

Exercises

1. Question

A uniform slab of dimension 10 cm × 10 cm × 1 cm is kept between two heat reservoirs at temperatures 10°C and 90°C. The larger surface areas touch the reservoirs. The thermal conductivity of the material is 0.80 W m^{-1} $°C^{-1}$. Find the amount of heat flowing through the slab per minute.

Answer

Given: Area of the uniform slab: A = 10× 10 cm^2 = 100 cm^2 = 0.01 m^2Height of the slab : x = 1 cm = 0.01 mTemperature difference of two heat reservoirs:ΔT = 90-10=80° CThe thermal conductivity of the material: K=0.80 W m^{-1} $°C^{-1}$.**Formula used:**Rate of amount of heat flowing is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred , ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness of the material.Substituting we get,$\frac{\Delta\theta}{\Delta t} = \frac{0.80 \times 0.01 \times 80}{0.01}$

$\therefore \frac{\Delta\theta}{\Delta t} = 64\, Js^{-1} \therefore \frac{\Delta\theta}{\Delta t} = 64 \times 60\, Jmin^{-1} \therefore \frac{\Delta\theta}{\Delta t} = 3840\ Jmin^{-1}$Hence, the amount of heat flowing through slab is 3840 J/min.

2. Question

A liquid-nitrogen container is made of a 1 cm thick Styrofoam sheet having thermal conductivity 0.025 J $s^{-1}m^{-1}°C^{-1}$. Liquid nitrogen at 80 K is kept in it. A total area of 0.80 m^2 is in contact with the liquid nitrogen. The atmospheric temperature is 300 K. Calculate the rate of heat flow from the atmosphere to the liquid nitrogen.

Answer

Given: Thickness of the container : x = 1 cm = 0.01 mThermal conductivity of the sheet : K= 0.025 J $s^{-1}m^{-1}°C^{-1}$Temperature of the liquid nitrogen: T_1 = 80 KArea of the container : A = 0.8 m^2Temperature of the atmosphere : T_2 = 300 K**Formula used:**Rate of amount of heat flowing is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness of the material. Here, ΔT = T_2- T_1 = 220 KSubstituting the values we get,$\therefore \frac{\Delta\theta}{\Delta t} = \frac{0.025 \times 0.8 \times 220}{0.01} \therefore \frac{\Delta\theta}{\Delta t} = 440\, Js^{-1}$Hence, the rate of heat flow from the atmosphere to the liquid nitrogen is 440 J/s.

3. Question

The normal body-temperature of a person is 97°F. Calculate the rate at which heat is flowing out of his body through the clothes assuming the following values. Room temperature = 47°F, surface of the body under clothes = 1.6 m^2, conductivity of the cloth = 0.04 J s–^{1} m^{-1} $°C^{-1}$, thickness of the cloth = 0.5 cm.

Answer

Given: Body Temperature : T_1 = 97 °F = 36.1 °CRoom temperature: T_2 = 47°F = 8.3 °CSurface area under clothes : A = 1.6 m^2Thermal conductivity of the cloth : K =0.04 J s–^{1} m^{-1} °C^{-1}Thickness of the cloth : x = 0.5 cm = 0.005 m**Formula used:**Rate of amount of heat flowing is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness of the material.Here, ΔT = T_1-T_2 = 36.1-8.3 = 27.8 °CSubstituting we get,∴ $\frac{\Delta\theta}{\Delta t} = \frac{0.04 \times 1.6 \times 27.8}{0.005}$∴ $\frac{\Delta\theta}{\Delta t} = 355.84\, Js^{-1}$ Hence, the rate at which heat is flowing out of his body through the clothes is 355.84 J/s.

4. Question

Water is boiled in a container having a bottom of surface area 25 cm^2, thickness 1.0 mm and thermal conductivity 50W m^{-1} °C^{-1}. 100g of water is converted into steam per minute in the steady state after the boiling starts. Assuming that no heat is lost to the atmosphere, calculate the temperature of the lower surface of the bottom. Latent heat of vaporization of water = 2.26 × 10^6 J kg^{-1}.

Answer

Given: Area of the bottom of container: A = 25 cm^2 = 0.0025 m^2.Thickness of the container: x = 1 mm = 0.001 mThermal conductivity of the container: K =50 W m^{-1} °C^{-1}.Latent heat of vaporization : L = 2.26 × 10^6 J kg^{-1}.Mass of the water converted to steam : m = 100 g = 0.1 kgTemperature of the water : T_2 = 100 ° C**Formula used:**Rate of amount of heat flowing is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness of the material.Also,Δθ = Q =L× mHere, Q is the amount of heat absorbed or released, L is the Latent heat and m is the mass of the substance.Also, Δt = 1 minute = 60 secondsLet temperature of the bottom of the container be T_1.Substituting we get,$\frac{L \times m}{\Delta t} = K \times \frac{A\Delta T}{x}$

$$\therefore \frac{2.26 \times 10^6 \times 0.1}{60} = \frac{50 \times 0.0025}{0.001} \times \Delta T \therefore \Delta T = \frac{2.26 \times 10^6 \times 0.1 \times 0.001}{50 \times 0.0025 \times 60} \therefore T1 - T2 = 30.13\,°C \therefore T1 = 30.13 + 100 \therefore T1 = 130.13\,°C$$

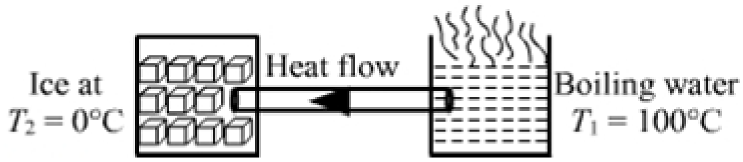
Hence, Temperature of the bottom of the container is130.13 ° C.

5. Question

One end of a steel rod (K = 46 J s^{-1} m^{-1} °C^{-1}) of length 1.0 m is kept in ice at 0°C and the other end is kept in boiling water at 100°C. The area of cross-section of the rod is 0.04 cm^2. Assuming no heat loss to the atmosphere, find the mass of the ice melting per second. Latent heat of fusion of ice = 3.36 × 10^5 J kg^{-1}.

Answer

Given: Thermal conductivity of the steel rod : K=46 J s^{-1} m^{-1} °C^{-1}.Length of the rod : x = 1 mAs heat flows from area of high temperature to low temperature,



Temperature of the end in water : T_1 = 100° CTemperature of the end in ice : T_2 = 0° CArea of cross section of the rod :A = 0.04 cm^2 = 0.04× 10^{-4} m^2.Latent heat of fusion of ice = 3.36 × 10^5 J kg^{-1}.**Formula used:**Rate of amount of heat flowing is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness of the material.Also,Δθ = Q =L× mHere, Q is the amount of heat absorbed or released, L is the Latent heat and m is the mass of the substance.Here, Δt = 1 second.Substituting,∴ $\frac{L \times m}{\Delta t} = K \times \frac{A\Delta T}{x}$∴ $m = \frac{K}{L} \times \frac{A\Delta T}{x} \times \Delta T$

$\therefore m = \frac{46 \times 0.04 \times 10^{-4} \times (100-0)}{3.36 \times 10^5 \times 1} \times 1$ ∴ m = 5.47 × 10^{-8} kgHence, mass of the ice melting per second is 5.47 × 10^{-8} kg.

6. Question

An icebox almost completely filled with ice at 0°C is dipped into a large volume of water at 20°C. The box has walls of surface area 2400 cm^2, thickness 2.0 mm and thermal conductivity 0.06 W m^{-1} °C^{-1}. Calculate the rate at which the ice melts in the box. Latent heat of fusion of ice = 3.4 × 10^5 J kg^{-1}.

Answer

Given: Temperature of the water : T_1 = 20° CTemperature of the ice box : T_2= 0° CSurface area of the box : A = 2400 cm^2 = 0.24 m^2Thickness of the box : x = 2 mm = 0.002 mThermal conductivity of the box : K = 0.06 W m^{-1} °C^{-1}.Latent heat of fusion of ice = 3.4 × 10^5 J kg^{-1}.**Formula used:**Rate of amount of heat flowing is given as: $\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness of the material.Also,Δθ = Q =L× mHere, Q is the amount of heat absorbed or released, L is the Latent heat and m is the mass of the substance.We need to find rate at which ice melts, which means we need to calculate the decrease in mass of the ice per second: Δm/ΔtWhere Δm is the rate of change of mass.Now,$\frac{\Delta\theta}{\Delta t} = \frac{0.06 \times 0.24 \times (20-0)}{0.002}$∴ $\frac{\Delta\theta}{\Delta t} = 144\, Js^{-1}$

Substituting for Δθ we get$L \times \frac{\Delta m}{\Delta t} = 144$∴ $\frac{\Delta m}{\Delta t} = \frac{144}{3.4 \times 10^5}$

∴ $\frac{\Delta m}{\Delta t} = 4.23 \times 10^{-4}$ kg s^{-1} OR∴ $\frac{\Delta m}{\Delta t} = 1.52\, kg\, hr^{-1}$ Hence, the rate at which ice melts in the box is4.23× 10^{-4} kg/s or 1.52 kg/hr.

7. Question

A pitcher with 1 mm thick porous walls contains 10 kg of water. Water comes to its outer surface and evaporates at the rate of 0.1 g s^{-1}. The surface area of the pitcher (one side) = 200 cm^2. The room temperature = 45°C, latent heat of vaporization = 2.27×10^6 J kg^{-1}, and the thermal conductivity of the porous walls = 0.80 J $s^{-1}m^{-1}$ °C^{-1}. Calculate the temperature of water in the pitcher when it attains a constant value.

Answer

Given: Thickness of the pitcher: x = 1 mm = 0.001 mMass of the water in the pitcher : m = 10 kgRate at which water evaporates at it's outer surface:= 0.1 gs^{-1} = 0.1×10^{-3} kg s^{-1}.The surface area of the pitcher : A = 200 cm^2= 0.02 m^2.Room temperature : T_1 = 42 ° CLatent heat of vaporization: L = 2.27×10^6 J kg^{-1}.Thermal conductivity of the porous walls:K = 0.80 J $s^{-1}m^{-1}$ °C^{-1}.Let the constant temperature of the water in the pitcher be T_2.**Formula used:**Rate of amount of heat flowing is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness of the material.Also,Δθ = Q =L× mHere, Q is the amount of heat absorbed or released, L is the Latent heat and m is the mass of the substance.Now, 0.1×10^{-3} kg of water evaporates in 1 second. Thus by unitary method, 10 kg of water will evaporate in 10^5 seconds.Δt = 10^5 seconds.Substituting we get,$\frac{L \times m}{\Delta t} = K \times \frac{A\Delta T}{x}$

$$\therefore \frac{2.27 \times 10^6 \times 10}{10^5} = \frac{0.8 \times 0.02}{0.001} \times (42 - T_2)$$

$$\therefore 42 - T_2 = \frac{2.27 \times 10^6 \times 10 \times 0.001}{10^5 \times 0.8 \times 0.02}$$

∴ 42 – T_2 = 14.18 ∴ T_2 = 42 - 14.18 ∴ T_2 = 27.82 °CHence, the temperature of water in the pitcher when it attains a constant value is 27.82 °C.

8. Question

A steel frame (K = 45 W m^{-1} °C^{-1}) of total length 60 cm and cross-sectional area 0.20 cm^2, forms three sides of a square. The free ends are maintained at 20°C and 40°C. Find the rate of heat flow through a cross-section of the frame.

Answer

Given: Thermal conductivity of steel frame: K=45 W m^{-1} °C^{-1}.Length of the steel frame : x = 60 cm = 0.6 mArea of cross section : A = 0.20 cm^2 = 0.2×10^{-4} m^2.Temperature difference between the free ends :ΔT = 40-20= 20°C.**Formula used:**Rate of amount of heat flowing is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness of the material.Substituting the values we get,

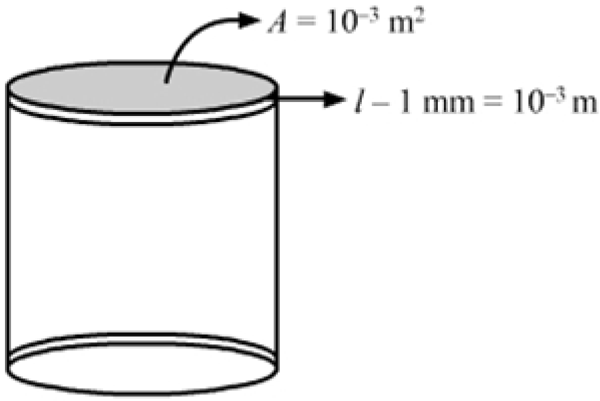
$\frac{\Delta\theta}{\Delta t} = \frac{45 \times 0.2 \times 10^{-4} \times 20}{0.6} \therefore \frac{\Delta\theta}{\Delta t} = 0.03\, Js^{-1}$Hence, the rate of heat flow through a cross-section of the frame is 0.03 J/s.

9. Question

Water at 50°C is filled in a closed cylindrical vessel of height 10 cm and cross-sectional area 10 cm^2. The walls of the vessel are adiabatic but the flat parts are made of 1 mm thick aluminium (K = 200 J s^{-1} m^{-1} $°C^{-1}$). Assume that the outside temperature is 20°C. The density of water is 1000 kg m^{-3}, and the specific heat capacity of water = 4200 J k^{-1} m^{-1} $°C^{-1}$. Estimate the time taken for the temperature to fall by 1.0°C. Make any simplifying assumptions you need but specify them.

Answer

Given: Temperature of the water : T_1 = 50 °CHeight of the vessel : h = 10 cm = 0.1 mCross section area of the vessel: A = 10 cm^2 = 0.001 m^2.Thickness of the flat parts: x = 1mm = 0.001 mThermal conductivity of the aluminium: K =200 J s^{-1} m^{-1} $°C^{-1}$Temperature outside: T_2 = 20 °CDensity of water : ρ = 1000 kg m^{-3}The specific heat capacity of water = 4200 J k^{-1} m^{-1} $°C^{-1}$.



Since the walls are adiabatic, no heat transfer would take place. Hence the heat would be transferred only via flat surfaces which are Up and Bottom surfaces.**Formula used:**Rate of amount of heat flowing is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness of the material.Rate of flow of heat from both the flat surfaces would be:$\therefore \frac{\Delta\theta}{\Delta t} = 2 \times \left(K \times \frac{A\Delta T}{x}\right)$

$\therefore \frac{\Delta\theta}{\Delta t} = 2 \times \frac{200 \times 0.001 \times (50-20)}{0.001} \therefore \frac{\Delta\theta}{\Delta t} = 12000\, J\, s^{-1} \ldots\ldots (1)$Hence, rate of heat flow from both the flat surfaces is 12000 J/s.We know that,ΔQ=Δθ = msΔT $\therefore \frac{\Delta\theta}{\Delta t} = \frac{ms\Delta T}{\Delta t} \ldots\ldots (2)$Where, ΔQ is the change in heat energy, m is the mass , s is the specific heat of the substance and ΔT is the change in temperature for the substance.Also,Mass = Density × Volume ∴ m = 1000 × 0.001× 0.1 ∴ m = 0.1 kgWe need to find time taken for the temperature to fall by 1°CThus, ΔT = 1°CSubstituting in equation (2) and using (1) we get,$12000 = \frac{0.1 \times 4200 \times 1}{\Delta t}$

$\therefore \Delta t = t = \frac{0.1 \times 4200}{12000} \therefore t = 0.035\ s$Hence, it took around 0.035 seconds for the temperature to drop by 1°C.

10. Question

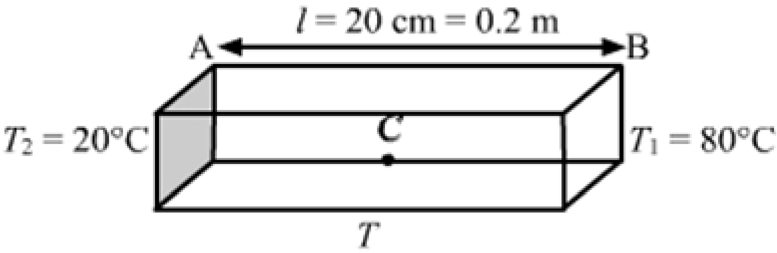
The left end of a copper rod (length = 20 cm, area of cross-section = 0.20 cm^2) is maintained at 20°C and the right end is maintained at 80°C. Neglecting any loss of heat through radiation, find

(a) the temperature at a point 11 cm from the left end and

(b) the heat current through the rod. Thermal conductivity of copper = 385 W m^{-1} °C^{-1}.

Answer

Given: Length of the rod: x = 20 cm =0.2 mArea of cross section of the rod: A = 0.2 cm^2 = 0.2× 10^{-4} m^2.Temperature at left end : T_2 = 20° CTemperature at right end : T_1 = 80° CThermal conductivity of copper: K = 385 W m^{-1} °C^{-1}.



Distance of point C from left end : x’ = 11cm = 0.11 m**Formula used:**(b)Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.Transfer of heat due to entire rod is,

$\frac{\Delta\theta}{\Delta t} = \frac{385 \times 0.2 \times 10^{-4} \times (80 - 20)}{0.2} \therefore \frac{\Delta\theta}{\Delta t} = 2.31\ J\ s^{-1}$Hence, heat current in the rod is 2.31 J/s(a)Let T be the temperature at point C.T>T_2 as heat flows from High temperature to low temperature.Substituting we get,$\frac{\Delta\theta}{\Delta t} = K \times \frac{A(T - 20)}{x'}$

$\therefore 2.31 = \frac{385 \times 0.2 \times 10^{-4}}{0.11} \times (T - 20) \therefore T - 20 = \frac{2.31 \times 0.11}{385 \times 0.2 \times 10^{-4}}$

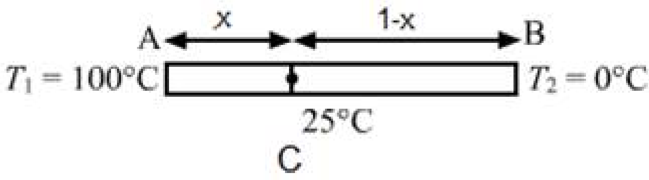
$\therefore T = 33 + 20 \therefore T = 53\ °C$ Hence, temperature at a distance of 11 cm from the left end is 53 °C.

11. Question

The ends of a metre stick are maintained at 100°C and 0°C. One end of a rod is maintained at 25°C. Where should its other end be touched on the metre stick so that there is no heat current in the rod in steady state?

Answer

Given: Temperature difference between the ends of the meter stick AB:$\Delta T = T_2 - T_1$ = 100-0 = 100 °CTemperature of one end of the rod: T_3 = 25 °CLength of the rod : l = 1 m



Here, C is the point at which the other end of the rod is placed.Distance between A and C = xDistance between C and B = 1-x**Formula used:**Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.Now, for zero heat current in the rod, the temperature difference must be zero: ΔT = 0.Since one end of the rod is maintained at 25 °C, the other end must be maintained at 25 °C.Hence heat current between A and C must be equal to the heat current between C and B$\left(\frac{\Delta\theta}{\Delta t}\right)_{AC} = \left(\frac{\Delta\theta}{\Delta t}\right)_{CB}$ $\therefore\ K \times \frac{A(\Delta T)_{AC}}{x} = K \times \frac{A(\Delta T)_{CB}}{1-x}$Here $(\Delta T)_{AC}$ and $(\Delta T)_{CB}$ is the temperature difference between AC and BC respectively.

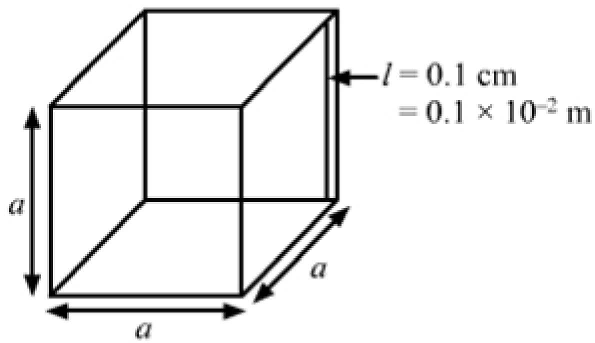
$\therefore \frac{100-25}{x} = \frac{25-0}{1-x}$ ∴ 75(1-x) = 25x ∴ 75 -75x = 25x ∴ 75 = 100x ∴ x = 75/100 ∴ x = 0.75 mHence, in order to have zero heat current through the rod the other end of the rod must be placed at a distance of 0.75 m from the end at 100 ° C.

12. Question

A cubical box of volume 216 cm^3 is made up to 0.1 cm thick wood. The inside is heated electrically by a 100 W heater. It is found that the temperature difference between the inside and the outside surface is 5°C in steady state. Assuming that the entire electrical energy spent appears as heat, find the thermal conductivity of the material of the box.

Answer

Given: Volume of the box : V = 216 cm^3 = 216× 10^{-6} m^3.Thickness of the box: x = 0.1 cm = 0.001 mPower of the heater : P = 100 WTemperature difference : ΔT = 5 °C



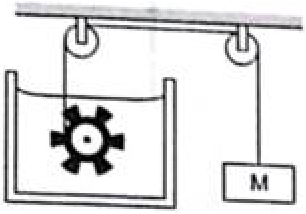
Formula used: Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.Volume of the cube is a^3. Where a is the side of the cube.∴ a = $(216\times 10^{-6})^{1/3}$ = 0.06 mAs heat will be transferred from all the sides of the cube,Surface area of the cube is : A = $6a^2$∴ A = 6× $(0.06)^2$ = 0.0216 m^2.We know that,Power = Energy per unit timeThus,$\frac{\Delta\theta}{\Delta t} = P = 100\ W$Substituting we get,

$100 = K \times \frac{0.0216 \times 5}{0.001}$∴ $K = \frac{100 \times 0.001}{0.0216 \times 5}$∴ $K = 0.9259\ W\,m^{-1}{}^{\circ}C^{-1}$

Hence, thermal conductivity of the box is 0.9529 W/m °C.

13. Question

Figure shows water in a container having 2.0 mm thick walls made of a material of thermal conductivity 0.50 W m^{-1} $°C^{-1}$. The container is kept in a melting ice bath at 0°C. The total surface area in contact with water is 0.05 m^2. A wheel is clamped inside the water and is coupled to a block of mass M as shown in the figure. As the block goes down, the wheel rotates. It is found that after some time a steady state is reached in which the block goes down with a constant speed of 10 cm s^{-1} and the temperature of the water remains constant at 1.0°C. Find the mass M of the block. Assume that the heat flows out of the water only through the walls in contact. Take g = 10 m s^{-2}.



Answer

Given: Thickness of the container: x = 2 mm = 0.002 mThermal conductivity of the container: K= 0.50 W m^{-1} $°C^{-1}$.Temperature of the water: T_1 = 1 °CTemperature of the ice bath: T_2 = 0 °CSurface are in contact with the water: 0.05 m^2Speed of the block: v = 10 cm/s = 0.1 m/sg = 10 m/s^2**Formula used:**Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.The effect of the block going down and the heat transfer has one identity in common, which is power.Power due to a block of mass M moving with constant velocity is given as$P = \frac{W}{t}$∴ $P = \frac{F.d}{t} = Fv$Here W is the work done by the block. ∴ P = (mg).v ∴ P = M× 10× 0.1 ∴ P = M× 1Also in terms of Heat Energy: P= Energy per unit time∴ $P = \frac{\Delta\theta}{\Delta t}$∴ $P = K \times \frac{A\Delta T}{x}$

∴ $M = \frac{0.50 \times 0.05 \times (1-0)}{0.002}$∴ $M = 12.5\ kg$ Hence, the mass of the block is 12.5 kg.

14. Question

On a winter day when the atmospheric temperature drops to −10°C, ice forms on the surface of a lake.

(a) Calculate the rate of increase of thickness of the ice when 10 cm of ice is already formed.

(b) Calculate the total time taken in forming 10 cm of ice. Assume that the temperature of the entire water reaches 0°C before the ice starts forming. Density of water = 1000 kg m^{-3}, latent heat of fusion of ice = 3.36×10^5 J kg^{-1} and thermal conductivity of ice = 1.7 W m^{-1} $°C^{-1}$. Neglect the expansion of water on freezing.

Answer

Given: Temperature of the water: T_1 = 0 °CTemperature of the atmosphere: T_2 = -10 °CChange in temperature : $\Delta T = T_1 - T_2$ = 10 °CLength of the ice formed : l = 10 cm = 0.1 mDensity of water: ρ = 1000 kg m^{-3}Latent heat of fusion of ice: L = 3.36×10^5 J kg^{-1}.Thermal conductivity of ice: K0. = 1.7 W m^{-1} $°C^{-1}$.**Formula used:**Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.Also,Δθ = Q =L× mHere, Q is the amount of heat absorbed or released, L is the Latent heat and m is the mass of the substance.And,Mass = Density × Volume ∴ m = ρ V = ρAlHere ρ is the density of water, A is the area and l is the length.(a)Let change in thickness be Δx,thus rate of change of thickness is:$\therefore \frac{\Delta x}{\Delta t} = K \times \frac{A\Delta T}{\Delta \theta}$$\therefore \frac{\Delta x}{\Delta t} = K \times \frac{A\Delta T}{mL}$

$\therefore \frac{\Delta x}{\Delta t} = K \times \frac{A\Delta T}{(Al \times \rho)L}$$\therefore \frac{\Delta x}{\Delta t} = \frac{1.7 \times (0 - (-10))}{0.1 \times 3.36 \times 1000 \times 10^5}$

$\therefore \frac{\Delta x}{\Delta t} = 5.05 \times 10^{-7}\ ms^{-1}$ Hence, the thickness of the ice increases at a rate of 5.05×10^{-7} m/s.(b)Consider time required to form thin layer of ice dx is dt.Using above formulations,Mass of dx: dm = Adx × ρAnd heat absorbed by the thin layer,dQ = dm× LNow, rate of Heat transfer due to thin layer becomes:$\therefore \frac{d\theta}{dt} = K \times \frac{A\Delta T}{x}$

$\therefore L . \frac{dm}{dt} = K \times \frac{A\Delta T}{x}$$\therefore LA\rho \frac{dx}{dt} = K \times \frac{A\Delta T}{x}$$\therefore \frac{L\rho}{K\Delta T} x dx = dt$ Integrating on both sides and setting the limit of ice formed x: 0 to 0.1.$\therefore \frac{L\rho}{K\Delta T} \int_0^{0.1} x dx = \int_0^t dt$

$\therefore \frac{L\rho}{K\Delta T} \left[\frac{x^2}{2}\right]_0^{0.1} = t$$\therefore t = \frac{3.36 \times 10^5 \times 1000}{1.7 \times 10} \times \frac{(0.1)^2}{2}$$\therefore t = 98823.52$ seconds

$\therefore t = \frac{98823.52}{3600}$$\therefore t = 27.45\ hours$ Hence, it took 27.45 hours to form 10 cm thick ice.

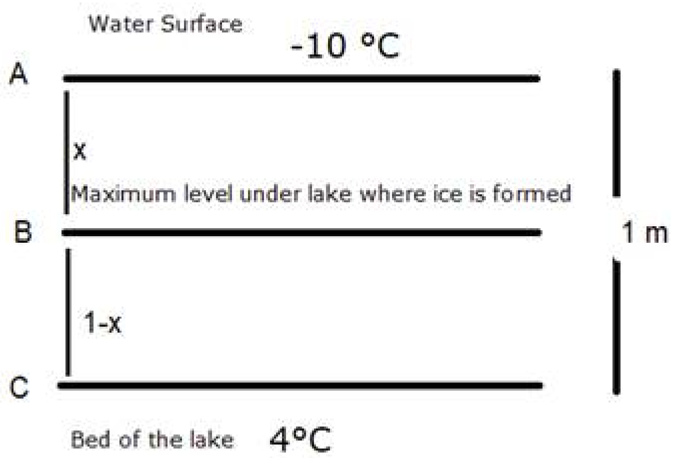
15. Question

Consider the situation of the previous problem. Assume that the temperature of the water at the bottom of the lake remains constant at 4°C as the ice forms on the surface (the heat required to maintain the temperature of the bottom layer may come from the bed of the lake). The depth of the lake is 1.0 m. Show that the thickness of the ice formed attains a steady state maximum value. Find this value.

The thermal conductivity of water = 0.50 Wm^{-1} $°C^{-1}$. Take other relevant data from the previous problem.

Answer

Given: Temperature at the bottom of the lake: T_1 = 4 °CTemperature above the surface : T_2 = - 10 °CDepth of the lake: d = 1 mThermal conductivity of water: K_W = 0.50 Wm^{-1} $°C^{-1}$.Thermal conductivity of ice: K_I = 1.7 W m^{-1} $°C^{-1}$.



Distance AB is : x_{AB} = x

mDistance CB is : x_{CB} = (1-x) m**Formula used:**Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.In the diagram, point B depicts the maximum level upto which ice can be formed inside the lake.Temperature at B : T_3 = 0°CThis ice attains a steady state maximum level. Steady state means that the temperature at any point remains unchanged.This means that the temperature difference between points A,B and C would be unchanged.$\left(\frac{\Delta\theta}{\Delta t}\right)_{CB} = \left(\frac{\Delta\theta}{\Delta t}\right)_{AB}$ This means that the rate of heat transfer between A and B equals the rate of heat transfer between B and C.

$\therefore\ K_W \times \frac{A(\Delta T)_{CB}}{x_{CB}} = K_I \times \frac{A(\Delta T)_{AB}}{x_{AB}}$$\therefore 0.50 \times \frac{4-0}{1-x} = 1.7 \times \frac{0+10}{x}$$\therefore \frac{2}{1-x} = \frac{17}{x}$

$\therefore 2x = 17 - 17x \therefore 19x = 17 \therefore x = 0.894\ m$Hence, after attaining steady state the thickness of the ice below the lake is 0.894 m.

16. Question

Three rods of lengths 20 cm each and area of cross-section 1 cm^2 are joined to form a triangle ABC. The conductivities of the rods are K_{AB} = 50 J s^{-1} m^{-1} $°C^{-1}$, K_{BC} = 200 J s^{-1} m^{-1} $°C^{-1}$ and K_{AC} = 400 J s^{-1} m^{-1} $°C^{-1}$. The junctions A, B and C are maintained at 40°C, 80°C and 80°C respectively. Find the rate of heat flowing through the rods AB, AC and BC.

Answer

Given: Length of all the rods: x= AB=BC=AC = 20 cm = 0.2 mArea of cross section of these rods: A = 1 cm^2 = 0.0001 m^2.Thermal conductivity of rod AB : K_{AB} = 50 J s^{-1}

m^{-1} °C^{-1}Thermal conductivity of rod BC : K_{BC} = 200 J s^{-1} m^{-1} °C^{-1}Thermal conductivity of rod AC : K_{AC} = 400 J s^{-1} m^{-1} °C^{-1}Temperature at A : T_1 = 40 °CTemperature at B : T_2 = 80 °CTemperature at C : T_3 = 80 °C**Formula used:**Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.(1)Rate of heat flowing in the rod AB is $\left(\frac{\Delta\theta}{\Delta t}\right)_{AB} = K_{AB} \times \frac{A(\Delta T)_{AB}}{x}$∴ $\left(\frac{\Delta\theta}{\Delta t}\right)_{AB} = \frac{50 \times 0.0001 \times (80-40)}{0.2}$

∴ $\left(\frac{\Delta\theta}{\Delta t}\right)_{AB} = 1\, J\, s^{-1}$Hence, rate of heat flowing through the rod Ab is 1 J/s or 1 W(2)Rate of heat flowing in the rod BC is $\left(\frac{\Delta\theta}{\Delta t}\right)_{BC} = K_{BC} \times \frac{A(\Delta T)_{BC}}{x}$

∴ $\left(\frac{\Delta\theta}{\Delta t}\right)_{BC} = \frac{200 \times 0.0001 \times (80-80)}{0.2}$∴ $\left(\frac{\Delta\theta}{\Delta t}\right)_{BC} = 0\, W$Hence, the rate of heat flowing through the rod BC is 0 as both the ends of the rods are maintained at same temperature.(3)Rate of heat flowing in the rod BC is $\left(\frac{\Delta\theta}{\Delta t}\right)_{AC} = K_{AC} \times \frac{A(\Delta T)_{AC}}{x}$

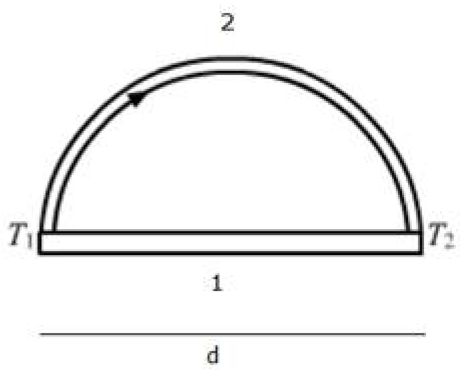
∴ $\left(\frac{\Delta\theta}{\Delta t}\right)_{AC} = \frac{400 \times 0.0001 \times (80-40)}{0.2}$∴ $\left(\frac{\Delta\theta}{\Delta t}\right)_{AC} = 8\, J\, s^{-1}$Hence, the rate of heat flowing through the rod AC is 8 J/s.

17. Question

A semicircular rod is joined at its end to a straight rod of the same material and the same cross-sectional area. The straight rod forms a diameter of the other rod. The junctions are maintained at different temperatures. Fid the ratio of the heat transferred through a cross-section of the semicircular rod to the heat transferred through a cross-section of the straight rod in a given time.

Answer

Given:



Temperature at junction 1: T_1Temperature at junction 2: T_2Length of the rod 1: x= d=2rWhere d is the diameter and r is the radius.Length of the rod 2: x' = circumference = πr**Formula used:**Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$

Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.As both the rods have same material, their thermal conductivity is the same.(1)Rate of heat flowing through rod 1$\left(\frac{\Delta\theta}{\Delta t}\right)_1 = K \times A \times \frac{T_1 - T_2}{2r}$ (2)Rate of heat flowing through rod 2 $\left(\frac{\Delta\theta}{\Delta t}\right)_2 = K \times A \times \frac{T_1 - T_2}{\pi r}$ Ratio of the rate of heat transferred from semi circular rod to straight rod is$\frac{\left(\frac{\Delta\theta}{\Delta t}\right)_2}{\left(\frac{\Delta\theta}{\Delta t}\right)_1} = \frac{K \times A \times \frac{T_1 - T_2}{\pi r}}{K \times A \times \frac{T_1 - T_2}{2r}} \therefore \frac{\left(\frac{\Delta\theta}{\Delta t}\right)_2}{\left(\frac{\Delta\theta}{\Delta t}\right)_1} = \frac{2r}{\pi r}$

$\therefore \left(\frac{\Delta\theta}{\Delta t}\right)_2 : \left(\frac{\Delta\theta}{\Delta t}\right)_1 = 2 : \pi$ Hence, the ratio of the heat transferred through a cross-section of the semicircular rod to the heat transferred through a cross-section of the straight rod in a given time is 2:π

18. Question

A metal rod of cross-sectional area 1.0 cm^3 is being heated at one end. At one time, the temperature gradient is 5.0°C cm^{-1} at cross-section A and is 2.5°C cm^{-3} at cross-section B. calculate the rate at which the temperature is increasing in the part AB of the rod. The heat capacity of the part AB = 0.40 J°C^{-1}, thermal conductivity of the material of the rod = 200 W m^{-1} °C^{-1}. Neglect any loss of heat to the atmosphere.

Answer

Given: Cross sectional area of the metal rod: A = 1 cm^2= 0.0001 m^2Temperature gradient at A :$(dT/dx)_A$ = 5.0 °C cm^{-1}= 500 °Cm^{-1}.Temperature gradient at B : $(dT/dx)_B$ = 250 °C m^{-1}.Heat capacity of the rod AB : C = 0.40 J°C^{-1}.Thermal conductivity of the material of the rod :K = 200 W m^{-1} °C^{-1}.**Formula used:**Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.Rate of heat flow is dθ/dtHence at cross section A:$\left(\frac{d\theta}{dt}\right)_A = KA \times \left(\frac{dT}{dx}\right)_A \therefore \left(\frac{d\theta}{dt}\right)_A = 200 \times 0.0001 \times 500$

$\therefore \left(\frac{d\theta}{dt}\right)_A = 10\ Js^{-1}$At cross section B:$\left(\frac{d\theta}{dt}\right)_B = KA \times \left(\frac{dT}{dx}\right)_B$

$\therefore \left(\frac{d\theta}{dt}\right)_B = 200 \times 0.0001 \times 250 \therefore \left(\frac{d\theta}{dt}\right)_B = 5\ Js^{-1}$Now, the rate of flow of heat throughout the rod AB is$\frac{\Delta\theta}{\Delta t} = \left(\frac{d\theta}{dt}\right)_A - \left(\frac{d\theta}{dt}\right)_B \therefore \frac{\Delta\theta}{\Delta t} = 10 - 5 = 5\ Js^{-1}$

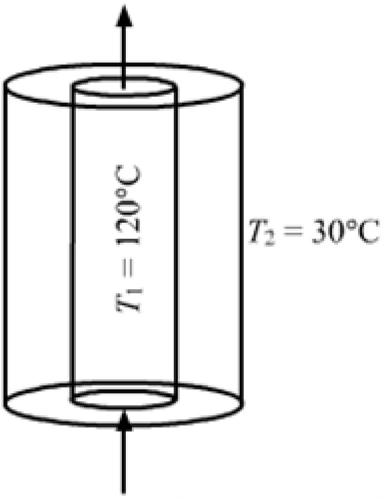
We know that, Δθ= Q = msΔThere, Q is the amount of heat, m is the mass of the material, s is the specific heat of the material and ΔT is the change in temperature.And Heat Capacity is : C = ms = Q/ΔT.$\therefore \frac{\Delta\theta}{\Delta t} = \frac{ms\Delta T}{\Delta t}$ Substituting we get,$5 = 0.4 \times \left(\frac{\Delta T}{\Delta t}\right) \therefore \frac{\Delta T}{\Delta t} = \frac{5}{0.4} \therefore \frac{\Delta T}{\Delta t} = 12.5\ °C/s$Hence, the rate at which temperature increases is 12.5 °C /s.

19. Question

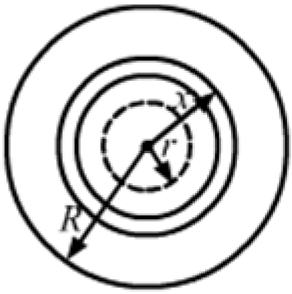
Steam at 120°C is continuously passed through a 50 cm long rubber tube of inner and outer radii 1.0 cm and 1.2 cm. The room temperature is 30°C. Calculate the rate of heat flow through the walls of the tube. Thermal conductivity of rubber = 0.15 J s^{-1} m^{-1} $°C^{-1}$.

Answer

Given: Steam temperature: T_1 = 120 °CLength of the tube : l = 50 cm = 0.5 mInner radii of the tube: r = 1 cm = 0.01 mOuter radii of the tube : R = 1.2 cm = 0.012 mRoom temperature: T_2 = 30 °CThermal conductivity of rubber: K = 0.15 J s^{-1} m^{-1} $°C^{-1}$



Formula used: Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, Δθ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.Consider an element dx at a distance of x from the center between r and R.



We will integrate this element dx to find total heat transferred from the tube.Heat flow can be given as q = Δθ / ΔtIn differential form:∴ $\frac{d\theta}{dt} = -\left(K \times \frac{AdT}{dx}\right)$Here we used negative sign because the heat flow decreases with increase in thickness dxAlso, Area of the tube formed due to element dx can be given asA = 2πxlHere, x is the radius of the tube due to dx and l is the length of the tube.∴ $\frac{d\theta}{dt} = -\left(K \times \frac{2\pi x l dT}{dx}\right)$∴ $q \times \frac{dx}{x} = -(2\pi l K)dT$

Now we integrate both the sides, taking temperature from tube to surrounding: T_1 to T_2 and radii from r to R∴ $q \int_r^R \frac{dx}{x} = -(2\pi l K) \int_{T_1}^{T_2} dT$

$\therefore q \times [\ln(x)]_r^R = -(2\pi l K) \times [T]_{T_1}^{T_2}$∴ $q \times \ln\left(\frac{R}{r}\right) = -(2\pi l K) \times (T_2 - T_1)$

$$\therefore q = \frac{(-(2\pi lK) \times (T_2 - T_1))}{\ln\left(\frac{R}{r}\right)} \therefore q = \frac{2 \times \pi \times 0.5 \times 0.15 \times (T_1 - T_2)}{\ln\left(\frac{0.012}{0.01}\right)}$$

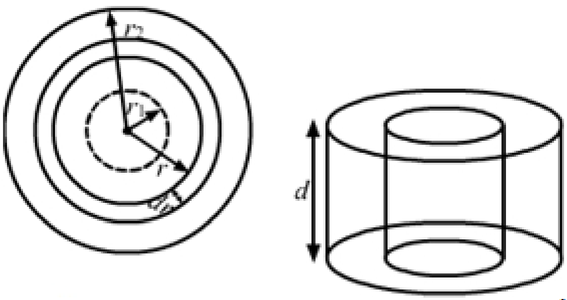
$\therefore q = 232.50\, Js^{-1}$ Hence, the rate of heat flow through the walls of the tube is 232.50 J/s.

20. Question

A hole of radius r_1 is made centrally in a uniform circular disc of thickness d and radius r_2. The inner surface (a cylinder of length d and radius r_1) is maintained at ta temperature θ_1 and the outer surface (a cylinder of length d and radius r_2) is maintained at a temperature $\theta_2(\theta_1 > \theta_2)$. The thermal conductivity of the material of the disc is K. Calculate the heat flowing per unit time through the disc.

Answer

Given: Radius of the inner cylinder: r_1Length of the cylinder= thickness of the disc: dRadius of the disc: r_2Temperature of inner cylinder: θ_1Temperature of outer surface: θ_2The thermal conductivity of the material of the disc : K



Formula used: Rate of amount of heat flowing or heat current is given as:$\frac{\Delta\theta}{\Delta t} = K \times \frac{A\Delta T}{x}$Here, $\Delta\theta$ is the amount of heat transferred, ΔT is the temperature difference, K is the thermal conductivity of the material, A is the area of cross section of the material and x is the thickness or length of the material.Consider an imaginary cylinder of radius r and thickness dr between r_1 and r_2.We will integrate considering this imaginary cylinder to get total heat transferred.In differential form heat flow is$\frac{d\theta}{dt} = -\left(K \times \frac{AdT}{dr}\right) = q$

Here q is the rate of heat flowing.Negative sign indicates the decrease in rate of heat flow with increase in the thickness of the imaginary tube.We know that area of the cylinder is:A = 2πrdWhere r is the radius of the cylinder and d is the length of the cylinder.Substituting we get,$q = -\frac{2\pi r dK dT}{dr} \therefore q \times \frac{dr}{r} = -(2\pi dK) \times dT$

Integrating both the sides we get the total rate of heat flow through the disc. Taking radius from r_1 to r_2 and temperature from θ_1 to θ_2.

$$\therefore q \int_{r_1}^{r_2} \frac{dr}{r} = -(2\pi dK) \int_{\theta_1}^{\theta_2} dT \therefore q \times [\ln(r)]_{r_1}^{r_2} = -(2\pi dK) \times [T]_{\theta_1}^{\theta_2}$$

$$\therefore q \times \ln\left(\frac{r_2}{r_1}\right) = -(2\pi dK) \times (\theta_2 - \theta_1) \therefore q = \frac{(-(2\pi dK) \times (\theta_2 - \theta_1))}{\ln\left(\frac{r_2}{r_1}\right)}$$

$\therefore q = \frac{(2\pi dK) \times (\theta_1 - \theta_2)}{\ln\left(\frac{r_2}{r_1}\right)}$ Hence, the heat flowing per unit time through the disc is q.

21. Question

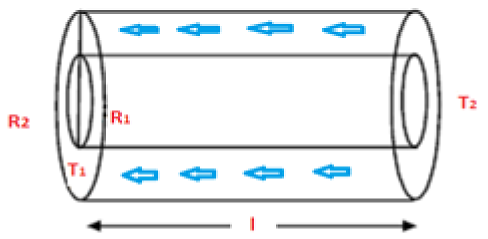
A hollow tube has a length ℓ, inner radius R_1 and outer radius R_2. The material has a thermal conductivity K. Find the heat flowing through the walls of the tube

(a) the flat ends are maintained at temperature T_1 and $T_2 (T_2 > T_1)$

(b) the inside of the tube is maintained at temperature T_1 and the outside is maintained at T_2.

Answer

Given data-



Length = l

Inner radius = R_1

Outer radius = R_2

Thermal conductivity = k

The corresponding diagram is shown in the fig.

a. When the flat ends are maintained at temperature T_1 and $T_2 (T_2 > T_1)$

Now, the area of cross- section through which heat is flowing is given by –

Area,

$A = \pi (R_2^2 - R_1^{\,2})$ (1)

Let q be the heat, then

Rate of flow of heat (H)–

$= \frac{dq}{dt}$ (2)

$H = \frac{k\,A.\Delta T}{l}$

Where

ΔT = is change in temperature between the two walls of the tube.

A= Area of cross section of the tube

K = thermal conductivity of the tube

L = length of the tube

Hence

From (1) and (2),

Rate of flow of heat -

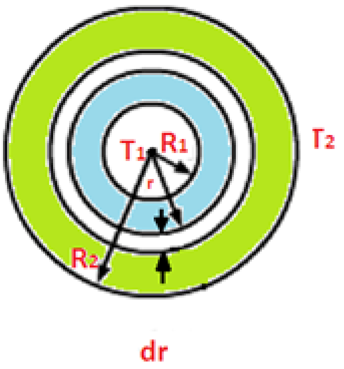
$H = k \times \pi(R_2^2 - R_1{}^2) \times (T_2 - T_1)$

b. When the inside of the tube is maintained at temperature T_1 and the outside is maintained at T_2.

Let's consider a small imaginary

cylinder of radius "r" of

differential radius "dr" as shown in fig.



Rate of flow of heat (H) –

$H = \frac{-k\,A\,(\frac{d\theta}{dt}]}{l}$ (1)

Where

K = thermal conductivity of the tube

A = area of cross section

L = length of the tube

$\frac{d\theta}{dt}$ = change in temperature between the two walls of the tube.

Negative sign since “r” increases, heat decreases.

Since, the cross-section of the tube is in cylindrical form

Hence Curved Surface Area of the Cylinder,

A =2πrl

Where

r = radius of the base

l = length of the tube

From (1), substituting the value of A,

$H = -2\pi rl\, k \frac{d\theta}{dt}$

⇒

$\frac{dt}{r} = -2\pi k \frac{d\theta}{H}$

Integrating both sides –

$\Rightarrow \int_{R_1}^{R_2} \frac{dt}{r} = -2\pi l\, k \int_{T_1}^{T_2} \frac{d\theta}{H}$

⇒

$\log_e \frac{R_2}{R_1} = -2\pi l\, k \frac{T_2 - T_1}{H}$

$\Rightarrow H = -2\pi l\, k \frac{T_2 - T_1}{\log_e \frac{R_2}{R_1}}$

22. Question

A composite slab is prepared by pasting two plates of thicknesses L_1 and L_2 and thermal conductivities K_1 and K_2. The slabs have equal cross-sectional area. Find the equivalent conductivity of the composite slab.

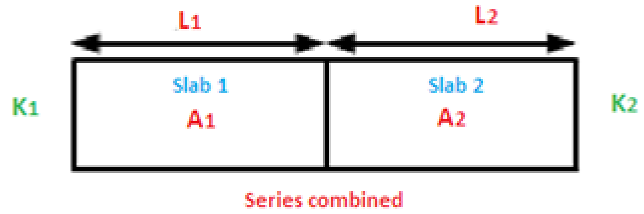
Answer

Here the slabs are placed in such a way that their conductivities are in series as shown in the fig below

Given

Thicknesses of the slabs as L_1 and L_2

Thermal conductivities as K_1 and K_2.



Hence, their equivalent conductivity is similar to 2 resistors connected in series.

From above fig.

Given thermal conductivities of the slabs as K_1 andK_2

The of rate thermal conduction through first slab is given by -

$Q_1 = k_1 \frac{A_1 (T - T_1)}{l_1}$ (1)

Where

k_1 = thermal conductivity of the first slab

A_1= area of first slab

l_1 = length of first slab

T_1= temperature of the first slab and

T= junction temperature

Similarly the rate thermal conduction through second slab is given by –

$Q_2 = k_2 \frac{A_2 (T_2 - T)}{l_2}$ (2)

Since they are connected end to end

$Q_{eqv} = Q_1 = Q_2$

Where Q_{eqv} is given by

$Q_{eqv} = Keqv\, A \frac{(T_2 - T_1)}{l_1 + l_2}$

Also, their area of cross section are equal ie,

$A_1 = A_2$ (3)

From (1) ,(2) and (3)

$Keqv\, A \frac{(T_2 - T_1)}{l_1 + l_2} = k_1 \frac{A_1 (T - T_1)}{l_1} = k_2 \frac{A_2 (T_2 - T)}{l_2}$

Solving for T

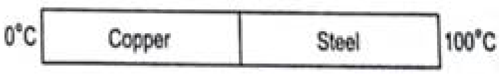
$$T = \frac{k_2\frac{(T_2)}{l_2} + k_1\frac{(T_1)}{l_1}}{\frac{k_1}{l_1}+\frac{k_2}{l_2}} \quad (4)$$

Substituting (4) in (1) and (2) –

$$K_{eqv} = \frac{l_1 + l_2}{\frac{k_1}{l_1}+\frac{k_2}{l_2}}$$

23. Question

Figure shows a copper rod joined to a steel rod. The rods have equal length and equal cross-sectional area. The free end of the copper rod is kept at 0°C and that of the steel rod is kept at 100°C. Find the temperature at the junction of the rods. Conductivity of copper = 390 W m^{-1} $°C^{-1}$ and that of steel = 46 W m^{-1} $°C^{-1}$.

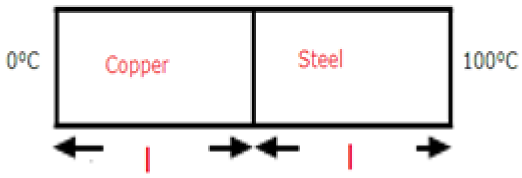


Answer

Given-

Conductivity of copper, K_1 = 390 W m^{-1} $°C^{-1}$

Steel, k_2 = 46 W m^{-1} $°C^{-1}$.



Let

length of rods = l and Area = A

Since rods are connected in series,

so the rate of flow of heat is same ie,

$Q_1 = Q_2$

Rate of flow of heat,

$$Q = \frac{dQ}{dt}$$

$$= \frac{\text{Temperature Difference}}{\text{Thermal resistance}}$$

Since for series connected rods, $Q_1 = Q_2$,

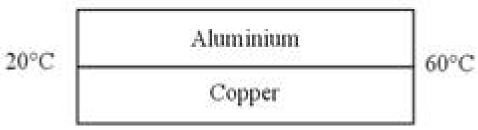
$\Rightarrow \frac{T-0}{\text{Resistance of cu}} = \frac{100-T}{\text{Rresistance of steel}}$

$\Rightarrow A\ K_1 \frac{T-0}{l} = A\ k_2 \frac{100-T}{l}$

$\Rightarrow T = 10.6°C$

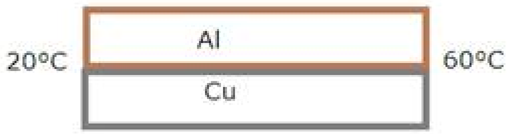
24. Question

An aluminium rod and a copper rod of equal length 1.0 m and cross-sectional area 1 cm^2 are welded together as shown in figure. One end is kept at a temperature of 20°C and the other at 60°C. Calculate the amount of heat taken out per second from the hot end. Thermal conductivity of aluminium = 200 $Wm^{-1}\ °C^{-1}$ and of copper = 390 $Wm^{-1}\ °C^{-1}$.



Answer

Given



Area of cross section

$A = 1\ cm^2$

$= 1 \times 10^{-4}\ m^2$

Thermal conductivity of aluminium, K_{Al} = 200 W/m°C

copper, K_{Cu} = 390 W/m°C

Now since these rods are connected in parallel,

So heat flowing per second

$= q_{Al} + q_{Cu}$

$= K_{Al} \times A \times \Delta T + K_{Cu} \times A \times \Delta T$

$= K_{Al}\ A \times (60-20) + K_{Cu}\ A \times (60-20)$

$= 1 \times 10^{-4}\ m^2 \times 40\ (200 + 390)$

$= 2.36\ W$

Heat drawn in 1 second = 2.36 W

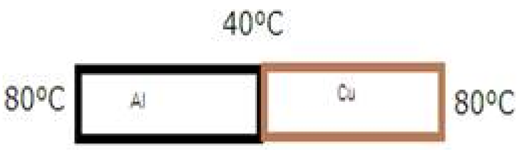
25. Question

Figure shows an aluminium rod joined to a copper rod. Each of the rods has a length of 20 cm and area of cross-section 0.20 cm^2. The junction is maintained at a constant temperature 40°C and the two ends are maintained at 80°C. Calculate the amount of heat taken out from the cold junction in one minute after the steady state is reached. The conductivities are K_{Al} = 200 W m^{-1}°C^{-1}andK_{Cu} = 400 W m^{-1} °C^{-1}.

Answer

Given –

Length of each rod of 20 cm = 0.2m



Area of cross-section 0.20 cm^2

= 2×10^{-5} m^2

Junction temperature = 40°C

End temperature = 80°C

The conductivities of Aluminum and copper,K_{Al}=200 W m^{-1} °C^{-1}

And K_{Cu} = 400 W m^{-1} °C^{-1}.

Now ,total heat drawn per second –

= Heat drawn due to copper rod + heat drawn due to Aluminium rod

= $Q_{Al} + Q_{Cu}$

We know, rate of heat absorption by the rod of length l, area A is given by

$Q = \frac{k\,A.\Delta T}{l}$

Where ΔT is the change in temperature

$= \frac{(200 \times 2 * 10{-}5\ (80-40)}{0.2} + \frac{400 \times 2 * 10{-}5\ (80-40)}{0.2}$

⇒ Q = 2.4 J

Heat drawn in 1 minute = 2.4 × 60 = 144J

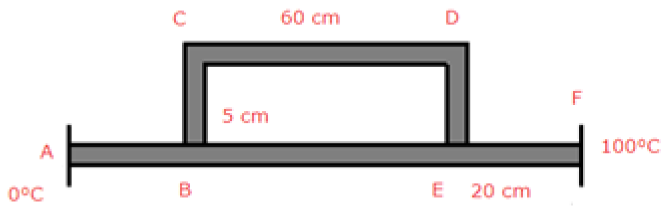
Hence, amount of heat taken out from the cold junction in one minute after the steady state is reached is 144J

26. Question

Consider the situation shown in figure. The frame is made of the same material and has a uniform cross-sectional are everywhere. Calculate the amount of heat flowing per second through a cross-section of the bent part if the total heat taken out per second from the end at 100°C is 130J.

Answer

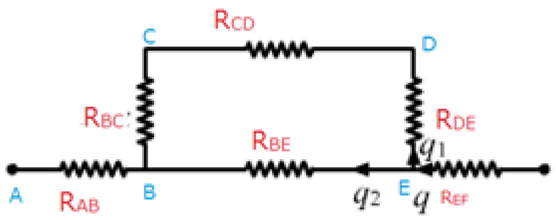
For the above frame, redrawing the fig,



(1)

The equivalent resistance network becomes –

Let R_{AB}, R_{BC}, R_{CD}, R_{DE}, R_{EF} and R_{BE} be the equivalent resistance across each cross-section as shown in fig. below –



(2)

Resistance in terms of conductivity

$R=\frac{l}{kA}$ (1)

From fig (1) and (2)and equation (1)

$R_{AB} = \frac{20}{kA}$, $R_{BC} = \frac{5}{kA}$, $R_{CD} = \frac{60}{kA}$, $R_{DE} = \frac{5}{kA}$, $R_{EF} = \frac{20}{kA}$ and $R_{BE} = \frac{60}{kA}$

Now, lets reduce the network into equivalent network .

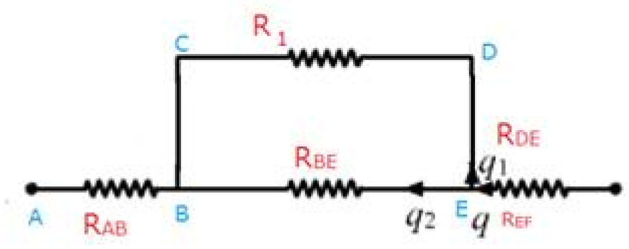
Since R_{BC} R_{CD} R_{DE} are connected in series, let R_1 be their equivalent resistance.

Then $R_1 = R_{BC} + R_{CD} + R_{DE}$

$= \frac{5}{kA} + \frac{60}{kA} + \frac{5}{kA}$

$= \frac{70}{kA}$

Now the circuit reduces to –



Now from Kirchhoff's current law(KCL), we know The algebraic sum of all currents entering and exiting a node must equal zero

Hence ,KCL at point E , since current $= \frac{\text{Charge (q)}}{\text{time(t)}}$-

$q = q_1 + q_2$

Now, since R_1 and R_{BE} are in parallel, so total heat across R_1 and R_{BE} will be same.

ie, $q_1 R_1 = q_2 R_{BE}$

$\Rightarrow q_1 \times \frac{70}{k_A} = q_2 \times \frac{60}{kA}$

$\Rightarrow q_2 = q_1 \times \frac{70}{k_A} \times \frac{k_A}{60}$

Now, $q = q_1 + q_2$

$= q_1 + \frac{7}{6} q_1$

Given q = 130 J, substituting above

$130 = \frac{13}{6} \times q_1$

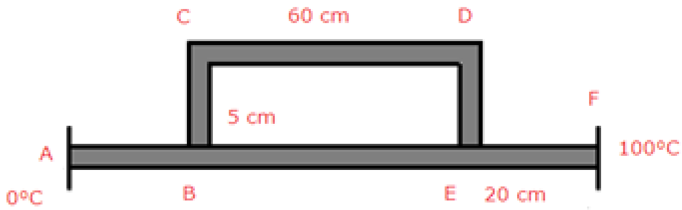
$\Rightarrow q_1 = 60$ J

27. Question

Suppose the bent part of the frame of the previous problem has a thermal conductivity of 780 J $s^{-1}m^{-1}$ °C^{-1}, whereas it is 390 J $s^{-1}m^{-1}$°$C^{-1}$for the straight part. Calculate the ratio of the rate of hat flow through the bent part to the rate of heat flow through the straight part.

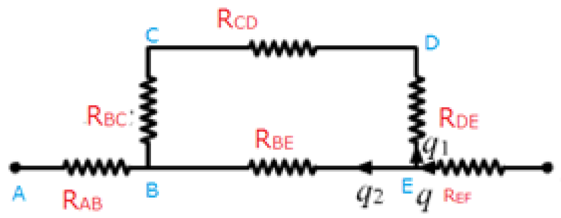
Answer

From previous question,–



Now, its given bent part has k = 780 J $s^{-1}m^{-1}$ $°C^{-1}$ and straight part has k =390 J $s^{-1}m^{-1}°C^{-1}$for

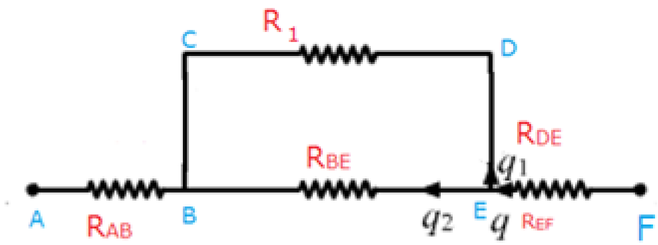
Also Resistance equivalent circuit was as follows-



Now,

$R_{AB} = \frac{20}{390 \times A}$, $R_{BC} = \frac{5}{780 \times A}$, $R_{CD} = \frac{60}{780 \times A}$ $R_{DE} = \frac{5}{780 \times A}$, $R_{EF} = \frac{20}{390 \times A}$ and $R_{BE} = \frac{60}{390 \times A}$

After reducing the equivalent resistance across B and E, our circuit becomes



Where

$R_1 = R_{BC} + R_{CD} + R_{DE}$

$= \frac{5}{780 \times A} + \frac{60}{780 \times A} + \frac{5}{780 \times A}$

$= \frac{70}{780 \times A}$

Since length is in cm and conductivity in meters, so multiply with 10^{-2}

$R_1 = \frac{70 \times 10^{-2}}{780 \times A}$

Again, since R_1 and R_{BE} are in parallel, so total heat across R1 and R_{BE} will be same.

ie, $q_1 \times R_1 = q_2 \times R_{BE}$

$$\Rightarrow q_1 \times \frac{70}{780 \times A} 10^{-2} = q_2 \times \frac{60}{390 \times A} 10^{-2}$$

$$\Rightarrow \frac{q1}{q2} = \frac{12}{7}$$

28. Question

A room has a window fitted with a single 1.0 m × 2.0 m glass of thickness 2 mm.

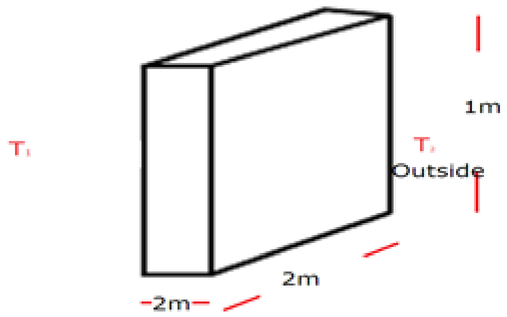
(a) Calculate the rate of heat flow through the closed window when the temperature inside the room is 32°C and that outside is 10°C.

(b) The glass is now replaced by two glass panes, each having a thickness of 1 mm and separated by a distance of 1 mm.

Calculate the rate of heat flow under the same conditions of temperature. Thermal conductivity of window glass = 1.0 J $s^{-1}m^{-1}$ °$C^{-1}$and that of air = 0.025 J $s^{-1}m^{-1}$ °C^{-1}.

Answer

The diagram is shown –



(a) Given

Thickness, l = 2 mm = 0.0002 m

Temperature inside the room = 32°C

outside = 10°C

Dimensions of wall = 1.0 m × 2.0 m

Rate of flow of heat

$$= \frac{k\,A\,\Delta T}{l}$$

Where,

ΔT = is change in temperature between the two sides of the window.

A= Area of cross section of the window

K = thermal conductivity of the window

L = length of the window

$= \frac{(1 \times 2 \times 1(40 - 32)}{(2 \times 10{-}3)}$

= 8000J/s

(b). Resistance of glass-

The equivalent circuit for the two glass panes and air becomes

Here resistance of glass $R_g = \frac{l}{Akg}$

And of air $R_{air} = \frac{l}{Aka}$

Since, these are connected in series, equivalent resistance becomes

$R_{eqv} = \frac{l}{A_{kg}} + \frac{l}{A_{ka}} + \frac{l}{A_{kg}}$

Thermal conductivity of window glass A_{kg}= 1.0 J $s^{-1}m^{-1}$ °C^{-1}

And of air, A_{kg} = 0.025 J $s^{-1}m^{-1}$ °C^{-1}.

Substituting values

$R_{eqv} = \frac{l}{A}(\frac{2}{kg} + \frac{1}{ka})$

$= \frac{1*10-3}{2}(\frac{2}{1} + \frac{1}{0.025})$

= 0.021

Now, rate of heat flow

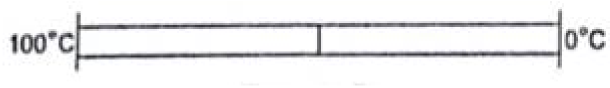
$\frac{Q}{t} = \frac{\Delta t}{R_{eqv}}$

$= \frac{(40-32)}{0.021}$

= 380.95

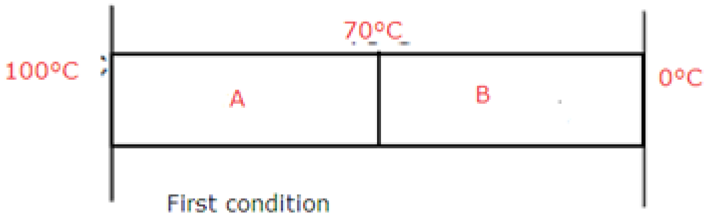
29. Question

The two rods shown in figure have identical geometrical dimensions. They are in contact with two heat baths at temperatures 100°C and 0°C. The temperature of the junction is 70°C. Find the temperature of the junction if the rods are interchanged.



Answer

Let's take first condition –



Since, rods are connected in series, rate of heat flow remains constant.

$\frac{Q}{t} = \frac{KA\Delta T}{l}$ = constant

Where

ΔT = is change in temperature between the two sides of the rods.

A= Area of cross section of the rods

K = thermal conductivity of the rods

L = length of the rod

Hence

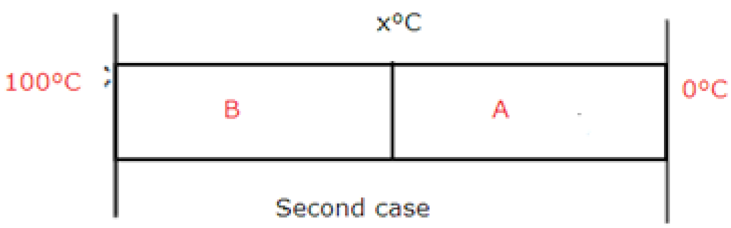
$\frac{KA \times A \times (100-70)}{l} = \frac{KB \times A \times (70-0)}{l}$

Where ,

K_A , K_B are thermal conductivity of rod A and B

$\Rightarrow 30\ K_A = 70\ K_B$ ....(1)

Now, in second condition, the rods are interchanged as shown in fig below –



Let x°C be the unknown temperature of the junction AB

Now, since rate of heat flow remains constant-

$$\frac{KB \times A \times (100-x)}{l} = \frac{KA \times A \times (0-x)}{l}$$

That gives

$100\ K_B - x\ K_B = K_A\ x$

Substituting for K_A from(1) and solving the above equation

$100 = \frac{7}{3} x + x$

⇒

$x = 30°C$

30. Question

The three rods shown in figure have identical geometrical dimensions. Heat flows from the hot end at a rate of 40 W in the arrangement

(a) Find the rates of heat flow when the rods are joined as in arrangement (b) and in (c). Thermal conductivities of alumimium and copper are 200 W m^{-1} $°C^{-1}$ and 400 W m^{-1} $°C^{-1}$ respectively.

Answer

(a).Let's redraw the diagram

Since all rods are connected in series,

$R_{eq} = R_{Al} + R_{Cu} + R_{Al}$

Given -

Temperature of the hot end ,T_1 = 100°C

cold end ,T_2 = 0°C

Substitution for R_{eq} –

$$R_{eq} = \frac{l}{AK_{Al}} + \frac{l}{AK_{Cu}} + \frac{l}{AK_{AL}}$$

Given Thermal conductivities of aluminium =200 W m−1 °C−1

Copper = 400 W m−1 °C−1

Therefore

$R_{eq} = \frac{i}{A}\left(\frac{1}{200} + \frac{1}{400} + \frac{1}{200}\right) = \frac{i}{A}\left(\frac{1}{80}\right)$

Now rate of flow of heat,

$\frac{dQ}{dt} = \frac{(T2-T1)}{R}$

$= \frac{(100-0)}{\frac{i}{A}\left(\frac{1}{80}\right)}$ (1)

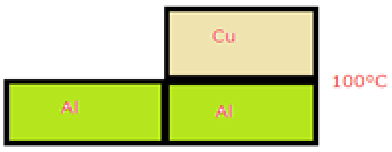
Given rate of flow of heat $\frac{dQ}{dt} = 40W$

From (1)

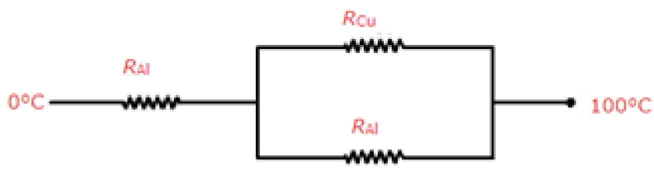
$40 = 80 \times 100 \times \frac{i}{A}$

$\Rightarrow \frac{i}{A} = \frac{1}{200}$ (2)

(b) Lets redraw the diagram –



The equivalent circuit in terms of thermal resistances becomes-



$R_{eq} = R_{Al} + \frac{1}{R_{Al}} + \frac{1}{R_{Cu}}$

$= \frac{l}{AK_{Al}} + \frac{AK_{Al}}{l} + \frac{AK_{Cu}}{l}$

$= \frac{\frac{1}{AK_{Al}} + \frac{1}{AK_{Al}} + \frac{1}{AK_{Cu}}}{\frac{1}{AK_{Al}} + \frac{1}{AK_{Cu}}}$

$= \frac{l}{A}\left(\frac{4}{600}\right)$

Now rate of flow of heat,

$\frac{dQ}{dt} = \frac{(T2-T1)}{R}$

$= \frac{100}{\frac{l}{A}\frac{4}{600}}$ (3)

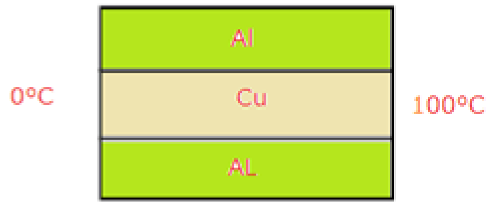
From (2) $\frac{A}{l} = \frac{1}{200}$

Substituting in (3)

$$\frac{dQ}{dt} = \frac{100}{200 * \frac{4}{600}}$$

$$\frac{dQ}{dt} = 75 \text{ W}$$

(c) Lets redraw the diagram-



Since the thermal resistors are connected in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_{Al}} + \frac{1}{R_{Cu}} + \frac{1}{R_{Al}}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{\frac{1}{AK_{Al}}} + \frac{1}{\frac{1}{AK_{Al}}} + \frac{1}{\frac{1}{AK_{Cu}}}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{A}{l}(K_{Al} + K_{Al} + K_{Cu}) = \frac{A}{l}(200+200+400)$$

$$\Rightarrow R_{eq} = \frac{A}{l} * (800)$$

$$\Rightarrow \frac{dQ}{dt} = \frac{(T_2 - T_1)}{R_{eq}}$$

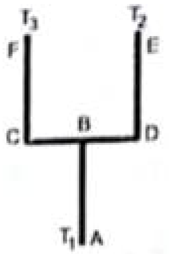
From (2) $\frac{A}{l} = \frac{1}{200}$

$$\Rightarrow \frac{dQ}{dt} = \frac{100 * 800}{(200)}$$

$$\Rightarrow \frac{dQ}{dt} = 400\text{W}$$

31. Question

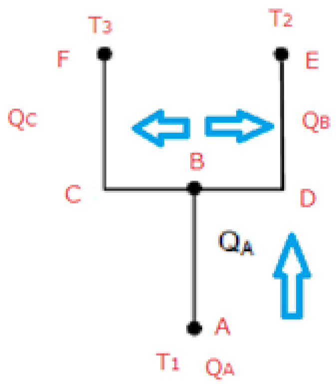
Four identical rods AB, CD, CF and DE are joined as shown in figure. The length cross-sectional area and thermal conductivity of each rod are ℓ, A and K respectively. The ends A, E and F are maintained at temperatures T_1, T_2 and T_3 respectively. Assuming no loss of heat to the atmosphere, final the temperature at B.



Answer

Let's redraw the diagram

Let the temperature at junction B be T.



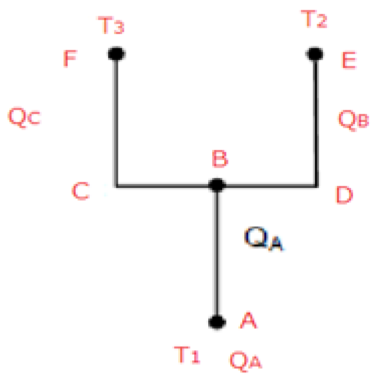
Let Q_A, Q_C and Q_B be the heat currents, i.e. rate of flow of heat per unit time in AB, BCE and BDF, respectively.

From fig.

At point B

$Q_A = Q_C + Q_B$

Now, rate of flow of heat is given by –



$$\frac{Q_A}{t} = \frac{Q_C}{t} + \frac{Q_B}{t}$$

$$\Rightarrow \frac{K_A (T_1 - T)}{l} = \frac{K_A (T_3 - T)}{\frac{3l}{2}} + \frac{K_A (T - T_2)}{\frac{3}{2}l}$$

$\Rightarrow (T_1 - T) = \frac{(T_3 - T)}{\frac{3}{2}} + \frac{(T - T_1)}{\frac{3}{2}}$

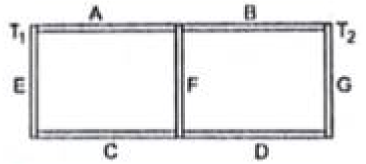
$\Rightarrow T = \frac{3T_1 + 2(T_2 + T_3)}{7}$

32. Question

Seven rods A, B, C, D, E, F and G are joined as shown in figure. All the rods have equal cross-sectional area A and length ℓ. The thermal conductivities of the rods are $K_A = K_C = K_D$, $K_D = 2K_D$, $K_E = 3K_D$, $K_F = 4K_D$ and $K_D = 5K_D$. The rod E is kept at a constant temperature T_1 and the rod G is kept at a constant temperature $T_2(T_2 > T_1)$.

(a) Show that the rod F ahs a uniform temperature $T = (T_1 + 2T_2)/3$.

(b) Find the rate of heat flowing from the source which maintains the temperature T_2.

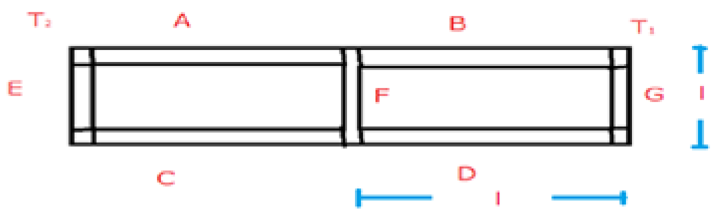


Answer

Given thermal conductivity of the respective rods as follows-

$K_A = K_C = K_0 K_B = K_D = 2K_0 K_E = 3K_0, K_F = 4K_0 K_9 = 5K_0$

Also, length of each rod is *l*



At steady state, temperature at the ends of rod F will be same.

Let T be the temperature of rod F

(a)

Rate of heat flow through rod A + rod C

= Rate of heat flow through rod B + rod D

$$\frac{Q}{t}(c) + \frac{Q}{t}(A) = \frac{Q}{t}(B) + \frac{Q}{t}(D)$$

$$\Rightarrow \frac{k_A(T_1 - T) \times A}{l} + \frac{k_C(T_1 - T) \times A}{l} = \frac{k_B(T_1 - T) \times A}{l} + \frac{k_D(T_1 - T) \times A}{l}$$

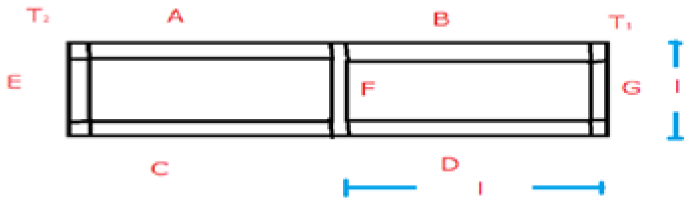
Substituting the values in terms of k_0 –

$$\Rightarrow \frac{k0(T_1-T)\times A}{l}+\frac{k0(T_1-T)\times A}{l}=\frac{2k0(T_1-T)\times A}{l}+\frac{2k0(T_1-T)\times A}{l}$$

$\Rightarrow 2k_0\,(T_1\text{-}T) = 2 \times 2\,k_0\,(T\text{–}T_2)$

$\Rightarrow T = \frac{T1+2T2}{3}$

(b) To find the rate of flow of heat from rod G, which is at Temperature T_2



Looking into the above diagram, we can say that it forms a balanced Wheatstone bridge.Also, as the ends of rod F are maintained at the same temperature, no heat current flows through rod F.

Hence we can remove the F for simplification

From above diagram, we can see that R_A and R_B are connected in series.

$\Rightarrow R_{AB} = R_A + R_B$

And R_C and R_D are connected in series

$\Rightarrow R_{CD} = R_C + R_D$

Then, R_{AB} and R_{CD} are connected in parallel

Now ,

$R_A = \frac{l}{K_0A}$, $R_B = \frac{l}{2K_0A}$, $R_C = \frac{l}{K_0A}$, $R_D = \frac{l}{2K_0A}$

Since R_A , R_B are connected in series

$R_{AB} = \frac{3l}{2K_0A}$ and $R_{CD} = \frac{3l}{2K_0A}$

Since R_{AB} R_{CD} are in parallel

$$R_{eqv} = \frac{1}{R_{AB}} + \frac{1}{R_{CD}}$$

$$= \frac{3l}{2K_0A} + \frac{3/}{2K_0A}$$

$$= \frac{3l}{4K_0A}$$

Now, rate of flow of heat from the source rod

$q = \frac{\Delta T}{R_{eqv}} = \frac{(T_1 - T_2)}{\frac{3l}{4K_0A}}$

$= \frac{4K_0A\,(T_1 - T_2)}{3l}$

Hence, rate of flow of heat from the source rod is given by

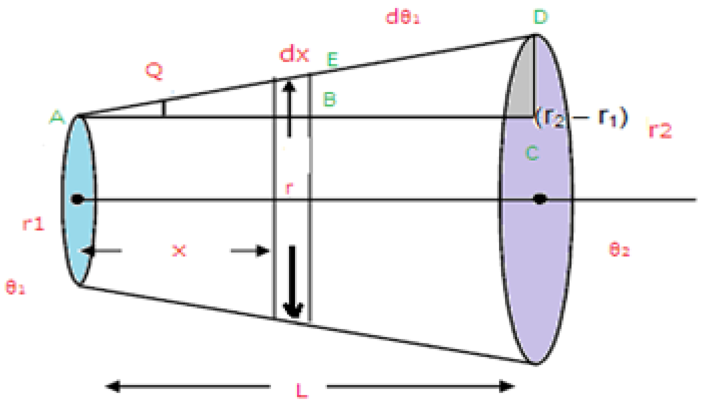
$q = \frac{4K_0A\,(T_1 - T_2)}{3l}$

33. Question

Find the rate of heat flow through a cross-section of the rod shown in figure ($\theta_2 > \theta_1$). Thermal conductivity of the material of the rod is K.

Answer

Let's redraw the diagram



From the above diagram we can say that ΔABE is similar to ΔACD.

By the property of similar triangles-

$\frac{x}{L} = \frac{r - r_1}{r_2 - r_1}$

$\Rightarrow x = r_1 + (r_2 - r_1)\frac{x}{L}$

Lets assume-

$a = \frac{(r_2 - r_1)}{L}$

$\Rightarrow r = ax + r_1$ (1)

Thermal resistance is given by –

$dR = \frac{dx}{K \cdot A}$

Now area $A = \pi r^2$

$$dR = \frac{dx}{K \cdot \pi r^2}$$

$$dR = \frac{dx}{K \cdot \pi (ax + r1)^2}$$

$$\int_0^R dR = \frac{1}{K \cdot A} \int_0^L \frac{dx}{(ax + r_1)^2}$$

Solving above integral

$$R = \left(\frac{-1}{K a \cdot \pi (ax + r_1)} \right)_0^L$$

$$\Rightarrow R = \frac{L}{K \cdot \pi r_1 \, r2}$$

–

Rate of heat flow = $\frac{\Delta Q}{R}$

$$q = \frac{Q_2 - Q_1}{L} K \pi r_1 r_2$$

34. Question

A rod of negligible heat capacity has length 20 cm, area of cross-section 1.0 cm^2 and thermal conductivity 200 W m^{-1} °C^{-1}. The temperature of one end is maintained at 0°C and that of the other end is slowly and linearly varied from 0°C to 60°C in 10 minutes. Assuming no loss of heat through the sides, find the total heat transmitted through the rod in these 10 minutes.

Answer

Given-Length of the rod, *l* = 20 cm = 0.2 mArea of cross section of the rod, *A* = 1.0 cm^2 = 1.0×10^{-4} m^2

Thermal conductivity of the rod, k = 200 W m-1°C-1

Also, the temperature of one end is maintained at 0°C and that of the other end is varied from 0°C to 60°C in 10 minutes.

Hence rate of increase of the temperature at one end is 0.1°C per second.

$$\Rightarrow \frac{d\theta}{dt} = \frac{(\Delta T)}{t} = \frac{60}{60 \times 10} = 0.1°C/s$$

Now, rate of flow of heat –

$$\frac{dQ}{dt} = (\theta 1 - \theta 2) \frac{kA}{l}$$

$$= \frac{kA}{l}(0.1) + \frac{kA}{d}(0.2) + \frac{kA}{d}(0.3) \ldots\ldots + \frac{kA}{d}(60) \text{ (since } \frac{d\theta}{dt} = 0.1°C/s)$$

$= \frac{kA}{l}(0.1 + 0.2 + 0.3 + \cdots + 60)$

We know Arithmetic Progressions –

For a series given by

a+2a+..................na

Sum of n terms is given by

Sum= $\frac{n}{2}\ (2a + (n - 1)d)$, where a is the first term, d is the difference between first and second term and n is the number of terms.

Substituting the values, for 10 minutes –

Here a= 0,1. d = 0.1 and n = 60 for 10mins,

$$\frac{dQ}{dt} = \frac{kA}{l} \times \frac{600\ (2 * 0.1 + (600 - 1)0.1}{2}$$

$$= \frac{(200 * 1 * 10 - 4)}{0.2} \times (18000)$$

= 1800J

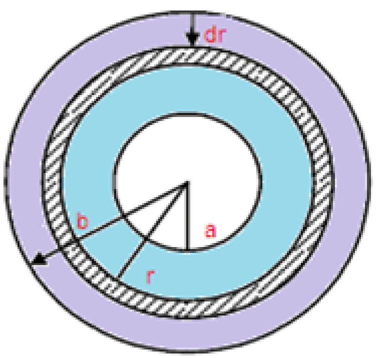
Hence , the total heat transmitted through the rod in these 10 minutes is 1800J

35. Question

A hollow metallic sphere of radius 20 cm surrounds a concentric metallic sphere of radius 5 cm. The space between the two spheres in filled with a nonmetallic material. The inner and outer spheres are maintained at 50°C and 10°C respectively and it is found that 100 J of heat passes from the inner sphere to the outer sphere per second. Find the thermal conductivity of the material between the spheres.

Answer

Let's redraw the circuit –



Let-Radius of the inner sphere =*a*Radius of the outer sphere = *b*

Given –

$a = r_1 = 5cm = 0.05m$

$b = r_2 = 20cm = 0.2m$

$\theta_1 = T_1 = 50°C$

$\theta_2 = T_2 = 10°$

Consider an imaginary shell of radii r and thickness dr.

Area, $A = \pi r^2$

Now, rate of flow of heat –

$q = -kA\left(\frac{dt}{dr}\right)$

dt = is change in temperature.

A= Area of cross section of the tube

K = thermal conductivity of the tube

d_r = change in length

Here, the negative sign is for decrease in temperature with increase in radius.

$q = -k\pi r^2\left(\frac{dt}{dr}\right)$

Taking integral on both sides –

$\int_a^b \frac{k\pi}{q} dt = -\int_{\theta_1}^{\theta_2} \frac{dr}{r^2}$

Solving above integral –

$q = \frac{dQ}{dt} = k\left(\frac{4\pi ab(\theta_1 - \theta_2)}{(b-a)}\right) = 100$ (given)

Substituting the values,

$K = \frac{15}{4\times\pi\times4\times10^{-1}} = 2.8 = 3$ W m-1°C-1

36. Question

Figure shows two adiabatic vessels, each containing a mass m of water at different temperatures. The ends of a metal rod of length L, area of cross-section A and thermal conductivity K, are inserted in the water as shown in the figure. Find the time taken for the difference between the temperatures in the vessels to become half of the original value. The specific heat capacity of water is s. Neglect the heat capacity of the rod and the container and any loss of heat to the atmosphere.

Answer

Given length of metal rod = L

specific heat capacity of water = s

Mass of water =m

Rate of transfer of heat –

$$\frac{Q}{t} = kA\frac{d\theta}{L}$$

$d\theta$ = is change in temperature.

A= Area of cross section of the tube

K = thermal conductivity of the tube

L= length

In time Δt, the heat transfer from the rod will be given by

$\Delta Q = \frac{KA(T_1 - T_2)\Delta t}{L}$ (1)

Now, heat loss by water at temperature T_1 is equal to the heat gain by water at temperature T_2

So, heat loss by water at temperature T_1 in time Δt is -

$\Delta Q = ms(T_1 - T_1')$ (2)

Where

m = mass of water

S = specific heat of water

From (1) and (2)

$\Rightarrow ms(T_1 - T_1') = KA(T_1 - T_2)\Delta t$

$$T_1' = T_1 - \frac{KA(T_1 - T_2)\Delta t}{(L \times m \times s)}$$

This is the fall in temperature of water at temperature T_1.

Similarly, rise in temperature of water at temperature T_2

$$T_2' = T_2 + \frac{KA(T_1 - T_2)\Delta t}{(L \times m \times s)}$$

Finally, change in temperature is given by-

$(T_1' - T_2') = T_1 - \frac{KA(T_1 - T_2)\Delta t}{(L \times m \times s)} - T_2 - \frac{KA(T_1 - T_2)\Delta t}{(L \times m \times s)}$

$\Rightarrow \{(T_1' - T_2') - (T_1 - T_2)\} = -2 \times \frac{KA(T_1 - T_2)\Delta t}{(L \times m \times s)}$

$\Rightarrow \frac{dT}{dt} = -2 \times \frac{KA(T_1 - T_2)\Delta t}{(L \times m \times s)}$

Where $\frac{dT}{dt}$ is the rate of change of temperature difference.

Taking integral on both sides-

$\int_{(T_1 - T_2)}^{(T_1 - T_2)} \frac{dT}{(T_1 - T_2)} = -\int 2 \frac{KA\Delta t}{(L \times m \times s)} dt$

$\Rightarrow t = \ln_2 \frac{kA}{m \times s}$

Hence the time taken for the difference between the temperatures in the vessels to become half of the original value is

$t = ln\,(2)\,\frac{L \times m \times s}{kA}$

37. Question

Two bodies of masses m_1 and m_2 and specific heat capacities s_1 and s_2 are connected by a rod of length ℓ, cross-sectional area A, thermal conductivity K and negligible heat capacity. The whole system is thermally insulated. At time t = 0, the temperature of the firs body is T_1 and the temperature of the second body is $T_2 (T_2 > T_1)$. Find the temperature difference between the two bodies at time t.

Answer

Given-

Masses of body = m_1 and m_2

Specific heat capacities = s_1 and s_2

Rod of length= ℓ,

Cross-sectional area = A

Thermal conductivity = K

Rate of transfer of heat from the rod is given by –

$\frac{\Delta Q}{\Delta t} = \frac{KA(T_2 - T_1)}{\ell}$ (1)

Where, $T_1\,and\,T_2$ = temperature of first and second body.

A= Area of cross section of the

K = thermal conductivity of the

L= length

Heat transfer from the rod in time Δt –

$\Delta Q=\frac{KA(T_2-T_1)\Delta t}{l}$ (2)

Heat loss by the body at temperature T_2 is equal to the heat gain by the body at temperature T_1.

Heat loss by the body at temperature T_2 in time Δt is –

$\Delta Q=m_2 s_2 \times (T_2'-T_2)$ (3)

From (1) and (2)

$m_2 s_2 \times (T_2'-T_2) =\frac{KA(T_2-T_1)\Delta t}{l}$

$\Rightarrow T_2'=T_2- \frac{KA(T_2-T_1)l(m_2 s_2)}{l(m_2 s_2)}\Delta t$

This is the fall in the temperature of the body at temperature T_2.

Similarly, rise in temperature of water at temperature T_1 is –

$T_1'=T_1+ \frac{KA(T_2-T_1)}{l(m_1 s_1)}\Delta t$

Change in the temperature

$(T_2'-T_1')$

$= T_2- \frac{KA(T_2-T_1)l(m_2 s_2)}{l(m_2 s_2)}\Delta t - T_1- \frac{KA(T_2-T_1)}{l(m_1 s_1)}\Delta t$

$\Rightarrow \{(T_2'-T_1')-(T_2-T_1)\}=- \frac{KA(T_2-T_1)l(m_2 s_2)}{l(m_2 s_2)}\Delta t - \frac{KA(T_2-T_1)}{l(m_1 s_1)}\Delta t$

$\Rightarrow \frac{\Delta T}{\Delta t}= -\frac{KA(T_2-T_1)}{l}(\frac{1}{m_1 s_1}+\frac{1}{m_2 s_2})\Delta t$

Where $\frac{\Delta T}{\Delta t}$ is the rate of change of temperature difference

$\Rightarrow \frac{1}{T_2-T_1}\Delta T= \frac{-KA}{l}(\frac{m_1 s_1+m_2 s_2}{m_1 s_1 m_2 s_2})$

Integrating both the sides -

$\int \frac{1}{T_2-T_1}\Delta T =\int \frac{-KA}{l}\left(\frac{m_1 s_1+m_2 s_2}{m_1 s_1 m_2 s_2}\right)dt$

$\Rightarrow \ln|T_2-T_1| = -\frac{-KA}{l}(\frac{m_1 s_1+m_2 s_2}{m_1 s_1 m_2 s_2})t$

Taking the anti-log

$\Rightarrow (T_2 - T_1) = e^{-\lambda t}$

Where

$\lambda = \frac{m_1 s_1 + m_2 s_2}{m_1 s_1 m_2 s_2}$

38. Question

An amount *n* (in moles) of a monatomic gas at an initial temperature T_0 is enclosed in a cylindrical vessel fitted with a light piston. The surrounding air has a temperature $T_s (> T_0)$ and the atmospheric pressure is p_a. Heat may be conducted between the surrounding and the gas through the bottom of the cylinder. The bottom has a surface area A, thickness x and thermal conductivity K. Assuming all changes to be slow, find the distance moved by the piston in time t.

Answer

Given,

In time *dt,* heat transfer through the bottom of the cylinder is given by-

$\frac{dQ}{dt} = \frac{KA(Ts - T0)}{x}$ (1)

In case of monoatomic gas, pressure remains constant.

Hence the heat content at constant pressure(enthalpy) is given by

$dQ = nC_p dT$ (2)

where,

dQ=change in heat

n = number of molecules

dT = change in temperature

C_p = amount of heat required to raise the temperature of a substance of 1Kg mass by one degree Celsius at constant pressure.

Comparing above equations-

$\frac{(nC_p dT)}{dt} = \frac{KA(T_s - T_0)}{x}$

For a monoatomic gas, C_p=52 R

$\Rightarrow \frac{n \times 5 \times RdT}{2dt} = \frac{KA(T_s - T_0)}{x}$

$\Rightarrow \frac{5nR}{2} \frac{dT}{dt} = \frac{KA(T_s - T_0)}{x}$

$$\Rightarrow \frac{dT}{T_s - T_0} = -\frac{2KAdt}{5nR \times x}$$

Integrating both the sides, we get

$$\int_{T_0}^{T_s} \frac{dT}{T_s - T_0} = \int \frac{2KAdt}{5nR \times x}$$

$$\ln(T_s - T_0)_{T_0}^{T} = -\frac{2KA \times t}{5nR \times x}$$

$$\Rightarrow \ln\left(\frac{(T_s - T)}{(T_s - T_0)}\right) = -\frac{2KAt}{5nR \times x}$$

Taking antilog

$$\Rightarrow T_s - T = (T_s - T_0) \times e^{\frac{2KAt}{5nRx}}$$

$$\Rightarrow T = T_s - (T_s - T_0) \times e^{\frac{2KAt}{5nRx}}$$

Rewriting

$$\Rightarrow T - T_0 = (T_s - T_0) \times \left(1 - e^{\frac{2KAt}{5nRx}}\right) \ (1)$$

Now, we know the gas equation given by

$$= \frac{PaAl}{nR}$$

Substituting in (1)

$$\frac{PaAl}{nR} = T - T_0 = (T_s - T_0) \times \left(1 - e^{\frac{2KAt}{5nRx}}\right)$$

Solving for the length/distance,

$$l = \frac{nR}{PaA}(T_s - T_0) \times \left(1 - e^{\frac{2KAt}{5nRx}}\right)$$

39. Question

Assume that the total surface area of a human body is 1.6 m^2 and that it radiates like an ideal radiator. Calculate the amount of energy radiated per second by the body if the body temperature is 37°C. Stefan constant σ is 6.0×10^{-8} W m^{-2} K^{-4}.

Answer

Given-Area of the body, $A = 1.6\ m^2$

Temperature of the body, $T = 310$ KWe know from Stefan-Boltzmann law, we have-

$$\frac{Energy\ Radiated}{Time} = \sigma AT^4$$

Where,

A is the area of the body

σ is the Stefan-Boltzmann constant

$\sigma = 6 \times 10^{-8}$ W m^{-2} K^{-4}.

Therefore,

Energy radiated per second = $1.6 \times 6 \times 10^{-8} \times (310)^4 = 886.58$

≈887 J

40. Question

Calculate the amount of heat radiated per second by a body of surface area 12 cm^2 kept in thermal equilibrium in a room at temperature 20°C. The emissivity of the surface = 0.80 and $\sigma = 6.0 \times 10^{-8}$ W m^{-2} K^{-4}.

Answer

Given

Area of the body, $A = 12 \times 10^{-4}$ m^2

Temperature of the body, T =20°C

= (273 + 20) K

= 293 K

Emissivity of the surface, e = 0.80

Stefan-Boltzmann constant $\sigma = 6.0 \times 10^{-8}$ W m^{-2} K^{-4}.

Now

Rate of emission of heat is given by-

$R = Ae\sigma T^4$

Where

A = Area of the surface

e = Emissivity of the surface

σ = Stefan-Boltzmann constant

And T = temperature

Substituting the values -

$\Rightarrow R = 12 \times 10^{-4} \times 0.80 \times 6.0 \times 10^{-8} \times (293)^4$

$\Rightarrow R = 0.42$ J