CBSE Sample Paper-01 (solved)
SUMMATIVE ASSESSMENT –I
MATHEMATICS
Class – IX

Time allowed: 3 hours | Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper comprises of 31 questions divided into four sections A, B, C and D. You are to attempt all the four sections.
c) Questions 1 to 4 in section A are one mark questions. These are MCQs. Choose the correct option.
d) Questions 5 to 10 in section B are two marks questions.
e) Questions 11 to 20 in section C are three marks questions.
f) Questions 21 to 31 in section D are four marks questions.
g) There is no overall choice in the question paper. Use of calculators is not permitted.

Section A

Q1. The value of $\sqrt{p^{-1}q}.\sqrt{q^{-1}r}.\sqrt{r^{-1}p}$ is

a) 0
  a. -1
  b. 2
  c. 1

Q2. If $x^2+\frac{1}{x^2}=14$, the value of $x+\frac{1}{x}$ is

a) $\sqrt{14}$
b) 7
c) 4
d) 196

Q3. The vertex angle of an isosceles triangle is 70°. The measure of one of the base angles is

a) 55°
b) 100°
c) 70°
d) 35°

Q4. Point $(-2,0)$ lies

a) On the negative part of the x - axis.
b) In the second quadrant
c) On the negative part of y - axis
d) In the third quadrant

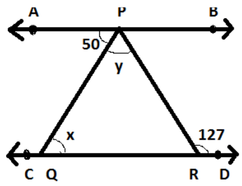
Section B

Q5. Is the product of two irrationals irrational? Justify your answer.

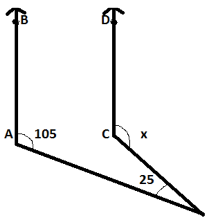
Q6. Find the remainder by using remainder theorem, when $p(x)=4x^3-12x^2+14x-3$ is divided by $g(x)=2x-1$.

Q7. Prove that two distinct lines cannot have more than one point in common.

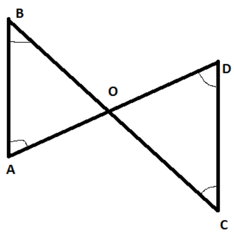
Q8. In the given figure, if $AB \parallel CD, \angle APQ=50°$ and $\angle PRD=127°$, find x and y.



Q9. In the given figure, $AB \parallel CD$. Find the value of x.



Q10. In the given figure, $\angle B<\angle A$ and $\angle C<\angle D$. Prove that $AD<BC$.



Section C

Q11. Prove that $\sqrt{5}$ is not a rational number.

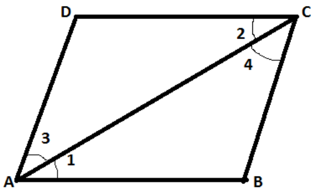
Q12. Express $0.2\overline{45}$ as a fraction in simplest form.

Q13. Check whether the following values of x are the zeroes of the polynomial.

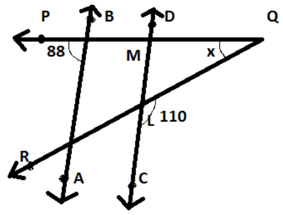
$$p(x)=3x^2-1; x=\frac{-1}{\sqrt{3}},\frac{2}{\sqrt{3}}$$

Q14. Without actual division, prove that $2x^4-6x^3+3x^2+3x-2$ is exactly divisible by x^2-3x+2.

Q15. In the given figure, it is given that $\angle 1=\angle 4$ and $\angle 3=\angle 2$. By which Euclid's axiom, it can be shown that if $\angle 2=\angle 4$ then $\angle 1=\angle 3$.



Q16. In the given figure, $AB \parallel CD$. Find the value of x.



Q17. Let OA, OB, OC and OD be the rays in the anticlock wise direction starting from OA such that $\angle AOB = \angle COD = 100°, \angle BOC = 82°$ and $\angle AOD = 78°$. Is it true that AOC and BOD are straight lines? Justify your answer.

Q18. In $\Delta PQR, \angle P = 70°, \angle R = 30°$. Which side of this triangle is the longest? Give reasons for your answer.

Q19. Plot the points $(3,2),(0,3),(5,0),(4,-2),(-2,4),(-2,-4)$ in the Cartesian plane.

Q20. A park in the shape of a quadrilateral $ABCD$, has $\angle C = 90°, AB = 9m, BC = 12m, CD = 5m$ and $AD = 8m$. How much area does it occupy?

Section D

Q21. a) If $a = 9 - 4\sqrt{5}$, find the value of $a - \frac{1}{a}$.

b) If $x = 3 + 2\sqrt{2}$, find the value of $x^2 + \frac{1}{x^2}$

Q22. Represent $\sqrt{3.5}$ on the number line.

Q23. Let p and q be the remainders, when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^3 - 12x + 6$ are divided by $x+1$ and $x-2$ respectively. If $2p + q = 6$, find the value of a.

Q24. Factorise: $x^{12} - y^{12}$

Q25. Prove that: $(x+y)^3 + (y+z)^3 + (z+x)^3 - 3(x+y)(y+z)(z+x) = 2(x^3 + y^3 + z^3 - 3xyz)$

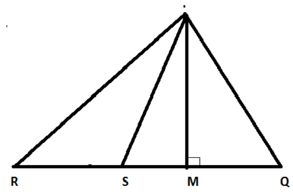
Q26. If $\left(x - \frac{1}{3}\right)$ and $(x-3)$ are factors of $ax^2 + 5x + b$, prove that $a = b$.

Q27. If D is the midpoint of the hypotenuse AC of a right angled ΔABC, prove that $BD = \frac{1}{2}AC$.

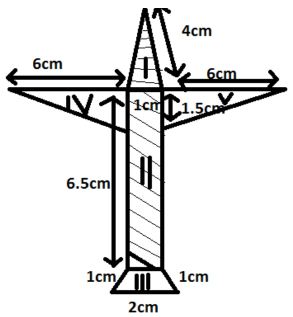
Q28. In a triangle, prove that the greater angle has the longer side opposite to it.

Q29. If the arms of one angle are respectively parallel to the arms of another angle, show that the two angles are either equal or supplementary.

Q30. In the given figure, PS is the bisector of $\angle QPR$, $PT \perp RQ$ and $\angle Q > \angle R$. Show that $\angle TPS = \frac{1}{2}(\angle Q - \angle R)$



Q31. Radha made a picture of an aeroplane with colored paper as shown in the following figure



Find the total area of the paper used.

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ANSWER KEY

1. D

2. C

3. A

4. A

5. Product of two irrationals may or may not be irrational.

   e.g 1. $\sqrt{5}$ and $\sqrt{5}$ both are irrational.

   But $\sqrt{5}*\sqrt{5}=5$, which is rational.

   2. $2+\sqrt{3}$ and $\sqrt{7}$ both are irrational.

   But $2+\sqrt{3}*\sqrt{7}=2\sqrt{7}+\sqrt{21}$, which is irrational.

6. By remainder theorem, we know that when $p(x)$ is divided by $g(x)$, it gives remainder equal to $p\left(\frac{1}{2}\right)$.

   Remainder = $p\left(\frac{1}{2}\right)=4\left(\frac{1}{2}\right)^3-12\left(\frac{1}{2}\right)^2+14\left(\frac{1}{2}\right)-3$

   $=4*\frac{1}{8}-12*\frac{1}{4}+14*\frac{1}{2}-3$

   $=\frac{1}{2}-3+7-3=\frac{3}{2}$

   Hence, required remainder $=\frac{3}{2}$

7. Here, we are given two lines p and q.

   We need to prove that lines p and q have only one point in common.

   For the time being, let us suppose that the two lines intersect in two distinct points say A and B. But this assumption contradicts with the above theorem that only one line can pass through two distinct points. So, the assumption that we started with that two lines can pass through two distinct points is incorrect.

Hence, are forced to conclude that two distinct lines cannot have more than one point in common.

8. $AB \parallel CD$ and PQ is a transversal.

$\Rightarrow x = 50°$ (alternate angles)

$AB \parallel CD$ and PR is a transversal.

$\therefore \angle APR = \angle PRD$ (alternate angles)

$50° + y = 127°$

$y = 127° - 50°$

$y = 77°$

9. From E, draw $EF \parallel AB \parallel CD$

Now, $EF \parallel CD$ and CE is the transversal.

$\therefore \angle DCE + \angle CEF = 180°$ (co-interior angles)

$\Rightarrow x + \angle CEF = 180°$

$\Rightarrow \angle CEF = 180° - x$

Again, $EF \parallel AB$ and AE is the transversal.

$\therefore \angle BAE + \angle AEF = 180°$ (co-interior angles)

$\Rightarrow 105° + \angle AEC + \angle CEF = 180°$

$\Rightarrow 105° + 25° + (180° - x) = 180°$

$\Rightarrow x = 130°$

Hence, $x = 130°$

10. In $\Delta AOB, \angle B < \angle A$

$\Rightarrow OA < OB$ (smaller angle has shorter side opposite to it) ………..(1)

In $\Delta OCD, \angle C < \angle D$

$\Rightarrow OD < OC$ (smaller angle has shorter side opposite to it) …………..(2)

Adding (1) and (2), we get

$OA + OD < OB + OC$

$AD < BC$

Hence proved

11. Let 5 be a whole number and not a perfect square. If possible, let the square root of 5 be a rational number $\frac{p}{q}$ which is in simplest form. This means that p and q have no common factor.

Now, $\frac{p}{q} = \sqrt{5}$

$\Rightarrow \frac{p^2}{q^2} = 5$

$\Rightarrow p^2 = 5q^2$

$\Rightarrow 5$ is a factor of p^2. ($\because 5$ is a factor of $5q^2$ and $p^2 = 5q^2$)

$\Rightarrow 5$ is a factor of p.

Let $p = 5m$ for some natural number m.

Then, $p = 5m$

$\Rightarrow p^2 = 5^2 m^2$

$\Rightarrow 5q^2 = 5^2 m^2$ ($\because p^2 = 5q^2$)

$\Rightarrow q^2 = 5m^2$

$\Rightarrow 5$ is a factor of q^2 ($\because 5$ is a factor of $5m^2$ and $q^2 = 5m^2$)

$\Rightarrow 5$ is a factor of q

But, 5 is factor of p and 5 is a factor of q means that 5 is a factor of both of p and q. This contradicts the assumption that p and q have no common factor. This means that our supposition is wrong.

Hence, $\sqrt{5}$ is an irrational number.

12. Let $x = 0.2\overline{45}$

$x = 0.2454545\ldots\ldots\ldots$

Multiplying both sides by 10 we have,

$10x = 2.454545\ldots\ldots$ ....(1)

Multiplying both sides by 100 we have,

$1000x = 245.454545\ldots\ldots\ldots$ ....(2)

Subtracting (1) from (2), we get

$1000x - 10x = 245.4545\ldots - 2.4545\ldots\ldots$

$990x = 243$

$x = \frac{243}{990}$

$x = \frac{27}{110}$

13. $p(x) = 3x^2 - 1$

$x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}},$

$p\left(\frac{-1}{\sqrt{3}}\right) = 3\left(\frac{-1}{\sqrt{3}}\right)^2 - 1$

$= 3 * \frac{1}{3} - 1$

$= 1 - 1 = 0$

$p\left(\frac{2}{\sqrt{3}}\right) = 3\left(\frac{2}{\sqrt{3}}\right)^2 - 1$

$= 3 * \frac{4}{3} - 1$

$= 4 - 1 = 3 \neq 0$

$\therefore \frac{-1}{\sqrt{3}}$ is a zero but $\frac{2}{\sqrt{3}}$ is not a zero of the given polynomial.

14. Let $p(x) = 2x^4 - 6x^3 + 3x - 2$ and $g(x) = x^2 - 3x + 2$

Then, $g(x) = x^2 - 3x + 2$

$= (x-1)(x-2)$

Clearly $(x-1)$ and $(x-2)$ are factors of $g(x)$

Let $x - 1 = 0 \Rightarrow x = 1$

$p(1) = 2(1)^4 - 6(1)^3 + 3(1)^2 + 3(1) - 2$

$= 2 - 6 + 3 + 3 - 2 = 0$

Let $x - 2 = 0 \Rightarrow x = 2$

$p(2) = 2(2)^4 - 6(2)^3 + 3(2)^2 + 3(2) - 2$

$= 32 - 48 + 12 + 6 - 2$

$= 50 - 50 = 0$

$\therefore (x-1)$ and $(x-2)$ are factors of $p(x)$

$\Rightarrow g(x)=(x-1)(x-2)$ is a factor of $p(x)$

Hence, $p(x)$ is exactly divisible by $g(x)$

15. By Euclid's 1axiom, which states that " things which are equal to the same thing are equal to one another ". Prove this statement youself.

16. $x=70°+88°=18°$ $(\because \angle QLM = 180°-110°=70°$ and $AB \| CD \Rightarrow \angle PML = 88°)$

17. Draw the figure

Given: OA, OB, OC and OD are rays in the anticlockwise direction such that

$\angle AOB = \angle COD = 100°$ and $\angle BOC = 82°, \angle AOD = 78°$

AOC is not a line.

Because, $\angle AOB + \angle BOC = 100° + 82° = 182°$, which is not equal to $180°$.

Similarly, BOD is not a line.

Because, $\angle COD + \angle AOD = 78° + 100° = 178°$, which is not equal to $180°$.

18. $\angle Q = 180° - (70° + 30°)$, which is largest

$\therefore$ longest side is PR.

19. Plot the points on the graph paper.

20. Draw the figure

Using pythagoras theorem, in right angled ΔBCD, $BD = 13m$

Using heron's formula, area of $\Delta ABD = 35.5 sq.m$

Using area of triangle $BCD = \frac{1}{2} * b * h = 30 sq.m$

Total area =65.5sq.m

21. a) $a = 9-4\sqrt{5} \Rightarrow \frac{1}{a} = \frac{1}{9-4\sqrt{5}} = \frac{9+4\sqrt{5}}{81-80} = 9+4\sqrt{5}$

$\therefore a - \frac{1}{a} = 9-4\sqrt{5}-9-4\sqrt{5} = -8\sqrt{5}$

b) $x = 3+2\sqrt{2} \Rightarrow x^2 = 9+8+12\sqrt{2} = 17+12\sqrt{2}$

$\frac{1}{x^2} = \frac{1}{17+12\sqrt{2}} = \frac{17-12\sqrt{2}}{289-288} = 17-12\sqrt{2}$

$$\therefore x^2+\frac{1}{x^2}=17+12\sqrt{2}+17-12\sqrt{2}=34$$

22. Do it yourself.

23. Let $p(x)=x^3+2x^2-5ax-7$ and $q(x)=x^3+ax^2-12x+6$

$p(-1)=p$ and $q(2)=q$

$\therefore p=-1+2+5a-7 \Rightarrow p=5a-6$

$q=8+4a-24+6=4a-10$

$\Rightarrow 2p+q=6$

$\Rightarrow 10a-12+4a-10=6$

$\Rightarrow 14a=28 \Rightarrow a=2$

24. $x^{12}-y^{12}=(x^6-y^6)(x^6+y^6)$

$=(x^3-y^3)(x^3+y^3)(x^2+y^2)(x^4+y^4-x^2y^2)$

$=(x-y)(x^2+y^2+xy)(x+y)(x^2+y^2-xy)(x^2+y^2)(x^4+y^4-x^2y^2)$

25. Let $x+y=p, y+z=q, z+x=r$

$LHS=p^3+q^3+r^3-3pqr$

$=(p+q+r)(p^2+q^2+r^2-pq-pr-rp)$

Now, $p+q+r=2(x+y+z)$

$p^2+q^2+r^2-pq-pr-rp=(x+y)^2+(y+z)^2+(z+y)(y+z)-(y+z)(z+x)-(z+x)(x+y)$

Solving we get, $=x^2+y^2+z^2-xy+yz-xz$

$\therefore (p+q+r)(p^2+q^2+r^2-pq-rq-rp)=2(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$=2(x^3+y^3+z^3-3xyz)$

26. Let $f(x)=ax^2+5x+b$

$f(3)=0 \Rightarrow 9a+15+b=0 \Rightarrow 9a+b=-15$ ....(1)

Similarly, $f\left(\frac{1}{3}\right)=0 \Rightarrow \frac{a}{9}+\frac{5}{3}+b=0 \Rightarrow \frac{a}{3}+b=-\frac{5}{3}$ ....(2)

$(1)=(2) \Rightarrow a=b$

27. **Given**: ΔABC in which $\angle B = 90°$ and D is the mid-point of AC.

To prove: $BD = \frac{1}{2}AC$

Construction: Produce BD to E so that $BD = DE$. Join EC

Proof: In ΔADB and ΔCDE, we have

$AD = DC$ ΔABC (Given)

$BD = DE$ (By construction)

$\angle ADB = \angle CDE$ (Vertically opp. Angles)

$\therefore \Delta ADB \cong \Delta CDE$ (By SAS congruence criterion)

$\Rightarrow AB = CE$ and $\angle CED = \angle ABD$ ....(1) (CPCT)

Thus, transversal BE cuts AB and CE such that the alternate angles $\angle CED$ and $\angle ABD$ are equal. So, $CE \parallel AB$

$\Rightarrow \angle ABC + \angle ECB = 180°$ (co-interior angles)

$\Rightarrow \angle ECB = 90°$ ($\because \angle ABC = 90°$)

Thus, in ΔABC and ΔECB, we have

$AB = AC$ (From (1))

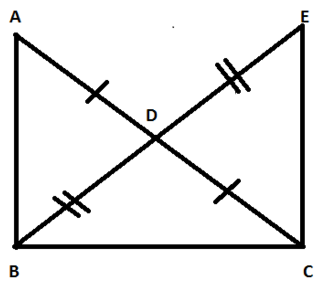
$BC = CB$ (common)

$\angle ABC = \angle ECB = 90°$

$\therefore \Delta ABC \cong \Delta ECB$ (By SAS)

$\Rightarrow AC = BE$ (CPCT)

$\Rightarrow \frac{1}{2}AC = \frac{1}{2}BE \Rightarrow \frac{1}{2}AC = BD$



28. **Given**: A triangle ABC in which $\angle ABC > \angle ACB$

To prove: $AC > AB$

Proof: There are three possibilities

(1) $AB > AC$

(2) $AB = AC$

(3) $AB < AC$

CASE 1: $AB > AC$

$\angle C > \angle B$**,** as the greater side has greater angle opposite to it.

It is not possible as we are given that $\angle B > \angle C$

CASE 2: $AB = AC$

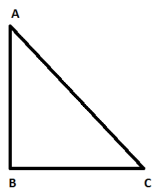
Then $\angle C = \angle B$ as angles opposite to equal sides are equal.

But $\angle B > \angle C$ is given. So it is also not possible.

CASE 3: $AB < AC$

As only one case is left, it has to be true.

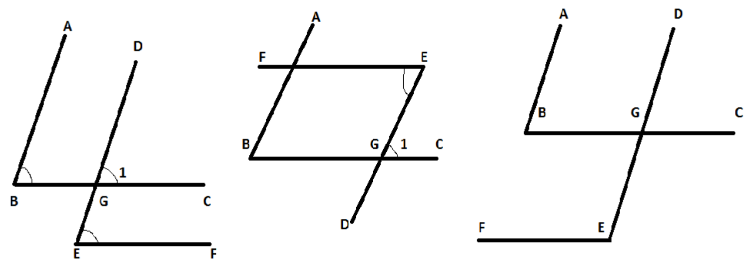
Hence, if two sides of a triangle are unequal, the greater side has greater angle opposite to it.



29. **Given**: Two angles $\angle ABC$ and $\angle DEF$ such that $BA \parallel ED$ and $BC \parallel EF$.

To prove: $\angle ABC = \angle DEF$ or $\angle ABC + \angle DEF = 180°$

Proof: The arms of the angles may be parallel in the same sense or in the opposite sense. So, three cases arise.



Case1: When both pairs of arms are parallel in the same sense.

In this case, $BA \parallel ED$ and BC is the transversal.

$\therefore \angle ABC = \angle 1$ (corresp. angles)

Again, $BC \parallel EF$ and DE is the transversal.

$\therefore \angle 1 = \angle DEF$ (corresp. Angles)

Hence, $\angle ABC = \angle DEF$

CASE 2: When both pairs of arms are parallel in opposite sense.

In this case, $BA \parallel ED$ and BC is transversal.

$\therefore \angle ABC = \angle 1$ (corresp. Angles)

Again, $FE \parallel BC$ and ED is the transversal

$\therefore \angle DEF = \angle 1$ (alternate int. angles)

Hence, $\angle ABC = \angle DEF$

CASE 3: When one pair of arms are parallel in same sense and other pair parallel in opposite sense.

In this case, $BA \parallel ED$ and BC is the transversal.

$\therefore \angle EGB = \angle ABC$ (alternate int. angles)

Now, $BC \parallel EF$ and DE is the transversal

$\therefore \angle DEF + \angle EGB = 180°$ (co-int. angles)

$\Rightarrow \angle DEF + \angle ABC = 180° \left(\because \angle EGB = \angle ABC\right)$

Hence, $\angle ABC$ and $\angle DEF$ are supplementary.

30. $\angle Q + \angle R = 180° - 2\angle QPS = 180° - 2\left(\angle QPT + \angle TPS\right)$

$= 180° - 2\left(90° - \angle 1 + \angle TPS\right)$

$\Rightarrow \angle 1 + \angle 2 = 2\angle 1 - 2\angle TPS$

$\Rightarrow \angle TPS = \frac{1}{2}\left(\angle 1 - \angle 2\right) = \frac{1}{2}\left(\angle Q - \angle R\right)$

31. ***Area of region 1***:

Region 1 is enclosed by a triangle of sides $a = 4cm, b = 5cm$ and $c = 1cm$

Let 2s be the perimeter of the triangle. Then,

$2s = 4 + 4 + 1 \Rightarrow s = \left(\frac{9}{2}cm\right)$

Using Heron's formula, area of region $1 = 19875 sq.cm$

Area of region 2:

Region 2 is a rectangle of length $65cm$ and breadth $1cm$

$\therefore$ Area of region $2 = 6.5 * 1$ sq.cm

Area of region 3:

Region 3 is an isos. Trapezium

Using pythagoras theorem for ΔABC, find BE

Area of region 3 $= 1.3 sq.cm$

Area of region 4 $= 4.5 sq.cm$ using area of triangle

Area of region 5: Region 4 and 5 are congruent, so , area $= 4.5 sq.cm$

Hence, the total area $= 18.7875 sq.cm$