Alternating Current

AC Voltage Applied to a Resistor, Inductor and Capacitor

Alternating Current (AC)

Alternating current is an electric current whose magnitude changes with time continuously and which reverses its direction periodically. It is mathematically represented as
$I = I_o \cos \omega t$ or $I = I_o \sin\omega t$
where, I_o = Peak value of AC and I = Instantaneous value of AC

Advantages of AC over DC

- Flexibility of converting from one value to other using transformer
- Can be transmitted over long distances economically as well as without much power loss

Mean Value or Average value of AC

It is defined as that value of steady current which sends the same amount of charge through a circuit in the time of half cycle (i.e. $T/2$) as is sent by AC through the same circuit in the same time.
Let, in small time dt, charge sent in the circuit due to AC, I=Io sinωtI=Io sinωt, is q. Thus,
$dq = Idt$

or, $q = \int_0^{\frac{T}{2}} I dt = \int_0^{\frac{T}{2}} I_o \sin \omega t dt$

or, $q = \frac{2I_o}{\omega} = \frac{I_o T}{\pi}$ .....(i)

If I_m is the average value of AC for first half cycle, then

$q = I_m \times \frac{T}{2}$ .....(ii)

From (i) and (ii), we have

$I_m = \frac{2}{\pi} I_o = 0.637 I_o$

Similarly, $V_m = \frac{2}{\pi} V_o = 0.637 V_o$

Root Mean Square Value (r.m.s.) of AC
It is defined as that value of steady current which generates the same amount of heat in a resistor in a given time as is done by AC through the same resistor in the same time.
If I=Io sinωtI=Io sinωt is flowing through a resistor of resistance R, then in small time dt, the heat generated in the resistance is
$dH = I^2 R dt$

or, $H = \int_0^T I^2 R dt = \int_0^T \left(I_o \sin \omega t\right)^2 R dt = \frac{I_o^2 RT}{2}$ .....(i)

If I_v be the r.m.s. value of AC, then heat produced by I_v in the same resistance is

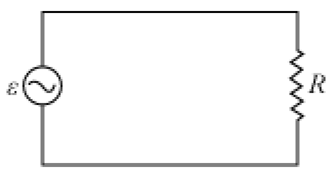
$H = I_v^2 \; Rdt$ .....(ii)

From (i) and (ii), we get

$I_v = \frac{I_o}{\sqrt{2}} = 0.707 I_o$

Similarly, $V_v = \frac{V_o}{\sqrt{2}} = 0.707 V_o$

AC Voltage Applied to a Resistor



$v = v_m \sin \omega t$ (ac voltage)

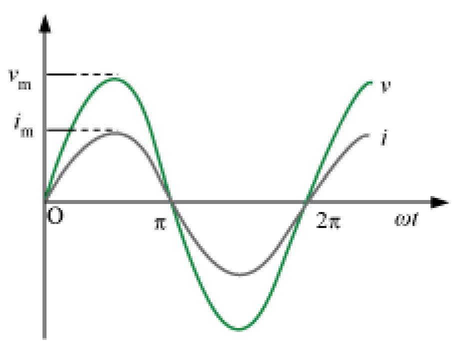
$v_m \sin \omega t = iR$

$i = \frac{v_m}{R} \sin \omega t$

$\because i_m = \frac{v_m}{R}$

$\therefore i = i_m \sin \omega t$

- In pure resistor, voltage and current are in phase with each other.



- Instantaneous power dissipated in the resistor is

$p = i^2 R$
$= i_m^2 R \sin^2 \omega t$

- Average power,

$$\overline{p} = \frac{1}{2} i_m^2 R$$

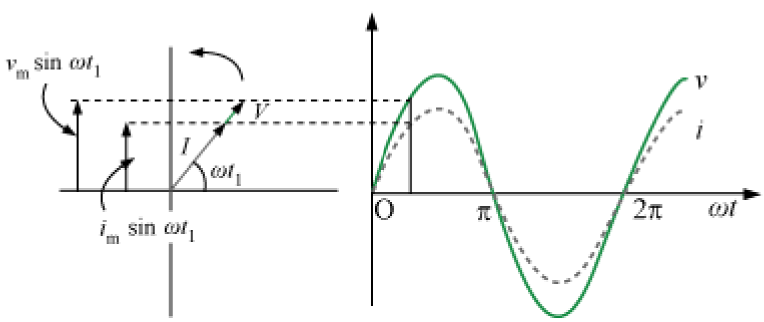
- Root mean square current, $I_{rms} = \sqrt{\overline{i^2}}$

$$= \sqrt{\frac{1}{2} i_m^2} = \frac{i_m}{\sqrt{2}}$$
$$= 0.707 i_m$$
$$V = \frac{v_m}{\sqrt{2}} = 0.707\, v_m$$
$$\therefore \frac{v_m}{\sqrt{2}} = \frac{i_m}{\sqrt{2}} R$$
$$V = IR$$

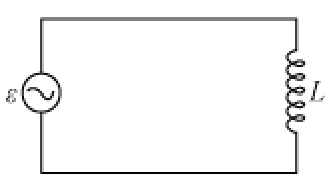
- Average power

$$P = \overline{P} = \frac{1}{2} i_m^2 R = I^2 R$$
$$P = V^2 / R$$
$$= IV$$

Phase diagram for the circuit:



AC voltage applied to an inductor



Source, $v = v_m \sin\omega t$

Using Kirchhoff's loop rule,

$$\sum \varepsilon(t) = 0$$

$$v - \frac{Ldi}{dt} = 0$$

$$\frac{di}{dt} = \frac{v}{L} = \frac{v_m}{L}\sin\omega t$$

Integrating *di/dt* with respect to time,

$$\int \frac{di}{dt}dt = \frac{v_m}{L}\int \sin(\omega t)dt$$

$$i = -\frac{v_m}{\omega L}\cos(\omega t) + \text{constant}$$

$$-\cos\omega t = \sin\left(\omega t - \frac{\pi}{2}\right)$$

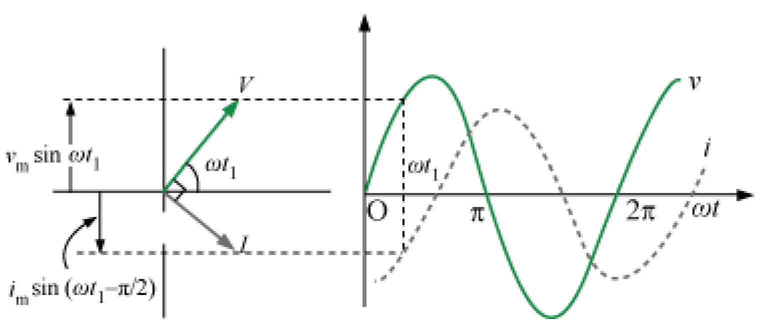
$$\therefore i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$\because$ Where, $i_m = \dfrac{v_m}{\omega L}$ is the amplitude of current

Inductive reactance, $X_L = \omega L$

$$i_m = \frac{v_m}{X_L}$$

Phase diagram for the circuit:



- Instantaneous power

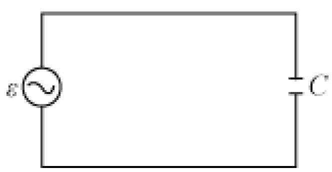
$$P_L = iv = i_m \sin\left(\omega t - \frac{\pi}{2}\right) \times v_m \sin(\omega t)$$
$$= -i_m v_m \cos\omega t \sin\omega t$$
$$= -\frac{i_m v_m}{2} \sin(2\omega t)$$

∴Average power

$$P_L = \left\langle -\frac{i_m v_m}{2} \sin(2\omega t) \right\rangle$$
$$= -\frac{i_m v_m}{2} \langle \sin(2\omega t) \rangle = 0$$

Do you know
A choke is a type of inductor designed in such a way that it blocks high frequency AC and allows low frequency AC or DC to pass through it. It has a magnetic core on which insulated coils are wound. Because of its low electric resistance, it allows low frequency signals to easily pass through it with low power loss.

AC voltage applied to a capacitor



ac voltage,

$$v = v_m \sin\omega t$$
$$\therefore v = \frac{q}{C}$$

Applying Kirchhoff's loop rule,

$$v_m \sin \omega t = \frac{q}{C}$$

$$i = \frac{dq}{dt}$$

$$\therefore i = \frac{d}{dt}(v_m C \sin \omega t)$$

$$= \omega C v_m \cos(\omega t)$$

$$\cos \omega t = \sin \omega t + \frac{\pi}{2}$$

$$\therefore i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i_m = \omega C v_m$$

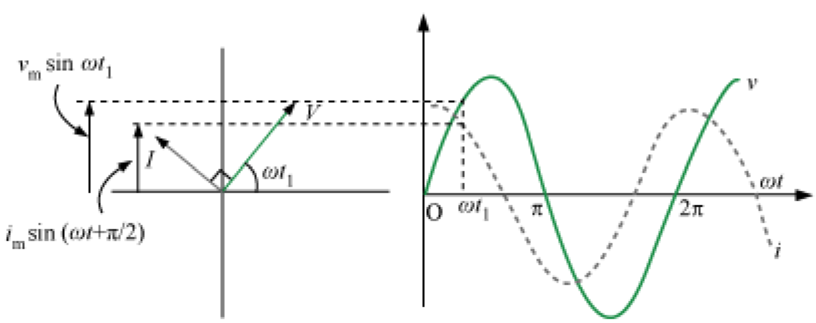
$$i_m = \frac{v_m}{\left(\frac{1}{\omega C}\right)}$$

Capacitive reactance

$$X_C = \frac{1}{\omega C}$$

$$i_m = \frac{v_m}{X_C}$$

Phase diagram for the circuit:



Instantaneous power

$$P_C = iv = i_m \cos(\omega t) v_m \sin(\omega t)$$

$$= i_m v_m \cos \omega t \sin \omega t$$

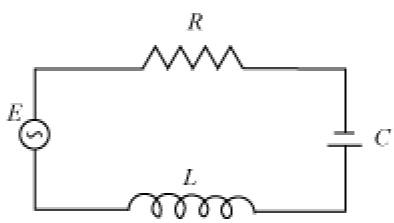
$$= \frac{i_m v_m}{2} \sin(2\omega t)$$

- Average power,

$$P_C = \left\langle \frac{i_m v_m}{2} \sin 2\omega t \right\rangle$$
$$= \frac{i_m v_m}{2} \langle \sin 2\omega t \rangle$$
$$= 0$$

(Since $\langle \sin 2\omega t \rangle$ = 0 over a complete cycle)

LCR Circuit



- An ac source (E) has a voltage of $v = v_m \sin\omega t$
- Let

q – Charge on the capacitor

i– Current

t – Time

Using Kirchhoff's loop rule in the above circuit, we obtain

$$L\frac{di}{dt} + iR + \frac{q}{C} = v$$

Analytical solution:

$$L\frac{di}{dt}+Ri+\frac{q}{C}=v_{\mathrm{m}}\sin\omega t$$
$$\because i=\frac{dq}{dt}$$
$$\therefore \frac{di}{dt}=\frac{d^2q}{dt^2}$$
$$\Rightarrow L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=v_{\mathrm{m}}\sin\omega t \qquad \ldots(1)$$

Let us assume,

$$q=q_{\mathrm{m}}\sin(\omega t+\theta)$$
$$\therefore \frac{dq}{dt}=q_{\mathrm{m}}\omega\cos(\omega t+\theta) \qquad \ldots(2)$$
$$\frac{d^2q}{dt^2}=-q_{\mathrm{m}}\omega^2\sin(\omega t+\theta) \qquad \ldots(3)$$

Putting the values of equations (2) and (3) in equation (1),

$$L\left(-q_{\mathrm{m}}\omega^2\sin\left(\omega t+\theta\right)\right)+R\left(q_{\mathrm{m}}\omega\cos\left(\omega t+\theta\right)\right)+\frac{q_{\mathrm{m}}\sin\left(\omega t+\theta\right)}{C}=v_{\mathrm{m}}\sin\omega t \qquad \ldots(4) \quad \because X_{\mathrm{C}}=\frac{1}{\omega C}$$

and $X_{\mathrm{L}} = \omega L$

∴ The above equation becomes $q_{\mathrm{m}}\omega\left[R\cos\left(\omega t+\theta\right)+\left(X_{\mathrm{C}}-X_{\mathrm{L}}\right)\sin\left(\omega t+\theta\right)\right]=V_{\mathrm{m}}\sin\omega t \qquad \ldots(5)$

Multiplying and dividing by $Z=\sqrt{R^2+\left(X_{\mathrm{C}}-X_{\mathrm{L}}\right)^2}$, we obtain

$$q_{\mathrm{m}}\omega Z\left[\frac{R}{Z}\cos\left(\omega t+\theta\right)+\frac{X_{\mathrm{C}}-X_{\mathrm{L}}}{Z}\sin\left(\omega t+\theta\right)\right]=v_{\mathrm{m}}\sin\omega t \qquad \ldots(6)$$

Let $\frac{R}{Z}=\cos\phi$

And, $\frac{X_{\mathrm{C}}-X_{\mathrm{L}}}{Z}=\sin\phi$

$$\therefore \tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{X_C - X_L}{R}$$

$$\therefore \phi = \tan^{-1}\frac{X_C - X_L}{R}$$

$$\therefore q_m \omega Z \cos(\omega t + \theta - \phi) = v_m \sin\omega t \quad ...(7)$$

Comparing the two sides of equation (7),

$$v_m = q_m \omega Z = i_m Z$$

Where,

$$i_m = q_m \omega$$

$$\theta - \phi = -\frac{\pi}{2}$$

$$\Rightarrow \theta = -\frac{\pi}{2} + \phi$$

∴ Current in the circuit is

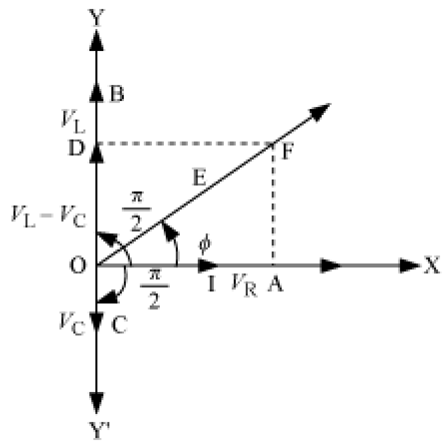
$$i = \frac{dq}{dt} = q_m \omega \cos(\omega t + \theta)$$

$$= i_m \cos(\omega t + \theta)$$

$$\Rightarrow i = i_m \sin(\omega t + \phi)$$

$$i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

And, $\phi = \tan^{-1}\frac{X_C - X_L}{R}$



Resonance

- Instantaneous current in the LCR circuit is

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

- At a particular value of the angular frequency of ac, ω_0, the inductive reactance and capacitive reactance are just equal.

$$\therefore \omega_0 L = \frac{1}{\omega_0 C}$$

- At $\omega = \omega_0$, the impedance of the LCR circuit is,

$$Z(at\omega_0) = \sqrt{R^2 + \left(\omega_0 L - \frac{1}{\omega_0 C}\right)^2}$$

$$Z(at\omega_0) = R$$

- $$I(at\omega_0) = \frac{E}{Z} = \frac{E}{R}$$

- A series LCR circuit which admits maximum current corresponding to a particular angular frequency ω_0 of the ac source is called series resonant circuit and the angular frequency ω_0 is called the resonant angular frequency.

- $\because \omega_0 L = \frac{1}{\omega_0 C}$

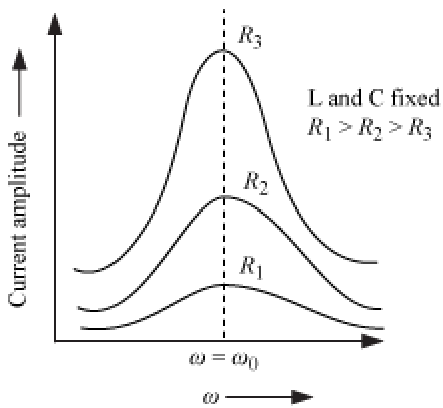
  $\therefore \omega_0 = \frac{1}{\sqrt{LC}}$

- Let f_0 be the resonance frequency.

  $\therefore f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$

Sharpness of resonance

- When the resistance of an LCR circuit is very low, a large current flows, and the angular frequency is close to the resonant frequency such as an LCR series circuit is said to be more selective or sharper.



- Suppose value of ω is such that the current in the circuit is $\frac{1}{\sqrt{2}}$ times the current amplitude of resonance.

- Two values are considered which are symmetrical about ω_0.

  $\omega_1 = \omega_0 + \Delta\omega$

  $\omega_2 = \omega_0 - \Delta\omega$

  i.e., $\omega_1 - \omega_2 = 2\Delta\omega$ is often called the bandwidth of the circuit

  $\omega_0/2\Delta\omega$ – Measure of the sharpness of resonance

Mathematical expression for sharpness of resonance

$$\frac{E}{\sqrt{R^2+\left(\omega_1 L-\frac{1}{\omega_1 C}\right)^2}}=\frac{1}{\sqrt{2}}\times I\left(at\omega_0\right)$$

$$\therefore \frac{E}{\sqrt{R^2+\left(\omega_1 L-\frac{1}{\omega_1 C}\right)^2}}=\frac{1}{\sqrt{2}}\times\frac{E}{R}$$

$$\left[\because I\left(at\omega_0\right)=\frac{E}{R}\right]$$

$$\Rightarrow \sqrt{R^2+\left(\omega_1 L-\frac{1}{\omega_1 C}\right)^2}=\sqrt{2}R$$

$$\Rightarrow R^2+\left(\omega_1 L-\frac{1}{\omega_1 C}\right)^2=2R^2$$

$$\Rightarrow \left(\omega_1 L-\frac{1}{\omega_1 C}\right)^2=R^2$$

$$\Rightarrow \left(\omega_1 L-\frac{1}{\omega_1 C}\right)=R$$

$$\Rightarrow \left(\omega_0-\Delta\omega\right)L-\frac{1}{\left(\omega_0-\Delta\omega\right)C}=R$$

$$\Rightarrow \omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right)-\frac{1}{\omega_0 C\left(1+\frac{\Delta\omega}{\omega_0}\right)}=R$$

$$\Rightarrow \omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right)-\frac{\omega_0 L}{\left(1+\frac{\Delta\omega}{\omega_0}\right)}=R$$

$$\left(\because \omega_0 C=\frac{1}{\omega_0 L}\right)$$

$$\Rightarrow \omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right)-\omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right)^{-1}=R$$

$$\Rightarrow \omega_0 L\left(1+\frac{\Delta\omega}{\omega_0}\right)-\omega_0 L\left(1-\frac{\Delta\omega}{\omega_0}\right)=R$$

$$\Rightarrow \omega_0 L\times\frac{2\Delta\omega}{\omega_0}=R$$

$$\Rightarrow \Delta\omega=\frac{R}{2L}$$

∴ Sharpness of resonance

$$\frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$$

The ratio $\frac{\omega_0 L}{R}$ is also called quality factor Q of the circuit.

$$\therefore Q = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \frac{L}{R}$$
$$= \frac{1}{R}\sqrt{\frac{L}{C}}$$

Power in AC Circuit

- A voltage $v = v_m \sin \omega t$ is applied to an LCR circuit, whichdrives a current in the circuit. This is given by $i = i_m \sin(\omega t + \Phi)$

  $$i_m = \frac{v_m}{Z}$$
  $$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

- Instantaneous power supplied by the source is

  $p = vi$

  $= (v_m \sin\omega t) \times (i_m \sin(\omega t + \Phi)$

  $$= \frac{v_m i_m}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$$

- The average power over a cycle is average of the two terms on the R.H.S of the above equation. The second term is time dependent, so its average is zero.

  $$\therefore P = \frac{v_m i_m}{2}\cos\phi$$
  $$= \frac{v_m}{\sqrt{2}} \frac{i_m}{\sqrt{2}}\cos\phi$$
  $$= VI\cos\phi$$

- $$P = I^2 Z\cos\phi$$

$\cos\phi$ is called the power factor.

Case I

For resistive circuit (containing only resistor),

$\Phi = 0$

$\therefore \cos\phi = 1$

Therefore, maximum power is dissipated.

Case II

For pure inductive circuit or pure capacitive circuit, the phase difference between current and voltage is $\frac{\pi}{2}$.

$$\therefore \phi = \frac{\pi}{2}, \cos\phi = 0$$

Therefore, zero power is dissipated. This current is sometimes referred to as watt-less current.

Case III

For LCR series circuit,

$$\phi = \tan^{-1}\frac{(X_C - X_L)}{R}$$

Therefore, power is dissipated only in the resistor.

Case IV

For power dissipated at resonance in an LCR circuit,

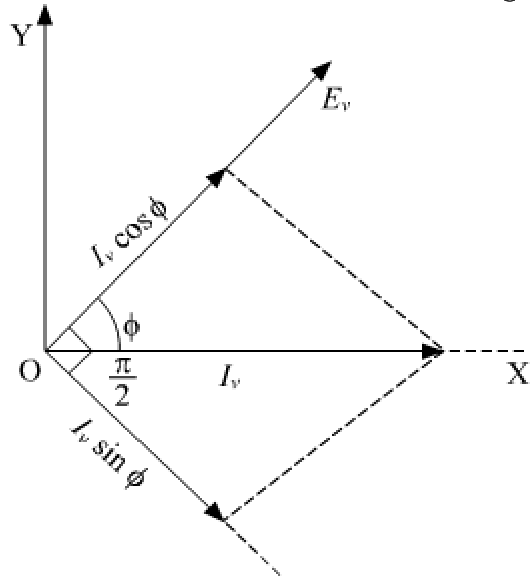
$$X_C - X_L = 0,\ \ \phi = 0$$
$$\therefore \cos\phi = 1$$

Therefore, maximum power is dissipated.

Wattless Current or Idle Current

It is that current which consumes no power for its maintenance in an electric circuit.

In an inductive circuit, the current lags behind the voltage which is shown in the figure below.



As shown in the figure, I_v can be resolved into two components. Also, the average power consumed per cycle in an inductive circuit is given as

$$P = E_v I_v \cos\varphi$$

Now, power consumed due to component $I_v \cos\varphi$ is

$$P_1 = E_v(I_v \cos\varphi) \cos 0 = E_v I_v \cos\varphi$$

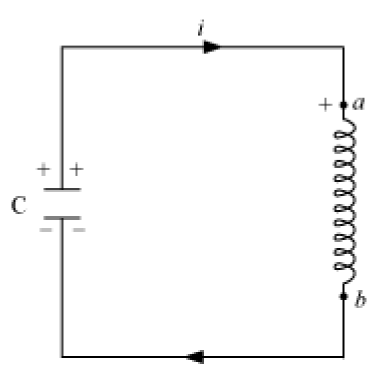
Now, power consumed due to component $I_v \sin\varphi$ is

$$P_2 = E_v\left(I_v \cos\varphi\right) \cos\frac{\pi}{2} = 0$$

Thus, the component I_v sinφφ makes no contribution to the consumption of power in the a.c. circuit. Hence, it is known as wattless current.

LC Oscillations

- When a capacitor is connected with an inductor, the charge on the capacitor and current in the circuit exhibit the phenomenon of electrical oscillations.



- Let at t = 0, the capacitor is charged q_m and connected to an inductor.

- Charge in the capacitor starts decreasing giving rise to current in the circuit.
- Let

  $q \rightarrow$ Charge

  $t \rightarrow$ Time

  $i \rightarrow$ Current

  According to Kirchhoff's loop rule,

  $$\frac{q}{C} - L\frac{di}{dt} = 0$$
  $$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0\left(\because i = -\frac{dq}{dt}\right)$$

  This equation is of the form of a simple harmonic oscillator equation.

- The charge oscillates with a natural frequency of $\omega_0 = \frac{1}{\sqrt{LC}}$ and it varies sinusoidally with time as

  $$q = q_m \cos(\omega_0 t + \phi)$$

  Where,

  $q_m \rightarrow$ Maximum value of q

  $\Phi \rightarrow$ Phase constant

- At

  $t = 0$

  $q = q_m$,

  we have $\cos\Phi = 1$ or $\Phi = 0$

  $\therefore q = q_m \cos(\omega_0 t)$

  $$i = -\frac{dq}{dt}$$

$\therefore i = i_m \sin\omega_0 t$

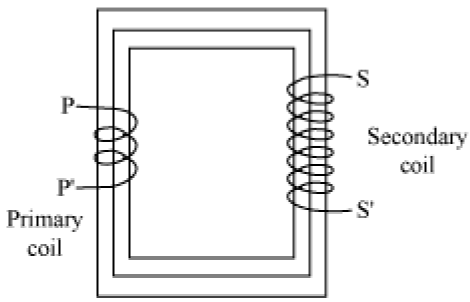
Where, $i_m = \omega_0 q_m$

- *LC* oscillations are similar to the mechanical oscillation of a block attached to a spring.

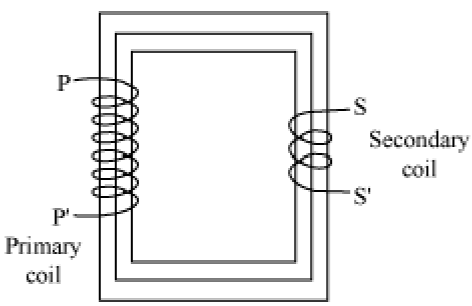
Transformers

Principle – It works on the principle of electromagnetic induction. When current in one circuit changes, an induced current is set up in the neighbouring circuit.

Construction



Step-up transformer



Step-down transformer

Working

- Alternating *emf* is supplied to the primary coil PP'. The resulting current produces an induced current in secondary.
- Magnetic flux linked with primary is also linked with the secondary. The induced *emf* in each turn of the secondary is equal to that induced in each turn of the primary.
- Let

E_P – Alternating *emf* applied to primary

n_P – Number of turns in the primary

$\frac{d\phi}{dt}$ – Rate of change of flux through each turn of primary coil

$$\therefore E_p = -n_p \frac{d\phi}{dt} \qquad ...(1)$$

E_S– Alternating *emf* of secondary

n_S – Number of turns in secondary

$$\therefore E_s = -n_s \frac{d\phi}{dt} \qquad ...(2)$$

Dividing equation (2) by (1),

$$\frac{E_s}{E_p} = \frac{n_s}{n_p} = k$$

- For step-up transformer, $K > 1$

  $\therefore E_s > E_p$

- For step-down transformer, $K < 1$

  $\therefore E_s < E_p$

- According to law of conservation of energy,

  Input electrical power = Output electrical power

  $E_pI_p = E_sI_s$

  $$\therefore \frac{E_s}{E_p} = \frac{I_p}{I_s}$$

- Transformers are used in telegraph, telephone, power stations, etc.
- Losses in transformer:

- Copper loss – Heat in copper wire is generated by working of a transformer. It can be diminished using thick copper wires.
- Iron loss – Loss is in the bulk of iron core due to the induced eddy currents. It is minimized by using thin laminated core.
- Hysteresis loss – Alternately magnetizing and demagnetizing, the iron core cause loss of energy. It is minimized using a special alloy of iron core with silicon.
- Magnetic loss – It is due to the leakage of magnetic flux.