Linear Equations in One Variable

Mathematical Expressions Of Word Problems

Suppose Rahul has Rs 2100 with him. He goes to a market and purchases five shirts. Now the money left with him is Rs 100.

How can we write this situation mathematically?

Let us look at some more examples now.

Example:

Write the following statements in the form of a linear equation.

1. The sum of two consecutive even numbers is 46.

2. One-fourth of a number plus 5 is 30.

3. When 20 is subtracted from *m*, the result is 16.

4. The perimeter of an equilateral triangle is 27 cm.

5. Mohit is 5 years older than Rohit and the sum of their ages is 35.

Solution:

1. Let one even number be $2x$.
   Then, the other even number will be $(2x + 2)$.
   The linear equation is
   $2x + (2x + 2) = 46$
   $\Rightarrow 4x + 2 = 46$

2. Let the number be z.
   One-fourth of the number = $\frac{z}{4}$
   According to the given statement,
   $\frac{z}{4} + 5 = 30$

3. The difference between m and 20 is 16.
   Therefore,
   $m - 20 = 16$

4. We know that in an equilateral triangle, all sides are equal in length.
   Let the length of one side be x.
   The perimeter is the sum of all sides of the triangle.
   $\Rightarrow x + x + x = 27$
   $\Rightarrow 3x = 27$

5. Let the age of Rohit be x years.
   Mohit's age will be $(x + 5)$ years.
   The sum of their ages is 35,
   $\therefore x + (x + 5) = 35$
   $\Rightarrow 2x + 5 = 35$

Solution of Linear Equations That Contain Linear Expressions on One Side and Numbers on the Other Side

In our daily life, we come across many types of linear equations. Let us take an example.

Suppose, there are 30 students in a class and they are collecting money to donate for a charity cause. If each of the students pays the same amount and their teacher pays Rs 100, then the total amount collected will be Rs 1000.

Now can we find out the amount that each student contributed?

Let us now solve some more equations which contain expressions on one side and numbers on the other side.

Example 1:

Solve the following linear equations.

I. $\frac{x}{2} - \frac{5}{3} = \frac{10}{3}$

II. $12x - 13 = 131$

Solution:

I. $\frac{x}{2}-\frac{5}{3}=\frac{10}{3}$

On adding $\frac{5}{3}$ to both the sides, we obtain

$$\frac{x}{2}-\frac{5}{3}+\frac{5}{3}=\frac{10}{3}+\frac{5}{3}$$
$$\Rightarrow\frac{x}{2}=\frac{10}{3}+\frac{5}{3}=\frac{15}{3}=5$$

On multiplying both the sides with 2, we obtain

$x = 10$

We can also check the answer by putting the value of x on LHS of the given equation.

Check: $\frac{10}{2}-\frac{5}{3}=\frac{30-10}{6}=\frac{20}{6}=\frac{10}{3}$ = R.H.S.

Hence, the answer is verified.

II. $12x - 13 = 131$

On adding 13 to both the sides, we obtain

$12x - 13 + 13 = 131 + 13$

$\Rightarrow 12x = 144$

On dividing by 12, we obtain

$x = \frac{144}{12}$

$\Rightarrow x = 12$

Example 2:

For the equation $2(x + 11) = 44$, find the value of x.

Solution:

The given equation is $2(x + 11) = 44$.

On dividing both the sides by 2, we obtain

$$x + 11 = \frac{44}{2} = 22$$

On subtracting 11 from both the sides, we obtain

$x + 11 - 11 = 22 - 11$

$\Rightarrow x = 11$

Example 3:

The sum of two consecutive numbers is 20. Find the numbers.

Solution:

Let the first number be x, then the other number will be $x + 1$.

It is given that the sum of the numbers is 20.

$\therefore x + (x + 1) = 20$

$\Rightarrow 2x + 1 = 20$

On subtracting 1 from both the sides, we obtain

$2x + 1 - 1 = 20 - 1$

$\Rightarrow 2x = 19$

On dividing both the sides by 2, we obtain

$$x = \frac{19}{2}$$

$$x + 1 = \frac{19}{2} + 1 = \frac{21}{2}$$

Thus, the numbers are $\frac{19}{2}$ and $\frac{21}{2}$.

Example 4:

If the area of a rectangle is 40 cm^2 and its length is 10 cm, then find its width.

Solution:

Area of a rectangle = length × width

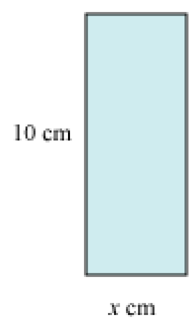
We know the area and length and we have to find the width of the rectangle.

Let the width of the rectangle be x cm.

It is given that the area is 40 cm^2.

$\therefore 40 = 10 \times x$

$10x = 40$



On dividing both the sides by 10, we obtain

$x = \frac{40}{10} = 4$

Thus, the width of the rectangle is 4 cm.

Example 5:

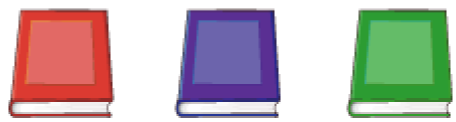
In a library, there are a total of 1500 books of Maths, Science, and English. The ratio of these books is 4: 5: 6. Find the total number of books of each subject.

Solution:

The ratio of the books is 4: 5: 6.

Let the number of Maths books, Science books and English books be $4x$, $5x$, and $6x$ respectively.

There are 1500 books in total.



$\therefore 4x + 5x + 6x = 1500$

$15x = 1500$

On dividing both the sides by 15, we obtain

$x = \frac{1500}{15} = 100$

$\therefore$ Number of Maths books = $4x = 400$

Number of Science books = $5x = 500$

Number of English books = $6x = 600$

Example 6:

In a two-digit number, the digit at tens place is greater than the digit at units place by 5. If the digits are interchanged and the number so formed is added to the original number, then we obtain the sum as 99. What is the original number?

Solution:

Let the digit at units place be b.

It is given that the digit at tens place is greater than the digit at units place by 5.

Therefore, the digit at tens place is $b + 5$.

Thus, the two-digit number can be written as $10(b + 5) + b = 11b + 50$

If the digits are interchanged, then the resulting number will be = $10b + (b + 5) = 11b + 5$

It is given that the sum of these two numbers is 99.

$\therefore (11b + 50) + (11b + 5) = 99$

$22b + 55 = 99$

On subtracting 55 from both the sides, we obtain

$22b = 99 - 55 = 44$

On dividing by 22, we obtain

$$b = \frac{44}{22}$$

$\Rightarrow b = 2$

Thus, the units digit is 2 and tens digit is 7. Hence, the number is 72.

Solution of Linear Equations That Contains Linear Expressions on Both Sides

Suppose two children Arpit and Amit are playing a game. Arpit thinks of a number and says to Amit that if he subtracts 2 from that number, then the answer obtained is same as half of the original number. He asks Amit to find the original number, which he had thought in the beginning.

Can Amit find it? Is it possible to find the original number from the given situation?

Yes, it is. By using the concept of linear equations in one variable, we can find the solution for the above problem.

Now, we will first form a linear equation with the help of the given situation.

Let the original number be *x*. It is given that if Arpit subtracts 2 from the original number, then the answer obtained will be half of the original number.

i.e, $x - 2 = \frac{x}{2}$

In this equation, there is a variable on both the sides. How will we solve such an equation?

Solve the following equations.

1. $\mathbf{5z+\frac{17}{6}=3z+5}$

2. $\mathbf{3(z-3)=\frac{z}{2}-2}$

3. $\mathbf{2(x+20)=\frac{x}{3}+65}$

Solution:

1. $5z+\frac{17}{6}=3z+5$

Our first step is to rearrange the terms of the equation such that the variables are on one side and numbers are on the other side.

Therefore, on subtracting 5*z* from both the sides, we obtain

$$5z+\frac{17}{6}-5z=3z+5-5z$$
$$\Rightarrow 5-2z=\frac{17}{6}$$

Now, on subtracting 5 from both the sides, we obtain

$$5-2z-5=\frac{17}{6}-5$$
$$\Rightarrow -2z=\frac{17-30}{6}$$
$$\Rightarrow -2z=\frac{-13}{6}$$
$$\Rightarrow 2z=\frac{13}{6}$$

On dividing both the sides by 2, we obtain

$$z=\frac{13}{12}$$

2. $3(z-3)=\frac{z}{2}-2$

$3z-9=\frac{z}{2}-2$

On subtracting $3z$ from both the sides, we obtain

$\Rightarrow 3z-9-3z=\frac{z}{2}-2-3z$

$\Rightarrow -9=\frac{-5z}{2}-2$

On adding 2 to both the sides, we obtain

$\Rightarrow -9+2=\frac{-5z}{2}-2+2$

$\Rightarrow -7=\frac{-5z}{2}$

$\Rightarrow \frac{5z}{2}=7$

On multiplying both the sides by $\frac{2}{5}$, we obtain $z=\frac{14}{5}$

3. $2(x+20)=\frac{x}{3}+65$

$2x+40=\frac{x}{3}+65$

On subtracting $2x$ from both the sides, we obtain

$\Rightarrow 2x+40-2x=\frac{x}{3}+65-2x$

$\Rightarrow 40=\frac{-5x}{3}+65$

On subtracting 65 from both the sides, we obtain

$\Rightarrow 40-65=\frac{-5x}{3}+65-65$

$\Rightarrow -25=\frac{-5x}{3}$

$\Rightarrow \frac{5x}{3}=25$

On multiplying both the sides by $\frac{3}{5}$, we obtain

$$x = 25 \times \frac{3}{5}$$

$\Rightarrow x = 5 \times 3$

$\Rightarrow x = 15$

Example 2:

The difference between the ages of Raj and Mohini is 20 years. Ten years later, Raj's age will be half of Mohini's age. Find their present ages.

Solution:

Let Raj's age be x. The difference between their ages is 20 years. Now, we are given that Raj's age after 10 years will be half of Mohini's age. Therefore, Mohini is older than Raj.

We can write Mohini's age as $20 + x$.

After 10 years, Raj's age will be $x + 10$ and Mohini's age will be $20 + x + 10 = 30 + x$

According to the given condition, we obtain

$$x + 10 = \frac{1}{2}(30 + x)$$

On multiplying both the sides by 2, we obtain

$2(x + 10) = 30 + x$

$\Rightarrow 2x + 20 = 30 + x$

On subtracting $2x$ from both the sides, we obtain

$2x + 20 - 2x = 30 + x - 2x$

$\Rightarrow 20 = 30 - x$

On subtracting 30 from both the sides, we obtain

$20 - 30 = 30 - x - 30$

$\Rightarrow -10 = -x$

$\Rightarrow x = 10$

∴ Raj's age = 10 years

Mohini's age = 20 + 10 = 30 years

Reducing Equations To Simpler Form

We know how to solve linear equations, in which the variable is on one side or on both sides. However, some linear equations are in a very complex form. Therefore, before solving those equations, firstly we simplify them. Let us consider such an equation.

$$\frac{5x+8}{3} - \frac{x}{2} = 3\frac{1}{2}$$

To solve this equation, let us try to reduce it to a simpler form.

$$\frac{5x+8}{3} - \frac{x}{2} = \frac{7}{2}$$

Firstly, we multiply both sides of the equation by 6 as 6 is the L.C.M. of 3 and 2.

$$6\left(\frac{5x+8}{3}\right) - 6\left(\frac{x}{2}\right) = 6\left(\frac{7}{2}\right)$$
$$\Rightarrow 2(5x+8) - 3(x) = 3 \times 7$$
$$\Rightarrow 10x + 16 - 3x = 21 \qquad \text{(Opening the brackets)}$$
$$\Rightarrow 7x + 16 = 21$$

Now, the equation is in a simple form.

$$\Rightarrow 7x = 21 - 16$$
$$\Rightarrow 7x = 5$$
$$\Rightarrow x = \frac{5}{7}$$

Now, let us verify our answer.

$$\text{L.H.S} = \frac{5x+8}{3} - \frac{x}{2}$$

$$\Rightarrow \frac{5\left(\frac{5}{7}\right)+8}{3} - \frac{\left(\frac{5}{7}\right)}{2}$$

$$\Rightarrow \frac{\frac{25}{7}+8}{3} - \frac{5}{14}$$

$$\Rightarrow \frac{\frac{25+56}{7}}{3} - \frac{5}{14}$$

$$\Rightarrow \frac{81}{21} - \frac{5}{14}$$

$$\Rightarrow \frac{162-15}{42}$$

$$\Rightarrow \frac{147}{42}$$

$$\Rightarrow \frac{7}{2} = \text{R.H.S}$$

Thus, $\frac{5}{7}$ is the solution of the given linear equation.

Let us try to solve some more linear equations by simplifying them.

Example 1:

Simplify and solve the linear equation

0.2 (2*p* + 3) + 0.8 (2*p* – 0.5) = 42.2

Solution:

The given equation is

$0.2\ (2p + 3) + 0.8\ (2p - 0.5) = 42.2$

Opening the brackets, we obtain

$0.4\ p + 0.6 + 1.6\ p - 0.4 = 42.2$

$2.0p + 0.2 = 42.2$

$2.0\,p = 42.2 - 0.2$

$2.0\,p = 42$

$$p = \frac{42}{2}$$
$$p = 21$$

Thus, $p = 21$ is the solution of the given linear equation.

Example 2:

Solve the following linear equation.

$$2x + 5 - \frac{3x}{4} = \frac{x}{5} - \frac{6x}{3}$$

Solution:

The numbers in the denominators are 4, 5, and 3.

The L.C.M. of 4, 5, and 3 is 60.

Multiplying the given equation by 60 on both sides, we obtain

$$60\left(2x + 5 - \frac{3x}{4}\right) = 60\left(\frac{x}{5} - \frac{6x}{3}\right)$$
$$\Rightarrow 120x + 300 - 45x = 12x - 120x \qquad \text{(Opening the brackets)}$$
$$\Rightarrow 75x + 300 = -108x$$
$$\Rightarrow 75x + 108x = -300$$
$$\Rightarrow 183x = -300$$
$$\Rightarrow x = -\frac{300}{183}$$
$$\Rightarrow x = -\frac{100}{61}$$

Thus, $x = -\frac{100}{61}$ is the solution of the given equation.

Equations Reducible To Linear Form

Some equations may not seem to be linear on first observation. However, it may be possible to reduce them to a linear form.

Let us solve some more examples involving equations reducible to linear form.

Example 1:

Solve the following equations.

(a) $\frac{7x-45}{4x}=-2$

(b) $\frac{3x+1}{x-3}=5$

Solution:

(a)

$$\frac{7x-45}{4x}=-2$$
$$\Rightarrow 7x-45=-8x$$
$$\Rightarrow 7x+8x=45$$
$$\Rightarrow 15x=45$$
$$\Rightarrow x=3$$

(b)

$$\frac{3x+1}{x-3}=5$$
$$\Rightarrow 3x+1=5(x-3)$$
$$\Rightarrow 3x+1=5x-15$$
$$\Rightarrow 3x-5x=-15-1$$
$$\Rightarrow -2x=-16$$
$$\Rightarrow x=8$$

Example 2:

Solve the following equations.

(a) $\frac{15y+7}{9y+8}=\frac{3}{2}$

(b) $\frac{2z-11}{5} = 9 - z$

Solution:

(a)

$$\frac{15y+7}{9y+8} = \frac{3}{2}$$

$$\Rightarrow 2(15y+7) = 3(9y+8)$$

$$\Rightarrow 30y + 14 = 27y + 24$$

$$\Rightarrow 30y - 27y = 24 - 14$$

$$\Rightarrow 3y = 10$$

$$\Rightarrow y = \frac{10}{3}$$

(b)

$$\frac{2z-11}{5} = 9 - z$$

$$\Rightarrow 2z - 11 = 5(9 - z)$$

$$\Rightarrow 2z - 11 = 45 - 5z$$

$$\Rightarrow 2z + 5z = 45 + 11$$

$$\Rightarrow 7z = 56$$

$$\Rightarrow z = 8$$

Example 3:

The present ages of Ravi and Meena are in the ratio 2:3. After seven years, the ratio of their ages will be 3:4. Find the present ages of Ravi and Meena.

Solution:

Let the present ages of Ravi and Meena be $2x$ years and $3x$ years respectively.

After seven years,

Ravi's age = $(2x + 7)$ years

Meena's age = $(3x + 7)$ years

Ratio of their ages after seven years $= \frac{2x+7}{3x+7}$

This ratio is given as 3:4.

$$\frac{2x+7}{3x+7} = \frac{3}{4}$$

By cross multiplication, we obtain

$4(2x + 7) = 3(3x + 7)$

$\Rightarrow 8x + 28 = 9x + 21$

$\Rightarrow 9x - 8x = 28 - 21$

$\Rightarrow x = 7$

Thus, present age of Ravi = $2x = 2 \times 7 = 14$ years

Present age of Meena = $3x = 3 \times 7 = 21$ years

Example 4:

The numerator of a rational number is less than its denominator by 3. If the numerator is increased by 12 and the denominator is increased by 3, then the number obtained is $\frac{7}{4}$. Find the rational number.

Solution:

Let the denominator of the rational number be x.

Then, numerator = $x - 3$

When the numerator is increased by 12 and denominator by 3, the rational number obtained is $\frac{x-3+12}{x+3}$.

According to the question,

$$\frac{x-3+12}{x+3} = \frac{7}{4}$$

$\Rightarrow \frac{x+9}{x+3} = \frac{7}{4}$

By cross-multiplication, we obtain

$4(x + 9) = 7(x + 3)$

$\Rightarrow 4x + 36 = 7x + 21$

$\Rightarrow 7x - 4x = 36 - 21$

$\Rightarrow 3x = 15$

$\Rightarrow x = \frac{15}{3}$

$\Rightarrow x = 5$

Thus, numerator of the rational number, $x - 3 = 5 - 3 = 2$

Denominator = $x = 5$

Thus, the required rational number is $\frac{2}{5}$.