CBSE Test Paper 04

CH-11 Conic Sections

1. The equation of the normal to the parabola $y^2 = 8$ x having slope 1 is
   a. x + y – 6 = 0
   b. x + y + 6 = 0
   c. x – y + 6 = 0
   d. x – y – 6 = 0
2. The eccentricity of the hyperbola $x^2 - y^2 = 9$ is
   a. less than 1
   b. none of these
   c. $\sqrt{2}$
   d. 1
3. Circumcentre of the triangle, whose vertices are (0, 0), (6, 0) and (0, 4) is
   a. (0, 3)
   b. (3, 0)
   c. (3, 2)
   d. (2, 0)
4. The equation $x^2 + y^2 = 0$ represents
   a. a degenerate circle.
   b. an ellipse.
   c. pairs of straight line.
   d. an equilateral hyperbola.
5. Which one of the following lines is farthest from the centre of the circle $x^2 + y^2 = 10$?
   a. 3 x + 4 y – 15 = 0
   b. 12x + 5 y + 26 = 0
   c. $x + \sqrt{3}\ y + 7 = 0$
   d. x + y = 1
6. Fill in the blanks:

   The points at which the hyperbola intersects the transverse axis are called the ________ of the hyperbola.

7. Fill in the blanks:

   A ________ is defined as the locus of a point in a plane, which moves in a plane such that its distance from a fixed point in that plane is always constant.

8. Find the equation of the parabola that satisfies the given conditions: Focus (6, 0) directrix x = = - 6

9. Find the equation of the parabola that satisfies the given conditions: Vertex (0, 0) Focus (3, 0)

10. At what point of the parabola $x^2 = 9y$ is the abscissa three times that of ordinate?

11. Find the coordinate of the focus of the parabola $x^2 = -16y$.

12. Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum. $y^2 = 10x$

13. Find the equation of the set of all points, the sum of whose distance from the points (3, 0) and (9, 0) is 12.

14. Find the equation of ellipse having Ends of major axis $(\pm 3, 0)$, ends of minor axis $(0, \pm 2)$

15. Find the equation of hyperbola, when foci are at (±5, 0) and transverse axis is of length 8.

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Solution

1. (d) x – y – 6 = 0

   Explanation: slope form of normal is $y=mx-2am-am^3$

   for the given parabola a = 2 and m = 1

   therefore y = x -4 -2

   i.e; x - y - 6 = 0

2. (c) $\sqrt{2}$

   Explanation:

   $x^2 - y^2 = 9$

   above equation can be written as,

   $\frac{x^2}{(3)^2} - \frac{y^2}{(3)^2} = 1$

   comparing it with the standard equation we get a=3 and b=3

   as c=$\sqrt{a^2 + b^2}$

   we get c= 3 $\sqrt{2}$

   and as e = $\frac{c}{a}$

   we get e = $\sqrt{2}$

3. (c) (3, 2)

   Explanation:

   circumcentre of a right-angled triangle ABC right angled at A is $\frac{b+c}{2}$ as circumcentre of right-angled triangle lies on the midpoint of the hypotenuse.

so midpoint of BC=($\frac{6+0}{2}$,$\frac{0+4}{2}$) i.e.(3,2)

4. (a) a degenerate circle.
   Explanation: The above circle can be written as $(x-0)^2+(y-0)^2=(0)^2$

   Here the center is (0,0) and radius is also 0 units.

   So it is a degenerate circle as degenerate circle is a circle(a point) where radius is zero units.

5. (c) $x+\sqrt{3}\,y+7=0$

   Explanation:

   $x^2+y^2=10$

   after completing the square, we get

   $(x-0)^2+(y-0)^2=(\sqrt{10})^2$

   so center is (0,0) and equation of given line is $x+\sqrt{3}\,y+7=0$

   by applying formula of distance of a point from a given line i.e.,$\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}$

   where(x_1,y_1) are the coordinates of point and A and B are coefficient of x,y in given line so d = $\frac{7}{2}$ which is greatest.

6. vertices

7. circle

8. The required equation of parabola is
   $y^2 = 4 \times 6x \Rightarrow y^2 = 24x$

9. The vertex of the parabola is at (0, 0) and focus is at (3, 0),
   $\Rightarrow$ y = 0 $\Rightarrow$ The axis of parabola is along x-axis
   So the parabola is of the form $y^2 = 4ax$.

The required equation of parabola is

$y^2 = 4 \times 3x \Rightarrow y^2 = 12x$

10. Let the ordinate of the required point is y.

$\therefore$ abscissa = 3y

$\therefore$ the coordinates of the points are (3y, y).

These points lies on the parabola $x^2 = 9y$.

$\therefore (3y)^2 = 9y$

$\Rightarrow 9y^2 = 9y$

$\Rightarrow 9y^2 - 9y = 0$

$\Rightarrow 9y(y - 1) = 0$

$\Rightarrow y - 1 = 0 \ [\because y \neq 0]$

$\Rightarrow y = 1$

$\therefore$ abscissa = $3 \times y = 3$

Hence, the required point is (3,1).

11. Given equation of parabola is $x^2 = -16y$.

On comparing with the parabola $x^2 = -4ay$, we get

a = 4

As equation of parabola is $x^2 = -16y$.

It means focus lies on the negative direction of y-axis.

$\therefore$ Coordinate of focus = (0,- a) = (0, - 4)

12. The given equation of parabola is $y^2 = 10x$ which is of the form $y^2 = 4ax$

$\therefore 4a = 10 \Rightarrow a = \frac{10}{4} \Rightarrow a = \frac{5}{2}$

$\therefore$ Coordinates of focus are $\left(\frac{5}{2}, 0\right)$

Axis of parabola is y = 0

Equation of the directrix is $x = \frac{-5}{2} \Rightarrow 2x + 5 = 0$

Length of latus rectum $= \frac{4 \times 5}{2} = 10$

13. Let the given points be A (3, 0) and B (9, 0). And P (x, y) be any point on the curve.

Given, the sum of distances from point P (x, y) to the points A (3, 0) and B (9, 0) is equal to 12.

$\therefore$ PA + PB = 12

$\Rightarrow \sqrt{(x-3)^2+y^2}+\sqrt{(x-9)^2+y^2}$ = 12 [$\because$ distance, d = $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$]

$\Rightarrow \sqrt{(x-3)^2+y^2}$ = 12 - $\sqrt{(x-9)^2+y^2}$

On squaring both sides, we get

$(x - 3)^2 + y^2 = 144 + (x - 9)^2 + y^2 - 24\sqrt{(x-9)^2+y^2}$

$\Rightarrow x^2 + 9 - 6x = 144 + x^2 + 81 - 18x - 24\sqrt{(x-9)^2+y^2}$

$\Rightarrow 12x - 216 = - 24\sqrt{(x-9)^2+y^2}$

$\Rightarrow x - 18 = - 2\sqrt{(x-9)^2+y^2}$ [dividing both sides by 12]

Again squaring both sides, we get

$x^2 + 324 - 36x = 4 (x^2 + 81 - 18x + y^2)$

$\Rightarrow 3x^2 + 4y^2 - 36x = 0$

14. Ends of major axis $(\pm 3, 0)$ lie on x-axis.

    So the equation of ellipse in standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

    Now ends of major axis (± a, 0) is (± 3, 0) ⇒ a = 3

    Ends of minor axis $(0, \pm b)$ is $(0, \pm 2) \Rightarrow$ b = 2

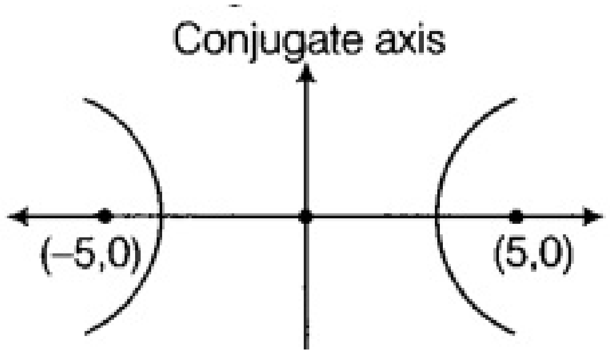
    Thus equation of required ellipse is

    $\frac{x^2}{9}+\frac{y^2}{4}=1$

15. Here, foci are at (±5, 0)

    $\therefore$ (± c,0) = (± 5,0)

    $\Rightarrow$ c = 5



    And length of transverse

    axis = 2a = 8 $\Rightarrow$ a = 4

    Also, we know that, $c^2 = a^2 + b^2$

$\Rightarrow$ 25 = 16 + b^2 [$\because$ a = 4, c = 5]

$\Rightarrow$ b^2 = 9

Since, the foci lie on X-axis. Therefore, the equation of hyperbola is of the form

$\frac{x^2}{a^2} - \frac{y^2}{b^2}$ = 1

On putting the values of a^2 and b^2, we get

$\frac{x^2}{16} - \frac{y^2}{9}$ = 1

which is the required equation of hyperbola.