# Radioactivity (Part - 1)

**Q.214. Knowing the decay constant λ of a nucleus, find: (a) the probability of decay of the nucleus during the time from 0 to t; (b) the mean lifetime ζ of the nucleus.**

**Ans.** (a) The probability of survival (i.e. not decaying) in time t is $e^{-\lambda t}$. Hence the probability of decay is $1 - e^{-\lambda t}$

(b) The probability that the particle decays in time dt around time t is the difference

$$e^{-\lambda t} - e^{-\lambda(t+dt)} = e^{-\lambda t}[1 - e^{-e\lambda dt}] = \lambda e^{-\lambda t} dt$$

Therefore the mean life time is

$$T = \int_0^\infty t\lambda e^{-\lambda t} dt \Big/ \int_0^\infty \lambda e^{-\lambda t} dt = \frac{1}{\lambda}\int_0^\infty x e^{-x} dx \Big/ \int_0^\infty e^{-x} dx = \frac{1}{\lambda}$$

**Q.215. What fraction of the radioactive cobalt nuclei whose halflife is 71.3 days decays during a month?**

**Ans.** We calculate λ first

$$\lambda = \frac{\ln 2}{T_{1/2}} = 9{\cdot}722 \times 10^{-3} \text{ per day}$$

Hence

fraction decaying in a month $= 1 - e^{-\lambda t} = 0{\cdot}253$

**Q.216. How many beta-particles are emitted during one hour by 1.0μg of $Na^{24}$ radionuclide whose half-life is 15 hours?**

**Ans.** Here

$$N_0 = \frac{1\,\mu g}{24\,g} \times 6{\cdot}023 \times 10^{23} = 2.51 \times 10^{16}$$

Also $\lambda = \frac{\ln 2}{T_{1/2}} = 0{\cdot}04621$ per hour

So the number of β rays emitted in one hour is
No $(1 - e^{-\lambda t}) = 1.13 \times 10^{15}$

**Q.217. To investigate the beta-decay of $Mg^{23}$ radionuclide, a counter was activated at the moment t = 0. It registered $N_1$ beta-particles by a moment $t_1 = 2.0$s, and by a moment $t_2 = 3t_1$ the number of registered beta-particles was 2.66 times greater. Find the mean lifetime of the given nuclei.**

**Ans.** If $N_0$ is the number of radionuclei present initially, then

$$N_1 = N_0 (1 - e^{-t_1/\tau})$$

$$\eta N_1 = N_0 (1 - e^{-t_2/\tau})$$

where $\eta = 2{\cdot}66$ and $t_2 = 3t_1$. Then

$$\eta = \frac{1 - e^{-t_2/\tau}}{1 - e^{-t_1/\tau}}$$

or $\eta - \eta e^{-t_1/\tau} = 1 - e^{-t_2/\tau}$

Substituting the values

$$1{\cdot}66 = 2{\cdot}66\, e^{-2/\tau} - e^{-6/\tau}$$

Put $e^{-2/\tau} = x$. Then

$$x^3 - 2{\cdot}66\, x + 1{\cdot}66 = 0$$
$$(x^2 - 1)\, x - 1{\cdot}66\, (x - 1) = 0$$

or $(x - 1)(x^2 + x - 1{\cdot}66) = 0$

Now $x \neq 1$ so $x^2 + x - 1{\cdot}66 = 0$

$$x = \frac{-1 \pm \sqrt{1 + 4 \times 1{\cdot}66}}{2}$$

Negative sign has to be rejected as x >0.
Thus   x = 0.882

This gives $\tau = \frac{-2}{\ln 0{\cdot}882} = 15{\cdot}9 \text{ sec}.$

**Q.218. The activity of a certain preparation decreases 2.5 times after 7.0 days. Find its half-life.**

**Ans.** If the half-life is T days

$$(2)^{-7/T} = \frac{1}{2{\cdot}5}$$

Hence $\frac{7}{T} = \frac{\ln 2{\cdot}5}{\ln 2}$

or $T = \frac{7 \ln 2}{\ln 2{\cdot}5} = 5{\cdot}30 \text{ days.}$

**Q.219. At the initial moment the activity of a certain radionuclide totalled 650 particles per minute. What will be the activity of the preparation after half its half-life period?**

**Ans.** The activity is proportional to the number of parent nuclei (assuming that the daughter is not radioactive). In half its half-life period, the number of parent nucli decreases by a factor

$$(2)^{-1/2} = \frac{1}{\sqrt{2}}$$

So activity decreases to $\frac{650}{\sqrt{2}} = 460$ articles per minute.

**Q.220. Find the decay constant and the mean lifetime of $Co^{55}$ radionuclide if its activity is known to decrease 4.0% per hour. The decay product is nonradioactive.**

**Ans.** If the decay constant (in $(\text{hour})^{-1}$ ) is λ., then the activity after one hour will decrease by a factor $e^{-\lambda}$Hence

$0.96 = e^{-\lambda}$

or $\lambda = 1{\cdot}11 \times 10^{-5} s^{-1}$ = 0.0408 per hour he mean life time is 24.5 hour

**Q.221. A $U^{238}$ preparation of mass 1.0 g emits $1.24.10^4$ alphaparticles per second. Find the half-life of this nuclide and the activity of the preparation.**

**Ans.** Here

$$N_0 = \frac{1}{238} \times 6{\cdot}023 \times 10^{23}$$

$= 2.531 \text{ X } 10^{21}$
The activity is A = $1.24 \times 10^4$ dis/sec .

Then $\lambda = \frac{A}{N_0} =$ $4.90 \times 10^{18}$ per sec .

Hence the half life is

$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} =$ $4.49 \times 10^9$ years

**Q.222. Determine the age of ancient wooden items if it is known that the specific activity of $C^{14}$ nuclide in them amounts to 3/5 of that in lately felled trees. The half-life of $C^{14}$ nuclei is 5570 years.**

**Ans.** in old wooden atoms the number of $C^{14}$ nuclei steadily decreases because of radioactive decay. (In live trees biological processes keep replenishing $C^{14}$ nuclei maintaining a balance. This balance starts getting disrupted as soon as the tree is felled.)

If $T_{1/2}$ is the half life of $C^{14}$ then $e^{-t \times \frac{\ln 2}{T_{1/2}}} = \frac{3}{5}$

Hence $t = T_{1/2} \frac{\ln 5/3}{\ln 2}$ = 4105 years ≈ $4.1 \times 10^3$ years

**Q.223. In a uranium ore the ratio of $U^{238}$ nuclei to $Pb^{206}$ nuclei is $\eta = 2.8$. Evaluate the age of the ore, assuming all the lead $Pb^{206}$ to be a final decay product of the uranium series. The half-life of $U^{238}$ nuclei is $4.5.10^9$ years.**

**Ans.** What this implies is that in the time since the ore was formed $\frac{\eta}{1+\eta} U^{238}$ nuclei have remained undecayed. Thus

$$\frac{\eta}{1+\eta} = e^{-t \times \frac{\ln 2}{T_{1/2}}}$$

Or

$$t = T_{\frac{1}{2}} \frac{\ln \frac{1+\eta}{\eta}}{\ln 2}$$

Substituting $T_{\frac{1}{2}} = 4{\cdot}5 \times 10^9$ years, η = 2.8

we get t = 1.98 x $10^9$ years.

**Q.224. Calculate the specific activities of $Na^{24}$ and $U^{235}$ nuclides whose half-lifes are 15 hours and 7.1.$10^8$ years respectively.**

**Ans.** The specific activity of $Na^{24}$ is

$$\lambda \frac{N_A}{M} = \frac{N_A \ln 2}{M T_{\frac{1}{2}}}$$ = 3.22 x $10^{17}$ dis/(gm.sec)

Here M = molar w eight of $N_a^{24} = 24\ \text{gm}$, $N_A$ is Avogadro number & $T_{\frac{1}{2}}$ is the half-life of $Na^{24}$

Sim ilarly the specific activity of $U^{235}$ is

$$\frac{6{\cdot}023 \times 10^{23} \times \ln 2}{235 \times 10^8 \times 365 \times 86400}$$

= 0.793 x $10^5$ dis/(gm-s)

**Q.225. A small amount of solution containing $Na^{24}$ radionuclide with activity A = 2.0.$10^3$disintegrations per second was injected in the bloodstream of a man. The activity of 1 $cm^3$of blood sample taken t = 5.0 hours later turned out to be A' = 16 disintegrations per minute per $cm^3$. The half-life of the radionuclide is T = 15 hours. Find the volume of the man's blood.**

**Ans.** Let V = volume of blood in the body of the human being. Then the total activity of the blood is A' V. Assuming all this activity is due to the injected $Na^{24}$ and taking account of the decay of this radionuclide, we get

$$VA' = A e^{-\lambda t}$$

Now $\lambda = \frac{\ln 2}{15}$ per hour, t = 5 hour

Thus $V = \frac{A}{A'} e^{-\ln 2/3} = \frac{2{\cdot}0 \times 10^3}{(16/60)} e^{-\ln 2/3}$ cc = 5.99 litre

**Q.226. The specific activity of a preparation consisting of radioactive $Co^{58}$ and nonradioactive $Co^{59}$ is equal to $2.2.10^{12}$ dis/(s•g). The half-life of $Co^{58}$ is 71.3 days. Find the ratio of the mass of radioactive cobalt in that preparation to the total mass of the preparation (in per cent).**

**Ans.** We see that
Specific activity of the sample

$= \frac{1}{M + M'}$ {Activity of M gm of $Co^{58}$ in the sample}

Here M and At are the masses of $Co^{58}$ and $Co^{59}$ in the sample. Now activity of M gm of $Co^{58}$

$$= \frac{M}{58} \times 6{\cdot}023 \times 10^{23} \times \frac{\ln 2}{71{\cdot}3 \times 86400} \text{ dis/sec}$$

$= 1.168 \times 10^{15} M$

Thus from the problem

$$1{\cdot}168 \times 10^{15} \frac{M}{M + M'} = 2.2 \times 10^{12}$$

Or $\frac{M}{M + M'} = 1{\cdot}88 \times 10^{-3}$

**Q.227. A certain preparation includes two beta-active components with different half-lifes. The measurements resulted in the following dependence of the natural logarithm of preparation activity on time t expressed in hours:**

| $t$ | 0 | 1 | 2 | 3 | 5 | 7 | 10 | 14 | 20 |
|---|---|---|---|---|---|---|---|---|---|
| $\ln A$ | 4.10 | 3.60 | 3.10 | 2.60 | 2.06 | 1.82 | 1.60 | 1.32 | 0.90 |

**Find the half-lifes of both components and the ratio of radioactive nuclei of these components at the moment t = 0.**

**Ans.** Suppose $N_1$ $N_2$ are the initial number of component nuclei whose decay constants are $\lambda_1$ , $\lambda_2$ ( in $(hour)^{-1}$ Then the activity at any instant is

$$A = \lambda_1 N_1 e^{-\lambda_1 t} + \lambda_2 N_2 e^{-\lambda_2 t}$$

The activity so defined is in units dis/hour. We assume that data In A given is of its natural logarithm. The daughter nuclei are assumed nonradioactive.

We see from the data that at large t the change in In A per hour of elapsed time is constant and equal to - 0.07. Thus

$\lambda_2 = 0.07$ per hour
We can then see that the best fit to data is obtained by

$$A(t) = 51{\cdot}1\, e^{-0.66 t} + 10{\cdot}0\, e^{-0{\cdot}07 t}$$

[To get the fit we calculate $A(t)\, e^{0{\cdot}07 t}$. We see that it reaches the constant value 10.0 at t = 7, 10, 14, 20 very nearly. This fixes the second term. The first term is then obtained

by subtracting out the constant value 10.0 from each value of $A(t)\, e^{0{\cdot}07 t}$ in the data for small t ]

Thus we get $\lambda_1 =$ 0.66 per hour

$$\left.\begin{aligned} T_1 &= 1{\cdot}05 \text{ hour} \\ T_2 &= 9{\cdot}9 \text{ hours} \end{aligned}\right\} \text{half–lives}$$

Ratio $\dfrac{N_1}{N_2} = \dfrac{51{\cdot}1}{10{\cdot}0} \times \dfrac{\lambda_2}{\lambda_1} = 0{\cdot}54$

The answer given in the book is misleading.

**Q.228. A $P^{32}$ radionuclide with half-life T = 14.3 days is produced in a reactor at a constant rate q = $2.7.10^9$ nuclei per second. How soon after the beginning of production of that radionuclide will its activity be equal to A = $1.0.10^9$ dis/s?**

**Ans.** Production of the nucleus is governed by the equation

$$\frac{dN}{dt} = \underbrace{g}_{\text{supply}} - \underbrace{\lambda N}_{\text{decay}}$$

We see that N will approach a constant value $\frac{g}{\lambda}$ This can also be proved directly. Multiply by $e^{\lambda t}$ and write

$$\frac{dN}{dt} e^{\lambda t} + \lambda e^{\lambda t} N = g e^{\lambda t}$$

Then

$$\frac{d}{dt}(N e^{\lambda t}) = g e^{\lambda t}$$

or $$N e^{\lambda t} = \frac{g}{\lambda} e^{\lambda t} + \text{const}$$

At t = 0 when the production is starteed, N = 0

$$0 = \frac{g}{\lambda} + \text{constant}$$

Hence

$$N = \frac{g}{\lambda}(1 - e^{-\lambda t})$$

Now the activity is

$$A = \lambda N = g(1 - e^{-\lambda t})$$

From the problem

$$\frac{1}{2{\cdot}7} = 1 - e^{-\lambda t}$$

This gives λt = 0.463

so $$t = \frac{0{\cdot}463}{\lambda} = \frac{0{\cdot}463 \times T}{0{\cdot}693} = 9{\cdot}5 \text{ days} .$$

Algebraically $t = -\frac{T}{\ln 2} \ln\left(1 - \frac{A}{g}\right)$

**Q.229. A radionuclide $A_1$ with decay constant $\lambda_1$ transforms into a radionuclide $A_2$ with decay constant $\lambda_2$. Assuming that at the initial moment the preparation contained only the radionuclide $A_1$, find:**
**(a) the equation describing accumulation of the radionuclide $A_2$ With time;**
**(b) the time interval after which the activity of radionuclide $A_2$ reaches the maximum value.**

**Ans.** (a) Suppose $N_1$ and $N_2$ are the number of two radionuclides $A_1$, $A_2$ at time t. Then

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \qquad (1)$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \qquad (2)$$

From (1)

$$N_1 = N_{10} e^{-\lambda_1 t}$$

Where $N_{10}$ is the initial number of nuclides $A_1$ at time t = 0

From (2)

$$\left(\frac{dN_2}{dt} + \lambda_2 N_2\right) e^{\lambda_2 t} = \lambda_1 N_{10} e^{-(\lambda_1 - \lambda_2) t}$$

or $(N_2 e^{\lambda_2 t}) = \text{const} \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2} e^{-(\lambda_1 - \lambda_2) t}$

since $N_2 = 0$ at t = 0

Constant $N_2 = \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2}$

Thus $= \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2} (e^{-\lambda_2 t} - e^{-\lambda_1 t})$

(b) The activity of nuclide $A_2$ is $\lambda_2 N_2$. This is maximum when $N_2$ is maximum. That happens when

$$\frac{dN_2}{dt} = 0$$

This requires

$$\lambda_2 \, e^{-\lambda_2 t_m} = \lambda_1 e^{-\lambda_1 t_m}$$

Or

$$t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$$

**Q.230. Solve the foregoing problem if $\lambda_1$ = $X_2$ = X.**

**Ans. (**a) This case can be obtained from the previous one on putting

$$\lambda_2 = \lambda_1 - \varepsilon$$

where ε is very small and letting ε → 0 at the end. Then

$$N_2 = \frac{\lambda_1 N_{10}}{\varepsilon}(e^{\varepsilon t} - 1)\, e^{-\lambda_1 t} = \lambda_1 t\, e^{-\lambda_1 t} N_{10}$$

or dropping the subscript 1 as the two values are equal

$$N_2 = N_{10} \lambda t\, e^{-\lambda t}$$

(b) This is maximum when

$$\frac{dN_2}{dt} = 0 \quad \text{or} \quad t = \frac{1}{\lambda}$$

# Radioactivity (Part - 2)

**Q.231. A radionuclide $A_1$ goes through the transformation chain $A_1 \rightarrow A_2 \rightarrow A_3$ (stable) with respective decay constants $\lambda_1$ and $\lambda_2$. Assuming that at the initial moment the preparation contained only the radionuclide $A_1$ equal in quantity to $N_{10}$ nuclei, find the equation describing accumulation of the stable isotope $A_3$.**

**Ans.** Here we have the equations

$$\frac{dN_1}{dt} = -\lambda_1 N_1$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \quad \text{and} \quad \frac{dN_3}{dt} = \lambda_2 N_2$$

From problem 229

$$N_1 = N_{10} e^{-\lambda_1 t}$$
$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_1 - \lambda_2}\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right)$$

Then $$\frac{dN_3}{dt} = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} N_{10}\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right)$$

Or $$N_3 = \text{Const} - \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}\left(\frac{e^{-\lambda_2 t}}{\lambda_2} - \frac{e^{-\lambda_1 t}}{\lambda_1}\right) N_{10}$$

since $N_3 = 0$ initially

$$\text{Const} = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} N_{10}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

So $$N_3 = \frac{\lambda_1 \lambda_2 N_{10}}{\lambda_1 - \lambda_2}\left[\frac{1}{\lambda_2}\left(1 - e^{-\lambda_2 t}\right) - \frac{1}{\lambda_1}\left(1 - e^{-\lambda_1 t}\right)\right]$$

$$= N_{10}\left[1 + \frac{\lambda_1 e^{-\lambda_2 t} - \lambda_2 e^{-\lambda_1 t}}{\lambda_2 - \lambda_1}\right]$$

**Q.232. A Bi21° radionuclide decays via the chain**

$$Bi^{210} \xrightarrow[\lambda_1]{} Po^{210} \xrightarrow[\lambda_2]{} Pb^{206} \text{ (stable)},$$

where the decay constants are $\lambda_1 = 1.60 \cdot 10^{-6}$ $s^{-1}$, $\lambda_2 = 5.80.10^{-8}$ $s^{-1}$. Calculate alpha- and beta-activities of the $Bi^{210}$ preparation of mass 1.00 mg a month after its manufacture.

**Ans** We have the chain

$$B_i^{210} \xrightarrow[\lambda_1]{\beta} P_0^{210} \xrightarrow[\lambda_2]{\alpha} P_b^{206}$$

$A_1$ $A_2$ $A_3$

of the previous problem initially

$$N_{10} = \frac{10^{-3}}{210} \times 6{\cdot}023 \times 10^{23} = 2{\cdot}87 \times 10^{18}$$

A month after preparation
$N_1 = 4.54 \times 10^{16}$
$N_2 = 2.52 \times 10^{18}$

using the results of the previous problem.

Then

$A\beta = \lambda_1 N_1 = 0.725 \times 10^{11}$ dis/sec
$A\alpha = \lambda_2 N_2 = 1.46 \times 10^{11}$ dis/sec

**Q.233. (a) What isotope is produced from the alpha-radioactive $Ra^{228}$ as a result of five alpha-disintegrations and four β-disintegrations?**
**(b) How many alpha- and β-decays does $U^{238}$ experience before turning finally into the stable $Pb^{206}$ isotope?**

**Ans.** (a) Ra has Z - 88, A - 226 After 3 a emission and 4 p (electron) emission
A - 206
Z - 88 + 4-5x2 - 82
The product is $^{82}pb^{206}$

(b) We require
- ΔZ = 10 = 2 n - m
- ΔA = 32 = ft x 4

Here n = no. of α emissions
m = no. of β emissions
Thus n = 8, m = 6

**Q.234. A stationary $Pb^{200}$ nucleus emits an alpha-particle with kinetic energy $T_\alpha$ = 5.77 MeV. Find the recoil velocity of a daughter nucleus. What fraction of the total energy liberated in this decay is accounted for by the recoil energy of the daughter nucleus?**

**Ans.** The momentum of the α-particle is

$\sqrt{2 M_\alpha T}$ This is also the recoil m om entum o f the daughter nuclear in opposite direction. The recoil velocity of the daughter ntideus is

$$\frac{\sqrt{2 M_\alpha T}}{M_d} = \frac{2}{196}\sqrt{\frac{2T}{M_p}} = 3.39\text{x}10^5 \text{ m/S}$$

The eneigy of the daughter nucleus is $\frac{M_\alpha}{M_d} T$ and this represents a fraction

$$\frac{M_\alpha / M_d}{1 + \frac{M_\alpha}{M_d}} = \frac{M_\alpha}{M_\alpha + M_d} = \frac{4}{200} = \frac{1}{50} = 0{\cdot}02$$

of total eneigy. Here Md is the mass of the daughter nudeus.

**Q.235. Find the amount of heat generated by 1.00 mg of a $Po^{210}$ preparation during the mean lifetime period of these nuclei if the emitted alpha-particles are known to possess the kinetic energy 5.3 MeV and practically all daughter nuclei are formed directly in the ground state.**

**Ans.** The number of nuclei initially present is

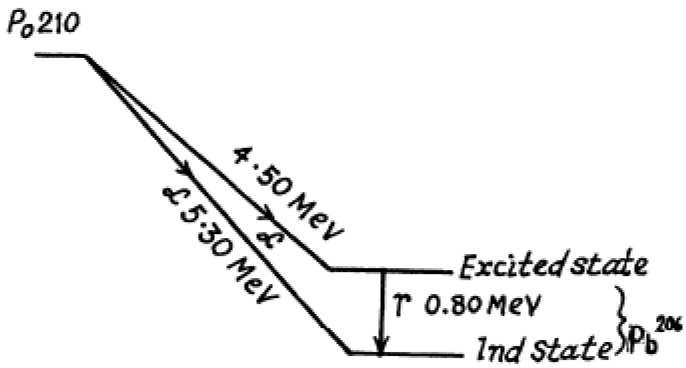
$$\frac{10^{-3}}{210} \times 6{\cdot}023 \times 10^{23}$$

In the mean life time of these nuclei the number decaying is the

fraction $1 - \frac{1}{e} = 0{\cdot}632.$ Thus the eneigy released is $2.87 \times 10^{18} \times 0.632 \times 5.3 \times 1.602 \times 10^{-13}$ J = 1.54MJ

**Q.236. The alpha-decay of $Po^{210}$ nuclei (in the ground state) is accompanied by emission of two groups of alpha-particles with kinetic energies 5.30 and 4.50 MeV. Following the emission of these particles the daughter nuclei are found in the ground and excited states. Find the energy of gamma-quanta emitted by the excited nuclei.**

**Ans.** We neglect all recoil effects. Then the following diagram gives the eneigy of the gamma ray




**Q.237. The mean path length of alpha-particles in air under standard conditions is defined by the formula $R = 0.98.10^{-27}\ v^3_0$ cm, where $v_0$ (cm/s) is the initial velocity of an alpha-particle. Using this formula, find for an alpha-particle with initial kinetic energy 7.0 MeV:**
**(a) its mean path length;**
**(b) the average number of ion pairs formed by the given alphaparticle over the whole path R as well as over its first half, assuming the ion pair formation energy to be equal to 34 eV.**

**Ans.** (a) For an alpha particle with initial K.E. 7.0 MeV, the initial velocity is

$$v_0 = \sqrt{\frac{2T}{M_\alpha}}$$

$$= \sqrt{\frac{2 \times 7 \times 1{\cdot}602 \times 10^{-6}}{4 \times 1{\cdot}672 \times 10^{-24}}}$$

$= 1.83 \times 10^9$an/sec
Thus R = 6.02an
(b) Over the whole path the number of ion pairs is

$$\frac{7 \times 10^6}{34} = 2{\cdot}06 \times 10^5$$

Over the first half of the path We write the formula for the mean path as $R \alpha E^{1/2}$ where E is the initial energy. Thus if the energy of the a-particle after traversing the first half of the path is $E_1$ then

$$R_0 E_1^{3/2} = \frac{1}{2} R_0 E_0^{3/2} \quad \text{or} \quad E_1 = 2^{-2/3} E_0$$

Hence number of ion pairs formed in the first half of the path length is

$$\frac{E_0 - E_1}{34\, e\, V} = (1 - 2^{-2/3}) \times 2{\cdot}06 \times 10^5 = 0{\cdot}76 \times 10^5$$

**Q.238. Find the energy Q liberated in $\beta^-$- and $\beta^+$-decays and in K-capture if the masses of the parent atom $M_P$, the daughter atom $M_d$ and an electron m are known.**

**Ans.** In $\beta^-$ decay

$$Z^{X^A} \rightarrow {}_{Z+1}Y^A + e^- + Q$$

$$Q = (M_x - M_y - m_e)\, c^2$$

$$= [(M_x + Z\, m_e) - (M_y + Z\, m_e + m_e)]\, c^2$$

$$= (M_p - M_d)\, c^2$$

since $M_p$, $M_d$ are the masses of the atoms. The binding energy of the electrons in ignored. In K capture

$$e_K^- + {}_ZX^A \rightarrow {}_{Z-1}Y^A + Q$$

$$Q = (M_X - M_Y)\, c^2 + m_e\, c^2$$

$$= (M_x^{c^2} + Z\, m_e\, c^2) - (M_Y\, c^2 + (Z-1)\, m_e\, c^2)$$

$$= c^2 (M_p - M_d)$$

In $\beta^+$ decay $\quad {}_ZX^A \rightarrow {}_{Z-1}Y^A + e^+ + Q$

Then $Q = (M_x - M_y - m_e)\, c^2$

$$= [M_x + Z\, m_e]\, c^2 - [M_y + (Z-1)\, m_e]\, c^2 - 2\, m_e\, c^2$$

$$= (M_p - M_d - 2\, m_e)\, c^2$$

**Q.239. Taking the values of atomic masses from the tables, find the maximum kinetic energy of beta-particles emitted by $Be^{10}$ nuclei and the corresponding kinetic energy of recoiling daughter nuclei formed directly in the ground state.**

**Ans.** The reaction is $Be^{10} \rightarrow B^{10} + e^- + \bar{\nu}_e$

For maximum K.E. of electrons we can put the energy of $\bar{\nu}_e$ to be zero. The atomic masses are

$Be^{10}$ = 10.016711 amu
$B^{10}$ = 10.016114 amu

So the K.E. of electrons is (see previous problem)
597 x $10^{-6}$ amu x $c^2$ = 0.56 MeV

The momentum of electrons with this K.E. is 0.941 $\frac{MeV}{c}$
and the recoil energy of the daughter is

$$\frac{(0{\cdot}941)^2}{2 \times M_d\, c^2} = \frac{(0{\cdot}941)^2}{2 \times 10 \times 938}\,\text{MeV} = 47{\cdot}2\ \text{eV}$$

**Q.240. Evaluate the amount of heat produced during a day by a $\beta^-$-active $Na^{24}$ preparation of mass m = 1.0 mg. The beta-particles are assumed to possess an average kinetic energy equal to 1/3 of the highest possible energy of the given decay. The half-life of $Na^{24}$ is T = 15 hours.**

**Ans.** The masses are
$Na^{24}$ = 24 - 0.00903 amu and $Mg^{24}$ = 24 - 0.01496 amu
The reaction is

$$Na^{24} \rightarrow Mg^{24} + e^- + \bar{\nu}_e$$

The maximum K.E. of electrons is
0.00593 x 93 MeV = 5.52 MeV

Average K.E. according to the problem is then $\frac{5\cdot52}{3}$ = 1.84 MeV
The initial number of $Na^{24}$ is

$$\frac{10^{-3} \times 6\cdot023 \times 10^{23}}{24} = 2\cdot51 \times 10^{19}$$

The fraction decaying in a day is
$1 - (2)^{-24/15} = 0.67$
Hence the heat produced in a day is
$0.67 \times 2.51 \times 10^{19} \times 1.84 \times 1.602 \times 10^{-13}$ Joul = 4.95MJ

**Q.241. Taking the values of atomic masses from the tables, calculate the kinetic energies of a positron and a neutrino emitted by $C^{11}$ nucleus for the case when the daughter nucleus does not recoil.**

**Ans.** We assume that the parent nucleus is at rest Then since the daughter nucleus does not recoil, we have

$$\vec{p} = -\vec{p}_\nu$$

i.e. positron & v mometum are equal and opposite. On the other hand

$\sqrt{c^2 p^2 + m_e^2 c^4} + c\,p = Q =$ total energy released. (Here we have used the fact that energy of the neutrino is $c\,|\vec{p}_\nu| = c\,p$)

Now $Q = [\,(\text{Mass of } C^{11} \text{ nucleus}) - (\text{Mass of } B^{11} \text{nucleus})\,]\,c^2$

$$= [\,\text{Mass of } C^{11} \text{ atom} - \text{Mass of } B^{11} \text{ atom} - m_e\,]\,c^2$$

$$= 0\cdot00213 \text{ amu} \times c^2 - m_e c^2$$

$$= (0\cdot00213 \times 931 - 0\cdot511)\text{ MeV} = 1\cdot47\text{ MeV}$$

Then $c^2 p^2 + (0\cdot511)^2 = (1\cdot47 - c\,p)^2 = (1\cdot47)^2 - 2\cdot94\,c\,p + c^2 p^2$

Thus c p = 0.646 M eV = energy of neutrino

Also K.E. of electron = 1.47 - 0.646 - 0.511 = 0.313 MeV

**Q.242. Find the kinetic energy of the recoil nucleus in the positronic decay of a Nn nucleus for the case when the energy of positrons is maximum.**

**Ans.** The K.E. of the positron is maximum when the energy of neutrino is zero. Since the recoil energy of the nucleus is quite small, it can be calculated by successive approximation.
The reaction is

$$N^{13} \rightarrow C^{13} + e^{+} + \nu_e .$$

The maximum energy available to the positron (including its rest energy) is

$c^2$ (Mass of $N^{13}$ nucleus - Mass of $C^{13}$ nucleus)

= $c^2$ (Mass of $N^{13}$ atom - Mass of $C^{13}$ atom - $m_e$)

= 0.00239 $c^2$ - $m_e c^2$

= (0.00239 x 931 - 0.511) MeV

= 1.71 MeV

The momentum corresponding to this energy is 1.636 MeV/c .
The recoil energy of the nucleus is then

$$E = \frac{p^2}{2M} = \frac{(1 \cdot 636)^2}{2 \times 13 \times 931}$$ = 111 eV = 0.111 keV

on using $Mc^2$ - 13 x 931 MeV

**Q.243. From the tables of atomic masses determine the velocity of a nucleus appearing as a result of K-capture in a Bel atom provided the daughter nucleus turns out to be in the ground state.**

**Ans.** The process is

$$e_k^- + Be^7 \rightarrow Li^7 + \nu$$

The energy available in the process is
Q = $c^2$ (Mass of $Be^7$ atom - Mass of $Li^{-7}$ atom)
= 0.00092 x 931 MeV = 0.86 MeV

The momentum of a K electron is negligible. So in the rest frame of the $Be^7$ atom, most of the energy is taken by neutrino whose momentum is very nearly 0.86MeV/c The momentum of the recoiling nucleus is equal and opposite. The velocity of recoil is

$$\frac{0{\cdot}86\ \text{MeV}/c}{M_{Li}} = c \times \frac{0{\cdot}86}{7 \times 931} = 3.96 \times 10^6 \text{ cm/s}$$

**Q.244. Passing down to the ground state, excited $Ag^{109}$ in nuclei emit either gamma quanta with energy 87 keV or K conversion electrons whose binding energy is 26 keV. Find the velocity of these electrons.**

**Ans.** In internal conversion, the total energy is used to knock out K electrons. The KE. of these electrons is energy available-B.E. of K electrons
= (87 - 26) = 61 keV

The total energy including rest mass of electrons is 0.511 + 0.061 = 0.572 MeV
The momentum corresponding to this total energy is

$$\sqrt{(0{\cdot}572)^2 - (0{\cdot}511)^2}\ /c = 0257 \text{ MeV/c.}$$

The velocity is then = $\frac{c^2 p}{E} = c \times \frac{0{\cdot}257}{0{\cdot}572} = 0{\cdot}449\,c$

**Q.245. A free stationary $Ir^{191}$ nucleus with excitation energy E = 129 keV passes to the ground state, emitting a gamma quantum. Calculate the fractional change of gamma quanta energy due to recoil of the nucleus.**

**Ans.** With recoil neglected, the y-quantram will have 129 keV eneigy. To a first approximation, its momentrum will be 129 keV/c and the energy of recoil will be

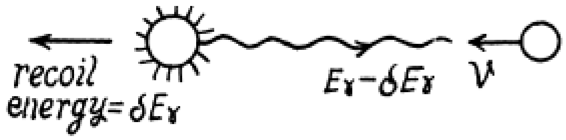
$$\frac{(0{\cdot}129)^2}{2 \times 191 \times 931} \text{MeV} = 4.18 \times 10^{-8} \text{ MeV}$$

In the next approximation we therefore write $E_\gamma \simeq {\cdot}129 - 8{\cdot}2 \times 10^{-8}\ \text{MeV}$

Therefore $\frac{\delta E_\gamma}{E_\gamma} = 3{\cdot}63 \times 10^{-7}$

**Q.246. What must be the relative velocity of a source and an absorber consisting of free $Ir^{191}$ nuclei to observe the maximum absorption of gamma quanta with energy ε = 129 keV?**

**Ans.** For maximum (resonant) absorption, the absorbing nucleus must be moving with enough speed to cancel the momentum of the oncoming photon and have just right eneigy ($\varepsilon$ = 129 keV) available for transition to the excited state.




Since $\delta E_\gamma \simeq \frac{\varepsilon^2}{2Mc^2}$ and momentum of the photon is $\frac{\varepsilon}{c}$, these condition can be satisfied if the velocity of the nucleus is

$$\frac{\varepsilon}{Mc} = c\frac{\varepsilon}{Mc^2} =$$ 218m/s = 0.218km/s

**Q.247. A source of gamma quanta is placed at a height h = 20m above an absorber. With what velocity should the source be displaced upward to counterbalance completely the gravitational variation of gamma quanta energy due to the Earth's gravity at the point where the absorber is located?**

**Ans.** Because of the gravitational shift the frequency of the gamma ray at the location of the absorber is increased by

$$\frac{\delta\omega}{\omega} = \frac{gh}{c^2}$$

For this to be compensated by the Doppler shift (assuming that resonant absorption is possible in the absence of gravitational field) we must have

$$\frac{gh}{c^2} = \frac{v}{c} \quad \text{or} \quad v = \frac{gh}{c} = 0{\cdot}65\ \mu\text{m/s}$$

**Q.248. What is the minimum height to which a gamma quanta source containing excited $Zn^{67}$ nuclei has to be raised for the gravitational displacement of the Mossbauer line to exceed the line width itself, when registered on the Earth's surface? The registered gamma quanta are known to have an energy $\varepsilon$ = 93 keV and appear on transition of $Zn^{67}$ nuclei to the ground state, and the mean lifetime of the excited state is $\zeta$ = 14μS.**

**Ans.** The natural life time is

$\Gamma = \frac{\hbar}{\tau} = 4{\cdot}7 \times 10^{-10}\,\text{eV}$

Thus the condition $\delta E_\gamma \geq \Gamma$ implies $\frac{g h}{c^2} \geq \frac{\Gamma}{\varepsilon} = \frac{\hbar}{\tau \varepsilon}$

$h \geq \frac{c^2 \hbar}{\tau \varepsilon g}$ = 4.64 metre

(h here is height of the place, not planck's constant.)