CHAPTER : 16

ELECTRIC POTENTIAL AND CAPACITORS

In modules 2 and 3, you learnt about the direction of flow of fluids and thermal energy. You may recall that the level of water in a container determines the direction in which it flows. If the level of water in one container is higher than that in the other, water will flow from higher level to lower level, irrespective of the quantity of water in the containers. Temperature plays a similar role in case of flow of thermal energy from one object to another. Thermal energy always flows from a body at higher temperature to the one at lower temperature. Here also, the direction of flow does not depend on the quantity of thermal energy possessed by an object.

Electric potential plays a similar role in the flow of charges from one point to another. The positive charge always moves from a point at higher potential to a point at lower potential. A positive test charge, when left free in an electric field, moves in the direction of the electric field. From this behaviour of a positive test charge, you may be tempted to say that the **electric field (E)** and **electric potential** (*V*) are closely related. In this lesson, you will learn to establish a relation between these physical quantities. You will also learn about a device called capacitor, which is used to store charge, filter alternating current and finds wide applications in electronic circuitory as well as power transmission.

OBJECTIVES

After studying this lesson, you should be able to :

- *explain the meaning of electric potential at a point and potential difference;*
- *derive expressions for electric potential due to a point charge and a dipole;*
- *explain the principle of capacitors and state their applications;*
- *derive an expression for the capacitance of a parallel plate capacitor;*
- *obtain equivalent capacitance in grouping of capacitors;*

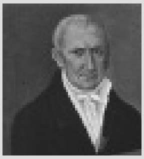
- *calculate the energy stored in a capacitor; and*
- *explain polarization of dielectric materials in an electric field.*

16.1 LECTRIC POTENTIAL AND POTENTIAL DIFFERENCE

When a charged particle is made to move in an electrostatic field in a direction opposite to the direction of the field, work is done by an external agency. This work is stored as potential energy of charge in accordance with the law of conservation of energy. So, we can say that an electric charge placed at a point in an electric field has potential energy, which is a function of its position. We can visualize the potential energy of charge in the field as a scalar function of position and for a unit charge call it potential. It means that different points in an electric field would be at different potentials. And if a positively charged particle is placed in an electric field, it will tend to move from higher to lower potential to minimize its potential energy. In the next lesson, you will learn how the concept of potential difference leads to flow of current in electric circuits.

*The **electric potential** at any point in an electric field is equal to the work done against the electric force in moving a unit positive charge from outside the electric field to that point.* Electric potential is a scalar quantity, as it is related to work done.

Alessandro, Conte Volta (1745-1827)



Born at Como, Italy, Volta was a professor at Pavia for more than 20 years. A well travelled man, he was known to many famous men of his times. He decisively proved that animal electricity observed by Luigi Galvani in frog muscles was a general phenomenon taking place between two dissimilar metals seperated by acidic or salt solutions. On the basis of this observation, he invented first electro-chemical cell, called voltaic cell. The unit of potential difference is named volt in his honour.

The potential at a point is taken positive when work is done against the field by a positive charge but negative when work is done by the electric field in moving the unit positive charge from infinity to the point in the field.

Consider two points A and B in an electric field (Fig. 16.1). If a test charge q_0 is moved from point A to point B along any path by an external force, the amount of work done by the external force is given by

$$W_{AB} = q_0 (V_B - V_A) \qquad (16.1)$$

Thus, potential difference between points A and B will be

$$V_{AB} = V_B - V_A = \frac{W_{AB}}{q_0} \qquad (16.2)$$

Here V_A and V_B are potentials at points A and B, respectively.

A potential difference is said to exist between two points in an electric field, if work is done against the electric force in moving a positive test charge from one point to the other. Note that this work is independent of the path. (*For this reason, the electric field is said to be a conservative field*). The SI unit of potential and potential difference is **volt** :

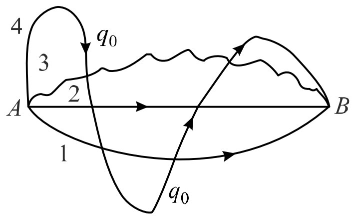


Fig. 16.1 : The work done in moving a test charge from one point to another in an electric field is independent of the path followed.

1 **volt** = 1 **joule**/1 **coulomb**

If one joule of work is done in taking a test charge of one coulomb from one point to the other in an electric field, the potential difference between these points is said to be one volt. If one joule of work is done in bringing a test charge of one coulomb from infinity to a point in the field, the potential at that point is one volt.

Note that potential at a point is not a unique quantity as its value depends on our choice of zero potential energy (infinity). However, the potential difference between two points in a stationary field will have a unique value. Let us now learn to calculate potential at a point due to a single charge.

16.1.1 Potential at a point due to a Point Charge

Suppose we have to calculate electric potential at point P due to a single point charge $+q$ situated at O (Fig. 16.2), where $OP = r$. The magnitude of electric field at P due to the point charge is given by

$$E_p = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2} \qquad (16.3)$$

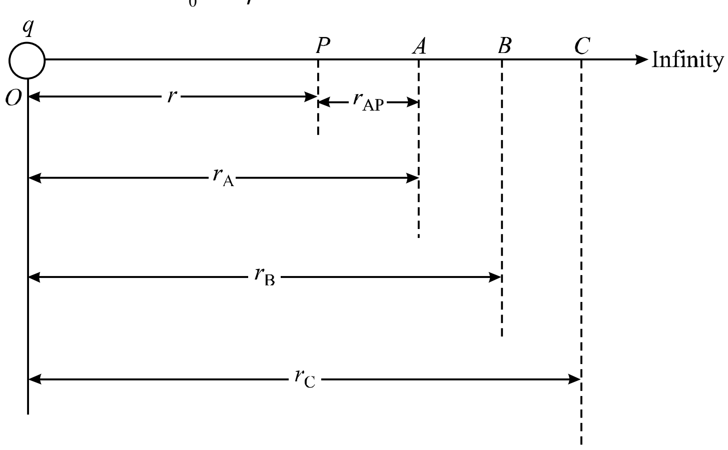


Fig. 16.2 : Work done per unit charge in moving a charge q_0 from infinity to a point P in an electric field E is the potential at that point.

Similarly, the electric field at point A will be

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_A^2} \tag{16.4}$$

If points P and A are very close, the average field E_{AP} between these points can be taken as the geometric mean of E_P and E_A :

$$E_{AP} = \sqrt{E_A \times E_p}$$

$$= \sqrt{\frac{1}{4\pi\varepsilon_o}\frac{q}{r_A^2} \times \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_A r} \tag{16.5}$$

Therefore, the magnitude of force experienced by a test charge q_0 over this region will be

$$F_{AP} = q_o E_{AP} = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r_A r} \tag{16.6}$$

and the work done in moving charge q_0 from A to P is given by

$$W_{AP} = F_{AP} \times r_{AP}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q\,q_o}{r_A r} \times (r_A - r)$$

$$= \frac{q\,q_0}{4\pi\varepsilon_0} \times \left(\frac{1}{r} - \frac{1}{r_A}\right) \tag{16.7}$$

where r_{AP} is the distance between points A and P.

Similarly, work done in moving this charge from B to A will be given by

$$W_{BA} = \frac{q\,q_0}{4\pi\varepsilon_0} \times \left(\frac{1}{r_A} - \frac{1}{r_B}\right) \tag{16.8a}$$

And work done in moving the test charge from C to B will be

$$W_{CB} = \frac{q\,q_0}{4\pi\varepsilon_0} \times \left(\frac{1}{r_B} - \frac{1}{r_C}\right) \tag{16.8b}$$

and so on. The total work done in moving the charge from infinity to the point P will be

$$W = \frac{q\,q_0}{4\pi\varepsilon_0} \times \left(\frac{1}{r} - \frac{1}{r_A} + \frac{1}{r_A} - \frac{1}{r_B} + \frac{1}{r_B} - \frac{1}{r_C} ... + ... - \frac{1}{\infty}\right)$$

$$= \frac{q\,q_0}{4\pi\varepsilon_0} \times \left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$= \frac{q\,q_0}{4\pi\varepsilon_0 r} \qquad (16.9)$$

By definition, potential at a point is given by

$$V_P = \frac{W}{q_0}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad (16.10)$$

Note that potential is inversely proportional to distance. It is positive or negative depending on whether q is positive or negative.

If there are several charges of magnitudes $q_1, q_2, q_3, \ldots$, the electric potential at a point is the scalar sum of the potentials due to individual charges (Fig.16.3) :

$$V = V_1 + V_2 + V_3 + ...$$

$$= \sum_{i=1}^{\infty} \frac{q_i}{4\pi\varepsilon_0 r_i} \qquad (16.11)$$

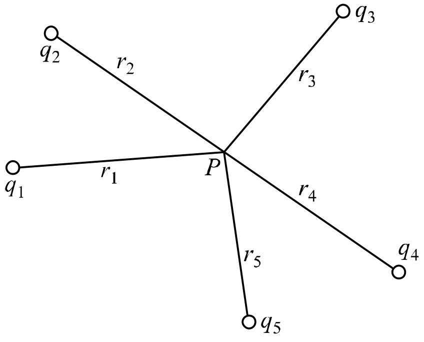


Fig. 16.3 : Potential at a point P due to a system of charges

16.1.2 Potential at a Point due to an Electric Dipole

Let us consider an electric dipole consisting of two equal and opposite point charges $-q$ at A and $+q$ at B, separated by a distance $2l$ with centre at O. We wish to calculate potential at a point P, whose polar co-ordinates are (r, θ); i.e. $OP = r$ and $\angle BOP = \theta$, as shown in Fig. 16.4. Here $AP = r_1$ and $BP = r_2$. We can easily calculate potential as P due to point charges at A and B using Eqn.(16.10) :

$$V_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{(-q)}{r_1}$$

and

$$V_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r_2}$$

Total potential at P due to both the charges of the dipole is given by

$$V = V_1 + V_2$$

That is,

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \qquad (16.12)$$

To put this result in a more convenient form, we draw normals from A and B on the line joining O and P. From Δ BOD, we note that $OD = l \cos\theta$ and from Δ OAC we can write $OC = l\cos\theta$. For a small dipole ($AB \ll OP$), from Fig. 16.4, we can take $PB = PD$ and $PA = PC$. Hence

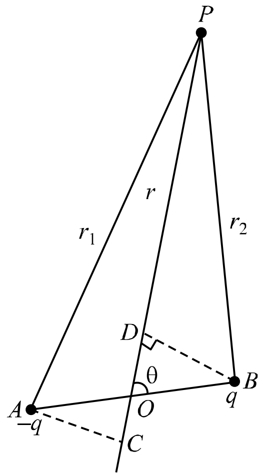


Fig. 16.4 : Electric potential at a point P due to an electric dipole.

$$r_1 = r + l\cos\theta$$

$$r_2 = r - l\cos\theta$$

Using these results in Eqn (16.12), we get

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r - \ell\cos\theta)} - \frac{1}{(r + \ell\cos\theta)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(2\ell\cos\theta)}{(r^2 - \ell^2\cos^2\theta)}\right]$$

$$= \frac{q \times 2\ell\cos\theta}{4\pi\varepsilon_0 r^2}$$

where we have neglected the term containing second power of l since $l \ll r$.

In terms of dipole moment ($p = q \times 2\ell$), we can express this result as

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \qquad (16.13)$$

This result shows that unlike the potential due to a point charge, the potential due to a dipole is inversely proportional to the square of the distance.

Let us now consider its special cases.

Special Cases

Case I : When point P lies on the axial line of the dipole on the side of positive charge, $\theta = 0$ and $\cos\theta = 1$. Then Eqn. (16.13) reduces to

$$V_{\text{AXIS}} = \frac{p}{4\pi\varepsilon_0 r^2} \qquad (16.14)$$

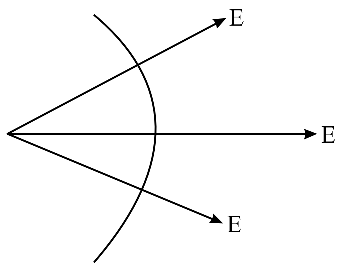
Case II : When point P lies on the axial line of the dipole but on the side of negative charge, $\theta = 180^\circ$ and $\cos\theta = -1$. Hence

$$V_{\text{AXIS}} = -\frac{p}{4\pi\varepsilon_0 r^2} \qquad (16.15)$$

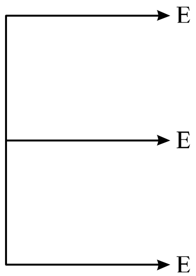
Case III : When point P lies on the equatorial line of the dipole (perpendicular bisector of AB), $\theta = 90^\circ$ and $\cos\theta = 0$. Then

$$V_{\text{equatorial}} = 0 \qquad (16.16)$$

That is, electric potential due to a dipole is zero at every point on the equatorial line of the dipole. When a dipole is kept in 3D space, the equatorial line will lie in the plane of the paper. The potential at all points in this plane will be same, i.e. zero. Such a surface is referred to as *equipotential surface*.The electric field is always perpendicular to an equipotential surface. No work is done in moving a charge from one point to another on the equipotential surface.



(**a**) Spherical equipotential surface



(**b**) Plane equipotential surface

Fig. 16.5 : Equipotential surfaces and electric field directions

16.1.3 Potential Energy of a System of Point Charges

The electric potential energy is the energy possessed by a system of point charges by virtue of their being in an electric field. When charges are infinite distance apart, they do not interact and their potential energy is zero. If we want to assemble a charge system, i.e. bring charges near each other, work will have to be done. This work is stored in the form of potential energy in the system of these charges. This is called the electric potential energy of the charge system. Hence, we can define *potential energy of a system of point charges as the total amount of work done in bringing various point charges of the system to their respective positions from infinitely large mutual separations.*

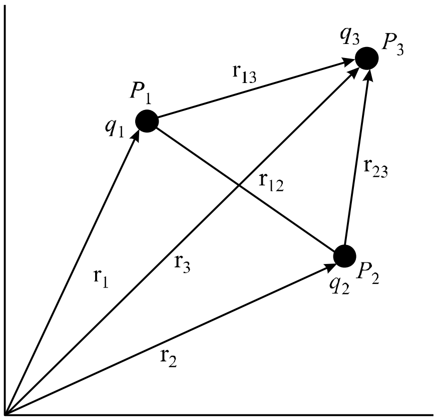


Fig. 16.6 : Potential energy of a system of point charges separated by a distance

Suppose that a point charge q_1 is located at a point P_1 with position vector $\mathbf{r}_1$ in space. Assume that point charge q_2 is at infinity. This is to be brought to the point P_2 having position vector $\mathbf{r}_2$ where $P_1P_2 = \mathbf{r}_{12}$, as shown in Fig. 16.6. We know that electric potential at P_2 due to charge q_1 at P_1 is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{|\mathbf{r}_{12}|} \tag{16.17}$$

From the definition of potential, work done in bringing charge q_2 from infinity to point P_2 is

$$W = (\text{Potential at } P_2) \times \text{value of charge}$$

This work is stored in the system of charges q_1 and q_2 in the form of electric potential energy U. Thus,

$$U = \frac{q_1 \times q_2}{4\pi\varepsilon_0 |\mathrm{r}_{12}|} \tag{16.18}$$

In case the two charges have same sign, work is done against the repulsive force to bring them closer and hence, electric potential energy of the system increases. On the other hand, in separating them from one another, work is done by the field. As a result, potential energy of the system decreases. If charges are of opposite sign, i.e. one is positive and the other is negative, the potential energy of the charge system decreases in bringing the charges closer and increases in separating them from one another.

For a three point charge system (Fig. 16.6), Eqn. (16.18) can be written as

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{\mathrm{r}_{12}} + \frac{q_1q_3}{\mathrm{r}_{13}} + \frac{q_2q_3}{\mathrm{r}_{23}}\right) \tag{16.19}$$

Proceeding in the same way, we can calculate the potential energy of a system of any number of charges.

By combining Eqns. (16.3) and (16.13), the potential energy of a dipole in a uniform electric field can be written as

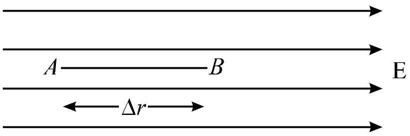
$$U_\theta = -pE\cos\theta = -\mathbf{p}.\mathbf{E} \tag{16.20}$$

where $\mathbf{p}$ is the dipole moment in electric field $\mathbf{E}$ and θ is the angle between $\mathbf{p}$ and $\mathbf{E}$.

16.2 RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

Consider two points A and B in a uniform electric field $\mathbf{E}$, separated by a small distance Δr. By definition, potential difference ΔV between A and B is equal to the work done in moving a unit positive test charge from A to B:

ΔV = (Force on unit positive charge) × (*AB*)
$= \mathbf{E}.\ \Delta\mathbf{r} = E(\Delta r)\cos 180^{\circ}$
$= -\mathbf{E}\,\Delta\mathbf{r}$



or $$\mathbf{E} = -\frac{\Delta V}{\Delta r} \qquad (16.21)$$

The negative sign indicates that work is done against the electric field.

Hence, at any point, the electric field is equal to negative rate of change of potential with distance (called ***potential gradient***) at that point in the direction of field. Remember that electric potential is a scalar quantity but electric potential gradient is a vector as it is numerically equal to electric field.

From the above relation, for a uniform electric field, we can write

$$E = \frac{V_A - V_B}{d} \qquad (16.22)$$

Here V_A and V_B are potentials at points *A* and *B,* respectively separated by a distance *d*.

Example 16.1 : In a 10 volt battery, how much work is done when a positively charged particle having charge 1.6×10^{-19} C is moved from its negative terminal to the positive terminal?

Solution : According to Eqn. (16.2)

$$V_{AB} = W_{AB} / q_O$$

Since V_{AB}= 10 V and $q_0 = 1.6 \times 10^{-19}$ C, we get

$$W_{AB} = (10\text{V}) \times (1.6 \times 10^{-19}\text{C})$$
$$= 1.6 \times 10^{-18}\ \text{J}$$

Example 16.2 : A point charge *q* is at the origin of Cartesian co-ordinate system. The electric potential is 400 V and the magnitude of electric field is 150 N C^{-1} at a point *x.* Calculate *x* and *q*.

Solution : The electric field

$$E = \frac{V}{x}$$

On inserting the numerical values, we get

$$150 = \frac{400}{x}$$

or $$x = 2.67\ \text{m}$$

Recall that electric field is given by the expression

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

We subsitute $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ N C}^{-2} \text{ m}^2$, $E = 150 \text{ N C}^{-1}$ and $x = 2.67$ m and obtain

$$q = \frac{(150 \text{ N C}^{-1}) \times (2.67 \text{m})^2}{9 \times 10^9 \text{ N C}^{-2}}$$

$$= 11.9 \times 10^{-8} \text{ C}$$

INTEXT QUESTIONS 16.1

1. A metallic sphere of radius R has a charge $+q$ uniformly distributed on its surface. What is the potential at a point r ($> R$) from the centre of the sphere?
2. Calculate the work done when a point charge is moved in a circle of radius r around a point charge q.
3. The electric potential V is constant in a region. What can you say about the electric field **E** in this region ?
4. If electric field is zero at a point, will the electric potential be necessarily zero at that point.
5. Can two equipotential surfaces intersect?

On the basis of charge conduction, substances are broadly classified as **conductors** and **insulators**. In solids, conduction of electricity usually takes place due to free electrons, whereas in fluids, it is due to ions. Conductors have free charge carriers through which electric currents can be established on applying an electric field. Metals are good conductors. Substances having no free charge carriers are called ***insulators***. The common insulators are wood, ebonite, glass, quartz, mica etc. Substances which have electrical conductivity in between those of conductors and insulators are called ***semiconductors***. The ratio of electrical conductivities of good conductors and good insulators is of the order of 10^{20}. Let us now learn how conductors behave in an electric field.

16.2.1 Behaviour of Conductors in an Electric Field

Conductors have electrons which are not bound tightly in their atoms. These are free to move within the conductor. However, there is no net transfer of electrons (charges) from one part of the conductor to the other in the absence of any applied electric field. The conductor is said to be in electrostatic equilibrium.

Refer to Fig. 16.7(a) which shows a conductor placed in an external electric field **E**. The free electrons are accelerated in a direction opposite to that of the electric field. This results in build up of electrons on the surface *ABCD* of the conductor. The surface *FGHK* becomes positively charged because of removal of electrons. These charges (-ve on surface *ABCD* and +ve on surface *FGHK*) create their own fields, which are in a direction opposite to **E**. The transfer of electrons from *FGHK* to *ABCD* continues till **E** becomes equal to $\mathbf{E}_1$. Such a state of electrostatic equilibrium is reached usually in 10^{-16} s. We then say that equilibrium is reached almost instantaneously. If there is a cavity inside a conductor, the electric field inside the cavity is zero (Fig. 16.7(b)).

These results are true for a charged conductor or when charges are induced on a neutral conductor by an external electric field.

This property of a conductor is used in **Electrostatic Shielding** — a phenomenon of protecting a certain region of space from external electric fields. To protect delicate instruments from external electric fields, they are enclosed in hollow conductors. That is why in a thunder storm accompanied by lightning, it is safer to be inside a car or a bus than outside. The metallic body of the car or bus provides electrostatic shielding from lightning.

Conductors in electrostatic equilibrium exhibit the following properties :

- There is no electric field inside a conductor.
- The electric field outside a charged conductor is perpendicular to the surface of the conductor, irrespective of the shape of the conductor.
- Any charge on the conductor resides on the surface of the conductor.

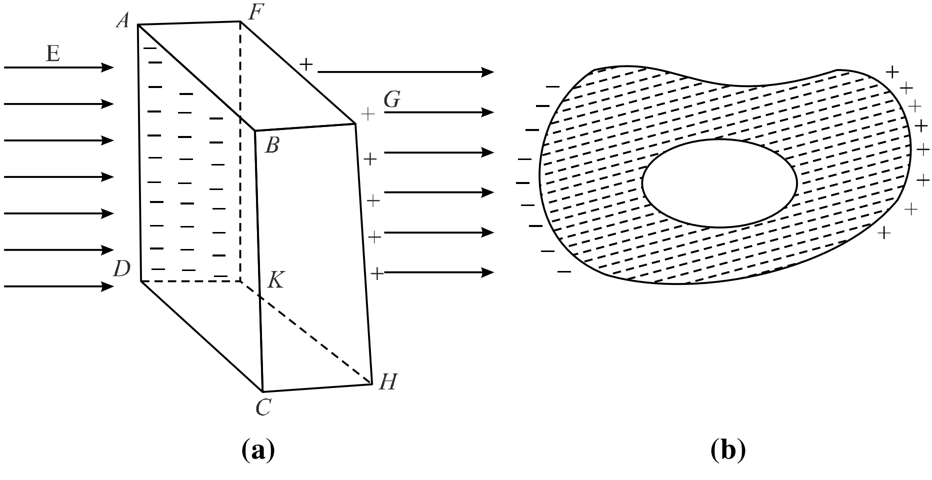


Fig.16.7 : Electrostatic shielding: **(a)** External electric field E pulls free electrons on the surface *ABCD*. The surface *FGHK,* which is deficient in electrons, becomes positively charged;the net field inside the conductor is zero. **(b)** If there is a cavity inside a conductor, the field inside the cavity is zero.

16.3 CAPACITANCE

Let us consider two conductors having equal but opposite charges $+Q$ and $-Q$ on them. There is a potential difference V between them. Such a system of conductors is called a ***capacitor***. Experimentally it is found that the potential difference is directly proportional to charge on a conductor. As charge increases, the potential difference between them also increases but their ratio remains constant. This ratio is termed as capacitance of the capacitor:

$$C = Q / V \tag{16.23}$$

The capacitance is defined as the ratio between the charge on either of the conductors and the potential difference between them. It is a measure of the capability of a capacitor to store charge.

In SI system of units, capacitance is measured in farad (F). The capacitance is one farad, if a charge of one coulomb creates a potential difference of one volt :

$$1 \text{ farad} = \frac{1\text{ coulomb}}{1\text{ volt}} \tag{16.24}$$

You may recall from the previous unit that coulomb is a very large unit of charge. It means that farad is also a very large unit of capacitance. Usually we use capacitors of values in microfarad or picofarad:

$$1 \text{ microfarad} = 10^{-6} \text{ farad, written as } \mu\text{F}$$

$$1 \text{ picofarad} = 10^{-12} \text{ farad, written as pF}$$

In an electrical circuit, a capacitor is represented by two parallel lines.

16.3.1 Capacitance of a Spherical Conductor

Suppose that a sphere of radius r is given charge q. Let the potential of the sphere be V. Then

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Since $C = q/V$, we find that

$$C = \frac{q}{q/4\pi\varepsilon_0 r} = 4\pi\varepsilon_0 r = \frac{r}{9\times10^9} \tag{16.25}$$

This shows that capacitance of a spherical conductor is directly proportional to its radius. In fact, it is numerically equal to its radius divided by 9×10^9, where radius is taken in metre. For example, the capacitance of a sphere of radius 0.18 m is

$$C = \frac{0.18}{9} \times 10^{-9}\text{ F} = 20 \text{ pF}$$

16.3.2 Types of Capacitors

You will come across many types of capacitors in your physics laboratory. The power supply system of your city also uses capacitors. These also form important components of devices such as radio, T.V., amplifires and oscillators. A capacitor essentially consists of two conductors, one charged and the other usually earthed. To understand the principle of a capacitor, let us consider an insulated metal plate A and give it positive charge (q) till its potential (V) becomes maximum. (Any further charge given to it would leak out.) The capacitance of this plate is equal to q/V.

Now bring another insulated metal plate B near plate A. By induction, negative charge is produced on the nearer face of B and equal positive charge develops on its farther face (Fig. 16.8a). The induced negative charge tends to decrease whereas induced positive charge tends to increase the potential of A. If plate B is earthed (Fig.16.8b), the induced positive charge on it, being free, flows to earth. (In reality, it is the negative charge that flows from the earth to the plate. Positive charges in the plate are immobile.) But negative charge will stay as it is bound to positive charge on A. Due to this induced negative charge on B, the potential of A decreases and its capacitance increases.

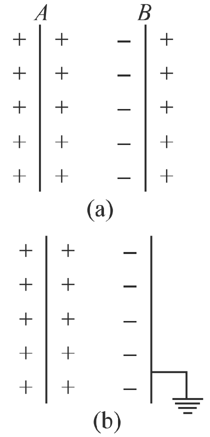


Fig.16.8 : Working principle of a capacitor

Hence, we can say that *capacitance of an insulated conductor can be increased by bringing near it an uncharged earthed conductor.* This is the basic principle of a capacitor. Capacitors are used for storing large amounts of electric charge and hence electrical energy in a small space for a small interval of time.

A Parallel Plate Capacitor

A parallel plate capacitor is one of the simplest capacitors in which two parallel metallic plates, each of area A, are separated from one another by a small distance d. An insulating medium like air, paper, mica, glass etc separates the plates. The plates are connected to the terminals of a battery, as shown in Fig. 16.9. Suppose that these plates acquire $+q$ and $-q$ charge when the capacitor is fully charged. These charges set up a uniform electric field **E** between the plates. When the separation d is small compared to the size of the plates, distortion of electric field at the boundaries of the plates can be neglected.

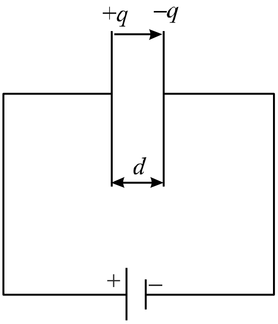


Fig. 16.9 : Working principle of a capacitor

If σ is surface charge density on either plate, the magnitude of electric field between the plates is given by

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$$

and the potential difference between the plates is given by

$$V = Ed$$

Hence, capacitance of a parallel plate capacitor, whose plates are separated by d and have air in-between them is given by

$$C_0 = \frac{q}{V} = \frac{q}{qd/\varepsilon_0 A}$$

$$= \frac{\varepsilon_0 A}{d} \qquad (16.26)$$

It shows that capacitance of a parallel plate capacitor is directly proportional to the area of the plates and inversely proportional to their separation. It means that to obtain high capacitance, area of the plates should be large and separation between them should be small.

If the plates of a capacitor are separated by a dielectric material other than air or vacuum, the capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon A}{d} = \frac{k\,\varepsilon_0 A}{d}$$

where ε is called permittivity of the medium. Therefore, we find that capacitance of a dielectric filled parallel plate capacitor becomes K times the capacitance with air or vacuum as dielectric :

$$C = KC_0 \qquad (16.27)$$

16.3.3 Relative Permittivity or Dielectric Constant

We can also define dielectric constant by calculating the force between the charges. According to Coulomb's law, the magnitude of force of interaction between two charges q_1 and q_2 separated by a distance r in vacuum is :

$$F_V = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad (16.28)$$

where ε_0 is the permittivity of free space.

If these charges are held at the same distance in a material medium, the force of interaction between them will be given by

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2} \qquad (16.29)$$

On combining Eqns. (16.28) and (16.29), we get

$$\frac{F_v}{F_m} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r \qquad (16.30)$$

where ε_r (or K) is relative permittivity. It is also termed as dielectric constant of the medium. Note that it is the ratio of permittivity of the material medium to the permittivity of free space. We can also define the dielectric constant of a medium as the ratio of the electrostatic force of interaction between two point charges held at certain distance apart in air or vacuum to the force of interaction between them held at the same distance apart in the material medium.

The dielectric constant can also be expressed as

$$K = \frac{\text{Capacitance with dielectric between the plates}}{\text{Capacitance with vacuum between the plates}}$$

$$= \frac{C_m}{C_0}$$

Thus

$$C_m = KC_0 \tag{16.31}$$

For metals, $K = \infty$, for mica $K \approx 6$, and for paper $K = 3.6$.

16.4. GROUPING OF CAPACITORS

Capacitors are very important elements of electrical and electronic circuits. We need capacitors of a variety of capacitances for different purposes. Sometimes a capacitance of a proper value may not be available. In such situations, grouping of capacitors helps us to obtain desired (smaller or larger) value of capacitance with available capacitors. Two most common capacitor groupings are :

- Series grouping, and
- Parallel grouping

Let us learn about these now.

16.4.1 Parallel Grouping of Capacitors

In parallel grouping, one plate of each capacitor is connected to one terminal and the other plate is connected to another terminal of a battery, as shown in Fig. 16.10. Let V be the potential difference applied to the combination between points A and B. Note that *in parallel combination, potential difference across each capacitor is the same.* Therefore, charge on these will be different, say q_1, q_2 and q_3 such that

$$q_1 = C_1V$$

$$q_2 = C_2V$$

$$q_3 = C_3V \tag{16.32}$$

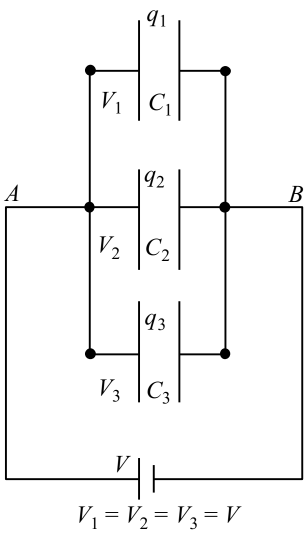


Fig. 16.10 : Capacitors joined in parallel

Total charge on all the capacitors of the combination is :

$$q = q_1 + q_2 + q_3$$

$$q = (C_1 + C_2 + C_3 + \ldots)V \tag{16.33}$$

Let C_p be the equivalent capacitnace in parallel combination. Then

$$q = C_p V$$

From these relations, we get

$$q = C_p V = (C_1 + C_2 + C_3)V$$

In general, we can write

$$C_p = C_1 + C_2 + C_3 = \sum_{i=1}^{n} C_i \tag{16.34}$$

Thus, we see that *equivalent capacitance of a number of capacitors joined in parallel is equal to the sum of the individual capacitances.*

Remember that in parallel combination, all the capacitors have the same potential difference between their plates but charge is distributed in proportion to their capacitances. Such a combination is used for charge accumulation.

16.4.2 Series Grouping of Capacitors

In the series combination of capacitors, the first plate of the first capacitor is connected to
the electrical source. The second plate of the first capacitor is connected to the first plate of the second capacitor. The second plate of second capacitor is connected to first plate of the next capacitor of the combination and so on. The second plate of last capacitor of the combination is connected to the electrical source, as shown in Fig.16.11. Let $+q$ unit of charge be given to the first plate of capacitor C_1 from the source. Due to electrical induction, as expalined in the principle of capacitor, $-q$ charge appears on the inner side of right plate of C_1 and $+q$ charge develops on the outer side of the second plate of C_1. The $+q$ unit of charge flows to the first plate of C_2 and so on. Thus, each capacitor receieves the same charge of magnitude q. As their capacitances are different, potential difference across these capacitors will be

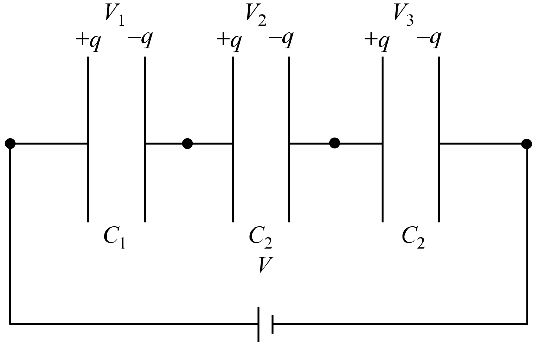


Fig.16.11 : Capacitors in series grouping. The amount of charge on each capacitor plate is same.

$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3} \tag{16.35}$$

If C_s is the total capacitance of the series grouping, then

$$V = \frac{q}{C_s}$$

and $$V = V_1 + V_2 + V_3 \quad (16.36)$$

Hence $$\frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

or $$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad (16.37)$$

For n capacitors joined in series, we can write

$$\frac{1}{C_s} = \sum_{i=1}^{n} \frac{1}{C_i}$$

Types of Capacitors

There are three common varieties of capacitors in commercial use. Their schematic diagrams are shown in Fig.16.12.

1. **Paper capacitor:** Several large thin sheets of paraffin impregnated paper or mylar are cut in proper size (rectangular). Several sheets of metallic foils are also cut to the same size. These are spread one over the other alternately. The outer sheet is mylar, then over it a sheet of metal foil, again over it a sheet of mylar and then a sheet of metal foil and so on. The entire system is then rolled in the form of a cylinder to form a small device.

2. **Metal plate capacitors:** A large number of metals are alternately joined to two metal rods as shown in Fig.16.12 (b). The entire plate system is immersed in silicon oil which works as dielectric material between the plates. High voltage capacitors are usually of this type. Variable capacitors of micro farad capacitance are usually of this type and use air as dielectric. One set of plates is fixed and the other set is movable. The movable plates, when rotated, change their effective area, thereby changing the capacitance of the system. You might see such capacitors in a radio receiver. Variable capacitance helps in tuning to different radio stations.

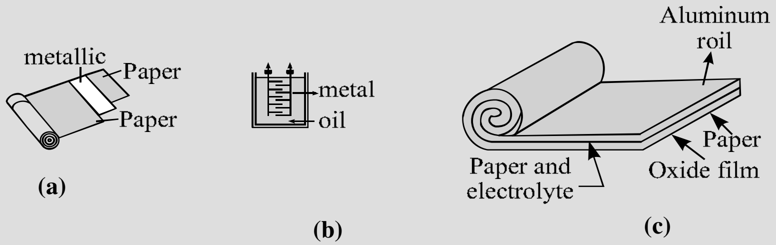


Fig.16.12 : Different types of capacitors : **a)** paper capacitor, **b)** variable capacitor, and **c)** electrolytic capacitor

3. **Electrolytic capacitor:** An electrolytic capacitor is shown Fig. 16.12(c). A metal foil is rolled in the shape of a cylinder with increasing diameter so that there is always a space between one surface and the other. The system is immersed in an electrolyte in the form of a solution. This solution is conducting because of ions in the solution. A voltage is applied between the electrolyte and the metallic foil. Because of the conducting nature of the electrolyte, a thin layer of metal oxide, which is an insulator, is formed on the foil. The oxide layer works as dielectric material. Since the dielectric layer is extremely thin, the system provides a very high value of capacitance. It is important in this type of capacitor to mark the **positive** and **negative** terminals. A wrong connection of positive and negative terminals removes the oxide layer. (The capacitor then starts conducting.) This type of capacitor is used in storing large amount of charge at low voltage.

Thus, *the reciprocal of equivalent capacitance of any number of capacitors connected in series is equal to the sum of the reciprocals of individual capacitances*. From the above relation, you will agree that C_s is less than the least of C_1, C_2, and C_3.

Note that *all the capacitors in series grouping have the same amount of charge but the potential difference between their plates are inversely proportional to their capacitances.* It means that the capacitor with minimum capacitance of the combination will have maximum potential difference between its plates.

Example 16.3 : The capacitance of a parallel plate air capacitor is 22.0 μF. The separation between the plates is d. A dielectric slab of thickness $d/2$ is put in-between the plates. Calculate the effective capacitance, if the dielectric constant $K = 5$.

Solution: The Capacitance of the air capacitor is given by

$$C_0 = \frac{\varepsilon_0 A}{d} = 22.0\ \mu\text{F}$$

The new system can be considered as a series combination of two capacitors:

$$C_1 = \frac{K\varepsilon_0 A}{d/2} = \frac{2K\varepsilon_0 A}{d} = 2\,KC_0$$

and

$$C_2 = \frac{\varepsilon_0 A}{d/2} = \frac{2\varepsilon_0 A}{d} = 2C_0$$

The effective capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C = \frac{C_1\, C_2}{C_1 + C_2}$$

$$= \frac{2KC_0 \times 2C_0}{2KC_0 + 2C_0}$$

$$= \frac{2KC_0}{K+1}$$

$$= \frac{10 \times 22 \times 10^{-6}\text{F}}{6}$$

$$= 36.7\ \mu\text{F}$$

INTEXT QUESTIONS 16.2

1. Write the dimensions of capacitance.
2. What is the potential difference between two points separated by a distance d in a uniform electric field $\mathbf{E}$?
3. The usual quantities related with an air capacitor are C_0, E_0 and V_0. How are these related with C, $\mathbf{E}$ and V of the same capacitor filled with dielectric constant K ?
4. Calculate the area of air filled capacitor plate when the separation between the plates is 50 cm and capacitance is 1.0 μF .

16.4.3 Energy Stored in a Capacitor

The charging of a capacitor can be visualized as if some external agent, say a battery, pulls electrons from the positive plate of a capacitor and transfers them to the negative plate. Some work is done in transferring this charge, which is stored in the capacitor in the form of electrostatic potential energy. This energy is obtained from the battery (stored as chemical energy). When this capacitor is discharged through a resistor, this energy is released in the form of heat.

Let us assume that an uncharged capacitor, when connected to a battery, develops a maximum charge q. The charging takes place slowly. The initial potential difference between the capacitor plates is zero and the final potential difference is V. The average potential difference during the entire process of charging is

$$\frac{0+V}{2} = \frac{V}{2} = \frac{q}{2C}$$

The work done during charging is given by

$$W = \text{Charge} \times \text{potential difference}$$

$$= q\,\frac{q}{2C} = \frac{1}{2}\frac{q^2}{C}$$

Hence potential energy

$$U = \frac{1}{2}qV = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}CV^2 \qquad (16.38)$$

This energy is stored in the electric field between the plates. The stored energy is directly proportional to the capacitance. It also increases as potential difference increases. However, every capacitor can store only a limited amount of energy. An automatic discharge will take place when the potential difference becomes more than its threshold value.

It is dangerous to touch the plates of a charged capacitor. The capacitor may get discharged through your body resulting in an electric shock. Such a shock could be fatal for high value capacitors when fully charged.

16.5 DIELECTRICS AND DIELECTRIC POLARIZATION

We know that dielectrics are insulating materials, which transmit electric effects without conducting. Dielectrics are of two types : **non-polar** and **polar**. We now learn about these.

(a) Non-polar dielectrics

In the molecules of non–polar dielectrics, the centre of positive charge coincides with the centre of negative charge. Each molecule has zero dipole moment in its normal state. These molecules are mostly symmetrical such as nitrogen, oxygen, benzene, methane, CO_2, etc.

(b) Polar dielectrics

Polar dielectrics have asymmetric shape of the molecules such as water, NH_3, HCl etc. In such molecules, the centres of positive and negative charges are separated through a definite distance and have finite permanent dipole moment.

When a non-polar dielectric is held in an external electric field, the centre of positive charge in each molecule is pushed in the direction of **E** and the centre of negative charge is displaced in the direction opposite to **E**. Because of external electric field, centres of positive and negative charges in the non-polar dielectric molecules are separated. Dielectric is then said to be polarized and a tiny dipole moment develops in each molecule. In fact, the force due to external electric field pulling the charge centres apart balances the force of mutual attraction between the centres (i.e. equilibrium is set) and the molecule is said to be polarised. Induced dipole moment **p** acquired by the molecule may be written as

$$\mathbf{p} = \alpha\,\varepsilon_0\,\mathbf{E}$$

where α is constant of proportionality and is called atomic/molecular polarizability.

Let us now consider a non-polar slab *ABCD* placed in an electric field **E** maintained between the plates of a capacitor. As shown in Fig.16.13, the dielectric slab gets polarised. The nuclei of dielectric molecules are displaced towards the negative plate and electrons towards the positive plate. Because of polarisation, an electric field $\mathbf{E_p}$ is produced within the dielectric, which is opposite to **E**. Hence, due to the presence of a non-polar dielectric, the field between the plates is reduced, i.e. effective electric field in a polarised dielectric is given by

$$\mathbf{E}(\text{effective}) = \mathbf{E} - \mathbf{E_p} \qquad (16.39)$$

Thus, the potential difference between the capacitor plates is correspondingly reduced (as $V = Ed$), increasing the value of capacitance of the capacitor (as $C = q/V$).

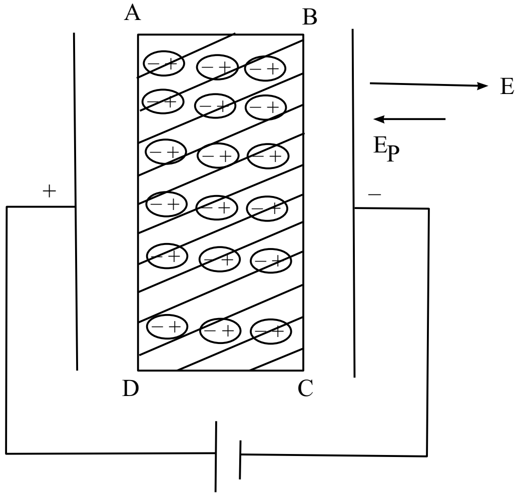


Fig.16.13 : A dielectric slab between the charged capacitor plates.

Applications of Electrostatics

Electrostatics provides basis for the theory of electromagnetics, apart from useful assistance in many fields of science and technology.

- Capacitors are essential parts of most electronic and electrical circuitry. These play a very crucial role in power transmission.
- Gold leaf electroscope – the simple device used for detecting charge, paved the way for cosmic ray research.
- Lightning conductor devised by Benjamin Franklin is still used to protect sky-scrappers from the strokes of lightning and thunder.
- The working of photocopiers, so common these days, is based on the principle of electrostatics.

INTEXT QUESTIONS 16.3

1. Two capacitors C_1= 12 mF and C_2 = 4 mF are in group connections. Calculate the effective capacitance of the system when they are connected (a) in series (b) in parallel.

2. Four capacitors are connected together as shown in Fig.16.14. Calculate the equivalent capacitance of the system.

3. An air capacitor C = 8 mF is connected to a 12V battery. Calculate

   (a) the value of Q when it is fully charged?

   (b) the charge on the plates, when slab of dielectric constant K = 5 fills the gap between the plates completely.

   (c) potential difference between the plates; and

   (d) capacitance of the new capacitor

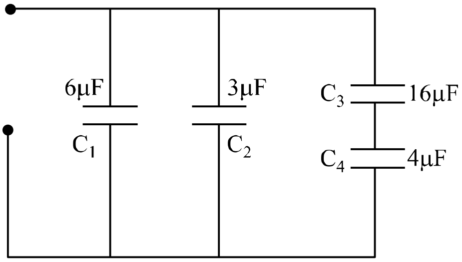


Fig.16.14 : Grouping of capacitors

4. A parallel plate capacitor of capacitance C_0 is connected to a battery and charged to a potential difference V_0. After disconnecting the battery, the gap between the plates is completely filled with a slab of dielectric constant K. How much energy is stored in the capacitor (a) in the first state? (b) in the second state? and (c) which one is larger and why?

WHAT YOU HAVE LEARNT

- The potential at any point in an electric field is equal to the work done against the electric field in moving a unit charge from infinity to that point.
- Work done in transferring a charge from one point to another in an electrostatic field is path independent.
- If one joule of work is done in bringing a test charge of one coulomb from infinity to a point in the field, we say that potential at that point is one volt.
- Electric potential due to a dipole is zero at every point on the equatorial line of the dipole.
- In an equipotential surface, every point has same electric potential.
- At any point in an electric field, the negative rate of change of potential with distance (called potential gradient) gives the field.
- Electrostatic shielding is the phenomenon of protecting a region of space from electric field.

- Capacitance of a conductor depends on its shape, size and nature of medium, rather than its material.
- The capacitance of a dielectric filled parallel plate capacitor becomes K times the capacitance with air or vacuum as dielectric.
- Relative permittivity is the ratio of capacitance with dielectric between the plates to the capacitance with air or vacuum between the plates.
- In series combination of capacitors, the equivalent capacitance is less than the least of any of the individual capacitances.
- In parallel combination of capacitors, the equivalent capacitance is equal to the sum of individual capacitances.
- Due to the presence of a non-polar dielectric, the field between the plates of a capacitor is reduced.

ANSWERS TO INTEXT QUESTIONS

16.1

1. The potential at r ($r > R$)

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

2. The field around a point charge possesses spherical symmetry. Thus every point on the surface of the sphere is equipotential. And no work is done when a charge moves on an equipotantial surface

3. $E = -\frac{dV}{dr}$ Since V is constant, E is zero.

   We can obtain the same result using Eqn. (16.22) :

   $E = \frac{V_A - V_B}{d}$. Since $V_A = V_B, E$ is zero

4. No. Not necessarily. When $E = 0$, the potential is either constant or zero.

5. Two equipotential surfaces never intersect. If they do so, at the point of intersection we can draw two normals giving directions of electric field.

16.2

1. $$C = \frac{Q}{V} = \frac{\dfrac{Q}{\text{Work done}}}{\text{Charge}} = \frac{Q \times Q}{\text{Work done}}$$

   $$= \frac{Q^2}{\text{N.m.}}$$

   The basic unit is

   $$A = \frac{C}{s}$$

$\therefore$ $C^2 = A^2\,s^2$ and newton = mass $\times$ acc = kg $\frac{m}{s^2}$

$$\text{Capacitance} = \frac{C^2}{Nm} = \frac{A^2s^2}{kg\,\dfrac{m^2}{s^2}} = \frac{A^2s^4}{kg\,m^2}$$

$$= A^2\,s^4\,(kg\,m^2)^{-1}$$

2. In a capacitor, E is uniform between this plates. Potential difference between the plates

   $V_A - V_B = E \times d.$

3. C_0, E_0, V_0 for air capacitor and
C, E, V for dielectric capacitor. Then

$$k = \frac{C}{C_0},\ k = \frac{V_0}{V}\ \ k = \frac{E_0}{E}.$$

4. $C = 1.0\ \mu\text{ F} = 1.0 \times 10^{-6}$ F.
$d = 50$ cm $= 0.5$m.

$$C = \frac{\varepsilon_0 A}{d}$$

$\therefore$ $A = \dfrac{Cd}{\varepsilon_0}$. Since $\varepsilon_0 = 8.85 \times 10^{-12}$,

$$A = \frac{1.0 \times 10^{-6} \times 0.5}{8.85 \times 10^{-12}}$$

$$= \frac{5 \times 10^{-7}}{8.85 \times 10^{-12}}$$

$$= 0.56 \times 10^{5}\ \text{m}^2$$

16.3

1 (a) 3 mF (b) 16 mF 2. 12.2μF

3. (a) 96mC (b) 0.480 C (c) 12 v (d) 40 mF

4. (a) $\frac{1}{2}\ C_0 V_0^2$ (b) $\frac{1}{2}\frac{(C_0V_0)^2}{C_0R} = \frac{1}{2\text{k}}C_0V_0^2$

(c) The energy in the first case is more, because same energy is used up for sucking in the dielectric slab.