HOTS (Higher Order Thinking Skills)

Que 1. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys can finish it in 14 days. Find the time taken by one man alone and that by one boy alone to finish the work.

Sol. Let one men alone can finish the work in x days and one boy alone can finish the work in y days. Then,

One day work of one man = $\frac{1}{x}$, One day work of one boy = $\frac{1}{y}$

$\therefore$ One day work of 8 men = $\frac{8}{x}$, One day work of 12 boys = $\frac{12}{y}$,

Since 8 men and 12 boys can finish the work in 10 days

$$10\left(\frac{8}{x}+\frac{12}{y}\right)=1 \quad \Rightarrow \quad \frac{80}{x}+\frac{120}{y}=1 \qquad \ldots.(i)$$

Again, 6 men and 8 boys can finish the work in 14 days

$$\therefore \quad 14\left(\frac{6}{x}+\frac{8}{y}\right)=1 \quad \Rightarrow \quad \frac{84}{x}+\frac{112}{y}=1 \qquad \ldots..(ii)$$

Put $\frac{1}{x}=u$ and $\frac{1}{y}=v$ in equation (i) and (ii), we get

$$80u + 120v - 1 = 0 \quad \text{and} \quad 84u + 112v - 1 = 0$$

By using cross-multiplication, we have

$$\frac{u}{120\times-1-112\times-1}=\frac{-v}{80\times-1-84\times-1}=\frac{1}{80\times112-84\times120}$$

$$\Rightarrow \quad \frac{u}{-120+112}=\frac{-v}{-80+84}=\frac{1}{80\times112-84\times120}$$

$$\Rightarrow \quad \frac{u}{-8}=\frac{-v}{4}=\frac{1}{-1120}$$

$$\Rightarrow \quad u=\frac{-8}{-1120}=\frac{1}{140} \quad \text{and} \quad v=\frac{-4}{-1120}=\frac{1}{280}$$

We have, $u=\frac{1}{140} \quad \Rightarrow \quad \frac{1}{x}=\frac{1}{140} \quad \Rightarrow \quad x=140$

and $v=\frac{1}{280} \quad \Rightarrow \quad \frac{1}{y}=\frac{1}{280} \quad \Rightarrow \quad y=280.$

Hence, one man alone can finish the work in 140 days and one boy alone can finish the work in 280 days.

Que 2. A boat covers 25 km upstream and 44 km downstream in 9 hours. Also, it covers 15 km upstream and 22 km downstream in 5 hours. Find the speed of the boat in still water and that of the stream.

Sol. Let the speed of the boat in still water be x km/h and that of the stream be y km/h.
Then,
Speed upstream = (x – y) km/h
Speed downstream = (x + y) km/h

Now, time taken to cover 25 km upstream = $\frac{25}{x-y} hours$

Time taken to cover 44 km downstream = $\frac{44}{x+y} hours$

The total time of journey is 9 hours

$\therefore \quad \frac{25}{x-y} + \frac{44}{x+y} = 9 \quad \ldots(i)$

Time taken to cover 15 km upstream = $\frac{15}{x-y}$

Time taken to cover 22 km downstream = $\frac{22}{x+y}$

In this case, total time of journey is 5 hours.

$\therefore \quad \frac{15}{x-y} + \frac{22}{x+y} = 5 \quad \ldots.(ii)$

Put $\frac{1}{x-y} = u$ and $\frac{1}{x+y} = v$ in equations (i) and (ii), we get

$25u + 44v = 9 \quad \Rightarrow \quad 25u + 44v - 9 = 0 \quad \ldots.(iii)$
$15u + 22v = 5 \quad \Rightarrow \quad 15u + 22v - 5 = 0 \quad \ldots.(iv)$

By cross-multiplication, we have

$$\frac{u}{44\times(-5)-22\times(-9)} = \frac{-v}{25\times(-5)-15\times(-9)} = \frac{1}{25\times22-15\times44}$$

$$\Rightarrow \quad \frac{u}{-220+198} = \frac{-v}{-125+135} = \frac{1}{550-660}$$

$$\Rightarrow \quad \frac{u}{-22} = \frac{-v}{10} = \frac{1}{-110} \quad \Rightarrow \quad \frac{u}{22} = \frac{v}{10} = \frac{1}{110}$$

$$\Rightarrow \quad \frac{u}{22} = \frac{1}{110} \quad \text{and} \quad \frac{v}{10} = \frac{1}{110}$$

$$\Rightarrow \quad u = \frac{22}{110} = \frac{1}{5} \quad \text{and} \quad v = \frac{1}{11}$$

We have, $u = \frac{1}{5} \quad \Rightarrow \quad \frac{1}{x-y} = \frac{1}{5} \quad \Rightarrow \quad x - y = 5 \quad \ldots.(v)$

and $v = \frac{1}{11} \quad \Rightarrow \quad \frac{1}{x+y} = \frac{1}{11} \quad \Rightarrow \quad x + y = 11 \quad \ldots.(vi)$

Solving equations (v) and (vi), we get x = 8 and y = 3.
Hence, speed of the boat in still water is 8 km/h and speed of the stream is 3 km/h.

Que 3. Students of a class are made to stand in rows. If one student is extra in a row, there would be 2 rows less. If one student is less in a row, there would be 3 rows more. Find the number of students in the class.

Sol. Let total number of roes be y

and total number of students in each row be x.

∴ Total number of students = xy

Case I: If one student is extra in a row, there would be two rows less.

Now, number of rows = (y – 2)

Number of students in each row = (x + 1)

Total number of students = Number of rows × Number of students in each row

$xy = (y - 2)(x + 1) \quad \Rightarrow \quad xy = xy + y - 2x - 2$

$\Rightarrow \quad xy - xy - y + 2x = -2 \quad \Rightarrow \quad 2x - y = -2 \qquad \ldots(i)$

Case II: If one student is less in a row, there would be 3 rows more.

Now, number of rows = (y + 3)

and number of students in each row = (x – 1)

Total number of students = Number of rows × Number of students in each row

$\therefore \quad xy = (y + 3)(x - 1) \quad \Rightarrow \quad xy = xy - y + 3x - 3$

$xy - xy + y - 3x = -3 \quad \Rightarrow \quad -3x + y = -3 \qquad \ldots.(ii)$

On adding equation (i) and (ii), we have

$$\begin{array}{r} 2x - y = -2 \\ -3x + y = -3 \\ \hline -x = -5 \end{array}$$

Or $x = 5$

putting the value of x in equation (i), we get

$2(5) - y = -2 \quad \Rightarrow \quad 10 - y = -2$

$-y = -2 - 10 \quad \Rightarrow \quad -y = -12$

or $y = 12$

∴ Total number of students in the class = 5 × 12 = 60.

Que 4. Draw the graph of 2x + y = 6 and 2x – y + 2 = 0. Shade the region bounded by these lines and x-axis. Find the area of the shaded region.

Sol. We have, $2x + y = 6 \quad \Rightarrow \quad y = 6 - 2x$

when x = 0, we have $y = 6 - 2 \times 0 = 6$

when x = 3, we have $y = 6 - 2 \times 3 = 0$

when x = 2, we have $y = 6 - 2 \times 2 = 2$

Thus, we get the following table:

x	0	3	2
y	6	0	2

Now, we plot the points A (0, 6), B (3, 0) and C (2, 2) on the graph paper. We join A, B

and C and extend it on both sides to obtain the graph of the equation 2x + y = 6.

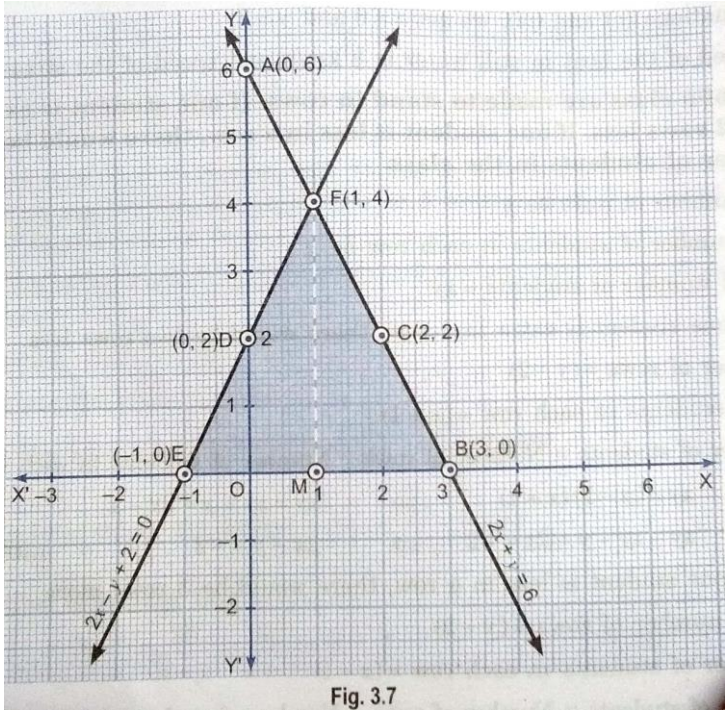

Fig. 3.7

We have, $2x - y + 2 = 0 \Rightarrow y = 2x + 2$

When x = 0, we have $y = 2 \times 0 + 2 = 2$

When x = – 1, we have $y = 2 \times (-1) + 2 = 0$

When x = 1, we have $y = 2 \times 1 + 2 = 4$

Thus, we have the following table:

x	0	-1	1
y	2	0	4

Now, we plot the points D (0, 2), E (– 1, 0) and F (1, 4) on the same graph paper. We join D, E and F and extend it on both sides to obtain the graph of the equation 2x – y + 2 = 0. It is evident from the graph that the two lines intersect at point F (1, 4). The area enclosed by the given lines and x-axis is shown in Fig. 3.7.

Thus, x = 1, y = 4 is the solution of the given system of equations. Draw FM perpendicular from F on x-axis.

Clearly, we have

FM = y-coordinates of point F (1, 4) = 4 and BE = 4

$\therefore$ Area of the shaded region = Area of ΔFBE

$\Rightarrow$ Area of the shaded region = $\frac{1}{2}(Base \times Height) = \frac{1}{2}(BE \times FM)$

$$= \left(\frac{1}{2} \times 4 \times 4\right) sq.units = 8\, sq.units.$$

Que 5. The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differ by 30 years. Find the ages of Ani and Biju.

Sol. Let the ages of Ani and Biju be x and y years respectively. Then $x - y = \pm 3$

Age of Dharam = 2x years

Age of Cathy = $\frac{y}{2} years$

Clearly, Dharam is older than Cathy.

$\therefore \quad 2x - \frac{y}{20} = 30 \quad \Rightarrow \quad \frac{4x-y}{2} = 30 \quad \Rightarrow \quad 4x - y = 60$

Thus, we have following two systems of linear equations

x – y = 3 ….(i)

4x – y = 60 ….(ii)

and x – y = – 3 ….(iii)

4x – y = 60 ….(iv)

Subtracting equation (i) from (ii), we get

$$\begin{array}{r} 4x - y = 60 \\ -x + y = -3 \\ \hline 3x = 57 \end{array} \quad \Rightarrow \quad x = 19$$

Putting $x = 19$ in equation (i), we get

$$19 - y = 3 \quad \Rightarrow \quad y = 16$$

Now, subtracting equation (iii) from (iv)

$$\begin{array}{r} 4x - y = 60 \\ -x \mp y = \mp 3 \\ \hline 3x = 63 \end{array} \quad \Rightarrow \quad x = 21$$

Putting x = 21 in equation (ii), we get

21 – y = – 3 y = 24

Hence, age of Ani = 19 years and age of Biju = 16 years or age of Ani = 21 years and age of Biju = 24 years