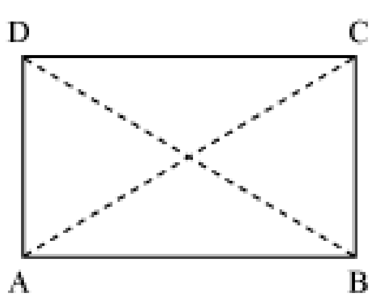
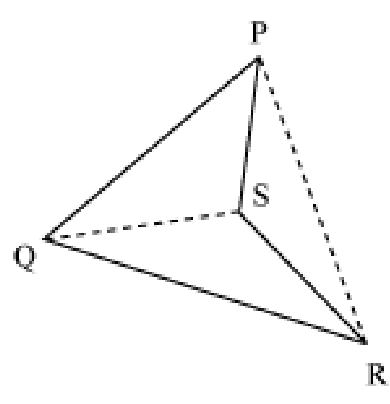
Rectilinear Figures

- **Polygons**
  - A simple closed curve made up of line segments only is called a **polygon.**
  - **Polygons can be classified according to their number of sides (or vertices).**

Number of side/vertices	**Classification**
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
.	.
.	.
n	*n* – gon

- The line segment connecting two non-consecutive vertices of a polygon are called **diagonals.**





For polygon ABCD, AC and BD are diagonals and for polygon PQRS, QS and PR are diagonals.

- The polygon, none of whose diagonals lie in its exterior, is called a **convex polygon.** In the given figure, ABCD is a convex polygon.

  The polygon whose atleast one of the diagonals lie in its exterior is called a **concave polygon.** PQRS is a concave polygon.

- The sum of all the interior angles of an n-sided polygon is given by, $(n - 2) \times 180°$.

  Example: What is the number of sides of a polygon whose sum of all interior angles is 720°?

  Solution: It is known that,

  $$(n-2)180° = 720°$$

  $$\Rightarrow (n-2) = \frac{720°}{180°} = 4$$

  $$\Rightarrow n = 6$$

  Thus, the required polygon is six-sided.

- A polygon, which is both equiangular and equilateral, is called a **regular polygon.** Otherwise, it is an **irregular polygon.**

  Example: Square is a regular polygon but rectangle is an irregular polygon.

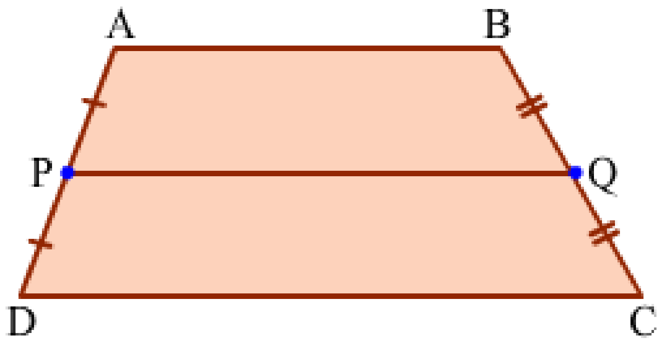
Properties of trapeziums:

1. If the mid-points of non-parallel sides of a trapezium are joined together, then the obtained line segment is:

(i) parallel to the parallel sides

(ii) half the sum of lengths of the parallel sides

In the given figure, ABCD is a trapezium with P and Q as the mid-points of sides AD and BC respectively.



Thus, we have

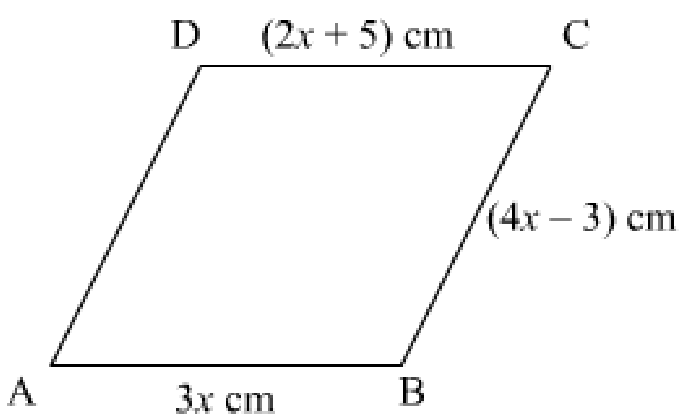
1. PQ || AB || DC

2. $PQ = \frac{1}{2}(AB + CD)$

Diagonals of an **isosceles trapezium** are congruent (equal).

- Opposite sides in a parallelogram are equal. Conversely, in a quadrilateral, if each pair of opposite sides are equal then the quadrilateral is a parallelogram.

  Example:
  In the following figure, ABCD is a parallelogram. Find the length of each sides.

  Solution:

  We know, the opposite sides of a parallelogram are equal in length.

  Therefore, AB = CD

  $3x = 2x + 5$

$\Rightarrow 3x - 2x = 5$

$\therefore x = 5$

Thus, AB = $3x = 3 \times 5 = 15$ cm

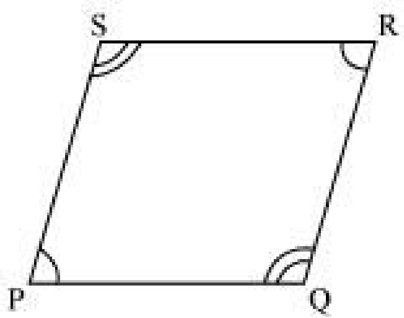
BC = $4x - 3 = 4 \times 5 - 3 = 17$ cm

CD = $2x + 5 = 2 \times 5 + 5 = 15$ cm

Also, BC = AD [opposite sides of parallelogram]

$\therefore$ AD = 17 cm

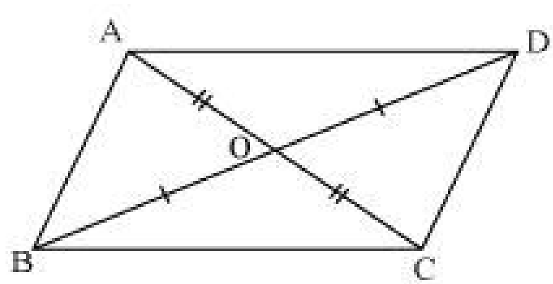
- In a parallelogram, opposite angles are equal. Conversely in a quadrilateral, if pair of opposite angles is equal, then the quadrilateral is a parallelogram.



If in the quadrilateral PQRS, $\angle P = \angle R$ and $\angle Q = \angle S$ as shown in the above figure, then the quadrilateral is a parallelogram.

- The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

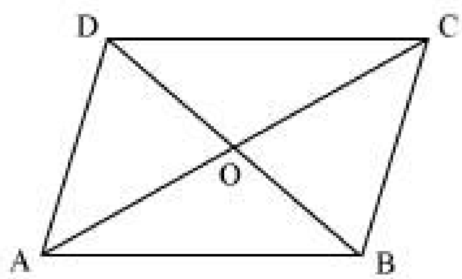
  Suppose ABCD is a quadrilateral. The diagonals of the quadrilateral intersect at O such that AO = OC and DO = OB



Therefore, ABCD is a parallelogram.

Example:

In the given figure, ABCD is a parallelogram. If OD = $(3x - 2)$ cm and OB = $(2x + 3)$ cm, then find x and length of diagonal BD.



Solution:

We know that the diagonals of a parallelogram bisect each other.

$\therefore$ OD = OB

$\Rightarrow 3x - 2 = 2x + 3$

$\Rightarrow 3x - 2x = 3 + 2$

$\Rightarrow x = 5$

Thus, the value of x is 5.

Length of BD = OD + OB

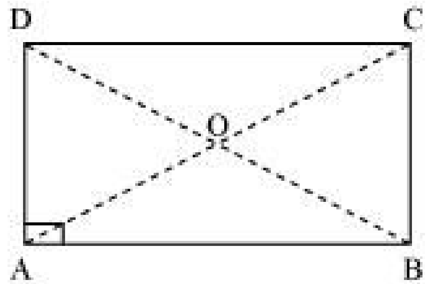
$= (3x - 2) + (2x + 3)$

$= (3\times5 - 2) + (2\times5 + 3)$

$= 13 + 13$

$= 26$ cm

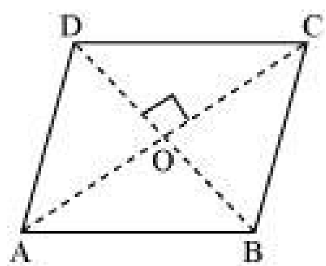
- **Rectangle:** A parallelogram whose each interior angle is a right angle.
  - Its diagonals are equal and bisect each other.



In rectangle ABCD, AC = BD. Also, OA = OC and OB = OD

- A parallelogram is a rectangle if its diagonals are equal.

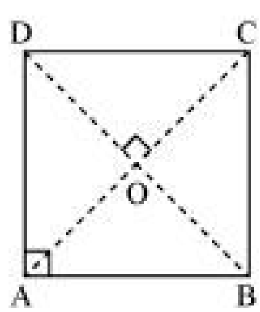
  - **Rhombus:** A quadrilateral whose opposite sides are parallel and all sides are of equal lengths.
    - Its opposite angles are of equal measure.
    - Its diagonals are perpendicular bisectors of one another.



In rhombus ABCD, OA = OC and OB = OD. Also, AC ⊥ BD.

- A quadrilateral is a rhombus if its diagonals bisect each other at right angles.

  - **Square:** A square is a rectangle with equal sides.
    - Its diagonals are equal and are perpendicular bisectors of each other.



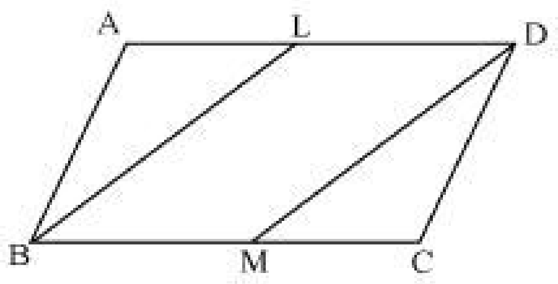
In square ABCD, AC = BD and AC ⊥ BD. Also, OA = OC and OB = OD.

- A quadrilateral is a square, if its diagonals are equal and bisect each other at right angles.

- A quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel.

Example:

In the given figure, ABCD is a parallelogram and L and M are the mid-points of AD and BC respectively. Prove that BMDL is a parallelogram.



Solution:

As L and M are the mid-points of AD and BC respectively.

Therefore, $BM = \frac{1}{2}BC \text{ and } LD = \frac{1}{2}AD$ … (1)

As BC = AD (Since ABCD is a parallelogram)

$\Rightarrow \frac{1}{2}BC = \frac{1}{2}AD$

$\Rightarrow$ BM = LD … (2) (From (1))

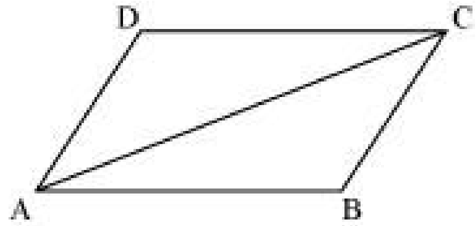
Also, BC || AD

$\Rightarrow$ BM || LD

Hence, BMDL is a parallelogram.

- Diagonal of a parallelogram divides it into two congruent triangles.

In the given figure, if ABCD is a parallelogram and AC is its diagonal then ΔABC $\cong$ ΔCDA.

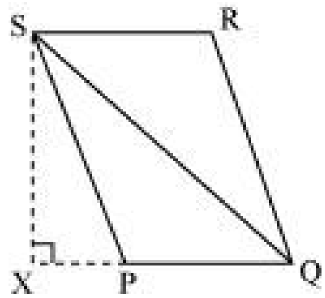


Example:

The area of the parallelogram PQRS is 120 cm^2. Find the distance between the parallel sides PQ and SR, if the length of the side PQ is 10 cm.

Solution:

Let us draw a diagonal SQ of parallelogram PQRS and a perpendicular SX on the extended line PQ as shown in the figure.



We know that a diagonal of a parallelogram divides it into two congruent triangles. Also, congruent figures are equal in area.

$\therefore$ area (ΔPQS) = area (ΔQRS)

Area of parallelogram PQRS = area (ΔPQS) + area (ΔQRS)

= 2 × area (ΔPQS)

$\Rightarrow$area (ΔPQS) = $\frac{1}{2}$(area of parallelogram PQRS) = $\frac{120}{2}$ cm^2 = 60 cm^2

Also, area (ΔPQS) = $\frac{1}{2}$ (PQ)(SX) = 60 cm^2

$\Rightarrow$ (PQ) (SX) = 120 cm^2

$\Rightarrow$ SX = $\frac{120}{10}$ cm^2

$\Rightarrow$ SX = 12 cm

Thus, the distance between the parallel sides PQ and SR is 12 cm.