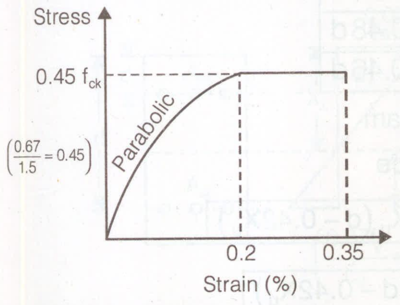
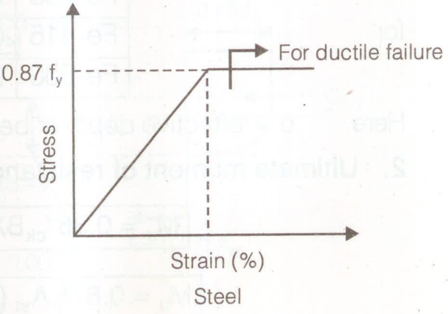
3. LIMIT STATE METHOD

DESIGN STRESS STRAIN CURVE AT ULTIMATE STATE





- **Design value of strength**

  For concrete

$$\boxed{f_d = \frac{f}{\gamma_m}} \rightarrow \boxed{f_d = \frac{0.67 f_{ck}}{1.5}} \rightarrow \boxed{f_d = 0.45 f_{ck}}$$

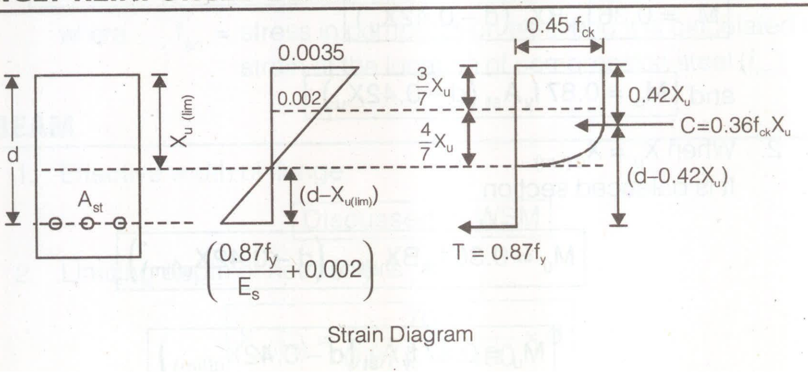
Here, γ_{mc} = Partial factor of safety for concrete = 1.5

f_d = design value of strength

For steel

$$\boxed{f_d = \frac{f_y}{1.15}} \rightarrow \boxed{f_d = 0.87 f_y}$$

SINGLY REINFORCED BEAM



Strain Diagram

1. **Limiting depth of neutral axis ($x_{u\,(lim)}$)**

$$X_{u(lim)} = \frac{700}{0.87 f_y + 1100} \times d$$

for

	$X_{u(lim)}$
Fe-250	0.53 d
Fe-415	0.48 d
Fe-500	0.46 d

Here d = effective depth of beam

2. **Ultimate moment of resistance**

$$M_u = 0.36 f_{ck} B X_u (d - 0.42 X_u)$$

$$M_u = 0.87 f_y A_{st} (d - 0.42 X_u)$$

3. **Actual depth of neutral axis (X_u)**

$$C = T \Rightarrow X_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} B}$$

Remember: In LSM actual depth of NA is found by equating total compressive and tensile force.

4. **Lever arm = d – 0.42 X_u**

- **Some special cases**

1. When $X_u < X_{u(lim)}$
   It is an under-reinforced section

$$M_u = 0.36 f_{ck} B X_u (d - 0.42 X_u)$$

and $$M_u = 0.87 f_y A_{st} (d - 0.42 X_u)$$

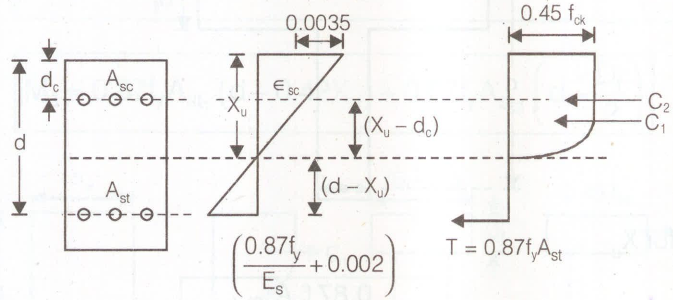
2. When $X_u = X_{u\,(lim)}$
   It is balanced section

$$M_u = 0.36 f_{ck} B X_{u(lim)} \left(d - 0.42 X_{u(lim)}\right)$$

$$M_u = 0.87 f_y A_{st} \left(d - 0.42 X_{u(lim)}\right)$$

3. When $X_u > X_{u(lim)}$
   It is over reinforced section. In this case keep X_u limited to $X_{u\,(lim)}$ and moment of resistance of the section shall be limited to limiting moment of resistance. ($M_{u\,(lim)}$).

DOUBLY REINFORCED SECTION



1. Limiting depth of neutral axis

$$X_{u(lim)} = \frac{700}{0.87 f_y + 1100} \times d$$

2. For actual depth of neutral axis (X_u)

$$C = T \Rightarrow C_1 + C_2 = T$$

$$\downarrow$$

$$0.36 f_{ck} B X_u + (f_{sc} - 0.45 f_{ck}) A_{sc} = 0.87 f_y A_{st}$$

3. Ultimate moment of resistance

$$M_u = 0.36 f_{ck} B X_u (d - 0.42 X_u) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d_c)$$

where f_{sc} = stress in compression steel and it is calculated by strain at the location of compression steel (f_{sc})

T-BEAM

1. Effective width of flange

Discussed in WSM

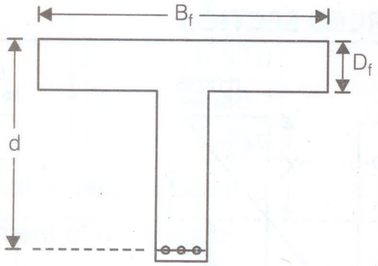
2. Limiting depth of neutral axis

$$X_{u(lim)} = \frac{700}{0.87 f_y + 1100} \times d$$

- **Singly reinforced T-Beam**

Case-1: When NA is in flange area

i.e., $X_u < D_f$



(a) for X_u

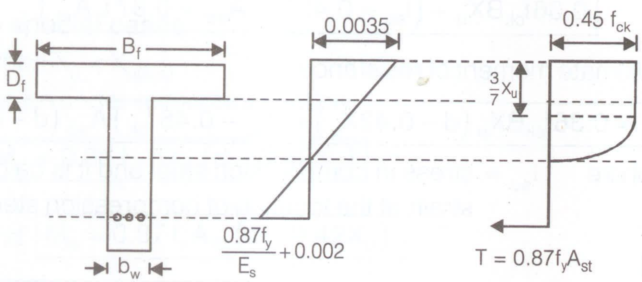
$$X_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} B_f} < D_f$$

(b) Ultimate moment of resistance

$$M_u = 0.36 f_{ck} B_f X_u (d - 0.42 X_u)$$

$$M_u = 0.87 f_y A_{st} (d - 0.42 X_u)$$

Case-2: When NA is in web area ($X_u > D_f$)



Case (a) when $X_u > D_f$

and $D_f < \frac{3}{7} X_u$

i.e., depth of flange is less than the depth of rectangular portion of stress diagram.

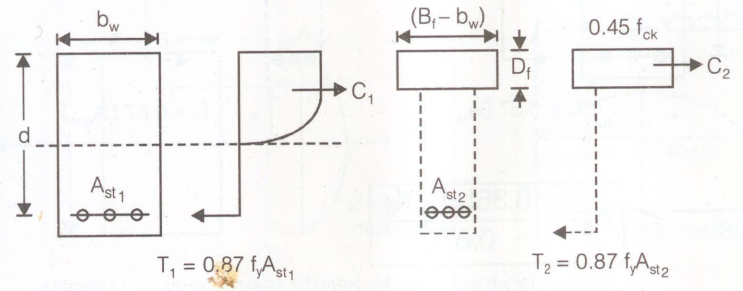
1. For actual depth of neutral ais

$$0.36 f_{ck} b_w X_u + 0.45 f_{ck} (B_f - b_w) D_f = 0.87 f_y Ast$$

2. Ultimate moment of resistance

$$M_u = 0.36 f_{ck} b_w X_u (d - 0.42 X_u) + 0.45 f_{ck} (B_f - b_w) D_f \left(d - \frac{D_f}{2}\right)$$

$$M_u = 0.87 f_y A_{st_1} (d - 0.42 X_u) + 0.87 f_y A_{st_2} \left(d - \frac{D_f}{2}\right)$$

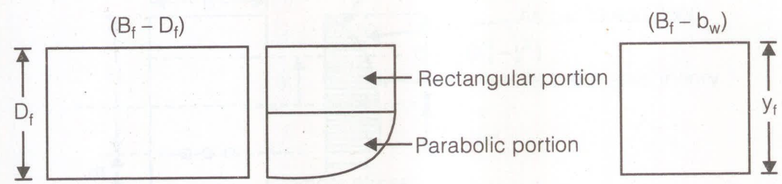


$$A_{st_1} = \frac{0.36 f_{ck} b_w X_u}{0.87 f_y}, \quad A_{st_2} = \frac{0.45 f_{ck} (B_f - b_w) D_f}{0.87 f_y}$$

- **Special Case (2):** When $X_u > D_f$

and $D_f > \frac{3}{7} X_u$

i.e., depth of flange is more than depth of rectangular portion of stress diagram.



As per IS: 456-2000

$(B_f - b_w)D_f$ portion of flange is converted into $(B_f - b_w)y_f$ section for which stress is taken constant throughout the section is 0.45 f_{ck}.

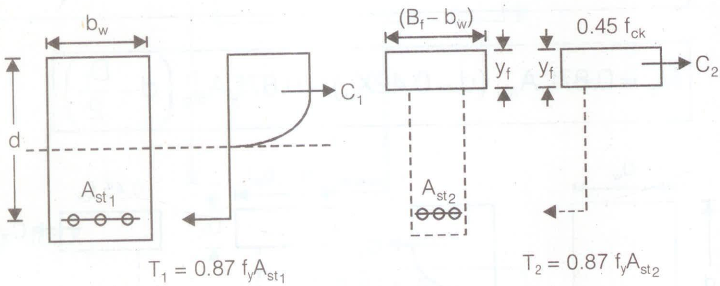
As per IS : 456-2000

$$y_f = 0.15 X_u + 0.65 D_f < D_f$$

1. For actual depth of neutral axis

$$0.36f_{ck}b_wX_u + 0.45f_{ck}(B_f - b_w)y_f = 0.87f_yA_{st_1} + 0.87f_yA_{st_2}$$

or $$0.36f_{ck}b_wX_u + 0.45f_{ck}(B_f - b_w)y_f = 0.87f_yA_{st}$$



Also $$A_{st_1} = \frac{0.36f_{ck}b_wX_u}{0.87f_y}$$

and $$A_{st_2} = \frac{0.45f_{ck}(B_f - b_w)y_f}{0.87f_y}$$

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