# CUET (UG)

## Mathematics Sample Paper - 07

## Solved

**Time Allowed: 50 minutes** **Maximum Marks: 200**

**General Instructions:**

1. There are 50 questions in this paper.
2. Section A has 15 questions. Attempt all of them.
3. Attempt any 25 questions out of 35 from section B.
4. Marking Scheme of the test:

a. Correct answer or the most appropriate answer: Five marks (+5).
b. Any incorrectly marked option will be given minus one mark (-1).
c. Unanswered/Marked for Review will be given zero mark (0).

### Section A

1. If A and B are square matrices of the same order, then (A + B) (A – B) is equal to **[5]**

a) $A^2 - B^2$

b) $A^2 - B^2 + BA - AB$

c) $A^2 - BA + B^2 + AB$

d) $A^2 - BA - AB - B^2$

2. Let $A = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$, then **[5]**

a) $A^2 = 0$

b) $A^2 = I$

c) $A^2 = -A$

d) $A^2 = A$

3. For any two matrices A and B, **[5]**

a) AB = BA is always true

b) Whenever AB exists, then BA exists

c) Sometimes AB = BA and sometimes AB $\neq$ BA

d) AB = BA is never true

4. The maximum value of $\frac{\log x}{x}$ in $0 < x < \infty$ is **[5]**

a) 0

b) none of these

c) – e

d) $\frac{1}{e}$

5. $f(x) = (x + 1)^3 (x - 3)^3$ is increasing in **[5]**

a) $(1, \infty)$ b) (-1, 3)

c) $(-\infty, 1)$ d) $(3, \infty)$

6. If the function f(x) = $x^2$ - kx + 5 is increasing on [2, 4], then **[5]**

a) $k \in (-\infty, 2)$ b) $k \in (2, \infty)$

c) $k \in (4, \infty)$ d) $k \in (-\infty, 4)$

7. If $I_{10} = \int_0^{\pi/2} x^{10} \sin x dx$, then value of $I_{10} + 90I_8$ is **[5]**

a) $10\left(\frac{\pi}{2}\right)^9$ b) $\left(\frac{\pi}{2}\right)^9$

c) $9\left(\frac{\pi}{2}\right)^8$ d) $9\left(\frac{\pi}{2}\right)^9$

8. $\int \frac{3x^2}{\sqrt{9-16x^6}} dx = ?$ **[5]**

a) $4\sin^{-1}\left(\frac{x^3}{4}\right) + C$ b) $\frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right) + C$

c) None of these d) $\frac{1}{4}\sin^{-1}\left(\frac{x^3}{3}\right) + C$

9. $\int (\sin(\log x) + \cos(\log x))$ dx is equal to **[5]**

a) log ( sin x - cos x) + c b) x sin (log x) +C

c) sin (log x) – cos (log x) + C d) x cos (log x) + C

10. The area bounded by the curve y = f (x), x-axis, and the ordinates x = l and x - b is (b - 1) sin (3b + 4). Then, f (x) is **[5]**

a) sin (3x + 4) b) sin (3x + 4) + 3 (x -1 ) cos (3x + 4)

c) None of these d) (x -1 ) cos(3x + 4)

11. The solution of the DE cos x(1 + cos y)dx - sin y(1 + sin x)dy = 0 is **[5]**

a) none of these b) 1 + sin x cos y = C

c) sin x cos y + cos x = C d) (1 + sin x)(1 + cos y) = C

12. The solution of the differential equation $\left(1 + x^2\right) \frac{dy}{dx} + 1 + y^2 = 0$, is **[5]**

a) $\tan^{-1} y - \tan^{-1} x = \tan^{-1} C$

b) $\tan^{-1} y + \tan^{-1} x = \tan^{-1} C$

c) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} C$

d) $\tan^{-1} y \pm \tan^{-1} x = \tan^{-1} C$

13. Minimize Z = 5x + 10 y subject to $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$ **[5]**

a) Minimum Z = 310 at (60, 0)

b) Minimum Z = 320 at (60, 0)

c) Minimum Z = 330 at (60, 0)

d) Minimum Z = 300 at (60, 0)

14. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X. **[5]**

a) $\frac{1}{3}$

b) $\frac{1}{6}$

c) $\frac{1}{4}$

d) $\frac{1}{5}$

15. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing? **[5]**

a) $\frac{13}{243}$

b) $\frac{17}{243}$

c) $\frac{9}{243}$

d) $\frac{11}{243}$

## Section B

## Attempt any 25 questions

16. The domain of function f : R → R defined by $f(x) = \sqrt{x^2 - 3x + 2}$ is **[5]**

a) $[2, \infty]$

b) $(-\infty, 1] \cup [2, \infty)$

c) $(-\infty, 1]$

d) [1, 2]

17. If $4 \cos^{-1} x + \sin^{-1} x = \pi$, then the value of x is **[5]**

a) $\frac{2}{\sqrt{3}}$

b) $\frac{1}{\sqrt{2}}$

c) $\frac{\sqrt{3}}{2}$

d) $\frac{3}{2}$

18. The matrix $P = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a **[5]**

a) none

b) diagonal matrix

c) square matrix | d) unit matrix

19. If $A = \begin{vmatrix} -1 & 2 & 4 \\ 3 & 1 & 0 \\ -2 & 4 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} -2 & 4 & 2 \\ 6 & 2 & 0 \\ -2 & 4 & 8 \end{vmatrix}$, then B is given by **[5]**

a) B = 4A | b) B = –4A

c) B = 6A | d) B = – A

20. The system of linear equations x + y + z = 2, 2x + y - z = 3, 3x + 2y - kz = 4 has a unique solution if , **[5]**

a) k = 0 | b) -1 < k < 1

c) -2 < k < 2 | d) $k \neq 0$

21. $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = ?$ **[5]**

a) 0 | b) -1

c) None of these | d) 1

22. If $y = \sin^{-1} x$ and $z = \cos^{-1}\sqrt{1-x^2}$, then $\frac{dy}{dz}$ is equal to (0 < x < 1) **[5]**

a) $\cos^{-1} x$ | b) $\sqrt{1-x^2}$

c) 1 | d) $\frac{1}{\sqrt{1-x^2}}$

23. If $f(x) = \left(\frac{x^l}{x^m}\right)^{l+m} \left(\frac{x^m}{x^n}\right)^{m+n} \left(\frac{x^n}{x^l}\right)^{n+l}$, then f'(x) is equal to **[5]**

a) $x^{l+m+n}$ | b) 0

c) none of these | d) 1

24. If f is derivable at x = a , then $\underset{x \to a}{Lt} \frac{xf(a) - af(x)}{x-a}$ is equal to **[5]**

a) af‘(9a) – f(a) | b) $f(a) - a\, f'(a)$

c) f ‘(a) | d) None of these

25. If $f(x) = \sqrt{x^2 - 10x + 25}$, then the derivative of f(x) in the interval [0, 7] is **[5]**

a) -1

b) 0

c) 1

d) none of these

26. If the function f(x) = $\begin{cases} \dfrac{1-\cos 4x}{8x^2}, x \neq 0 \\ k, x = 0 \end{cases}$ is continuous x = 0 then k = ? **[5]**

a) $\frac{-1}{2}$

b) $\frac{1}{2}$

c) 2

d) 1

27. The minimum value of x $\log_e$ x is equal to **[5]**

a) e

b) $\frac{1}{e}$

c) 2e

d) $-\frac{1}{e}$

28. If ax + $\frac{b}{x}$ $\geq$ c for all positive x where a, b, > 0, then **[5]**

a) ab < $\frac{c^2}{4}$

b) ab $\geq \frac{c^2}{4}$

c) ab $\geq \frac{c}{4}$

d) none of these

29. If f(x) = $\frac{1}{4x^2+2x+1}$, then its maximum value is **[5]**

a) 1

b) $\frac{4}{3}$

c) $\frac{3}{4}$

d) $\frac{2}{3}$

30. The equation of the normal to the curve y = sinx at (0, 0) is **[5]**

a) x – y = 0

b) x = 0

c) x + y = 0

d) y = 0

31. $\int_0^a \frac{\sqrt{x}}{(\sqrt{x}+\sqrt{a-x})} dx = ?$ **[5]**

a) $\frac{\sqrt{a}}{2}$

b) $\frac{a}{2}$

c) 2a

d) $\frac{2a}{3}$

32. $\int \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$ is equal to **[5]**

a) $-\cos\left(\frac{1}{x^2}\right)+C$

b) $\cos\left(\frac{1}{x^2}\right)+C$

c) $-\sin\left(\frac{1}{x}\right)+C$

d) $\cos\left(\frac{1}{x}\right)+C$

33. $\int\frac{(1+\tan x)}{(1-\tan x)}dx=?$ **[5]**

a) log |cos x – sin x| + C

b) log |cos x + sin x | + C

c) none of these

d) - log |cos x – sin x| + C

34. $\int\frac{x+\sin x}{1+\cos x}dx$ is equal to **[5]**

a) x.tan $\frac{x}{2}$ + C

b) x - tan $\frac{x}{2}$ + C

c) log|1 + cos x| + C

d) log |x + sin x| + C

35. The area bounded by the curves $y=\sqrt{5-x^2}$ and $y=\ |x-1|$ is **[5]**

a) $\frac{5\pi-5}{4}$ sq. units

b) $\frac{5\pi+2}{4}$ sq. units

c) $\frac{5\pi-2}{4}$ sq. units

d) $\frac{\pi-5}{4}$ sq. units

36. The solution of the DE $x\frac{dy}{dx}=\cot y$ is **[5]**

a) x sec y = C

b) x cos y = C

c) x tan y = C

d) none of these

37. The solution of the differential equation $y_1\ y_3=y_2^2$ is **[5]**

a) $2x=C_1e^{C_2y}+C_3$

b) x = $C_1e^{C_2y}+C_3$

c) y = $C_1e^{C_2x}+C_3$

d) None of these

38. Differential equation of the family of ellipses having foci on y-axis and centre at origin is **[5]**

a) $xyy'' + x(y')^2\ \ yy' = 0$

b) $xyy'' - x(y')^2 + yy' = 0$

c) $xy'' + x(y')^2\ \ yy' = 0$

d) $yy'' + x(y')^2\ \ yy' = 0$

39. If $\vec{a}+\vec{b}+\vec{c}=0$, then which of the following is/are correct? **[5]**

I. $\vec{a},\vec{b},\vec{c}$ are coplanar.

II. $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$.

Select the correct answer using the code given

a) Only II

b) Only I

c) Both I and II

d) Neither I nor II

40. Find $|\vec{a}|$ and $\left|\vec{b}\right|$, if $\left(\vec{a}+\vec{b}\right) \cdot \left(\vec{a}-\vec{b}\right) = 8$ and $|\vec{a}| = 8 \left|\vec{b}\right|$. **[5]**

a) $\frac{16\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{2}}{3\sqrt{7}}$

b) $\frac{19\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{5}}{3\sqrt{7}}$

c) $\frac{17\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{3}}{3\sqrt{7}}$

d) $\frac{21\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{6}}{3\sqrt{7}}$

41. If the vertices A, B, C of a triangle ABC are (1, 2, 3), (–1, 0, 0), (0, 1, 2), respectively, then find $\angle ABC$. [$\angle ABC$ is the angle between the vectors $\overrightarrow{BA}$ *and* $\overrightarrow{BC}$ ] **[5]**

a) $\cos^{-1}\left(\frac{13}{\sqrt{102}}\right)$

b) $\cos^{-1}\left(\frac{11}{\sqrt{102}}\right)$

c) $\cos^{-1}\left(\frac{15}{\sqrt{102}}\right)$

d) $\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$

42. The magnitude of the vector $6\hat{i} + 2\hat{j} + 3\hat{k}$ is **[5]**

a) 7

b) 5

c) 12

d) 1

43. If $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} = 0$, then **[5]**

a) $\vec{b} + \vec{c} = \overrightarrow{0}$

b) none of these

c) $\vec{b} = \vec{c}$

d) $\vec{b} = \overrightarrow{0}$

44. The angle between the lines $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{2}$ and $\frac{x-1}{-\sqrt{3}-1} = \frac{y-1}{\sqrt{3}-1} = \frac{z-1}{4}$ is **[5]**

a) $\frac{\pi}{3}$

b) $\frac{\pi}{4}$

c) $\frac{\pi}{6}$

d) $\cos^{-1}\left(\frac{1}{65}\right)$

45. The direction ratios of two lines are a, b, c and (b - c), (c - a), (a - b) respectively. The angle between these lines is **[5]**

a) $\frac{\pi}{2}$

b) $\frac{\pi}{4}$

c) $\frac{\pi}{3}$

d) $\frac{3\pi}{4}$

46. The projections of a line segment on X, Y and Z axes are 12, 4 and 3 respectively. The length and direction cosines of the line segment are **[5]**

a) $11; \frac{12}{11}, \frac{14}{11}, \frac{3}{11}$

b) $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

c) $19; \frac{12}{19}, \frac{4}{19}, \frac{3}{19}$

d) none of these

47. If $E_1$, $E_2$,.....,$E_n$ are mutually exclusive and exhaustive events associated with a samplespace, and A is any event of non zero probability, then **[5]**

a) $P\left(E_i|A\right) = \frac{P(E_i)P(A|E_i)}{\sum_{i=1}^{n} P(E_i)P(A|E_i)}$

b) $P\left(E_i|A\right) = \frac{P(E_i)P(A|E_i)}{\sum_{i=1}^{n} P(E_{i-1})P(A|E_i)}$

c) $P\left(E_i|A\right) = \frac{P(E_i)P(E_i|A)}{\sum_{i=1}^{n} P(E_i)P(A|E_i)}$

d) $P\left(E_i|A\right) = \frac{P(E_i)P(A|E_i)}{\sum_{i=1}^{n} P(E_i)P(A|E_{i-2})}$

48. In 4 throws of a pair of dice, what is the probability of throwing doublets at least twice? **[5]**

a) $\frac{19}{144}$

b) $\frac{7}{36}$

c) None of these

d) $\frac{17}{144}$

49. An unbiased coin is tossed until the first head appears or until four tosses are completed, whichever happens, earlier. Which of the following statement(s) is/are correct? **[5]**

i. The probability that no head is observed is $\frac{1}{16}$.

ii. The probability that the experiment ends with three tosses is $\frac{1}{8}$.

Select the correct answer using the code given below.

a) Only i

b) Both i and ii

c) Neither i nor ii

d) Only ii

50. Two men hit at a target with probabilities $\frac{1}{2}$ and $\frac{1}{3}$, respectively. What is the probability that exactly one of them hits the target? **[5]**

a) $\frac{1}{6}$

b) $\frac{1}{2}$

c) $\frac{1}{3}$

d) $\frac{2}{3}$

# Solutions

## Section A

1.

**(b)** $A^2 - B^2 + BA - AB$

**Explanation:** $(A + B)(A - B) = A(A - B) + B(A - B) = A^2 - AB + BA - B^2$

2. **(a)** $A^2 = 0$

**Explanation:** $\because\ A = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$,

$$A^2 = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

$\therefore$ A is a null matrix.

3.

**(c)** Sometimes AB = BA and sometimes AB $\neq$ BA

**Explanation:** If the two matrices A and B are of same order it is not necessary that in every situation AB = BA

AB = BA = I only when $A = B^{-1}$

$B = A^{-1}$

Other time AB $\neq$ BA

4.

**(d)** $\frac{1}{e}$

**Explanation:** Consider $f(x) = \frac{logx}{x}$

Then, $f'(x) = = \frac{x \cdot \frac{1}{x} - logx.1}{x^2} = \frac{1 - logx}{x^2}$

For maximum or minimum values of x we have f'(x) = 0

$f'(x) = 0 \Rightarrow (1 - logx) = 0$

$\Rightarrow logx = 1 \Rightarrow x = e.$

Now, $f'(x) = \frac{x^2 \cdot \frac{-1}{x} - (1 - logx)\, 2x}{x^4} = \left[\frac{-3 + 2logx}{x^3}\right]$

$f''(x)$ at ***at*** $x = e = \frac{-3}{e^3} < 0$

Therefore f(x) is maximum at x = e and the max. value = $\frac{\log e}{e} = \frac{1}{e}$

5. **(a)** $(1, \infty)$

**Explanation:** Given, function

$\Rightarrow$ $f(x) = (x + 1)^3 . (x - 3)^3$

$\Rightarrow$ $f'(x) = 3(x + 1)^2 (x - 3)^3 + 3(x - 3)^3 (x + 1)^3$

Put f'(x) = 0

$\Rightarrow$ $3(x + 1)^2 (x - 3)^3 = -3(x - 3)^2 (x + 1)^3$

$\Rightarrow$ $x - 3 = -(x + 1)$

$\Rightarrow$ $2x = 2$

$\Rightarrow$ $x = 1$

When x > 1 the function is increasing

x < 1 function is decreasing

Therefore f(x) is increasing in $(1, \infty)$.

6.

**(d)** $k \in (-\infty, 4)$

**Explanation:** $\because$ $f(x) = X^2 - kx + 5$ is increasing in $x \in [2, 4]$

$f'(x) = 2x - k$

$f'(x) > 0$, for increasing function

$2x - k > 0$

$k < 2x$ (k should be less than minimum value of 2x)

$k < 4$

$k \in (-\infty, 4)$

7. **(a)** $10\left(\frac{\pi}{2}\right)^9$

**Explanation:** We have,

$I_{10} = \int_0^{\pi/2} x^{10} \sin x dx\}$

$= \left[x^{10}(-\cos x)\right]_0^{\frac{\pi}{2}} - \int_0^{\pi/2}\left[10x^9 \int \sin x dx\right]dx$

$= \left[-x^{10}\cos x\right]_0^{\frac{\pi}{2}} - 10\int_0^{\pi/2} x^9(-\cos x)dx$

$= -\left[x^{10}\cos x\right]_0^{\frac{x}{2}} + 10\int_0^{\pi/2} x^9 \cos x dx$

$= -\left[x^{10}\cos x\right]_0^{\frac{\pi}{2}} + 10\left[x^9 \sin x\right]_0^{\frac{\pi}{2}} - 10\int_0^{\pi/2} 9x^8 \sin x dx$

$= -\left[\left(\frac{\pi}{2}\right)^{10} \times 0 - 0^{10}\cos 0\right] + 10\left[\left(\frac{\pi}{2}\right)^9 \times 1 - 0^9 \times 0\right] 90\int_0^{\pi/2} x^8 \sin x dx$

$$= 10\left[\left(\frac{\pi}{2}\right)^9 \times 1\right] - 90I_8$$

$$= 10\left(\frac{\pi}{2}\right)^9 - 90I_8$$

$$\therefore I_{10} + 90I_8 = 10\left(\frac{\pi}{2}\right)^9$$

8.

**(b)** $\frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right) + C$

**Explanation:** Put $x^3$ = t and $3x^2$ dx = dt

$$\therefore\ I = \int \frac{dt}{\sqrt{9-16t^2}} = \frac{1}{4}\int \frac{dt}{\sqrt{\frac{9}{16}-t^2}} = \frac{1}{4}\cdot\int \frac{dt}{\sqrt{\left(\frac{3}{4}\right)^2 - t^2}}$$

$$= \frac{1}{4}\sin^{-1}\frac{t}{(3/4)} + C = \frac{1}{4}\sin^{-1}\left(\frac{4x^3}{3}\right) + C$$

9.

**(b)** x sin (log x) +C

**Explanation:** $\int (sin(log\ x) + cos(log\ x))dx$

( Use By Part, Take 1 as II function )

$= \int \sin(log\ x).1dx + \int \cos(\log\ x)dx$

$= (\sin(\log x)).\ x - \int \cos(\log x)\frac{1}{x}.\ xdx + \int \cos(log\ x)dx.$

$= x\sin(\log x) + C$

10.

**(b)** sin (3x + 4) + 3 (x -1 ) cos (3x + 4)

**Explanation:** Given that area bounded by the curve x-axis,

x = 1 and x = b

$\Rightarrow \int_1^b ydx = \int_1^b f(x)dx$

$\Rightarrow \int_1^b ydx - [A]_1^b$

$$\Rightarrow \int_1^b f(x)dx = (b-1)\sin(3b+4)$$

$$\Rightarrow f(x) - \frac{d}{dx}[(x-1)\sin(3x+4)]$$

$\Rightarrow$ 3(x - 1) cos (3x + 4) + sin (3x + 4)

11.

**(d)** (1 + sin x)(1 + cos y) = C

**Explanation:** Given cos x (1+cos y) dx – sin y (1+sin x) dy = 0

Let 1 + cos y = t and 1 + sin x = u

On differentiating both equations, we obtain

-sin y dy = dt and cos x dx = du

Put this in the first equation

t du + u dt = 0

$$-\frac{du}{u} = \frac{dt}{t}$$

-log u = log t + C

log u + log t = C

log ut = C

ut = C

(1 + sin x)(1 + cos y) = C

12.

**(b)** $\tan^{-1} y + \tan^{-1} x = \tan^{-1} C$

**Explanation:** we have,

$$\left(1+x^2\right)\frac{dy}{dx} + 1 + y^2 = 0$$

$$\left(1+x^2\right)\frac{dy}{dx} = -\left(1+y^2\right)$$

$$\frac{dy}{1+y^2} = -\frac{dx}{1+x^2}$$

$$\int\frac{dy}{1+y^2} = -\int\frac{dx}{1+x^2}$$

$\tan^{-1} y = -\tan^{-1} x + \tan^{-1} C$

$\tan^{-1} y + \tan^{-1} x = \tan^{-1} C$

13.

**(d)** Minimum Z = 300 at (60, 0)

**Explanation:** Objective function is Z = 5x + 10 y ……………………(1).

The given constraints are : x + 2y ≤ 120, x + y ≥ 60, x – 2y ≥ 0, x, y ≥ 0 .

The corner points are obtained by drawing the lines x+2y =120, x+y = 60 and x-2y = 0. The points so obtained are (60,30),(120,0), (60,0) and (40,20)

| Corner points | Z = 5x + 10y |
|---|---|
| D(60 ,30 ) | 600 |
| A(120,0) | 600 |
| B(60,0) | 300……………………..(Min.) |
| C(40,20) | 400 |

Here , Z = 300 is minimum at ( 60, 0 ).

14. **(a)** $\frac{1}{3}$

**Explanation:** Let A = event of getting 6 on the dice.
A = {(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6)}
Let X is the random variable of " number of sixes ".
Therefore, X = 0, 1, 2.

P(X = 0) = $1 - \frac{11}{36} = \frac{25}{36}$; P( X = 1)= $\frac{10}{36}$ ; P(X = 2) =$\frac{1}{36}$

Therefore, the probability distribution is :

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | $\frac{25}{36}$ | $\frac{10}{36}$ | $\frac{1}{36}$ |

Therefore, Expectations of X :

$$E(X) = \sum_{i=1}^{n} X_i P\left(X_i\right) = 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36} = \frac{12}{36} = \frac{1}{3}$$

15.

**(d)** $\frac{11}{243}$

**Explanation:** Let x be the number of correct answers, then x has the binomial distribution with n = 5, p = $\frac{1}{3}$ and q = $\frac{2}{3}$.

Therefore , required probability = P (x ≥ 4).= P (x = 4) + P (x = 5).

$$= {}^5C_4\left(\frac{2}{3}\right)^{5-4}\left(\frac{1}{3}\right)^4 + {}^5C_5\left(\frac{2}{3}\right)^{5-5}\left(\frac{1}{3}\right)^5 = \frac{11}{243}$$

**Section B**

16.

**(b)** $(-\infty, 1] \cup [2, \infty)$

**Explanation:** ∵ f : R → R defined by

$f(x) = \sqrt{x^2 - 3x + 2}$

Here, $x^2 - 3x + 2 \geq 0$

$(x - 1)(x - 2) \geq 0$

$x \leq 1$ or $x \geq 2$

$\therefore$ Domain of $f = (-\infty, 1] \cup [2, \infty)$

17.

**(c)** $\frac{\sqrt{3}}{2}$

**Explanation:** $4\cos^{-1} x + \sin^{-1} x = \pi$

$\Rightarrow 3\cos^{-1} x + \cos^{-1} x + \sin^{-1} = \pi$

$\Rightarrow 3cos^{-1}x + \frac{\pi}{2} = \pi$

$\Rightarrow \cos^{-1}x = \frac{\pi}{6}$

$\Rightarrow x = \cos\frac{\pi}{6}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$

18.

**(c)** square matrix

**Explanation:** We know that as matrix is said to be square matrix if the number of rows is equal to the number of columns.

Therefore, the given matrix P = $\begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a square matrix.

19.

**(b)** B = –4A

**Explanation:** $\begin{vmatrix} -2 & 4 & 2 \\ 6 & 2 & 0 \\ -2 & 4 & 8 \end{vmatrix}$ = -2(16) - 4(48) + 2(56) = -168

$\begin{vmatrix} -1 & 2 & 4 \\ 3 & 1 & 0 \\ -2 & 4 & 2 \end{vmatrix}$ = -1(2) - 2(6) + 4(14) = 42

$\Rightarrow$ |B| = -168 = -4(42) = -4|A|.

Which is the required solution.

20.

**(d)** $k \neq 0$

**Explanation:** In the given question the system of linear equation has unique solution if:

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & -k \end{vmatrix} \neq 0 \Rightarrow 1(-k+2) - 1(-2k+3) + 1(4-3) \neq 0 \Rightarrow k \neq 0$$

21.

**(d)** 1

**Explanation:** Applying $R_2 \rightarrow (R_2 - 2R_1)$ and $R_3 \rightarrow (R_3 - 3R_1)$ and expanding by $C_1$, we get $\Delta = 1$.

22.

**(c)** 1

**Explanation:** $y = \sin^{-1}x, 0 < x < 1, 0 < x < 1$

$z = \cos^{-1}\sqrt{1-x^2}$

$= \cos^{-1}(\sqrt{1-\sin^2 y})$

$\left( \because y = \sin^{-1}x \Rightarrow (x = \sin y) \right)$

$\Rightarrow y = z \Rightarrow \frac{dy}{dz} = 1$

23.

**(b)** 0

**Explanation:** $f(x) = \left(\frac{x^l}{x^m}\right)^{l+m} \left(\frac{x^m}{x^n}\right)^{m+n} \left(\frac{x^n}{x^l}\right)^{n+l}$

$f(x) = \left(x^{l-m}\right)^{l+m}\left(x^{m-n}\right)^{m+n}\left(x^{n-l}\right)^{n+l}$

$f(x) = x^{l^2 - m^2 + m^2 - n^2 + n^2 - R^2}$

$f(x) = x^0$

$f(x) = 1$

$f'(x) = 0$

24.

**(b)** $f(a) - a f'(a)$

**Explanation:** $\lim_{x \to a} \frac{xf(a) - af(x)}{x-a}$

$$= \lim_{h \to 0} \frac{(a+h)f(a) - af(a+h)}{h} = \lim_{h \to 0} \left\{ \frac{hf(a)}{h} - \frac{af(a+h) - af(a)}{h} \right\} = f(a) - af'(a)$$

25.

**(d)** none of these

**Explanation:** $f(x) = \sqrt{x^2 - 10x + 25}$

$f(x) = \sqrt{(x-5)^2} = |x - 5|$

f(x) = x - 5 x $\geq$ 5
= -(x - 5) x < 5
f'(x) = x $\geq$ 5
= -1 x < 5
Hence, we can not define derivative of the function on [0, 7].

26.

**(d)** 1

**Explanation:** Here, given

$\Rightarrow f(x) = \frac{1-\cos 4x}{8x^2}$ is continuous at x = 0

$\Rightarrow f(x) = \lim_{x \to 0} \frac{2\sin^2 2x}{8x^2}$

$\Rightarrow f(x) = \lim_{x \to 0} \frac{2\sin^2 2x}{2 \times 4x^2}$

$\Rightarrow f(x) = \lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right)^2$

$\Rightarrow$ f(x) = 1
$\therefore$ k = 1

27.

**(d)** $-\frac{1}{e}$

**Explanation:** f(x) = x $\log_e$x
$\Rightarrow$ f'(x) = 1 + $\log_e$x
to find maxima or minima
f'(x) = 0
$\Rightarrow$ 1 + $\log_e$x = 0

$\Rightarrow$ x = $\frac{1}{e}$

f''(x) = $\frac{1}{x}$

f''$\left(\frac{1}{e}\right)$ = e > 0

x = $\frac{1}{e}$ is a local minima.

⇒ Minimum value of the function is

$$f\left(\frac{1}{e}\right) = \frac{1}{e}\log_e\left(\frac{1}{e}\right) = \frac{-1}{e}$$

28.

**(b)** ab $\geq \frac{c^2}{4}$

**Explanation:** f(x) = ax + $\frac{b}{x}$

⇒ f'(x) = a - $\frac{b}{x^2}$

f'(x) = 0

a - $\frac{b}{x^2}$ = 0

⇒ x = $\pm\sqrt{\frac{b}{a}}$

f''(x) = $\frac{2b}{x^3}$

$$f''\left(\sqrt{\frac{b}{a}}\right) = \frac{2b}{\left(\sqrt{\frac{b}{a}}\right)^3} > 0$$

⇒ x = $\sqrt{\frac{b}{a}}$ has a minima.

$$f\left(\sqrt{\frac{b}{a}}\right) = 2\sqrt{ab} \geq c$$

$$\frac{c}{2} \leq \sqrt{ab}$$

$$\Rightarrow \frac{c^2}{4} \leq ab$$

29.

**(b)** $\frac{4}{3}$

**Explanation:** f(x) = $\frac{1}{4x^2+2x+1}$

$\Rightarrow$ f'(x) = 8x + 2

For local minima or maxima we have

f'(x) = 8x + 2 = 0

$\Rightarrow$ x = $\frac{-1}{4}$

f''(x) = 8 > 0

$\Rightarrow$ function has maxima at x = $\frac{-1}{4}$

$f\left(\frac{-1}{4}\right) = \frac{4}{3}$

30.

**(c)** x + y = 0

**Explanation:** Since , $\frac{dy}{dx} = \cos x,$ therefore , slope of tangent at ( 0 , 0 ) = cos 0 = 1 and hence slope of normal at ( 0 , 0 ) is - 1 .

Equation of normal at (0,0) is,

y - 0 = slope of normal × ( x - 0)

y = -1(x)

x + y = 0

31.

**(b)** $\frac{a}{2}$

**Explanation:** Here,

$$f(x) = \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}$$

$$f(a-x) = \frac{\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}$$

We know that,

$$\therefore \int_0^a f(x) = \int_0^a f(a-x) = I$$

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}} dx$$

$$= \int_0^a dx$$

$$I = \frac{a}{2}$$

32.

**(d)** $\cos\left(\frac{1}{x}\right) + C$

**Explanation:** Let $\frac{1}{x} = t, -\frac{1}{x^2}dx = dt = \int \sin t(-dt) = \int(-\sin t)dt = \cos t + C.$

Which is the required solution.

33.

**(d)** - log |cos x – sin x| + C

**Explanation:**

The integral is $\int \frac{(1+\tan x)}{(1-\tan x)}dx$

since we know that, $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

sin (a + b) = sin a cos b + cos a sin b

$\int$cot x = log (sin x) + c

Therefore,

$\Rightarrow \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}dx$ (Rationalizing the denominator)

$\Rightarrow \int \frac{\cos x+\sin x}{\cos x-\sin x}dx$

Put cos x - sin x = t

(- sin x - cos x) dx = dt

(sin x + cos x) dx = -dt

$\int \frac{-dt}{t} = -\log t + c$

$\Rightarrow$ -log |cos x - sin x| + c

34. **(a)** x.tan $\frac{x}{2}$ + C

**Explanation:** Given: $\int \frac{x+\sin x}{1+\cos x}dx$

As we know

$\therefore$ sin 2x = $\frac{2\tan x}{1+\tan^2 x}$, 1 + tan$^2$ x = sec$^2$ x and cos 2x = $\frac{1+\tan^2 x}{1-\tan^2 x}$

$$\Rightarrow \frac{x+\sin x}{1+\cos x} = \frac{x + \dfrac{2\tan\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}}{1 + \dfrac{1+\tan^2\left(\frac{x}{2}\right)}{1-\tan^2\left(\frac{x}{2}\right)}}$$

$$= \frac{x + x\tan^2\left(\frac{x}{2}\right) + 2\tan\left(\frac{x}{2}\right)}{2}$$

$$\Rightarrow \int \frac{x+\sin x}{1+\cos x}dx = \int \frac{x + x\tan^2\left(\frac{x}{2}\right) + 2\tan\left(\frac{x}{2}\right)}{2}dx$$

Let $\frac{x}{2} = t$

$$\Rightarrow \frac{dx}{2} = dt$$

$$\Rightarrow \int \left(2t + 2t\tan^2 t + 2\tan t\right)dt = 2\int (t + t\tan 2t + \tan t)dt$$

$=2\int tdt + 2\int t\sec^2 tdt - 2\int tdt + 2\int \tan tdt$

$= 2\int tdt + 2\int t\sec^2 tdt - 2\int tdt + 2\int \tan tdt$

$= 2\int t\sec^2 tdt + 2\int \tan tdt$ .......(i)

Applying Integration by parts on $\int t\sec^2 tdt$

$$\Rightarrow \int t\sec^2 tdt = t\int \sec^2 tdt - \int \left(\frac{d}{dt}t\right)\left(\int \sec^2 tdt\right)dt$$

$\Rightarrow \int t\sec^2 tdt = t\tan t - \int \tan tdt$ ......(ii)

$\Rightarrow$ Put (ii) in (i)

$\Rightarrow 2(\text{t tant } - \int \text{ tan tdt}) + 2\int \text{ tant dt} = 2t \text{ tant } + c = x\tan\left(\frac{x}{2}\right) + C$

$\Rightarrow \int \frac{x+\sin x}{1+\cos x}dx = x\tan\left(\frac{x}{2}\right) + C$

35.

**(c)** $\frac{5\pi-2}{4}$ sq. units

**Explanation:** Required area :

$\int_{-1}^{2} \sqrt{5-x^2}dx - \int_{-1}^{1} (1-x)dx - \int_{1}^{2}(x-1)dx$

$= \left[\frac{x\sqrt{5-x^2}}{2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2}$

$-\left[x - \frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2}$

$= \left(\frac{5\pi-2}{4}\right) sq.\ units$

36.

**(b)** x cos y = C

**Explanation:** Given: $x\frac{dy}{dx} = \cot y$

Separating the variable, we obtain

$\frac{dy}{\cot y} = \frac{dx}{x}$

$\tan y \text{ dy} = \frac{\text{dx}}{\text{x}}$

Integrating both sides, we obtain

$\int \tan y\, dy = \int \frac{dx}{x}$

log sec y = log x + log c

C=xcos y

37.

**(c)** y = $C_1e^{C_2x} + C_3$

**Explanation:** We have ,

y1y3 = $y_2^2$

$$\Rightarrow \frac{\frac{dy}{dx}}{\frac{d^2y}{dx^2}} = \frac{\frac{d^2y}{dx^2}}{\frac{d^3y}{dx^3}}$$

$$\Rightarrow \frac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = \frac{\frac{d^3y}{dx^3}}{\frac{d^2y}{dx^2}}$$

$$\Rightarrow \frac{\frac{d^2y}{dx^2}}{\frac{dy}{dx}} = \int \frac{\frac{d^3y}{dx^3}}{\frac{d^2y}{dx^2}}$$

$$\Rightarrow \log\frac{dy}{dx} = \log\frac{d^2y}{dx^2} + \log C$$

$$\Rightarrow C\frac{dy}{dx} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \int C dx = \int \frac{\frac{d}{dx}\left(\frac{dy}{dx}\right)}{\frac{dy}{dx}}$$

$$\Rightarrow Cx + C_1 = \log\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = e^{Cx + C_1}$$

$$\Rightarrow \int dy = \int e^{Cx + C_1} dx$$

$$\Rightarrow y = C_4 e^{C_5 x} + C_3$$

$$\Rightarrow y = c_1 e^{c_2 x} + c_3$$

38. **(a)** $xyy'' + x(y')^2 \; yy' = 0$

**Explanation:** $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

$\frac{2x}{b^2} + \frac{2yy'}{a^2} = 0$

$\frac{x}{b^2} = -\frac{yy'}{a^2}$

$\frac{yy'}{x} = -\frac{a^2}{b^2}$

Differentiating both sides again,

$\frac{x(yy'' + y'^2 - yy')}{x^2} = 0$

$xyy'' + xy'^2 - yy' = 0$

39.

**(c)** Both I and II

**Explanation:** Given, $\vec{a} + \vec{b} + \vec{c} = 0$ ...(i)

I. Consider $[\vec{a}\vec{b}\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$

$= -(\vec{b} + \vec{c}) \cdot (\vec{b} \times \vec{c})$ ...[using Eq.(i)]

$= -(\vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{c} \cdot (\vec{b} \times \vec{c}))$

$= -([\vec{b}\vec{b}\vec{c}] + [\vec{c}\vec{b}\vec{c}])$

= -(0 + 0) = 0

Thus, the vectors are coplanar.

II. Consider,

$\vec{a} \times \vec{b} = \vec{a} \times (-\vec{a} - \vec{c})$

$= -(\vec{a} \times \vec{a} + \vec{c})$ [using Eq. (i)]

$= -(\vec{a} \times \vec{a} + \vec{a} \times \vec{c})$

$= -(\vec{0} + \vec{a} \times \vec{c}) = -(\vec{a} \times \vec{c})$

$= \vec{c} \times \vec{a}$ ...(ii)

[using Eq.] Similarly,

$\vec{b} \times \vec{c} = \vec{a} \times \vec{b}$

From Eqs. (ii) and (iii), we get

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

40. **(a)** $\frac{16\sqrt{2}}{3\sqrt{7}}, \frac{2\sqrt{2}}{3\sqrt{7}}$

**Explanation:** $(\vec{a} + \vec{b}).(\vec{a} - \vec{b}) = 8$

$\Rightarrow |\vec{a}|^2 - |\vec{b}|^2 = 8$

$\Rightarrow 64|\vec{b}|^2 - |\vec{b}|^2 = 8 \Rightarrow 63|\vec{b}|^2 = 8 \Rightarrow |\vec{b}| = \sqrt{\frac{8}{63}} = \frac{2\sqrt{2}}{3\sqrt{7}}$

$\Rightarrow |\vec{a}| = 8|\vec{b}| \Rightarrow |\vec{a}| = \frac{16\sqrt{2}}{3\sqrt{7}}$

41.

**(d)** $\cos^{-1}\left(\frac{10}{\sqrt{102}}\right)$

**Explanation:** Position vectors of the points A , B and C are $\hat{i} + 2\hat{j} + 3\hat{k}, -\hat{i}, and\ \hat{j} + 2\hat{k}$ respectively.

Then;

$cos\theta = \frac{\overrightarrow{BA}\overrightarrow{BC}}{|\overrightarrow{BA}||\overrightarrow{BC}|}$

$= \frac{(2\hat{i}+2\hat{j}+3\hat{k})\,.\,(\hat{i}+\hat{j}+2\hat{k})}{\sqrt{17}\sqrt{6}}$

$\Rightarrow cos\theta = \frac{10}{\sqrt{102}}$

$\Rightarrow \angle ABC = cos^{-1}(\frac{10}{\sqrt{102}})$

42. **(a)** 7

**Explanation:** 7

43.

**(c)** $\vec{b} = \vec{c}$

**Explanation:** $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$

$\Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0$ and $\vec{a} \times \vec{b} - \vec{a} \times \vec{c} = 0$

$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$ and $\vec{a} \times (\vec{b} - \vec{c}) = 0$

$\Rightarrow \vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$

Also,

$\Rightarrow |\vec{a}||\vec{b} - \vec{c}|\cos\theta = 0$ and $|\vec{a}||\vec{b} - \vec{c}|\sin\theta = 0$

$\Rightarrow$ If $\theta = \frac{\pi}{2} \Rightarrow \sin\theta = 1 \Rightarrow \vec{b} - \vec{c} = 0 \Rightarrow \vec{b} = \vec{c}$

44. **(a)** $\frac{\pi}{3}$

**Explanation:** We have,

$$\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{2}$$

$$\frac{x-1}{-\sqrt{3}-1}=\frac{y-1}{\sqrt{3}-1}=\frac{z-1}{4}$$

The direction ratios of the given lines are proportional to 1, 1, 2 and $-\sqrt{3}-1, \sqrt{3}-1, 4$.

So, The given lines are parallel to vectors $\overrightarrow{b_1}=\hat{i}+\hat{j}+2\hat{k}$ and

$\overrightarrow{b_2}=(-\sqrt{3}-1)\hat{i}+(\sqrt{3}-1)\hat{j}+4\hat{k}$.

Let $\theta$ be the angle between the given lines.

Now,

$$\cos\theta=\frac{\vec{b}_1\cdot\vec{b}_2}{\left|\vec{b}_1\right|\left|\vec{b}_2\right|}$$

$$=\frac{(\hat{i}+\hat{j}+2\hat{k})\cdot\{(-\sqrt{3}-1)\hat{i}+(\sqrt{3}-1)\hat{j}+4\hat{k}\}}{\sqrt{1^2+1^2+2^2}\sqrt{(-\sqrt{3}-1)^2+(\sqrt{3}-1)^2+4^2}}$$

$$=\frac{-\sqrt{3}-1+\sqrt{3}-1+8}{\sqrt{6}\sqrt{24}}$$

$$=\frac{6}{6\times 2}$$

$$=\frac{1}{2}$$

$$\Rightarrow \theta=\frac{\pi}{3}$$

45. **(a)** $\frac{\pi}{2}$

**Explanation:** Let's consider the first parallel vector to be $\vec{a}=a\hat{i}+b\hat{j}+c\hat{k}$ and second parallel vector be $\vec{b}=(b-c)\hat{i}+(c-a)\hat{j}+(a-b)\hat{k}$

For the angle, we can use the formula $\cos\alpha=\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|}$

For that, we need to find the magnitude of these vectors

$|\vec{a}| = \sqrt{a^2 + b^2 + (c)^2}$

$= \sqrt{a^2 + b^2 + c^2}$

$|\vec{b}| = \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}$

$= \sqrt{2\left(a^2 + b^2 + c^2 - ab - bc - ca\right)}$

$\Rightarrow \cos\alpha = \dfrac{(a\hat{i} + b\hat{j} + c\hat{k}) \cdot ((b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k})}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)} \times \sqrt{a^2+b^2+c^2}}$

$\Rightarrow \cos\alpha = \dfrac{ab-ac+bc-ba+ca-cb}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)} \times \sqrt{a^2+b^2+c^2}}$

$\Rightarrow \cos\alpha = \dfrac{0}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)} \times \sqrt{a^2+b^2+c^2}}$

$\Rightarrow \alpha = \cos^{-1}(0)$

$\therefore \alpha = \dfrac{\pi}{2}$

46.

**(b)** 13; $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

**Explanation:** 13; $\frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

If a line makes angles $\alpha, \beta$ and $\gamma$ with the axis, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ ....(i)
Let r be the length of the line segment. Then,
$r\cos\alpha = 12, r\cos\beta = 4, r\cos\gamma = 3$ ....(ii)

$\Rightarrow (r\cos\alpha)^2 + (r\cos\beta)^2 + (r\cos\gamma)^2 = 12^2 + 4^2 + 3^2$

$\Rightarrow r^2\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right) = 169$

$\Rightarrow r^2$ (1) = 169 [From (i)]

$\Rightarrow r = \sqrt{169}$

$\Rightarrow$ r = $\pm$ 13

$\Rightarrow$ r = 13 (since length cannot be negative)

Substituting r = 13 in (ii)
We get,

$\cos\alpha = \frac{12}{13}, \cos\beta = \frac{4}{13}, \cos\gamma = \frac{1}{13}$

Thus, the direction cosines of the line are
$\frac{12}{13}, \frac{4}{13}, \frac{1}{13}$

47. **(a)** $P\left(E_i|A\right) = \frac{P\left(E_i\right)P(A|E_i)}{\sum_{i=1}^{n} P\left(E_i\right)P(A|E_i)}$

**Explanation:** If $E_1, E_2, \ldots, E_n$ are mutually exclusive and exhaustive events associated with a sample space, and A is any event of non zero probability, then According to Bay's theorem:

$$P\left(E_i|A\right) = \frac{P\left(E_i\right)P(A|E_i)}{\sum_{i=1}^{n} P\left(E_i\right)P(A|E_i)}$$

48. **(a)** $\frac{19}{144}$

**Explanation:** In a single throw of a pair of dice, we have
n(S) = (6 x 6 )= 36.
Let E = {(1,1),(2,2),( 3, 3), (4,4), (5,5), (6,6)}.

Then, $P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}, P(\text{ not } E) = \left(1 - \frac{1}{6}\right) = \frac{5}{6}$

$\therefore p = \frac{1}{6}, q = \frac{5}{6}$ and n = 4.

Required probability = P( 2 or 3 or 4 successes)

$= {}^4C_2 \cdot \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^2 + {}^4C_3 \cdot \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right) + {}^4C_4 \cdot \left(\frac{1}{6}\right)^4$

$= \left(\frac{25}{216} + \frac{5}{324} + \frac{1}{1296}\right)$

$= \frac{171}{1296}$

$= \frac{19}{144}.$

49.

**(b)** Both i and ii

**Explanation:** $\because$ Probability of no head $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

The probability of the experiments end with three tosses, if TTH comes.

$\therefore$ Required probability = $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Hence, both statements are correct.

50.

**(b)** $\frac{1}{2}$

**Explanation:** Let A be the event that Mr. A hit the target and B be the event that Mr. B hit the target

$\therefore$ $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$

Now, P (exactly one o f them hits the target)

$= P(A \cap \bar{B}$ or $\bar{A} \cap B)$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B)$

$= P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B)$

$= \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{3}{6} = \frac{1}{2}$