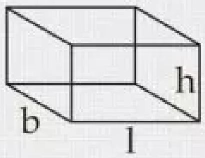
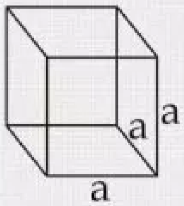
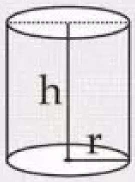
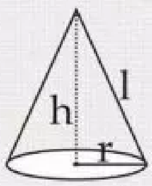
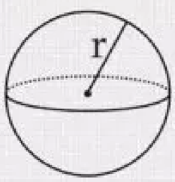
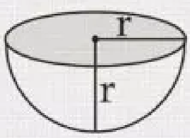
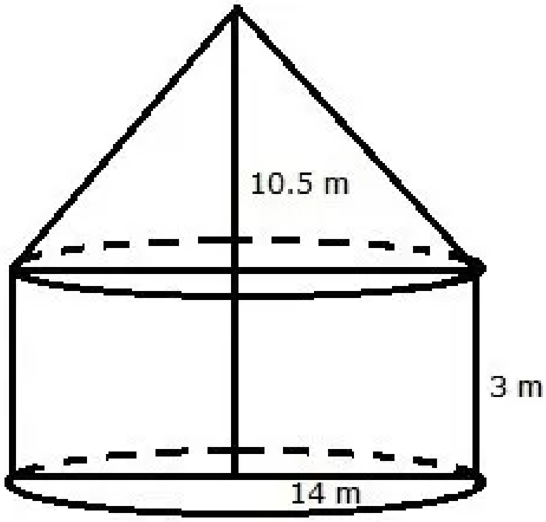
Volume & Surface Areas Of Solids

Exercise 19A

Name of the solid	Figure	Volume	Laterial/Curved Surface Area	Total Surface Area
Cuboid		lbh	$2lh + 2bh$ or $2h(l+b)$	$2lh+2bh+2lb$ or $2(lh+bh+lb)$
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3}\pi r^2 h$	πrl	$\pi rl + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$2\pi r^2+\pi r^2$ or $3\pi r^2$

Question 1:



Radius of the cylinder = 14 m
And its height = 3 m
Radius of cone = 14 m
And its height = 10.5 m
Let l be the slant height

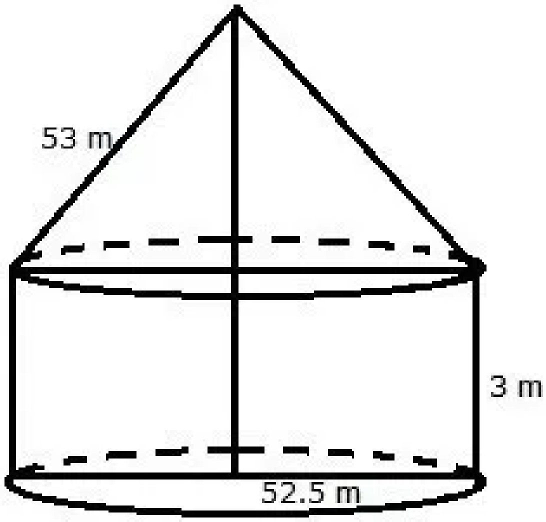
$$\therefore \quad l^2 = (14)^2 + (10.5)^2$$
$$l^2 = (196 + 110.25)\,m^2$$
$$l^2 = 306.25\ m^2$$
$$l = \sqrt{306.25}\ m$$
$$= 17.5\ m$$

Curved surface area of tent
= (curved area of cylinder + curved surface area of cone)

$$= 2\pi rh + \pi rl$$
$$= \left[\left(2 \times \frac{22}{7} \times 14 \times 3\right) + \left(\frac{22}{7} \times 14 \times 17.5\right)\right] m^2$$
$$= (264 + 770)\,m^2 = 1034\ m^2$$

Hence, the curved surface area of the tent = 1034
Cost of canvas = Rs.(1034 × 80) = Rs. 82720

Question 2:

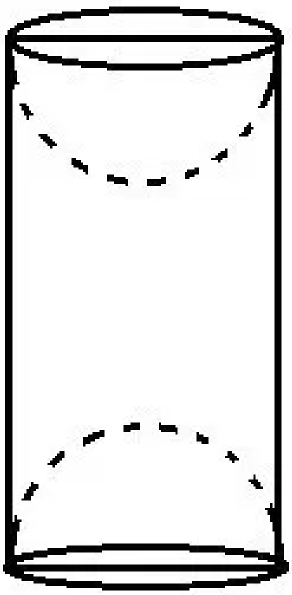


For the cylindrical portion, we have radius = 52.5 m and height = 3 m
For the conical portion, we have radius = 52.5 m
And slant height = 53 m
Area of canvas = 2rh + rl = r(2h + l)

$$= \left[\frac{22}{7} \times 52.5 \times (2 \times 3 + 53)\right] m^2$$

$$= \left(22 \times \frac{15}{2} \times 59\right) m^2 = 9735 m^2$$

Question 3:



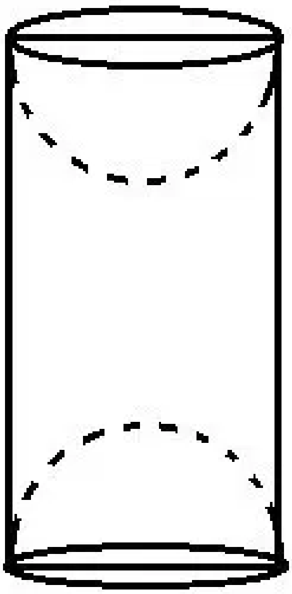
Height of cylinder = 20 cm
And diameter = 7 cm and then radius = 3.5 cm
Total surface area of article
= (lateral surface of cylinder with r = 3.5 cm and h = 20 cm)

$= \left[2\pi rh + 2 \times \left(2\pi r^2\right)\right]$ sq units

$= \left[\left(2 \times \frac{22}{7} \times \frac{7}{2} \times 20\right) + \left(4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right)\right] cm^2$

$= (440 + 154) cm^2 = 594\ cm^2$

Question 4:



Radius of wooden cylinder = 4.2 cm
Height of wooden cylinder = 12 cm
Lateral surface area

$= 2\pi rh$ sq cm

$= 2 \times \pi \times 4.2 \times 12 cm^2$

$= 100.8\pi\ cm^2$

Radius of hemisphere = 4.2 cm
Surface area of two hemispheres

$= 2 \times 2\pi r^2$ sq unit

$= 4\pi \times 4.2 \times 4.2\ cm^2$

$= 70.70\pi\ cm^2$

Total surface area = (100.8 + 70.56) π cm^2
= 538.56 cm^2
= 171.36 π
= 171.36 × $\frac{22}{7}$ cm^2
= 538.56 cm^2

Further, volume of cylinder = $\pi r^2 h = 4.2 \times 4.2 \times 12\ \pi\ cm^2$
$= 211.68\ \pi\ cm^2$

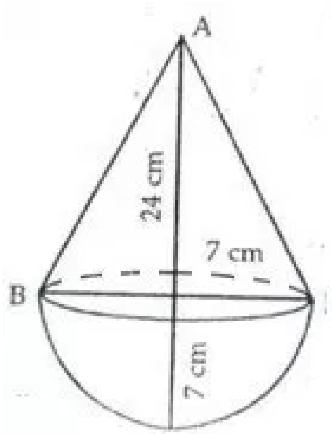
Volume of two hemispheres = $2 \times \frac{2}{3} \pi r^3$ cu.units
$= \frac{4}{3} \pi \times 4.2 \times 4.2 \times 4.2$
$= 98.784\ cm^3$

Volume of wood left = $(211.68 - 98.784)\ \pi$
$= 112.896\ \pi\ cm^3$
$= 112.896 \times \frac{22}{7}\ cm^3$
$= 354.816\ cm^3$

Question 5:
Radius o f cylinder = 2.5 m
Height of cylinder = 21 m
Slant height of cone = 8 m
Radius of cone = 2.5 m
Total surface area of the rocket = (curved surface area of cone + curved surface area of cylinder + area of base)

$$= \left(\pi rl + 2\pi rh + \pi r^2\right)$$
where $l = 8m$, $h = 21m$, $r = 2.5m$
$$= \left(\frac{22}{7} \times 2.5 \times 8 + 2 \times \frac{22}{7} \times 2.5 \times 21 + \frac{22}{7} \times 2.5 \times 2.5\right) m^2$$
$$= (62.85 + 330 + 19.64)\ m^2 = 412.5\ m^2$$

Question 6:



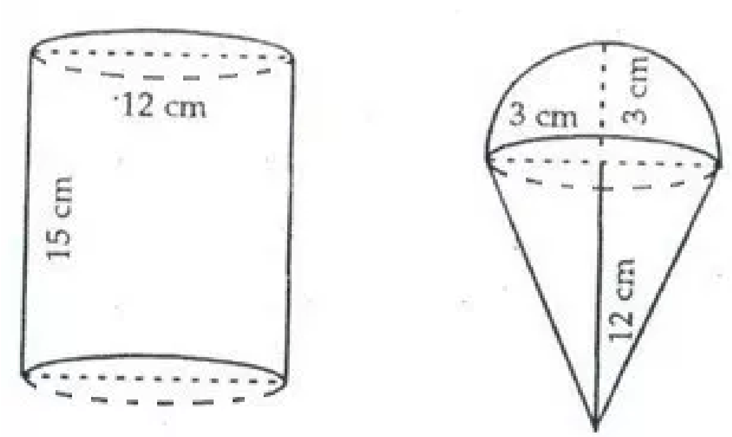
Height of cone = h = 24 cm
Its radius = 7 cm

$$\therefore \text{Slant height} = \sqrt{(24)^2 + 7^2}$$
$$= \sqrt{576 + 49}$$
$$= \sqrt{625} = 25 \text{ cm}$$

Total surface area of toy

$$= \left(\pi r l + 2\pi r^2\right)$$
$$= \pi r (l + 2r)$$
$$= \frac{22}{7} \times 7 \times (25 + 14)$$
$$= 22 \times 39 = 858 \text{ cm}^2$$

Question 7:



Height of cylindrical container h_1 = 15 cm
Diameter of cylindrical container = 12 cm

Volume of container = $\pi r_1^2 h_1 = \pi \times 6 \times 6 \times 15 = 540\pi \text{ cm}^2$
Height of cone r_2 = 12 cm
Diameter = 6 cm
Radius of r_2 = 3 cm

$$\text{Volume of cone} = \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3}\pi \times 3 \times 3 \times 12$$
$$= 36\pi \text{ m}^3$$

Radius of hemisphere = 3 cm

Volume of hemisphere = $\frac{2}{3}\pi r_2^3 = \frac{2}{3}\pi \times 3 \times 3 \times 3 = 18\pi$

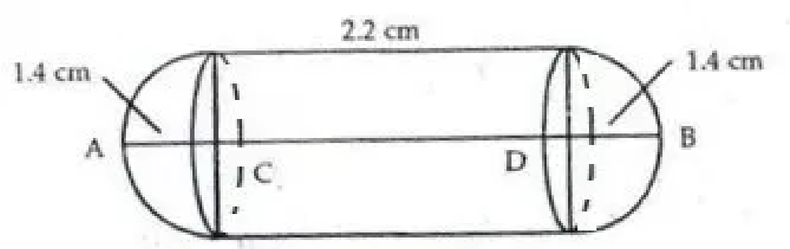
Volume of cone + volume of hemisphere
= 36π + 18π = 54π
Number of cones

$$= \frac{\text{Volume of container}}{\text{Volume of cone} + \text{Volume of hemisphere}}$$

$$= \frac{540\pi}{54\pi} = 10$$

Number of cones that can be filled = 10

Question 8:



Diameter of cylindrical gulabjamun = 2.8 cm
Its radius = 1.4 cm
Total height of gulabjamun = AC + CD + DB = 5 cm
1.4 + CD + 1.4 = 5
2.8 + CD = 5
CD = 2.2 cm

Height of cylindrical part h = 2.2 cm
Volume of 1 gulabjamun = Volume of cylindrical part + Volume of two hemispherical parts

$$= \pi r^2 h + \frac{2}{3}\pi r^2 + \frac{2}{3}\pi r^3$$

$$= \pi r^2 h + \frac{4}{3}\pi r^3 = \pi r^2\left(h + \frac{4}{3}r\right)$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \times \left(2.2 + \frac{4}{3} \times 1.4\right)$$

$$= 22 \times 0.2 \times 1.4 \times (2.2 + 1.87)$$

$$= 4.4 \times 1.4 \times 4.07 = 25.07\ cm^3$$

Volume of 45 gulabjamuns = 45 × 25.07 cm^3
Quantity of syrup = 30% of volume of gulabjamuns
= 0.3 × 45 × 25.07 = 338.46 cm^3

Question 9:
Diameter = 7cm, radius = = 3.5 cm
Height of cone = 14.5 cm – 3.5 cm = 11 cm

$$l = \sqrt{\left(\frac{7}{2}\right)^2 + (11)^2}\ \text{cm} = \sqrt{\frac{49}{4} + 121}\ \text{cm} = \sqrt{\frac{533}{4}}\ \text{cm}$$

$$l = \frac{23.08}{2}\ \text{cm} = 11.54\text{cm}$$

$$\text{Volume of toy} = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \left[\frac{1}{3}\pi r^2 (2r + h)\right]$$

$$\text{where } r = \frac{7}{2} \text{ and } h = 11$$

$$= \left[\frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left(2 \times \frac{7}{2} + 11\right)\right]\text{cm}^3$$

$$= (12.83 \times 18)\text{cm}^3 = 230.94\text{cm}^3$$

Total surface area of toy = $\left(2\pi r^2 + \pi r l\right)\text{cm}^2 = \pi r(2r + l)\text{cm}^2$

$$= \frac{22}{7} \times \frac{7}{2} \times \left(2 \times \frac{7}{2} + 11.54\right)\text{cm}^2$$

$$= (11 \times 18.54)\text{cm}^2 = 203.94\ \text{cm}^2$$

Question 10:
Diameter of cylinder = 24 m
Radius of cylinder = $\frac{24}{2}$ = 12 cm
Height of the cylinder = 11 m
Height of cone = (16 – 11) cm = 5 cm

Slant height of the cone l = $\sqrt{r^2 + h^2} = \sqrt{144 + 25}\ \text{m} = 13\ \text{m}$

Area of canvas required = (curved surface area of the cylindrical part) + (curved surface area of the conical part)

$$= (2\pi r h + \pi r l)\text{m}^2 = \pi r(2h + l)\text{m}^2$$

$$= \left[\frac{22}{7} \times 12 \times (2 \times 11 + 13)\right]\text{m}^2$$

$$= \left(\frac{22}{7} \times 12 \times 35\right)\text{m}^2 = 1320\ \text{m}^2$$

Question 11:
Radius of hemisphere = 10.5 cm
Height of cylinder = (14.5 – 10.5) cm = 4 cm
Radius of cylinder = 10.5 cm

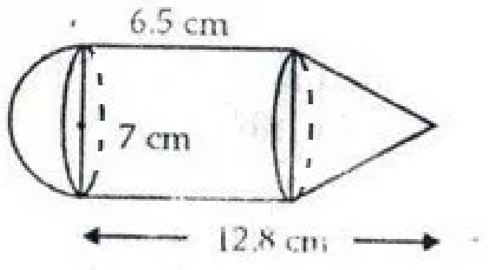
Capacity = Volume of cylinder + Volume of hemisphere

$$= \left(\pi r^2 h + \frac{2}{3}\pi r^3\right) cm^3 = \pi r^2 \left(h + \frac{2}{3} r\right) cm^3$$

$$= \left[\frac{22}{7} \times 10.5 \times 10.5 \times \left(4 + \frac{2}{3} \times 10.5\right)\right] cm^3$$

$$= (346.5 \times 11) cm^2 = 3811.5 cm^2$$

Question 12:



Height of cylinder = 6.5 cm
Height of cone = h_2 = (12.8-6.5) cm = 6.3 cm
Radius of cylinder = radius of cone
= radius of hemisphere
= $\frac{7}{2}$ cm
Volume of solid = Volume of cylinder + Volume of cone + Volume of hemisphere

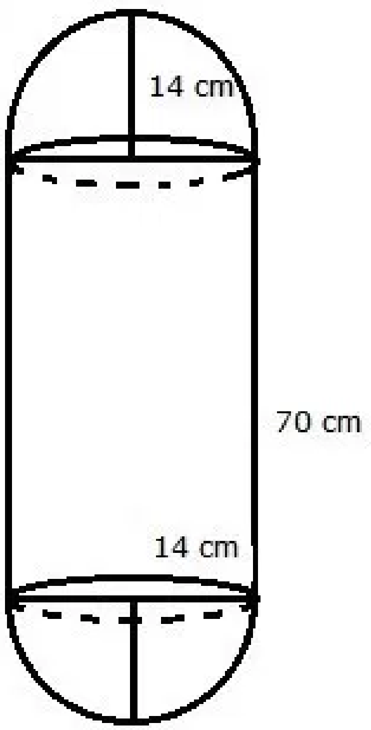
$$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 + \frac{2}{3}\pi r^3 = \pi r^2 \left(h_1 + \frac{1}{3} h_2 + \frac{2}{3} r\right)$$

$$= \left[\frac{22}{7} \times 3.5 \times 3.5 \times \left(6.5 + 6.3 \times \frac{1}{3} + \frac{2}{3} \times 3.5\right)\right]$$

$$= [(38.5) \times (6.5 + 2.1 + 2.33)] cm^3$$

$$= (38.5 \times 10.93) cm^3 = 420.80\ cm^3$$

Question 13:



Radius of each hemispherical end = $\frac{28}{2}$ = 14 cm
Height of each hemispherical part = Its Radius
Height of cylindrical part = (98 − 2 × 14) = 70 cm
Area of surface to be polished = 2(curved surface area of hemisphere) + (curved surface area of cylinder)

$= \left[2\left(2\pi r^2\right) + 2\pi rh\right]$ squnit

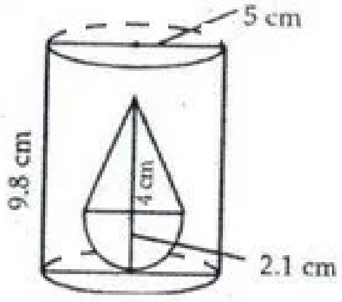
$= 2\pi r(2r + h)\,cm^2$

$= 2 \times \frac{22}{7} \times 14 \times [2 \times 14 + 70]\,cm^2$

$= (88 \times 98) = 8624\ cm^2$

Cost of polishing the surface of the solid
= Rs. (0.15 × 8624)
= Rs. 1293. 60

Question 14:



Radius of cylinder r_1 = 5 cm

And height of cylinder h_1 = 9.8 cm
Radius of cone r = 2.1 cm
And height of cone h_2 = 4 cm
Volume of water left in tub = (volume of cylindrical tub – volume of solid)

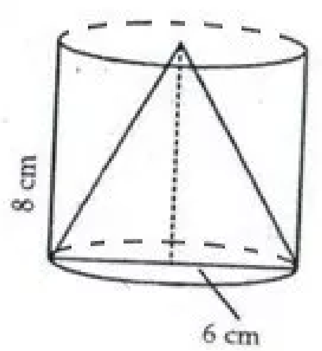
$$= \left(\pi r_1^2 h_1 - \frac{2}{3}\pi r^3 - \frac{1}{3}\pi r^2 h_2\right)$$

$$= \left(\frac{22}{7} \times 5 \times 5 \times 9.8 - \frac{2}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 - \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4\right)$$

$$= [(770 - 19.404) - 18.48]\, cm^3$$

$$= 732.116 cm^3$$

Question 15:
(i) Radius of cylinder = 6 cm
Height of cylinder = 8 cm



Volume of cylinder

$$\Rightarrow \pi r^2 \times 10800 = 972\pi$$

$$r^2 = \frac{972\pi}{10800\pi} = 0.09\ cm^2$$

$$r = \sqrt{0.09}\ cm = 0.3$$

Volume of cone removed

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 6 \times 6 \times 8\ cm^3$$

$$= 96\ \pi\ cm^3$$

(ii) Surface area of cylinder = 2π = 2π × 6 × 8 cm^2 = 96 π cm^2

Slant height of cone $= \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$ cm

$= \sqrt{100}$ cm $= 10$ cm

Curved surface area of cone $= \pi r l = \pi \times 6 \times 10 = 60\pi$

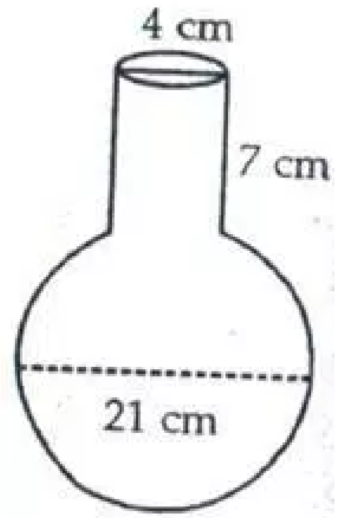
Area of base of cylinder $= \pi r^2 = \pi \times 6 \times 6 = 36\pi$

Total surface area of remaining solid

$= (96\pi + 60\pi + 36\pi)\,cm^2$

$= 192\pi\ cm^2 = 602.88\,cm^2$

Question 16:



Diameter of spherical part of vessel = 21 cm

Its radius $= \frac{21}{2}$ cm

Its volume $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}$

$= 11 \times 21 \times 21\ cm^3 = 4851\ cm^3$

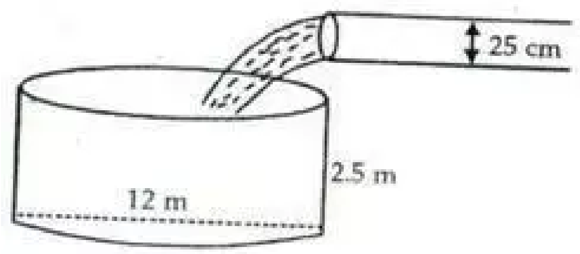
Volume of cylindrical part of vessel

$= \pi r^2 h = \frac{22}{7} \times 2 \times 2 \times 7\ cm^3$

$= 88\ cm^3$

$\therefore$ Volume of whole vessel $= (4851 + 88)\,cm^3 = 4939\ cm^3$

Question 17:



Height of cylindrical tank = 2.5 m
Its diameter = 12 m, Radius = 6 m

Volume of tank = $\pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 2.5 m^3 = \frac{1980}{7} m^3$

Water is flowing at the rate of 3.6 km/ hr = 3600 m/hr
Diameter of pipe = 25 cm, radius = 0.125 m
Volume of water flowing per hour

$$= \frac{22}{7} \times 0.125 \times 0.125 \times 3600 \text{ m}^3$$

$$= \frac{22 \times 3600}{7 \times 8 \times 8} \text{m}^3 = \frac{2475}{14} \text{m}^3$$

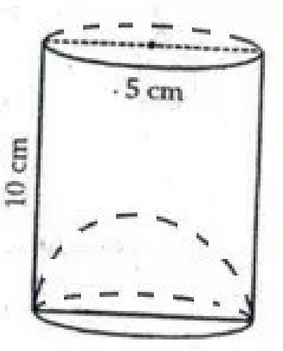
$$\text{Time taken to fill the tank} = \frac{1980}{7} \div \frac{2475}{14} \text{ hr}$$

$$= \frac{1980}{7} \times \frac{14}{2475} \text{hr} = \frac{792}{495} \text{hr}$$

$$= 1.36 \text{ hr} = 1 \text{ hr } 36 \text{ min.}$$

$$\text{Water charges} = \text{Rs.} \frac{1980}{7} \times 0.07 = \text{Rs.} 19.80$$

Question 18:



Diameter of cylinder = 5 cm
Radius = 2.5 cm
Height of cylinder = 10 cm
Volume of cylinder = $\pi r^2 h$ cu.units = 3.14 × 2.5 × 2.5 × 10 cm^3 = 196.25 cm^3
Apparent capacity of glass = 196.25
Radius of hemisphere = 2.5 cm
Volume of hemisphere

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times 3.14 \times 2.5 \times 2.5 \times 2.5 \text{ cm}^3$

$= 32.708 \text{ cm}^3$

Actual capacity of glass = (196.25 – 32.608) cm^3 = 163.54 cm^3

Exercise 19B

https://www.youtube.com/watch?v=6KpStN_0mjE

Question 1:

Radius of the cone = 12 cm and its height = 24 cm

Volume of cone = $\frac{1}{3}$ πr^3 h = (\frac { 1 }{ 3 } \times 12\times 12\times 24) π cm^3 = (48 × 24)π cm^3

$= (48 \times 24)\pi \text{ cm}^3$

$\text{Volume of each ball} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \times 3 \times 3 \times 3 = (36\pi) \text{cm}^3$

$\text{Number of balls formed} = \frac{\text{Volume of solid cone}}{\text{Volume of each ball}}$

$= \frac{(48 \times 24\pi)}{36\pi} = 32$

Question 2:

Internal radius = 3 cm and external radius = 5 cm

$\text{Volume of material in the shell} = \frac{2}{3}\pi \times \left[(5)^3 - (3)^3\right] \text{cm}^2$

$= \frac{2}{3} \times \frac{22}{7} \times 98 = \frac{616}{3} \text{cm}^3$

Radius of the cone = 7 cm

Let height of cone be h cm

$\text{Volume of cone} = \left(\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h\right) \text{cm}^3 = \frac{154h}{3} \text{cm}^3$

$\therefore \frac{154h}{3} = \frac{616}{3}$

$\Rightarrow h = \frac{616}{154} = 4 \text{ cm}$

Hence, height of the cone = 4 cm

Question 3:

Inner radius of the bowl = 15 cm

Volume of liquid in it =

$$\frac{2}{3}\pi r^3 = \left(\frac{2}{3}\pi \times (15)^3\right) cm^3$$

Radius of each cylindrical bottle = 2.5 cm and its height = 6 cm
Volume of each cylindrical bottle

$$= \pi r^2 h = \left(\pi \times \left(\frac{5}{2}\right)^2 \times 6\right) cm^2$$

$$= \left(\frac{25}{4} \times 6\pi\right) = \left(\frac{75\pi}{2}\right) cm^3$$

Required number of bottles = $\frac{\text{Volume of liquid}}{\text{Volume of each cylindrical bottle}}$

$$= \frac{\frac{2}{3} \times \pi \times 15 \times 15 \times 15}{\frac{75}{2} \times \pi} = 60$$

Hence, bottles required = 60

Question 4:

Radius of the sphere = $\frac{21}{2}$ cm

$$\text{Volume of the sphere} = \left(\frac{4}{3}\pi r^3\right) = \left[\frac{4}{3}\pi \times \left(\frac{21}{2}\right)^3\right] cm^3$$

Radius of cone = $\frac{7}{4}$ cm and height 3 cm

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \pi \times \left(\frac{7}{4}\right)^2 \times 3\right) cm^3$$

Let the number of cones formed be n, then

$$n \times \frac{1}{3}\pi \times \left(\frac{7}{4}\right)^2 \times 3 = \frac{4}{3}\pi \times \left(\frac{21}{2}\right)^3$$

$$n = \frac{4}{3}\pi \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \times \frac{3}{\pi} \times \frac{4}{7} \times \frac{4}{7} \times \frac{1}{3}$$

$$n = 504$$

Hence, number of cones formed = 504

Question 5:

Radius of the cannon ball = 14 cm

Volume of cannon ball = $\frac{4}{3}\pi r^3 = \left[\frac{4}{3}\pi \times (14)^3\right] cm^3$

Radius of the cone = $\frac{35}{2}$ cm
Let the height of cone be h cm

Volume of cone = $\left[\frac{1}{3}\pi \times \left(\frac{35}{2}\right)^2 \times h\right] cm^3$

$$\therefore \frac{4}{3}\pi \times (14)^3 = \frac{1}{3}\pi \times \left(\frac{35}{2}\right)^2 \times h$$

$$h = \frac{4}{3}\pi \times 14 \times 14 \times 14 \times \frac{3}{\pi} \times \frac{2}{35} \times \frac{2}{35}$$

$$= 35.84 \text{ cm}$$

Hence, height of the cone = 35.84 cm

Question 6:

Let the radius of the third ball be r cm, then,

Volume of third ball = Volume of spherical ball – volume of 2 small balls

$$\text{Volume of third ball} = \left[\frac{4}{3}\pi(3)^3 - \left\{\frac{4}{3}\pi\left(\frac{3}{2}\right)^3 + \frac{4}{3}\pi(2)^3\right\}\right]$$

$$= \left[36\pi - \left(\frac{9\pi}{2} + \frac{32\pi}{3}\right)\right] cm^3 = \frac{125\pi}{6} cm^3$$

$$\therefore \frac{4}{3}\pi r^3 = \frac{125\pi}{6}$$

$$r^3 = \frac{125\pi \times 3}{6 \times 4 \times \pi} = \frac{125}{8}$$

$$r = \left(\frac{5}{2}\right) cm = 2.5 \text{ cm}$$

Question 7:

External radius of shell = 12 cm and internal radius = 9 cm

Volume of lead in the shell = $\frac{4}{3}\pi\left[(12)^3 - (9)^3\right] cm^3$

Let the radius of the cylinder be r cm

Its height = 37 cm

Volume of cylinder = $\pi r^2 h$ = ($\pi r^2 \times 37$)

$$\therefore \frac{4}{3}\pi\left[(12)^3 - (9)^3\right] = \pi r^2 \times 37$$

$$\frac{4}{3} \times \pi \times 999 = \pi r^2 \times 37$$

$$r^2 = \frac{4}{3} \times \pi \times 999 \times \frac{1}{37\pi} = 36 \text{ cm}^2$$

$$r = \sqrt{36} \text{ cm}^2 = 6 \text{cm}$$

Hence diameter of the base of the cylinder = 12 cm

Question 8:

Volume of hemisphere of radius 9 cm

$$= \left(\frac{2}{3} \times \pi \times 9 \times 9 \times 9\right) cm^3$$

Volume of circular cone (height = 72 cm)

$$= \frac{1}{3}\left(\pi \times r^2 \times 72\right) cm$$

Volume of cone = Volume of hemisphere

$$\therefore \frac{1}{3} \times \pi r^2 \times 72 = \frac{2}{3} \pi \times 9 \times 9 \times 9$$

$$r^2 = \frac{2\pi}{3} \times 9 \times 9 \times 9 \times \frac{1}{24\pi} = 20.25$$

$$r = \sqrt{20.25} = 4.5 \text{ cm}$$

Hence radius of the base of the cone = 4.5 cm

Question 9:

Diameter of sphere = 21 cm

Hence, radius of sphere = $\frac{19}{2}$ cm

Volume of sphere = $\frac{4}{3}$ πr^3 = $\left(\frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}\right)$

Volume of cube = a3 = (1 × 1 × 1)

Let number of cubes formed be n

∴ Volume of sphere = n × Volume of cube

$$\therefore \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} = n \times 1$$

$$= (441 \times 11) = n$$

$$4851 = n$$

Hence, number of cubes is 4851.

Question 10:

Volume of sphere (when r = 1 cm) = $\frac{4}{3}$ πr^3 = (\frac { 4 }{ 3 } \times 1\times 1\times 1) π cm^3

Volume of sphere (when r = 8 cm) = $\frac{4}{3}$ πr^3 = (\frac { 4 }{ 3 } \times 8\times 8\times 8) π cm^3

Let the number of balls = n

$$n \times \left(\frac{4}{3} \times 1 \times 1 \times 1\right)\pi = \left(\frac{4}{3} \times 8 \times 8 \times 8\right)\pi$$

$$n = \frac{4 \times 8 \times 8 \times 8 \times 3}{3 \times 4} = 512$$

Question 11:

Radius of marbles = $\frac{Diameter}{2} = \frac{1.4}{2} cm$

$$\text{Volume of marbles} = \frac{4}{3}\pi r^3$$

$$= \left[\frac{4}{3} \times \pi \times \left(\frac{1.4}{2}\right) \times \left(\frac{1.4}{2}\right) \times \left(\frac{1.4}{2}\right)\right] cm^3$$

$$\text{Radius of beaker} = \left(\frac{7}{2}\right) cm$$

Volume of rising water in beaker

$$= \pi r^2 h = \left(\pi \times \left(\frac{7}{2}\right)^2 \times \left(\frac{56}{10}\right)\right) cm^3$$

Let the number of marbles be n

∴ n × volume of marble = volume of rising water in beaker

$$n \times \left(\frac{4}{3}\pi \times \frac{1.4}{2} \times \frac{1.4}{2} \times \frac{1.4}{2}\right) = \pi \times \frac{7}{2} \times \frac{7}{2} \times \frac{56}{10}$$

$$n = 150$$

Hence the number of marbles is 150

Question 12:

Radius of sphere = 3 cm

Volume of sphere = $\frac{4}{3}$ пr^3 = (\frac { 4 }{ 3 } \times 3\times 3\times 3) п cm^3 = 36п cm^3

Radius of small sphere = $\frac{0.6}{2}$ cm = 0.3 cm

Volume of small sphere = (\frac { 4 }{ 3 } \times 0.3\times 0.3\times 0.3) п cm^3

$$= \left(\frac{4}{3} \times \pi \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}\right) cm^3$$

$$= \left(\frac{4\pi}{3} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}\right) cm^2$$

Let number of small balls be n

$$n \times \left(\frac{4\pi}{3} \times \frac{3}{10} \times \frac{3}{10} \times \frac{3}{10}\right) = \frac{4}{3}\pi \times 3 \times 3 \times 3$$

$$n = 1000$$

Hence, the number of small balls = 1000.

Question 13:

Diameter of sphere = 42 cm

Radius of sphere = $\frac{42}{2}$ cm = 21 cm

Volume of sphere = $\frac{4}{3}$ пr^3 = (\frac { 4 }{ 3 } \times 21\times 21\times 21) п cm^3

Diameter of cylindrical wire = 2.8 cm

Radius of cylindrical wire = $\frac{2.8}{2}$ cm = 1.4 cm

Volume of cylindrical wire = $\pi r^2 h$ = (π × 1.4 × 1.4 × h) cm^3 = (1.96πh) cm^3
Volume of cylindrical wire = volume of sphere

$$\therefore 1.96\pi h = \frac{4}{3} \times \pi \times 21 \times 21 \times 21$$

$$h = \left(\frac{4}{3} \times \pi \times 21 \times 21 \times 21 \times \frac{1}{1.96} \times \frac{1}{\pi}\right) cm$$

$$h = 6300$$

$$h\left(\frac{6300}{100}\right) m = 63 \text{ m}$$

Hence length of the wire 63 m.

Question 14:
Diameter of sphere = 6 cm
Radius of sphere = $\frac{6}{2}$ cm = 3 cm
Volume of sphere = $\frac{4}{3}$ πr^3 = (\frac { 4 }{ 3 } \times 3\times 3\times 3) π cm^3 = 36π cm^3
Radius of wire = $\frac{2}{2}$ mm = 1 mm = 0.1 cm
Volume of wire = $\pi r^2 l$ = (π × 0.1 × 0.1 × l) cm^2 = (0.01 πl) cm^2
36π = 0.01 π l
$\therefore l = \frac{36}{0.01} = 3600$ cm
Length of wire = $\frac{3600}{100}$ m = 36 m

Question 15:
Diameter of sphere = 18 cm
Radius of copper sphere = $\frac{3600}{100}$ m = 36 m

$$\text{Volume of sphere} = \left(\frac{4}{3} \times \pi \times r^3\right) cm^3$$

$$= \left(\frac{4}{3}\pi \times 9 \times 9 \times 9\right) cm^3 = 972\pi cm^3$$

Length of wire = 108 m = 10800 cm
Let the radius of wire be r cm
= $\pi r^2 l$ cm^3 = (πr^2 × 10800) cm^3
But the volume of wire = Volume of sphere

$$\Rightarrow \pi r^2 \times 10800 = 972\pi$$

$$r^2 = \frac{972\pi}{10800\pi} = 0.09 \text{ cm}^2$$

$$r = \sqrt{0.09} \text{ cm} = 0.3$$

Hence the diameter = 2r = (0.3 × 2) cm = 0.6 cm

Question 16:
The radii of three metallic spheres are 3 cm, 4 cm and 5 cm respectively.

Sum of their volumes $= \frac{4}{3}\pi\left(3^3 + 4^3 + 5^3\right)\text{cm}^3$

$= \frac{4}{3}\pi(27 + 64 + 125) = \frac{4}{3}\pi \times 216$

Let r be the radius of sphere whose volume is equal to the total volume of three spheres.

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 216$$

$\Rightarrow \quad r^3 = 216$

$\therefore \quad r = 6\,\text{cm}$

$\therefore$ Diameter $= 6 \times 2 = 12$ cm

Exercise 19C

Question 1:

Here h = 42 cm, R = 16 cm, and r = 11 cm

Capacity $= \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)\text{cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times 42\left[(16)^2 + (11)^2 + 16 \times 11\right]\text{cm}^3$

$= (44 \times 553)\,\text{cm}^3 = 24332\text{cm}^3$

Question 2:

Here R = 33 cm, r = 27 cm and l = 10 cm

$\therefore h = \sqrt{l^2 - \left(R^2 - r^2\right)}\ \text{cm} = \sqrt{(10)^2 - (33 - 27)^2}\ \text{cm}$

$= \sqrt{(10)^2 - (6)^2} = \sqrt{64}\,\text{cm} = 8\,\text{cm}$

Capacity of the frustum

$= \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)\text{cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times 8\left[(33)^2 + (27)^2 + 33 \times 27\right]\text{cm}^3$

$= (8.38 \times 2709)\,\text{cm}^3 = 22701.4\ \text{cm}^3$

Total surface area

$= \left[\pi R^2 + \pi r^2 + \pi l(R+r)\right] cm^2$

$= \pi\left[R^2 + r^2 + l(R+r)\right] cm^2$

$= \frac{22}{7}\left[(33)^2 + (27)^2 + 10 \times (33+27)\right] cm^2$

$= \left(\frac{22}{7} \times 2418\right) cm^2 = 7599.43 cm^2$

Question 3:

Height = 15 cm, R = $\frac{56}{2}$ cm = 28 cm and r = $\frac{42}{2}$ cm = 21 cm

Capacity of the bucket =

$\frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right) cm^3$

$= \frac{1}{3} \times \frac{22}{7} \times 15\left[(28)^2 + (21)^2 + 28 \times 21\right] cm^3$

$= (15.71 \times 1831) cm^3$

$= (28482.23) cm^3$

Quantity of water in bucket = 28.49 litres

Question 4:

R = 20 cm, r = 8 cm and h = 16 cm

$\therefore\ l = \sqrt{h^2 + (R-r)^2} = \sqrt{(16)^2 + (20-8)^2}$

$= \sqrt{256 + 144} cm = 20 cm$

Total surface area of container = $\pi l (R+r) + \pi r^2$

$= \left[3.14 \times 20 \times (20+8) + 3.14 \times 8 \times 8\right] cm^2$

$= (3.14 \times 20 \times 28 + 3.14 \times 8 \times 8) cm^2$

$= (1758.4 + 200.96) cm^2$

$= 1959.36\ cm^2$

Cost of metal sheet used = Rs. $\left(1959.36 \times \frac{15}{100}\right)$ = Rs. 293.90

Question 5:

R = 15 cm, r = 5 cm and h = 24 cm

$\therefore\ l = \sqrt{h^2 + (R-r)^2} = \sqrt{(24)^2 + (10)^2} cm$

$= \sqrt{576 + 100} cm = \sqrt{676}\ cm = 26 cm$

(i) Volume of bucket =

$$\frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)$$

$$= \frac{1}{3} \times 3.14 \times 24 \times \left[(15)^2 + (5)^2 + 15 \times 5\right]$$

$$= (25.12 \times 325)\, cm^3$$

$$= 8164\ cm^3 = 8.164\ litres$$

Cost of milk = Rs. (8.164 × 20) = Rs. 163.28

(ii) Total surface area of the bucket

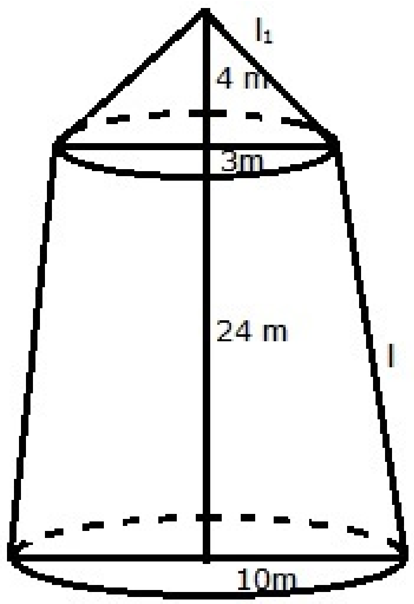
$$= \pi l(R + r) + \pi r^2$$

$$= (3.14 \times 26 \times 20 \times 3.14 \times 5 \times 5)\, cm^2$$

$$= 1711.3\ cm^2$$

Cost of sheet = $\left(1711.3 \times \frac{10}{100}\right)$ = Rs. 171.13

Question 6:



R = 10cm, r = 3 m and h = 24 m

Let l be the slant height of the frustum, then

$$l = \sqrt{h^2 + (R-r)^2}$$
$$= \sqrt{(24)^2 + (10-3)^2}$$
$$= \sqrt{(24)^2 + (7)^2}$$
$$= \sqrt{576 + 49}$$
$$= \sqrt{625}\ m = 25\ m$$

Let l_1 be the slant height of conical part

$r = 3$ m

and $h = 4$m

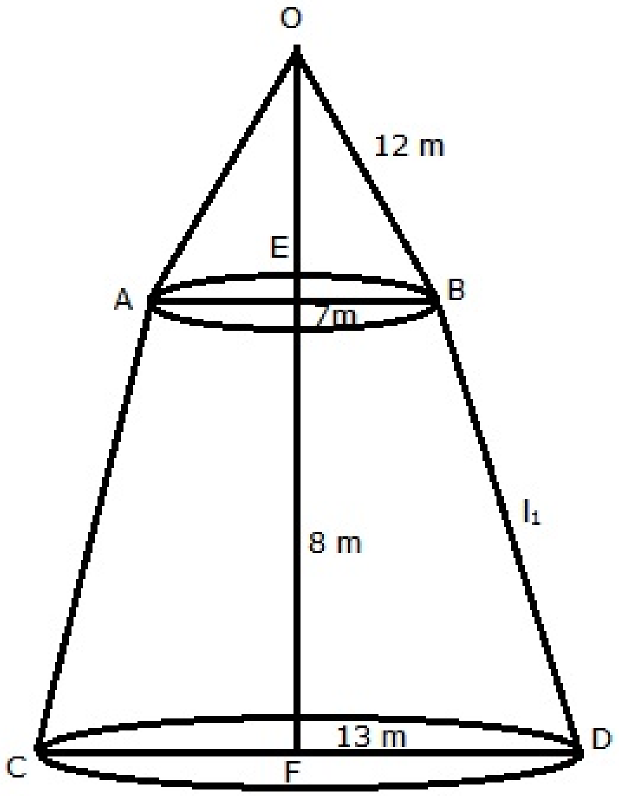
$\therefore \quad l_1 = \sqrt{3^2 + 4^2}$ m

$= \sqrt{25}$ m $= 5$ m

Quantity of canvas = (Lateral surface area of the frustum) + (lateral surface area of the cone)

$$= \left[\pi l(R+r) + \pi r l_1\right] m^2$$
$$= \pi[25 \times (10+3) + (3 \times 5)]m^2$$
$$= \frac{22}{7} \times \left[(25 \times 13) + (3 \times 5)\right] m^2$$
$$= 1068.57\ m^2$$

Question 7:



ABCD is the frustum in which upper and lower radii are EB = 7 m and FD = 13 m
Height of frustum = 8 m
Slant height l_1 of frustum

$= \sqrt{h^2 + (R - r)^2}$

$= \sqrt{8^2 + (13 - 7)^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100} = 10\,m$

Radius of the cone = EB = 7 m
Slant height l_2 of cone = 12 m
Surface area of canvas required

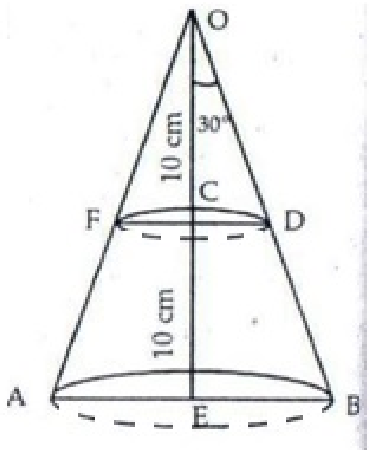
$= \pi(R + r)l_1 + \pi r l_2$

$= \pi[(13 + 7) \times 10 + 7 \times 12]$

$= \frac{22}{7} \times [200 + 84] = \frac{22}{7} \times 284\ m^2$

$= 892.6\ m^2$

Question 8:



In the given figure, we have
∠COD = 30°, OC = 10 cm, OE = 20 cm
Let CD = r cm and EB = R cm

$$\frac{CD}{OC} = \tan 30^\circ$$

$$\Rightarrow \frac{CD}{10} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow CD = \left(10 \times \frac{1}{\sqrt{3}}\right) cm$$

$$= \frac{10}{\sqrt{3}} cm$$

$$\frac{EB}{OE} = \tan 30^\circ = \frac{EB}{20} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow EB = \left(20 \times \frac{1}{\sqrt{3}}\right) cm \Rightarrow R = \frac{20}{\sqrt{3}} cm$$

Also, CE = 10 cm

Thus, ABDF is the frustum of a cone in which

$R = \frac{20}{\sqrt{3}} cm, \ r = \frac{10}{\sqrt{3}} cm$ and $h = 10 cm$

$$\text{Volume of frustum} = \frac{1}{3}\pi h\left(R^2 + r^2 + Rr\right)$$

$$= \frac{1}{3} \times \pi \times 10 \times \left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right)$$

$$= \left(\frac{\pi \times 10}{3} \times \frac{700}{3}\right) cm^3 = \left(\frac{7000\pi}{9}\right) cm^3$$

Volume of wire of radius r and length l

$$= \pi r^2 l = \pi\left[\frac{1}{32}\right]^2 l$$

Volume of wire = Volume of frustum

$$\pi\left(\frac{1}{32}\right)^2 l = \frac{7000\pi}{9}$$

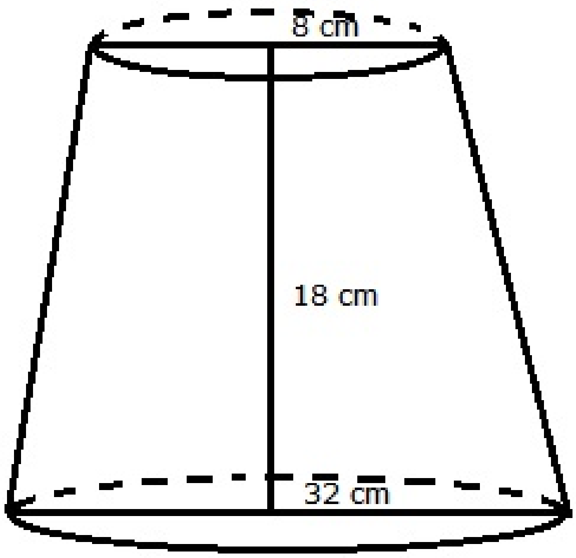
$$l = \frac{7000 \times 32 \times 32}{9} \ cm$$

$$= \frac{70 \times 32 \times 32}{9} m$$

$$= 7964.44 \ m$$

Length of the wire is 7964.44 m

Question 9:



Radii of upper and lower end of frustum are r = 8 cm, R = 32 cm
Height of frustum h = 18 cm

$$\text{Volume of frustum} = \frac{1}{3}\pi h\left[R^2 + r^2 + R \times r\right]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 18 \times \left[32^2 + 8^2 + 32 \times 8\right] cm^3$$

$$= \frac{22 \times 6}{7}\left[1024 + 64 + 256\right] cm^3$$

$$= \frac{132}{7} \times 1344 \ cm^3 = 25344 \ cm^3 = 25.344 \text{ litres}$$

Cost of milk at Rs 20 per litre = Rs. 25.344 × 20 = Rs. 506. 88