Long Answer Questions

Q. 1. Draw the graph showing the variation of binding energy per nucleon with the mass number for a large number of nuclei 2< A < 240. What are the main inferences from the graph? How do you explain the constancy of binding energy in the range 30 < A < 170 using the property that the nuclear force is short-ranged? Explain with the help of this plot the release of energy in the processes of nuclear fission and fusion.
[CBSE (AI) 2010, 2011, Chennai 2015, South 2016]

Ans. The variation of binding energy per nucleon versus mass number is shown in figure. Inferences from graph

(1) The nuclei having mass number below 20 and above 180 have relatively small binding energy and hence they are unstable.

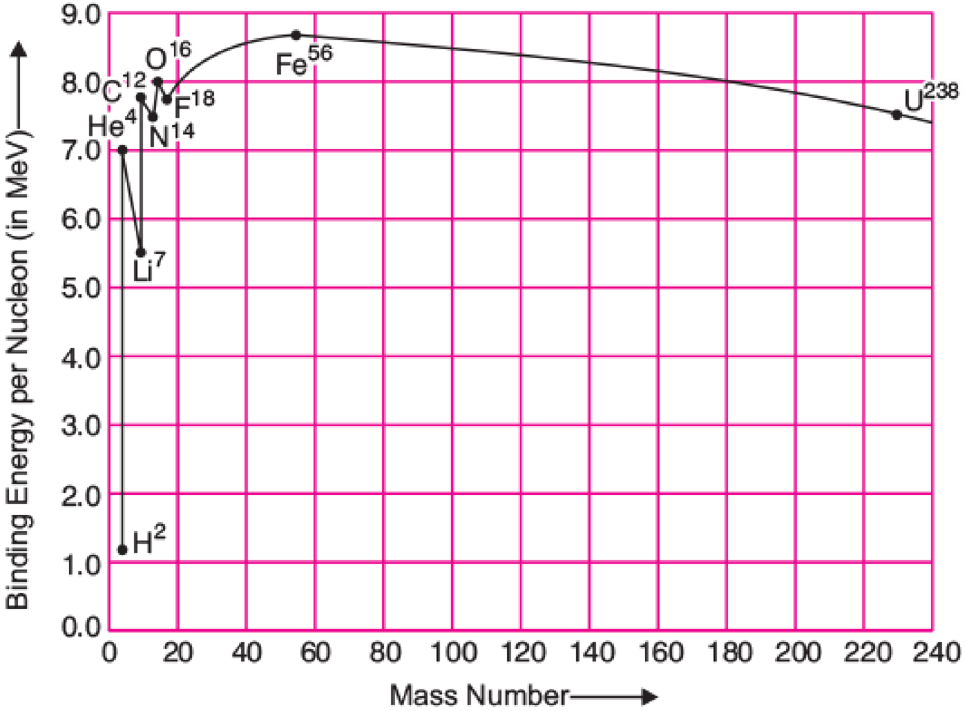
(2) The nuclei having mass number 56 and about 56 have maximum binding energy – 5·8 MeV and so they are most stable.

(3) Some nuclei have peaks, e.g., ${}_{2}^{4}\mathrm{He}$, ${}_{6}^{12}C$, ${}_{6}^{12}O$; this indicates that these nuclei are relatively more stable than their neighbours.

(i) Explanation of constancy of binding energy: Nuclear force is short ranged, so every nucleon interacts with its neighbours only, therefore binding energy per nucleon remains constant.

(ii) Explanation of nuclear fission: When a heavy nucleus (A ≥ 235 say) breaks into two lighter nuclei (nuclear fission), the binding energy per nucleon increases i.e, nucleons get more tightly bound. This implies that energy would be released in nuclear fission.

(iii) Explanation of nuclear fusion: When two very light nuclei (A ≤ 10) join to form a heavy nucleus, the binding is energy per nucleon of fused heavier nucleus more than the binding energy per nucleon of lighter nuclei, so again energy would be released in nuclear fusion.



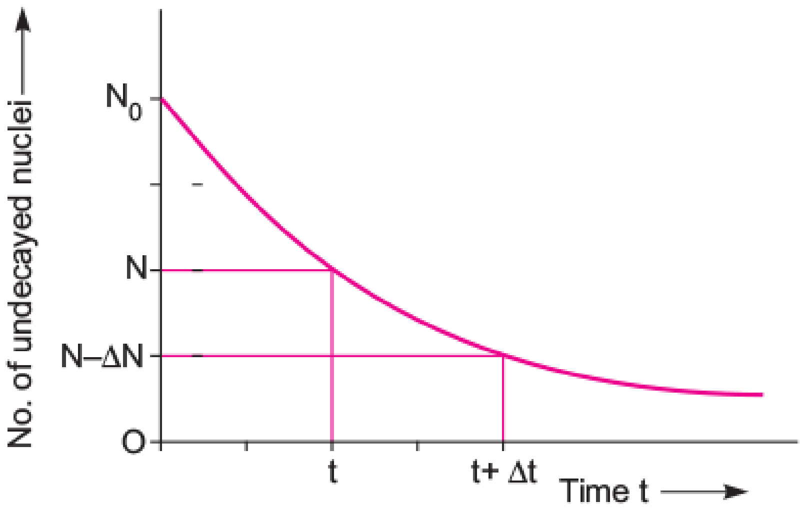
Q. 2. Derive the expression for the law of radiactive decay of a given sample having initially N_0 nuclei decaying to the number N present at any subsequent time t.

Plot a graph showing the variation of the number of nuclei versus the time t lapsed.

Mark a point on the plot in terms of $T_{1/2}$ value when the number present $N = N_0 /16$. [*CBSE Delhi 2014, (F) 2013*]

Ans. Radioactive decay Law: The rate of decay of radioactive nuclei is directly proportional to the number of undecayed nuclei at that time.

Derivation of formula



Suppose initially the number of atoms in radioactive element is N_0 and N the number of atoms after time *t*.

After time *t*, *dN* let be the number of atoms which disintegrate in a short interval *dt* then rate of disintegration will be $\frac{dN}{dt}$ this is also called the activity of the substance/element.

According to Rutherford-Soddy law

$$\frac{dN}{dt} \propto N \qquad \text{or} \qquad \frac{dN}{dt} = -\lambda N \qquad \ldots(i)$$

Where λ is a constant, called decay constant or disintegration constant of the element. Its unit is S^{-1}. Negative sign shows that the rate of disintegration decreases with increase of time. For a given element/substance λ is a constant and is different for different elements. Equation (*i*) may be rewritten as

Integrating $\log_e N = -\lambda t + C$ ...(*ii*)

Where *C* is a constant of integration.

At $t = 0, N = N_0$

∴ $\log_e N_0 = 0 + C \quad \Rightarrow \quad C = \log_e N_0$

∴ Equation (*ii*) gives $\log_e N = -\lambda t + \log_e N_0$

Or $\log_e N - \log_e N_0 = -\lambda t$

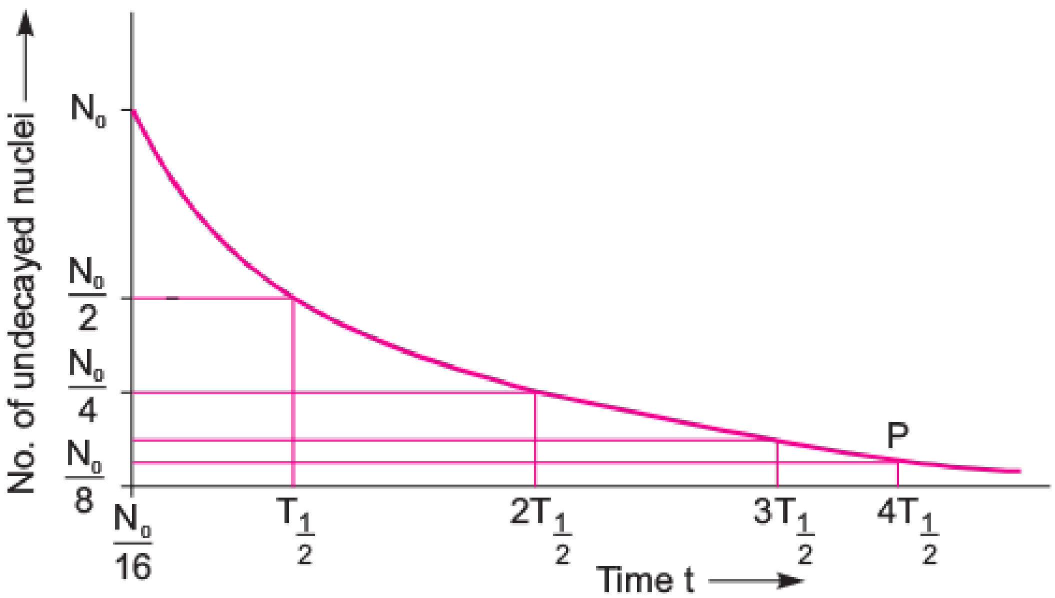
or $\log_e \frac{N}{N_0} = -\lambda t$

or $\frac{N}{N_0} = e^{-\lambda t}$

∴ $N = N_0 e^{-\lambda t}$ ...(iii)

According to this equation, the number of undecayed atoms/nuclei of a given radioactive element decreases exponentially with time (i.e., more rapidly at first and slowly afterwards).

Mark of $N = \frac{N_0}{16}$ in terms of $T_{1/2}$ is shown in fig.



Q. 3. Define the term: Half-life period and decay constant of a radioactive sample. Derive the relation between these terms. [CBSE Patna 2015]

Ans. Half-life period: The half-life period of an element is defined as the time in which the number of radiactive nuclei decay to half of its initial value.

Decay constant: The decay constant of a radioactive element is defined as the reciprocal of time in which the number of undecayed nuclei of that radioactive element falls to $\frac{1}{e}$ times of its initial value.

Relation between Half-life and Decay constant: The radioactive decay equation is

$$N = N_0 e^{-\lambda t} \qquad \ldots(i)$$

when $t = T,\ N = \frac{N_0}{2}$

$\therefore \qquad \frac{N_0}{2} = N_0 e^{-\lambda T}$

or $\qquad e^{-\lambda T} = \frac{1}{2} \qquad \ldots(ii)$

Taking log of both sides

$$-\lambda T \log_e e = \log_e 1 - \log_e 2$$

or $\qquad \lambda T = \log_e 2$

$\therefore \quad T = \frac{\log_e 2}{\lambda} = \frac{2.3026 \log_{10} 2}{\lambda} = \frac{2.3026 \times 0.3010}{\lambda} \qquad \ldots(iii)$

or $\quad T = \frac{0.6931}{\lambda}$

Q. 4. Derive expression for average life of a radio nuclei. Give its relationship with half-life. [CBSE (AI) 2010]

Ans. All the nuclei of a radioactive element do not decay simultaneously; but nature of decay process is statistical, *i.e.*, it cannot be stated with certainty which nucleus will decay when. *The time of decay of a nucleus may be between 0 and infinity. The mean of lifetimes of all nuclei of a radioactive element is called its mean life. It is denoted by* τ.

Expression for mean life

According to Rutherford - Soddy law, rate of decay of a radioactive element

$$R(t) = \left|\frac{dN}{dt}\right| = \lambda N$$

Therefore, the number of nuclei decaying in-between time t and $t + dt$ is

$dN = \lambda N dt$

If N_0 is the total number of nuclei at $t = 0$, then mean lifetime

$$\tau = \frac{\text{Total lifetime of all the nuclei}}{\text{Total number of nuclei}} = \frac{\sum t \ . \ dN}{N_0} = \frac{\sum t \ \lambda Ndt}{N_0}$$

Also we have $N = N_0 e^{-\lambda t}$

$$\therefore \qquad \tau = \frac{\sum t\lambda (N_0 e^{-\lambda t})\, dt}{N_0} = \lambda \sum t \ e^{-\lambda t} \ dt$$

As nuclei decay indefinitely, we may replace the summation into integration with limits from t = 0 to t = ∞ i.e.,

$$\tau = \lambda \int_0^{\infty} t \ e^{-\lambda t} \ dt.$$

Integrating by parts, we get

$$\tau = \lambda \left[\left\{ \frac{te^{-\lambda t}}{-\lambda} \right\}_0^{\infty} - \int_0^{\infty} 1 \left(\frac{e^{-\lambda t}}{-\lambda} \right) dt \right] = \lambda \left[0 + \frac{1}{\lambda} \left\{ \frac{e^{-\lambda t}}{-\lambda} \right\}_0^{\infty} \right]$$

$$= -\frac{1}{\lambda} [e^{-\lambda t}]_0^{\infty} = -\frac{1}{\lambda} [0 - 1] = \frac{1}{\lambda}$$

Thus, $\tau = \frac{1}{\lambda}$

i.e., the **mean lifetime of a radioactive element is reciprocal of its decay constant.**

Relation between mean life and half life

Half life $T = \frac{0.6931}{\lambda}$ ...(*i*)

Mean life $\tau = \frac{1}{\lambda}$ ...(*ii*)

Substituting value of λ from (*ii*) in (*i*), we get

T = 0.6931 τ ...(*iii*)

Q. 5. Answer the following questions [CBSE (F) 2014]

(1) Define the terms (i) half-life ($T_{1/2}$) and (ii) average life (τ). Find out their relationships with the decay constant (λ).

(2) A radioactive nucleus has a decay constant λ = 0.3465 (day)$^{-1}$. How long would it take the nucleus to decay to 75% of its initial amount?

Ans. (1) (i) Half-life ($T_{1/2}$) of a radioactive element is defined as the time taken by a radioactive nuclei to reduce to half of the initial number of radio nuclei.

(ii) Average life of a radioactive element is defined as the ratio of total life time of all radioactive nuclei, to the total number of nuclei in the sample.

Relation between half-life and decay constant is given by $T_{1/2} = \frac{0.693}{\lambda}$

Relation between average life and decay constant $\tau = \frac{1}{\gamma}$.

(2) Let N_0 = the number of radioactive nuclei present initially at time t= 0 in a sample of radioactive substance.

N = the number of radioactive nuclei present in the sample at any instant t.

$$\text{Here, } N = \frac{3}{4} N_0$$

$$\text{From the equation, } N = N_0 e^{-\lambda t}$$

$$\frac{3}{4} N_0 = N_0 e^{-0.3465t} \qquad \Rightarrow \qquad e^{0.3465t} = \frac{4}{3}$$

$$= 2.303\ (0.6020 - 0.4771) = 2.303 \times 0.1249$$

$$\therefore \qquad t = \frac{2.303 \times 0.1249}{0.3465} = 0.83 \text{ day.}$$

Q. 6. Compare and contrast the nature of α-, β- and γ-radiations.

Ans. Comparison of Properties of α-, β- and γ-rays.

	Property	α-particle	β-particle	γ-rays
1.	Nature	Nucleus of Helium	Very fast-moving electron (e^-)	electromagnetic wave of wavelength $\approx 10^{-2}$ Å
2.	Charge	$+2e$	$-e$	No charge
3.	Rest mass	6.6×10^{-27} kg	9.1×10^{-31} kg	zero
4.	Velocity	1.4×10^7 m/s to 2.2×10^7 m/s	0.3 c to 0.98 c	$c = 3 \times 10^8$ m/s
5.	Ionising Power	high, 100 times that of β-particle	100 times more than γ-rays	very small
6.	Penetrating Power	very small	high, 100 times more than α-particles	very high, 100 times more than β-particles

Q. 7. State Soddy-Fajan's displacement laws for radioactive transformations.

Ans. The atoms of radioactive element are unstable. When an atom of a radioactive element disintegrates, an entirely new element is formed. This new element possesses entirely new chemical and radioactive properties. The disintegrating element is called the parent element and the resulting product after disintegration is called the daughter element. Soddy and Fajan studied the successive product elements of disintegration of radioactive elements and gave the following conclusions:

(i) Alpha-Emission: a-particle is nucleus of a helium atom having atomic number 2 and atomic weight 4. It is denoted by $^{2}He_{4}$. Therefore when an α-particle is emitted from a radioactive parent atom (*X*), its atomic number is reduced by 2 and atomic weight is reduced by 4. Thus the daughter element has its place two groups lower in the periodic table. Thus the process of a-emission may be expressed as

$$_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + \underset{(\alpha\text{–particle})}{{}_{2}\mathrm{He}^{4}}$$

Examples:

(i) 92U238 90Th234 + 2He4

(ii) 80Ra226 86Rn222 + 2He4

2. **Beta-Emission:** β-particle is an electron (e) and is denoted by $_{-1}\beta^0$. When a β-particle is emitted from a parent atom (X), it s atomic number increases by 1, while atomic weight remains unchanged. As a result the daughter element (Y) has a place one group higher in the periodic table. Thus the process of β-emission may be expressed as

   $_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + {}_{-1}\beta^0 + \overline{\nu}$

   where $\overline{\nu}$ is a fundamental particle called antineutrino which is massless and chargeless.

   Example: $_{90}Th^{228} \rightarrow {}_{89}Ac^{228} + {}_{-1}b^0 + \overline{\nu}$

3. **Gamma-Emission:** The emission of λ-ray from a radioactive atom neither changes its atomic number nor its atomic weight. Therefore its place in periodic table remains undisplaced. In natural radioactivity λ-radiation is accompanied with either α or β-emission.