10

Thermal Properties of Matter

Multiple Choice Questions (MCQs)

Q. 1 A bimetallic strip is made of aluminium and steel ($\alpha_{Al} > \alpha_{steel}$). On heating, the strip will

(a) remain straight

(b) get twisted

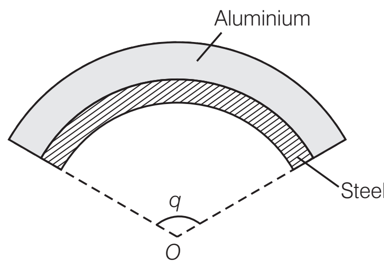
(c) will bend with aluminium on concave side

(d) will bend with steel on concave side

Thinking Process

The metallic strip with higher coefficient of linear expansion (α_{Al}) will expand more.

Ans. *(d)* As $\alpha_{Al} > \alpha$steal, aluminium will expand more. So, it should have larger radius of curvature. Hence, aluminium will be on convex side.



Q. 2 A uniform metallic rod rotates about its perpendicular bisector with constant angular speed. If it is heated uniformly to raise its temperature slightly

(a) its speed of rotation increases

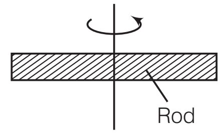
(b) its speed of rotation decreases

(c) its speed of rotation remains same

(d) its speed increases because its moment of inertia increases

Ans. ***(b)*** As the rod is heated, it expands. No external torque is acting on the system so angular momentum should be conserved.

$$L = \text{Angular momentum} = I\omega = \text{constant}$$



$\Rightarrow$ $$I_1\omega_1 = I_2\omega_2$$

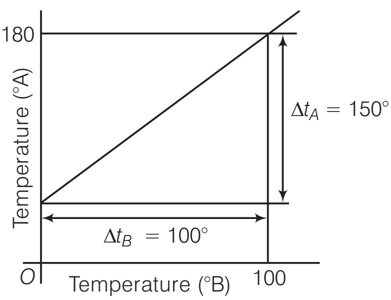
Due to expansion of the rod $I_2 > I_1$

$\Rightarrow$ $$\frac{\omega_2}{\omega_1} = \frac{I_1}{I_2} < 1$$

$\Rightarrow$ $$\omega_2 < \omega_1$$

So, angular velocity (speed of rotation) decreases.

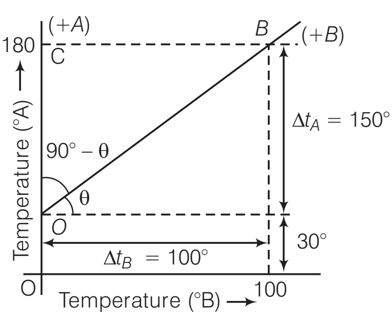
Q. 3 The graph between two temperature scales *A* and *B* is shown in figure between upper fixed point and lower fixed point there are 150 equal division on scale *A* and 100 on scale *B*. The relationship for conversion between the two scales is given by



(a) $\frac{t_A - 180}{100} = \frac{t_B}{150}$ (b) $\frac{t_A - 30}{150} = \frac{t_B}{100}$ (c) $\frac{t_B - 180}{150} = \frac{t_A}{100}$ (d) $\frac{t_B - 40}{100} = \frac{t_A}{180}$

Ans. ***(b)*** It is clear from the graph that lowest point for scale *A* is 30° and lowest point for scale *B* is 0^o. Highest point for the scale *A* is 180^o and for scale B is 100^o. Hence, correct relation is

$$\frac{t_A - (\text{LFP})_A}{(\text{UFP})_A - (\text{LFP})_A} = \frac{t_B - (\text{LFP})_B}{(\text{UFP})_B - (\text{LFP})_B}$$



$\Rightarrow$ $$\frac{t_A - 30}{180 - 30} = \frac{t_B - 0}{100 - 0}$$

$\Rightarrow$ $$\frac{t_A - 30}{150} = \frac{t_B}{100}$$

where, LFP $\rightarrow$ Lower fixed point
UFP $\rightarrow$ Upper fixed point

Q. 4 An aluminium sphere is dipped into water. Which of the following is true?

(a) Buoyancy will be less in water at 0°C than that in water at 4°C

(b) Buoyancy will be more in water at 0°C than that in water at 4°C

(c) Buoyancy in water at 0°C will be same as that in water at 4°C

(d) Buoyancy may be more or less in water at 4°C depending on the radius of the sphere

Thinking Process

Density of water is maximum at 4°C, this is because of anomalous expansion of water.

Ans. *(a)* Let volume of the sphere is V and ρ is its density, then we can write buoyant force

$$F = V\rho G \quad (g = \text{acceleration due to gravity})$$

$$\Rightarrow \quad F \propto \rho \quad (\because V \text{ and } g \text{ are almost constant})$$

$$\Rightarrow \quad \frac{F_{4^\circ C}}{F_{0^\circ C}} = \frac{\rho_{4^\circ C}}{\rho_{0^\circ C}} > 1 \quad (\because \rho_{4^\circ C} > \rho_{0^\circ C})$$

$$\Rightarrow \quad F_{4^\circ C} > F_{0^\circ C}$$

Hence, buoyancy will be less in water at 0°C than that in water at 4°C.

Q. 5 As the temperature is increased, the period of a pendulum

(a) increases as its effective length increases even though its centre of mass still remains at the centre of the bob

(b) decreases as its effective length increases even though its centre of mass still remains at the centre of the bob

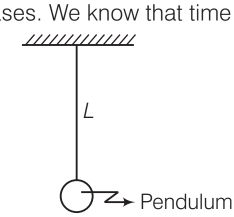
(c) increases as its effective length increases due to shifting to centre of mass below the centre of the bob

(d) decreases as its effective length remains same but the centre of mass shifts above the centre of the bob

Ans. *(a)* As the temperature is increased length of the pendulum increases. We know that time period of pendulum $T = 2\pi\sqrt{\frac{L}{g}}$

$$\Rightarrow \quad T \propto \sqrt{L}, \text{ as } L, \text{ increases.}$$

So, time period (T) also increases.



Q. 6 Heat is associated with

(a) kinetic energy of random motion of molecules

(b) kinetic energy of orderly motion of molecules

(c) total kinetic energy of random and orderly motion of molecules

(d) kinetic energy of random motion in some cases and kinetic energy of orderly motion in other

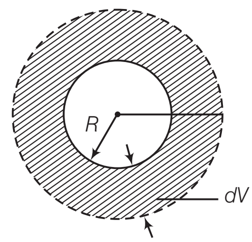
Ans. *(a)* We know that as temperature increases vibration of molecules about their mean position increases hence, kinetic energy associated with random motion of molecules increases.

Q. 7 The radius of a metal sphere at room temperature T is R and the coefficient of linear expansion of the metal is α. The sphere heated a little by a temperature ΔT so that its new temperature is $T + \Delta T$. The increase in the volume of the sphere is approximately.

(a) $2\pi R\alpha\,\Delta T$ (b) $\pi R^2\alpha\,\Delta T$

(c) $4\pi R^3\alpha\,\Delta T/3$ (d) $4\pi R^3\alpha\,\Delta T$

Ans. *(d)* Let the radius of the sphere is R. As the temperature increases radius of the sphere increases as shown.



Original volume $V_o = \frac{4}{3}\pi R^3$

Coefficient of linear expansion $= \alpha$

$\therefore$ Coefficient of volume expansion $= 3\alpha$

$\therefore$ $\frac{1}{V}\frac{dV}{dT} = 3\alpha \Rightarrow dV = 3V\alpha dt \simeq 4\pi R^3\alpha\Delta T$

$=$ Increase in the volume

Q. 8 A sphere, a cube and a thin circular plate, all of same material and same mass are initially heated to same high temperature.

(a) Plate will cool fastest and cube the slowest

(b) Sphere will cool fastest and cube the slowest

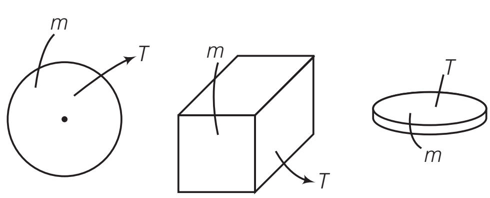
(c) Plate will cool fastest and sphere the slowest

(d) Cube will cool fastest and plate the slowest

Thinking Process

In this problem the cooling will be in the form of radiations that is according to Stefan's law. Since, emissive power directly proportional to surface. Here, for given volume, sphere have least surface area and circular plate of greatest surface area.

Ans. *(c)* Consider the diagram where all the three objects are heated to same temperature T. We know that density, $\rho = \frac{\text{mass}}{\text{volume}}$ as ρ is same for all the three objects hence, volume will also be same.



Sphere Cube Plate

As thickness of the plate is least hence, surface area of the plate is maximum.

We know that, according to Stefan's law of heat loss $H \alpha AT^4$

where, A is surface area of for object and T is temperature.

Hence, $H_{\text{sphere}} : H_{\text{cube}} : H_{\text{plate}}$

$= A_{\text{sphere}} : A_{\text{cube}} : A_{\text{plate}}$

As A_{plate} is maximum.

Hence, the plate will cool fastest.

As, the sphere is having minimum surface area hence, the sphere cools slowest.

Multiple Choice Questions (More Than One Options)

Q. 9 Mark the correct options

(a) A system X is in thermal equilibrium with Y but not with Z. The systems Y and Z may be in thermal equilibrium with each other.

(b) A system X is in thermal equilibrium with Y but not with Z.The systems Y and Z are not in thermal equilibrium with each other.

(c) A system X is neither in thermal equilibrium with Y nor with Z. The systems Y and Z must be in thermal equilibrium with each other.

(d) A system X is neither in thermal equilibrium with Y nor with Z. The systems Y and Z may be in thermal equilibrium with each other.

Ans. ***(b, d)***

According to question

$T_x = T_y$ ($\because$ x and y are in thermal equilibrium)

$T_x \neq T_z$ ($\because$ x is not in thermal equilibrium with z)

Clearly, $T_y \neq T_z$

Hence, y and z are not in thermal equilibrium.

(d) Given, $T_x \neq T_y$

and $T_x \neq T_z$

We cannot say about equilibrium of Y and Z, they may or may not be in equilibrium.

Q. 10 Gulab namuns (assumed to be spherical) are to be heated in an oven. They are available in two sizes, one twice bigger (in radius) than the other. Pizzas (assumed to be discs) are also to be heated in oven. They are also in two sizes, one twice bigger (in radius) than the other. All four are put together to be heated to oven temperature. Choose the correct option from the following.

(a) Both size gulab jamuns will get heated in the same time

(b) Smaller gulab jamuns are heated before bigger ones

(c) Smaller pizzas are heated before bigger ones

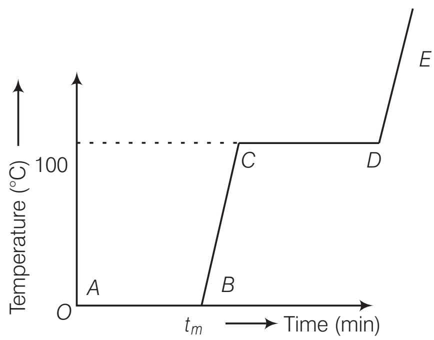
(d) Bigger pizzas are heated before smaller

Ans. ***(b, c)***

Smaller gulab jamuns are having least surface area hence, they will be heated first.
As in case of smaller gulab jamun heat radiates will be less

Similarly, smaller pizzas are heated before bigger ones because they are of small surface areas.

Q. 11 Refer to the plot of temperature *versus* time (figure) showing the changes in the state if ice on heating (not to scale). Which of the following is correct?



(a) The region AB represent ice and water in thermal equilibrium

(b) At B water starts boiling

(c) At C all the water gets converted into steam

(d) C to D represents water and steam in equilibrium at boiling point

Thinking Process

During phase change process, temperature of the system remains constant.

Ans. *(a, d)*

During the process AB temperature of the system is 0°C Hence, it represents phase change that is transformation of ice into water while temperature remains 0°C.

BC represents rise in temperature of water from 0°C to 100°C (at C).

Now, water starts converting into steam which is represent by CD.

Q. 12 A glass full of hot milk is poured on the table. It begins to cool gradually. Which of the following is correct?

(a) The rate of cooling is constant till milk attains the temperature of the surrounding

(b) The temperature of milk falls off exponentially with time

(c) While cooling, there is a flow of heat from milk to the surrounding as well as from surrounding to the milk but the net flow of heat is from milk to the surrounding and that is why it cools

(d) All three phenomenon, conduction, convection and radiation are responsible for the loss of heat from milk to the surroundings

Ans. *(b, c, d)*

When hot milk spread on the table heat is transferred to the surroundings by conduction, convection and radiation.

According to Newton's law of cooling temperature of the milk falls off exponentially. Heat also will be transferred from surroundings to the milk but will be lesser than that of transferred from milk to the surroundings.

Very Short Answer Type Questions

Q. 13 Is the bulb of a thermometer made of diathermic or adiabatic wall?

Ans. As diathermic walls alow exchange of heat energy between two systems and adiabatic walls do not, hence, diathermic walls are used to make the bulb of a thermometer.

Q. 14 A student records the initial length l, change in temperature ΔT and change in length Δl of a rod as follows

S. No.	l (m)	ΔT (°C)	Δl(m)
1.	2	10	4×10^{-4}
2.	1	10	4×10^{-4}
3.	2	20	2×10^{-4}
4.	3	10	6×10^{-4}

If the first observation is correct, what can you say about observation 2, 3 and 4.

Ans. From the 1st observation $\alpha = \frac{\Delta l}{l \Delta T} \Rightarrow \alpha = \frac{4 \times 10^{-4}}{2 \times 10} = 2 \times 10^{-5}\,°C^{-1}$

For 2nd observation $\Delta l = \alpha l \Delta T$

$= 2 \times 10^{-5} \times 1 \times 10 = 2 \times 10^{-4} \text{m} \neq 4 \times 10^{-4} \text{m}$ (Wrong)

For 3rd observation $\Delta l = \alpha l \Delta T$

$= 2 \times 10^{-5} \times 2 \times 20 = 8 \times 10^{-4} \text{m} \neq 2 \times 10^{-4} \text{m}$ (Wrong)

For 4th observation $\Delta l = \alpha l \Delta T$

$= 2 \times 10^{-5} \times 3 \times 10 = 6 \times 10^{-4} \text{m} = 6 \times 10^{-4} \text{m}$

[*i.e.*, observed value (Correct)]

Q. 15 Why does a metal bar appear hotter than a wooden bar at the same temperature? Equivalently it also appears cooler than wooden bar if they are both colder than room temperature.

Thinking Process

According to Kirchhoff's law, good radiator are good absorbers.

Ans. Due to difference in conductivity, metals having high conductivity compared to wood. On touch with a finger, heat from the surrounding flows faster to the finger from metals and so one feels the heat.

Similarly, when one touches a cold metal the heat from the finger flows away to the surroundings faster.

Q. 16 Calculate the temperature which has numeral value on Celsius and Fahrenheit scale.

Ans. Let Q be the value of temperature having same value an Celsius and Fahrenheit scale.

Now, we can write

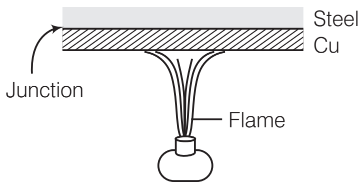
$$\frac{°F - 32}{180} = \frac{°C}{100}$$

$\Rightarrow$ Let $F = C = Q$

$$\Rightarrow \quad \frac{Q - 32}{180} = \frac{Q}{100} = Q = -40°C \text{ or } -40°F$$

Q. 17 **These days people use steel utensiles with copper bottom. This is supposed to be good for uniform heating of food. Explain this effect using the fact that copper is the better conductor.**

Ans. As copper is a good conductor of heat as compared to steel. The steel utensils with copper bottom absorbs heat more quickly than steel and give it to the food in utensil. As a result, of it, the food in utensil is heated uniformly and quickly.



Short Answer Type Questions

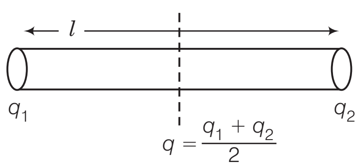
Q. 18 **Find out the increase in moment of inertia I of a uniform rod (coefficient of linear expansion α) about its perpendicular bisector when its temperature is slightly increased by ΔT.**

Thinking Process

As temperature increases length of the rod also increases hence, moment of inertia of the rod also increases.

Ans. Let the mass and length of a uniform rod be M and l respectively.

Moment of inertia of the rod about its perpendicular bisector. $(I) = \frac{Ml^2}{12}$



Increase in length of the rod when temperature is increased by ΔT, is given by

$$\Delta l = l.\alpha\Delta T \quad \ldots(i)$$

$\therefore$ New moment of inertia of the rod $(I') = \frac{M}{12}(l + \Delta l)^2 = \frac{M}{12}(l^2 + \Delta l^2 + 2I\,\Delta l)$

As change in length Δl is very small, therefore, neglecting $(\Delta l)^2$, we get

$$I' = \frac{M}{12}(l^2 + 2l\Delta l)$$

$$= \frac{Ml^2}{12} + \frac{MI\,\Delta l}{6} = I + \frac{MI\,\Delta l}{6}$$

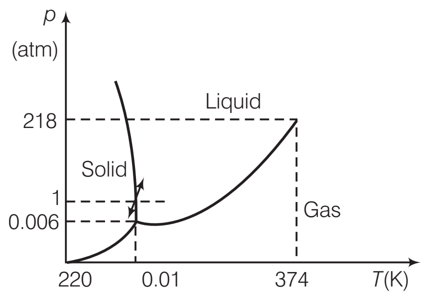
$\therefore$ Increase in moment of inertia

$$\Delta I = I' - I = \frac{Ml\Delta l}{6} = 2 \times \left(\frac{Ml^2}{12}\right)\frac{\Delta l}{l}$$

$$\Delta I = 2 \cdot I \alpha \Delta T \quad \text{[Using Eq. (i)]}$$

Q. 19 During summers in India, one of the common practice to keep cool is to make ice balls of crushed ice, dip it in flavoured sugar syrup and sip it. For this a stick is inserted into crushed ice and is squeezed in the palm to make it into the ball. Equivalently in winter in those areas where it snows, people make snow balls and throw around. Explain the formation of ball out of crushed ice or snow in the light of *p-T* diagram of water.

Ans. Refer to the *p-T* diagram of water and double headed arrow. Increasing pressure at 0°C and 1 atm takes ice into liquid state and decreasing pressure in liquid state at 0°C and 1 atm takes water to ice state.



When crushed ice is squeezed, some of it melts, filling up gap between ice flakes upon releasing pressure. This water freezes, binding all ice flakes and making the ball more stable.

Q. 20 100 g of water is supercooled to $-10°C$. At this point, due to some disturbance mechanised or otherwise some of it suddenly freezes to ice. What will be the temperature of the resultant mixture and how much mass would freeze? $[S_w = 1 \text{cal} / \text{g}/°\text{C}$ and $L^{w}_{\text{Fusion}} = 80 \text{ cal} / \text{g}]$

Ans. Given, mass of water $(m) = 100$

Change in temperature $\Delta T = 0 - (-10) = 10°C$

Specific heat of water $(S_w) = 1$ cal /g/°C

Latent heat of fusion of water $L^{w}_{\text{fusion}} = 80 \text{ cal} / \text{g}$

Heat required to bring water in super cooling from $-10°C$ to $0°C$,

$$Q = ms_w \Delta T$$
$$= 100 \times 1 \times 10 = 1000 \text{ cal}$$

Let m gram of ice be melted.

$\therefore$ $$Q = mL$$

or $$m = \frac{Q}{L} = \frac{1000}{80} = 12.5 \text{ g}$$

As small mass of ice is melted, therefore the temperature of the mixture will remain 0°C.

Note *To find the temperature of the mixture we must go through the two steps* $(Q = ms\,\text{D}T)$ *and* $(Q = mL)$*, we should not directly apply first one.*

Q. 21 One day in the morning. Ramesh filled up 1/3 bucket of hot water from geyser, to take bath. Remaining 2/3 was to be filled by cold water (at room temperature) to bring mixture to a comfortable temperature. Suddenly Ramesh had to attend to something which would take some times, say 5-10 min before he could take bath. Now, he had two options (i) fill the remaining bucket completely by cold water and then attend to the work, (ii) first attend to the work and fill the remaining bucket just before taking bath. Which option do you think would have kept water warmer? Explain

Thinking Process

We should apply logic in this problem in the context of Newton's law of cooling which gives a consequence about rate of fall of temperature of a body with respect to the difference of temperature of body and surroundings.

Ans. The first option would have kept water warmer because according to Newton's law of cooling, the rate of loss of heat is directly proportional to the difference of temperature of the body and the surrounding and in the first case the temperature difference is less, so, rate of loss of heat will be less.

Long Answers Type Questions

Q. 22 We would like to prepare a scale whose length does not change with temperature. It is proposed to prepare a unit scale of this type whose length remains, say 10 cm. We can use a bimetallic strip made of brass and iron each of different length whose length (both components) would change in such a way that difference between their length B remain constant. If $\alpha_{\text{iron}} = 1.2 \times 10^{-5}$ /K and $\alpha_{\text{brass}} = 1.8 \times 10^{-5}$ /K, what should we take as length of each strip?

Ans. According to question $l_{\text{iron}} - l_{\text{brass}} = 10\text{ cm} = \text{constant}$ at all temperatures

Let l_0 be length at temperature 0°C and l be the length after change in temperature of Δt.

Now, we can write $l_{\text{iron}} - l_{\text{brass}} = 10$ cm at all temperatures

$$l_{\text{iron}}(1+\alpha_{\text{iron}}\Delta t) - l_{\text{brass}}(1+\alpha_{\text{brass}}\Delta t) = 10\text{ cm}$$

$$l_{\text{iron}}\ \alpha_{\text{iron}} = l_{\text{brass}}\ \alpha_{\text{brass}}$$

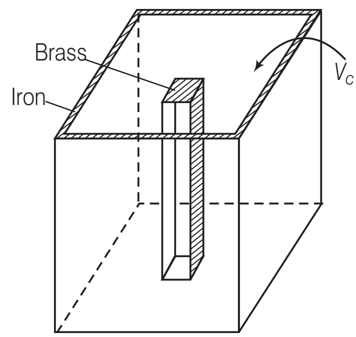
$$\therefore \quad \frac{l_{\text{iron}}}{l_{\text{brass}}} = \frac{1.8}{1.2} = \frac{3}{2}$$

$$\therefore \quad \frac{1}{2} l_{\text{brass}} = 10\text{ cm}$$

$$\Rightarrow \quad l_{\text{brass}} = 20\text{ cm and } l_{\text{iron}}\ 30\text{ cm}$$

Q 23 We would like to make a vessel whose volume does not change with temperature (take a hint from the problem above). We can use brass and iron ($\beta_{vbrass} = 6\times10^{-5}$ /K and $\beta_{viron} = 3.55\times10^{-5}$/K) to create a volume of 100 cc. How do you think you can achieve this?

Ans. In the previous problem the difference in the length was constant.
In this problem the difference in volume is constant.
The situation is shown in the diagram.



Let V_{io}, V_{bo} be the volume of iron and brass vessel at 0°C
V_i, V_b be the volume of iron and brass vessel at $\Delta\theta$°C,
γ_i, γ_b be the coefficicnt, of volume expansion of iron and brass.

As per question, $\quad V_{io} - V_{bo} = 100\text{ cc} = V_i - V_b \quad$ ...(i)

Now,

$$V_i = V_{io}(1 + \gamma_i \Delta\theta)$$
$$V_b = V_{bo}(1 + \gamma_b \Delta\theta)$$
$$V_i - V_b = (V_{io} - V_{bo}) + \Delta\theta(V_{io}\gamma_i - V_{bo}\gamma_b)$$

Since, $V_i - V_b$ = constant.

So,

$$V_{io}\gamma_i = V_{bo}\gamma_b$$

$$\Rightarrow \quad \frac{V_{io}}{V_{bo}} = \frac{\gamma_b}{\gamma_i} = \frac{\frac{3}{2}\beta_b}{\frac{3}{2}\beta_i} = \frac{\beta_b}{\beta_i} = \frac{6\times10^{-5}}{3.55\times10^{-5}} = \frac{6}{3.55}$$

$$\frac{V_{io}}{V_{bo}} = \frac{6}{3.55} \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$V_{io} = 244.9 \text{ cc}$$
$$V_{bo} = 144.9 \text{ cc}$$

Q. 24 Calculate the stress developed inside a tooth cavity filled with copper when hot tea at temperature of 57°C is drunk. You can take body (tooth) temperature to be 37°C and $\alpha = 1.7\times10^{-5}$/°C bulk modulus for copper $= 140\times10^{9}$ N/m^2.

Ans. Given, decrease in temperature (Δt) = 57 − 37 = 20°C
Coefficient of linear expansion (α) = 1.7×10^{-5}/°C
Bulk modulus for copper (B) = 140×10^{9} N/m^2
Coefficient of cubical expansion (γ) $= 3\alpha = 5.1\times10^{-5}$/°C
Let initial volume of the cavity be V and its volume increases by ΔV due to increase in temperature.

$$\therefore \qquad \Delta V = \gamma V \Delta t$$

$$\Rightarrow \qquad \frac{\Delta V}{V} = \gamma \Delta t \qquad \text{...(i)}$$

Thermal stress produced = $B \times$ Volumetric strain

$$= B \times \frac{\Delta V}{V} = B \times \gamma \Delta t$$

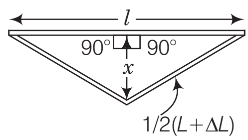
$$= 140 \times 10^9 \times (5.1 \times 10^{-5} \times 20)$$

$$= 1,428 \times 10^8 \text{ N/m}^2$$

This is about 10^3 times of atmospheric pressure.

Q. 25 A rail track made of steel having length 10 m is clamped on a railway line at its two ends (figure). On a summer day due to rise in temperature by 20° C. It is deformed as shown in figure. Find x (displacement of the centre) if $\alpha_{steel} = 1.2 \times 10^{-5}/° \text{C}$.

Ans. Consider the diagram.



Applying Pythagorus theorem in right angled triangle in figure.

$$\left(\frac{L + \Delta L}{2}\right)^2 = \left(\frac{L}{2}\right)^2 + x^2$$

$$\Rightarrow \qquad x = \sqrt{\left(\frac{L + \Delta L}{2}\right)^2 - \left(\frac{L}{2}\right)^2}$$

$$= \frac{1}{2}\sqrt{(L + \Delta L)^2 - L^2}$$

$$= \frac{1}{2}\sqrt{(L^2 + \Delta L^2 + 2L\Delta L) - L^2}$$

$$= \frac{1}{2}\sqrt{(\Delta L^2 + 2L\Delta L)}$$

As increase in length ΔL is very small, therefore, neglecting $(\Delta L)^2$, we get

$$x = \frac{1}{2} \times \sqrt{2L\Delta L} \qquad \text{...(i)}$$

But $\qquad \Delta L = L\alpha\Delta t \qquad$ ...(ii)

Substituting value of ΔL in Eq. (i) from Eq. (ii)

$$x = \frac{1}{2}\sqrt{2L \times L \alpha \Delta t} = \frac{1}{2} L\sqrt{2\alpha \Delta t}$$

$$= \frac{10}{2} \times \sqrt{2 \times 1.2 \times 10^{-5} \times 20}$$

$$= 5 \times \sqrt{4 \times 1.2 \times 10^{-4}}$$

$$= 5 \times 2 \times 1.1 \times 10^{-2} = 0.11 \text{m} = 11 \text{cm}$$

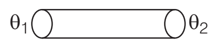
Note *Here we have assumed ΔL to be very small so that it can be neglected compared to L.*

Q. 26 A thin rod having length L_0 at 0°C and coefficient of linear expansion α has its two ends maintained at temperatures θ_1 and θ_2, respectively. Find its new length.

Thinking Process

When temperature of a rod varies linearly, temperature of the middle point of the rod can be taken as mean of temperatures at the two ends.

Ans. Consider the diagram



$$\theta = \frac{\theta_1 + \theta_2}{2}$$

Let temperature varies linearly in the rod from its one end to other end. Let θ be the temperature of the mid-point of the rod. At steady state,

Rate of flow of heat,

$$\left(\frac{dQ}{dt}\right) = \frac{KA(\theta_1 - \theta)}{(L_0/2)} = \frac{KA(\theta - \theta_2)}{(L_0/2)}$$

where, K is coefficient of thermal conductivity of the rod.

or ⇒ $$\theta_1 - \theta = \theta - \theta_2$$

or ⇒ $$\theta = \frac{\theta_1 + \theta_2}{2}$$

Using relation, $$L = L_0(1 + \alpha\,\theta)$$

or $$L = L_0\left[1 + \alpha\left(\frac{\theta_1 + \theta_2}{2}\right)\right]$$

Q. 27 According to Stefan's law of radiation, a black body radiates energy σT^4 from its unit surface area every second where T is the surface temperature of the black body and $\sigma = 5.67 \times 10^{-8}$ W/ m^2K^4 is known as Stefan's constant. A nuclear weapon may be thought of as a ball of radius 0.5 m. When detonated, it reaches temperature of 10^6K and can be treated as a black body.

(a) Estimate the power it radiates.

(b) If surrounding has water at 30°C, how much water can 10% of the energy produced evaporate in 1 s?

[S_w = 4186.0J / kg K and $L_v = 22.6 \times 10^5$ J / kg]

(c) If all this energy U is in the form of radiation, corresponding momentum is $p = U/c$. How much momentum per unit time does it impart on unit area at a distance of 1 km?

Ans. Given, $\sigma = 5.67 \times 10^{-8}$ W/m² kg

Radius, $= R = 0.5$ m, $T = 10^6$ K

(a) Power radiated by Stefan's law

$$P = \sigma A T^4 = (4\pi R^2) T^4$$
$$= (5.67 \times 10^{-4} \times 4 \times (3.14) \times (0.5)^2 \times (10^6)\ 4$$
$$= 1.78 \times 10^{17}\ \text{J/s} = 1.8 \times 10^{17}\ \text{J/s}$$

(b) Energy available per second, $U = 1.8 \times 10^{17}\ \text{J/s} = 18 \times 10^{16}\ \text{J/s}$

Actual energy required to evaporate water = 10% of 1.8×10^{17} J/s

$$= 1.8 \times 10^{16}\ \text{J/s}$$

Energy used per second to raise the temperature of m kg of water from 30°C to 100°C and then into vapour at 100°C

$$= m s_w\ \Delta\theta + m\,L_v = m \times 4186 \times (100 - 30) + m \times 22.6 \times 10^5$$
$$= 2.93 \times 10^5\ m + 22.6 \times 10^5\ m = 25.53 \times 10^5\ m\ \text{J/s}$$

As per question, $25.53 \times 10^5\ m = 1.8 \times 10^{16}$

or $$m = \frac{1.8 \times 10^{16}}{25.33 \times 10^5} = 7.0 \times 10^9\ \text{kg}$$

(c) Momentum per unit time,

$$p = \frac{U}{c} = \frac{U}{c} = \frac{1.8 \times 10^{17}}{3 \times 10^8} = 6 \times 10^8\ \text{kg-m/s}^2 \qquad \begin{bmatrix} P = \text{momentum} \\ V = \text{energy} \\ C = \text{velocity of Light} \end{bmatrix}$$

Momentum per unit time per unit

$$\text{area} \quad p = \frac{p}{4\pi R^2} = \frac{6 \times 10^8}{4 \times 3.14 \times (10^3)^2}$$

$$\Rightarrow \qquad d = 47.7\ \text{N/m}^2 \qquad [4\pi R^2 = \text{Surface area}]$$