Complex Numbers and Quadratic Equations

Complex Numbers

- The square root of −1 is represented by the symbol ***i***. It is read as iota.
  $i = \sqrt{-1}$ or $i^2 = -1$

- Any number of the form $a + ib$, where a and b are real numbers, is known as a complex number. A complex number is denoted by z.
  $z = a + ib$

- For the complex number $z = a + ib$, a is the real part and b is the imaginary part. The real and imaginary parts of a complex number are denoted by Re z and Im z respectively.

- For complex number $z = a + ib$, Re $z = a$ and Im $z = b$

- A complex number is said to be purely real if its imaginary part is equal to zero, while a complex number is said to be purely imaginary if its real part is equal to zero.

- For e.g., 2 is a purely real number and $3i$ is a purely imaginary number.

- Two complex numbers are equal if their corresponding real and imaginary parts are equal.

- Complex numbers $z_1 = a + ib$ and $z_2 = c + id$ are equal if $a = c$ and $b = d$.

- Let's now try and solve the following puzzle to check whether we have understood this concept.

Solved Examples

Example 1: Verify that each of the following numbers is a complex number.

$3+\sqrt{-7}, \sqrt{2}+\sqrt{5}$ and $1-5i$

Solution:

$3+\sqrt{-7}$ can be written as $3+i\sqrt{7}$, which is of the form $a + ib$. Thus, $3+\sqrt{-7}$ is a complex number.

$\sqrt{2}+\sqrt{5}$ is not of the form $a + ib$. But it is known that every real number is a complex number.

Thus, $\sqrt{2}+\sqrt{5}$ is a complex number.

$1 - 5i$ is of the form $a + ib$. Thus, $1 - 5i$ is a complex number.

Example 2: What are the real and imaginary parts of the complex number $-\sqrt{11}-\sqrt{-23}$?

Solution:

The complex number $-\sqrt{11}-\sqrt{-23}$ can be written as $-\sqrt{11}-i\sqrt{23}$, which is of the form $a + ib$.

Re $z = a =$ $-\sqrt{11}$ and Im $z = b = -$ $\sqrt{23}$

Example 3: For what values of x and y, $z_1 = (x + 1) - 10i$ and $z_2 = 19 + i(y - x)$ represent equal complex numbers?

Solution:

Two complex numbers are equal if their corresponding real and imaginary parts are equal.

For the given complex numbers,

$x + 1 = 19$ and $y - x = -10$

$\Rightarrow x = 18$ and $y - 18 = -10$

$\Rightarrow x = 18$ and $y = 8$

Thus, the values of x and y are 18 and 8 respectively.

Addition and Subtraction of Complex Numbers

- The addition of two complex numbers $z_1 = a + ib$ and $z_2 = c + id$ is defined as

  $z_1 + z_2 = (a + c) + i(b + d)$
  For example: $(4 + 3i) + (-2 + 6i) = (4 - 2) + i(3 + 6) = 2 + 9i$

- Several properties are exhibited by the addition of complex numbers.

- **Closure law**
  The addition of complex numbers satisfies closure property i.e., the sum of two complex numbers is a complex number.
  If z_1 and z_2 are any two complex numbers, then $z_1 + z_2$ is also a complex number.

- **Commutative law**
  The commutative law holds for the addition of complex numbers.
  If z_1 and z_2 are any two complex numbers, then $z_1 + z_2 = z_2 + z_1$.
  For example: $z_1 = 3 + 2i$ and $z_2 = -5 + 4i$
  $z_1 + z_2 = (3 + 2i) + (-5 + 4i) = -2 + 6i$
  $z_2 + z_1 = (-5 + 4i) + (3 + 2i) = -2 + 6i$
  $\therefore z_1 + z_2 = z_2 + z_1$

- **Associative law**
  The associative law holds for the addition of complex numbers.
  If z_1, z_2 and z_3 are any three complex numbers, then
  $(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$

- **Additive identity**
  The complex number $(0 + i0)$ is the additive identity. It is denoted by 0.
  For every complex number z, $z + 0 = z$

- **Additive inverse**
  The complex number $\{-a + i(-b)\}$is the additive inverse of the complex number $z = a + ib$.
  The inverse of a complex number z is denoted by $-z$.
  Also, $z + (-z) = 0$.
  For example: The inverse of the complex number $7 - 3i$ is $-7 + 3i$.

Difference of Complex Numbers

- The difference of complex numbers $z_1 = a + ib$ and $z_2 = c + id$ is defined as
  $z_1 - z_2 = z_1 + (-z_2) = (a + ib) + \{-(c + id)\}$
  $= (a + ib) + \{-c - id)\}$
  $= (a - c) + i(b - d)$
  For example: Let $z_1 = -1 + 3i$ and $z_2 = 7 + 4i$
  $z_1 - z_2 = (-1 + 3i) - (7 + 4i) = (-1 - 7) + i(3 - 4) = -8 - i$

- **Closure law**
  The difference of complex numbers satisfies the closure property i.e., the difference of two complex numbers is a complex number.
  If z_1 and z_2 are any two complex numbers, then $z_1 - z_2$ is also a complex number.

Solved Examples

Example1: If $Z_1 = 3 - i$ and $Z_2 = 1 + 2i$, then write the complex number $(Z_1 + 2Z_2 - 4)$ in the form $a + ib$ and determine the values of a and b.

Solution:

We have $Z_1 = 3 - i$ and $Z_2 = 1 + 2i$

$Z_1 + 2Z_2 - 4 = (3 - i) + 2\,(1 + 2i) - 4$

$= 3 - i + 2 + 4i - 4$

$= 1 + 3i$

Which is of the form $a + ib$

$\therefore a = 1$ and $b = 3$

Example 2: What is the additive inverse of $\left(-1+i\sqrt{3}\right)$?

Solution:

Let $Z = -1 + i\sqrt{3}$

Additive inverse of $Z = -(-1 + i\sqrt{3}) = 1 - i\sqrt{3}$

Multiplication of Complex Numbers

Multiplication of Complex Numbers and Their Properties

- The multiplication of two complex numbers $z_1 = a + ib$ and $z_2 = c + id$ is defined as

$z_1 z_2 = (a + ib) \times (c + id)$

$= a(c + id) + ib(c + id)$

$= ac + iad + ibc + i^2bd$ $\left[\because i = \sqrt{-1} \Rightarrow i^2 = -1\right]$

$= (ac - bd) + i\,(ad + bc)$

$\therefore \boxed{z_1 z_2 = (ac - bd) + i(ad + bc)}$

- For example:

  Let $z_1 = 1 + 2i$ and $z_2 = -3 + 4i$

  $z_1\, z_2 = (-3 - 8) + i\,\{4 + (-6)\} = -11 + i\,(-2) = -11 - 2i$

- The multiplication of complex numbers satisfies the following properties:

- **Closure law**

  The product of two complex numbers is a complex number.

  If z_1 and z_2 are any two complex numbers, then $z_1 z_2$ is a complex number.

- **Commutative law**

  Commutative law holds for the product of complex numbers i.e., for any two complex numbers z_1 and z_2, $z_1 z_2 = z_2 z_1$

- **Associative law**

  Associative law holds for the product of complex numbers.

  For any three complex numbers z_1, z_2 and z_3: $(z_1 z_2)\, z_3 = z_1\,(z_2 z_3)$

- **Distributive law**

  For any three complex numbers z_1, z_2 and z_3:

  $$z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$$
  $$(z_1 + z_2) z_3 = z_1 z_3 + z_2 z_3$$

- **Multiplicative identity**

  The complex number $1 + i0$ is the multiplicative identity of the complex number. It is denoted by 1. For any complex number z, $z \times 1 = z$.

- **Multiplicative inverse**

The complex number z_2 is said to be the multiplicative inverse of the complex number z_1 if $z_1 z_2 = 1$ (1 is the multiplicative identity). The multiplicative inverse of a complex number z is denoted by z^{-1}.

$$\because z \times \frac{1}{z} = 1$$

$\therefore \frac{1}{z}$ is the multiplicative inverse of z.

Multiplicative inverse of the complex number $z = a + ib$ is given by

$$z^{-1} = \frac{1}{z} = \frac{a}{a^2 + b^2} + i\frac{(-b)}{a^2 + b^2}$$

Powers of *i*

- $i = \sqrt{-1}$

  $i^2 = -1$

  $i^3 = i^2 \times i = (-1) \times i = -i$

  $i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$

  $i^5 = i^4 \times i = 1 \times i = i$
  $i^6 = i^4 \times i^2 = 1 \times -1 = -1$

  And so on…

- In general, we can write

  $i^{4k} = 1$
  $i^{4k+1} = i$
  $i^{4k+2} = -1$
  $i^{4k+3} = -i$

  Where k is any integer

For example: $i^{39} = i^{36+3} = i^{4\times 9+3}$

It is of the form i^{4k+3}, where $k = 9$

$\therefore i^{39} = -i$

Solved Examples

Example 1 Simplify the following:

$$\left[(-i)^{17} + \left(\frac{1}{i}\right)^{8}\right]$$

Solution:

$$\begin{aligned}
&(-i)^{17} + \left(\frac{1}{i}\right)^{8} \\
&= \left[(-1)\times i\right]^{17} + \left(\frac{1}{i}\right)^{8} \\
&= (-1)^{17}(i)^{17} + \left(i^{-1}\right)^{8} \\
&= -(i)^{16+1} + \left(i^{8}\right)^{-1} \\
&= -i + \left[i^{4\times 2}\right]^{-1} && \left[\because i^{4k+1} = i\right] \\
&= -i + (1)^{-1} && \left[\because i^{4k} = 1\right] \\
&= -i + 1 \\
&= (1-i)
\end{aligned}$$

Example 2 If $x + iy = (2 + 5i)(7 + i)$, then what are the values of x and y?

Solution:

$$\begin{aligned}
x + iy &= (2+5i)(7+i) \\
x + iy &= (2\times 7 - 5\times 1) + i(2\times 1 + 5\times 7) \\
x + iy &= (14-5) + i(2+35) \\
x + iy &= 9 + i(37)
\end{aligned}$$

On equating the real and imaginary parts, we obtain

x = 9 and y = 37

Example 3 What is the value of $\left(\sqrt{-25}\right)\left(\sqrt{-\frac{8}{49}}\right)$?

Solution:

We know that $\sqrt{-1}=i$

$$\begin{aligned}
\therefore \left(\sqrt{-25}\right)\left(\sqrt{\frac{-8}{49}}\right) &= \left(\sqrt{25}\times\sqrt{-1}\right)\left(\sqrt{-1}\sqrt{\frac{8}{49}}\right)\\
&= (5i)\left(\frac{2\sqrt{2}i}{7}\right)\\
&= \left(5\times\frac{2\sqrt{2}}{7}\right)i\times i\\
&= \frac{10\sqrt{2}}{7}i^2\\
&= \frac{10\sqrt{2}}{7}(-1) \qquad \left[\because i^2=-1\right]\\
&= \frac{-10\sqrt{2}}{7}
\end{aligned}$$

Note: Students may make mistakes while solving this question.

We know that $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$. However, when a and b are both negative, then $\sqrt{a}\times\sqrt{b}\neq\sqrt{ab}$.

Hence, this question cannot be solved as

$$\begin{aligned}
\left(\sqrt{-25}\right)\left(\sqrt{\frac{-8}{49}}\right) &= \sqrt{(-25)\left(-\frac{8}{49}\right)}\\
&= \sqrt{\frac{25\times 8}{49}}\\
&= \frac{10\sqrt{2}}{7}
\end{aligned}$$

Example 4 What is the multiplicative inverse of $5-9i$?

Solution:

Let $z = a + ib = 5 - 9i$

Accordingly, $a = 5$ and $b = -9$

We know that

$$z^{-1} = \frac{a}{a^2+b^2} + i\frac{-b}{a^2+b^2}$$
$$= \frac{5}{5^2+(-9)^2} + i\frac{9}{5^2+(-9)^2}$$
$$= \frac{5+9i}{106}$$

Thus, $\frac{5+9i}{106}$ is the multiplicative inverse of $5 - 9i$.

Division of Complex Numbers

- The division of two complex numbers z_1 and z_2 can be defined as $\frac{z_1}{z_2} = z_1 \times \frac{1}{z_2}$, where $\frac{1}{z_2}$ is the multiplicative inverse of z_2.

$$\frac{z_1}{z_2} = z_1 \times \text{ multiplicative inverse of } z_2$$

- To find the quotient of two complex numbers, find the product of the first number with the multiplicative inverse of the second number.

For example: If $z_1 = 1 + i$ and $z_2 = 2 - 3i$, then $\frac{z_1}{z_2} = \frac{1+i}{2-3i} = (1+i)\left(\frac{1}{2-3i}\right)$

We know that the multiplicative inverse of the complex number $z = a + ib$ is given by $\frac{1}{a+ib} = \frac{a}{a^2+b^2} + i\frac{(-b)}{a^2+b^2}$

$$\therefore \frac{1}{2-3i} = \frac{2}{2^2+(-3)^2} + i\frac{3}{2^2+(-3)^2} = \frac{2}{13} + i\frac{3}{13}$$

$$\text{Now, } \frac{z_1}{z_2} = (1+i)\left(\frac{2}{13}+i\frac{3}{13}\right) = \left(\frac{2}{13}-\frac{3}{13}\right)+i\left(\frac{2}{13}+\frac{3}{13}\right) = \frac{-1}{13}+i\frac{5}{13}$$

Solved Examples

Example 1 Write the complex number $\frac{2+\sqrt{3}i}{1-\sqrt{3}i}$ in the form of *a* + *ib*.

Solution:

$$\frac{2+\sqrt{3}i}{1-\sqrt{3}i} = \frac{2+\sqrt{3}i}{1-\sqrt{3}i} \times \frac{1+\sqrt{3}i}{1+\sqrt{3}i}$$

$$= \frac{\left(2+\sqrt{3}i\right)\left(1+\sqrt{3}i\right)}{\left(1-\sqrt{3}i\right)\left(1+\sqrt{3}i\right)}$$

$$= \frac{2+2\sqrt{3}i+\sqrt{3}i+3i^2}{(1)^2-\left(\sqrt{3}i\right)^2}$$

$$= \frac{2+2\sqrt{3}i+\sqrt{3}i+3\times(-1)}{1-3i^2} \qquad \left[i^2=-1\right]$$

$$= \frac{2+3\sqrt{3}i-3}{1-3\times(-1)} \qquad \left[i^2=-1\right]$$

$$= \frac{-1+3\sqrt{3}i}{1+3}$$

$$= \frac{-1+3\sqrt{3}i}{4}$$

$$= \frac{-1}{4}+\frac{3\sqrt{3}i}{4}$$

Identities of Complex Numbers

The identities for complex numbers are the same as the algebraic identities for real numbers. The identities which hold for complex numbers are as follows:

- $(z_1 + z_2)^2 = z_1^2 + z_2^2 + 2z_1z_2$
- $(z_1 - z_2)^2 = z_1^2 + z_2^2 - 2z_1z_2$
- $z_1^2 - z_2^2 = (z_1 + z_2)(z_1 - z_2)$

- $(z_1 + z_2)^3 = z_1^3 + z_2^3 + 3z_1^2z_2 + 3z_1z_2^2$
- $(z_1 - z_2)^3 = z_1^3 - z_2^3 - 3z_1^2z_2 + 3z_1z_2^2$

Modulus and Conjugate of a Complex Number

Modulus of a Complex Number

- The modulus of a complex number z = $a + ib$ is denoted by $|z|$ and defined as $|z| = \sqrt{a^2 + b^2}$.
- For example: The modulus of the complex number $z = 1 - \sqrt{3}i$ is $|z| = \sqrt{(1)^2 + \left(-\sqrt{3}\right)^2} = \sqrt{1+3} = \sqrt{4} = 2$
- The following results hold true for two complex numbers z_1 and z_2.
- $|z_1 z_2| = |z_1||z_2|$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, provided $|z_2| \neq 0$

Conjugate of a Complex Number

- The conjugate of a complex number $z = a + ib$ is denoted by $\bar{z}$ and defined as $\bar{z} = a - ib$.
- For example: The conjugate of the complex number $2 + \sqrt{-5}$ is $\bar{z} = 2 - \sqrt{-5} = 2 - i\sqrt{5}$
- The following results hold true for two complex numbers z_1 and z_2.
- $\overline{z_1 z_2} = \bar{z}_1 \, \bar{z}_2$
- $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$

- $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$
- $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$, provided $z_2 \neq 0$
- The modulus of a complex number and the modulus of its conjugate are equal.
  $|z| = |\overline{z}|$

Relation of Multiplicative Inverse with Modulus and Conjugate of a Complex Number

- The multiplicative inverse of a complex number $z = a + ib$ is given by
  $z^{-1} = \frac{a}{a^2 + b^2} + i\frac{-b}{a^2 + b^2}$
  $\Rightarrow z^{-1} = \frac{a - ib}{a^2 + b^2}$
  $\overline{z} = a - ib$ is the conjugate and $|z| = \sqrt{a^2 + b^2}$ is the modulus of the complex number z.
  $\therefore z^{-1} = \frac{\overline{z}}{|z|^2}$
  Or $z\overline{z} = |z|^2$ $\left(\because z^{-1} = \frac{1}{z}\right)$

  This is the required relation.

Solved Examples

Example 1: Determine the conjugate and multiplicative inverse of $3 + \sqrt{7}i$.

Solution:

Let $z = 3 + \sqrt{7}i$

Accordingly, conjugate, $\overline{z} = 3 - \sqrt{7}i$ and $|z|^2 = (3)^2 + \left(\sqrt{7}\right)^2 = 9 + 7 = 16$

Now, the multiplicative inverse is given by $z^{-1} = \frac{\overline{z}}{|z|^2}$

$z^{-1} = \frac{3-\sqrt{7}i}{16}$

Example 2: What is the conjugate of $\frac{(5+i)(1+2i)}{(3-4i)(1+i)}$?

Solution:

Let z = $\frac{(5+i)(1+2i)}{(3-4i)(1+i)}$

In order to find the conjugate of *z*, we first write it in the form of *a* + *ib*.

$z = \frac{3+11i}{7-i}$ (By the multiplication of complex numbers)

On multiplying the numerator and the denominator with $(7+i)$, we obtain

$$z = \frac{(3+11i)\times(7+i)}{(7-i)\times(7+i)}$$
$$= \frac{10+80i}{49+1}$$
$$= \frac{10+80i}{50}$$
$$= \frac{1+8i}{5}$$

Now, $\bar{z} = \frac{1-8i}{5}$

Thus, the conjugate of the given complex number is $\frac{1-8i}{5}$.

Example 3: What is the modulus of $z = (1+i)^{10}$?

Solution:

Modulus, $|z| = \left|(1+i)^{10}\right|$

It can be written as

$$|z| = \left|(1+i)(1+i)^9\right|$$

$$|z| = |(1+i)|\left|(1+i)^9\right| \qquad (\because |z_1 z_2| = |z_1||z_2|)$$

Continuing in this manner, we can write

$$|z| = |(1+i)||(1+i)|\ldots|(1+i)| \qquad (10 \text{ times})$$

$$|z| = |(1+i)|^{10}$$

Now, $|(1+i)| = \sqrt{1^2+1^2} = \sqrt{2}$

$$|z| = \left(\sqrt{2}\right)^{10} = 2^5$$

Quadratic Equations with Complex Roots

- Complex numbers are used for finding the roots of a quadratic equation whose discriminant is negative.
- The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by

  $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $b^2 - 4ac$ is the discriminant of the quadratic equation
- If the discriminant i.e., the value under the square root is negative, then the roots of the quadratic equation will be complex numbers.
-
- For example: For the equation $3x^2 + 7x + 6 = 0$, $a = 3$, $b = 7$ and $c = 6$

  ∴Discriminant = $b^2 - 4ac = (7)^2 - 4(3)(6) = 49 - 72 = -23$

  Thus, the roots of the quadratic equation are complex numbers.

Example 1 Solve the quadratic equation $x^2 - 2\sqrt{3}x + \sqrt{3} + 4 = 0$.

Solution:

The given quadratic equation is $x^2 - 2\sqrt{3}x + \sqrt{3} + 4 = 0$.

The discriminant of this equation is

$$b^2 - 4ac = \left(-2\sqrt{3}\right)^2 - 4(1)\left(\sqrt{3}+4\right) = 12 - 4\sqrt{3} - 16 = -\left(4+4\sqrt{3}\right)$$

Thus, the solution of the given equation is

$$\frac{-\left(-2\sqrt{3}\right) \pm \sqrt{-\left(4+4\sqrt{3}\right)}}{2} = \frac{2\sqrt{3} \pm \sqrt{4+4\sqrt{3}}\, i}{2}$$

Example 2 If the roots of the quadratic equation $ax^2 + bx + c = 0$ are imaginary, then what can we say about the signs of a and c?

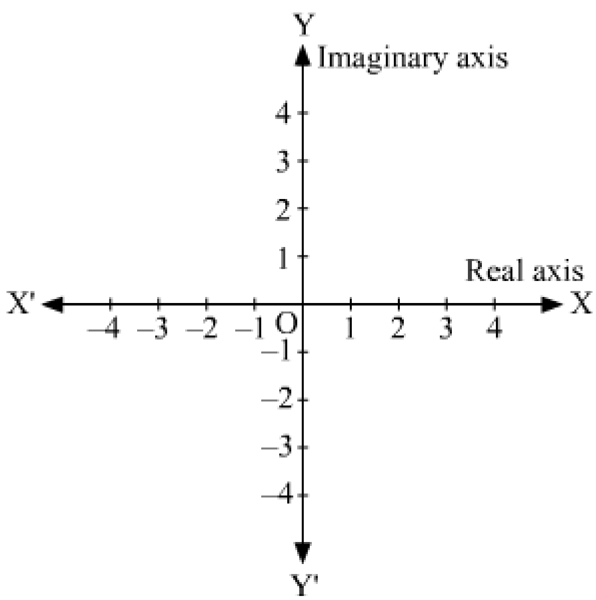
Solution:

The roots of quadratic equation $ax^2 + bx + c = 0$ are imaginary if the discriminant $b^2 - 4ac < 0$.

Here, b^2 is always positive whatever the sign of b is. Hence, the discriminant is negative if the product ac is positive. Thus, a and c must have the same signs.

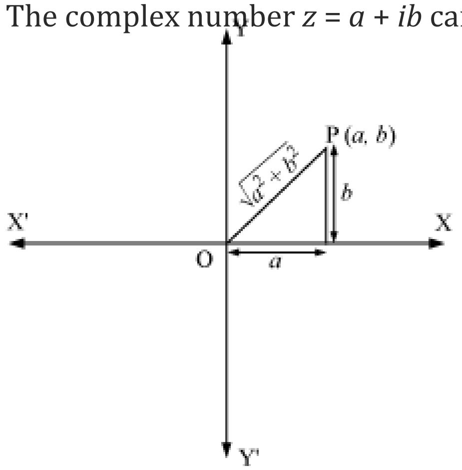
Concept of Argand Plane

Each complex number represents a unique point on **Argand plane**. An Argand plane is shown in the following figure.



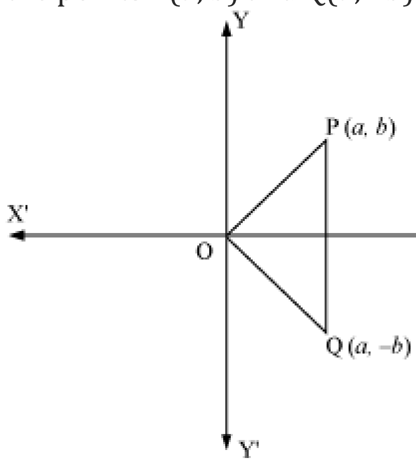
Here, x-axis is known as the **real axis** and y-axis is known as the **imaginary axis**.

- The complex number $z = a + ib$ can be represented on an Argand plane as



In this figure, OP = $\sqrt{a^2+b^2} = |z|$
Thus, the modulus of a complex number z = $a + ib$ is the distance between the point P(x, y) and the origin O.

- The conjugate of a complex number $z = a + ib$ is $\bar{z} = a - ib$. z and $\bar{z}$ can be represented by the points P(a, b) and Q(a, −b) on the Argand plane as



Thus, on the Argand plane, the conjugate of a complex number is the mirror image of the complex number with respect to the real axis.

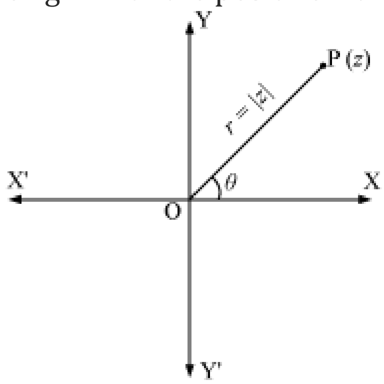
Polar Representation of Complex Numbers

- A complex number $z = a + ib$ can be written in the **polar form** as $\boldsymbol{z = r(\cos\theta + i\sin\theta)}$.

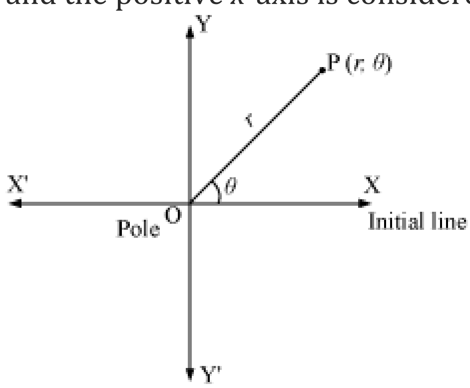
- Here, ***r*** is the **modulus** of the complex number and is given by $r = \sqrt{a^2+b^2}$

- θ is the **argument** of the complex number and is given by $\theta = \tan^{-1}\dfrac{b}{a}$

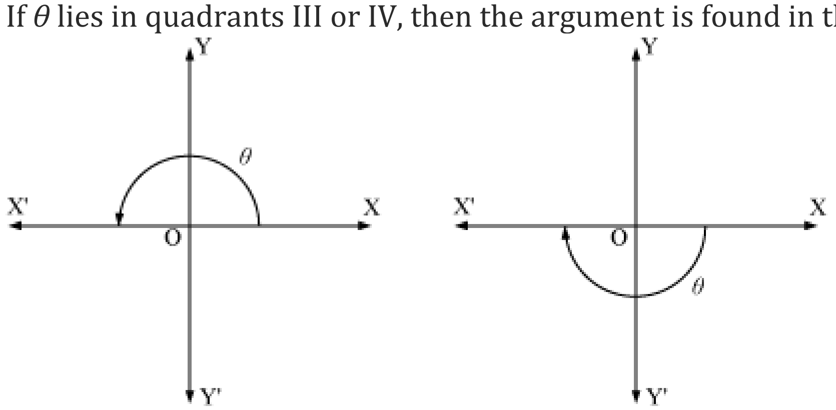
- Geometrically, r represents the distance of the point that represents the complex number from the origin, and θ represents the angle formed by the line joining the point and the origin with the positive x-axis.



- The polar coordinates of a complex number z are (r, θ). The origin is considered as the pole and the positive x-axis is considered as the initial line.



- The value of θ lying in the interval $-\pi < \theta \leq \pi$ is called the **principal argument** of the complex number z. In order to write the polar form of a complex number, we always find the principal argument.
- If θ lies in quadrants I or II, then the argument is found in the anticlockwise direction. If θ lies in quadrants III or IV, then the argument is found in the clockwise direction.



Example 1: Represent the complex number $\left(\sqrt{3}-i\right)$ in polar form.

Solution:

Let $z = r(\cos\theta + i\sin\theta)$ be the polar form of the complex number $\left(\sqrt{3} - i\right)$.

$\therefore r\cos\theta = \sqrt{3}$ and $r\sin\theta = -1$

On squaring and adding, we obtain

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = \left(\sqrt{3}\right)^2 + (-1)^2$$
$$r^2 = 3 + 1 = 4$$
$$r = \pm 2$$
$$r = 2 \qquad (r \text{ cannot be negative})$$

Now,

$\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = -\frac{1}{2}$

Here, $\cos\theta$ is positive and $\sin\theta$ is negative. Hence, θ lies in quadrant **IV**.

$\therefore \theta = -\frac{\pi}{6}$

Thus, the required polar form of the given complex number is

$$2\left\{\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right\}$$

Example 2: What are the modulus and the argument of the complex number $-\frac{1}{\sqrt{2}}(1+i)$?

Solution:

Let $r(\cos\theta + i\sin\theta) = \frac{-1}{\sqrt{2}}(1+i)$

Which gives,

$r\cos\theta = \frac{-1}{\sqrt{2}}$ and $r\sin\theta = \frac{-1}{\sqrt{2}}$

On squaring and adding, we obtain

$$r^2\left(\cos^2\theta + \sin^2\theta\right) = \left(\frac{-1}{\sqrt{2}}\right)^2 + \left(\frac{-1}{\sqrt{2}}\right)^2$$
$$r^2 = \frac{1}{2} + \frac{1}{2} = 1$$
$$r = \pm 1$$
$$r = 1 \left(\therefore r > 0\right)$$

Now, $\cos\theta = \frac{-1}{\sqrt{2}}$ and $\sin\theta = \frac{-1}{\sqrt{2}}$

Here, both $\cos\theta$ and $\sin\theta$ are negative.

Hence, θ lies in quadrant **III**.

$$\therefore \theta = \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$$

Thus, the modulus and argument of the given complex number are 1 and $-\frac{3\pi}{4}$ respectively.