



PUZZLER

This sky diver is falling at more than 50 m/s (120 mi/h), but once her parachute opens, her downward velocity will be greatly reduced. Why does she slow down rapidly when her chute opens, enabling her to fall safely to the ground? If the chute does not function properly, the sky diver will almost certainly be seriously injured. What force exerted on her limits her maximum speed?

(Guy Savage/Photo Researchers, Inc.)

chapter

6

Circular Motion and Other Applications of Newton's Laws

Chapter Outline


- 6.1** Newton's Second Law Applied to Uniform Circular Motion
- 6.2** Nonuniform Circular Motion
- 6.3** *(Optional)* Motion in Accelerated Frames
- 6.4** *(Optional)* Motion in the Presence of Resistive Forces
- 6.5** *(Optional)* Numerical Modeling in Particle Dynamics

In the preceding chapter we introduced Newton's laws of motion and applied them to situations involving linear motion. Now we discuss motion that is slightly more complicated. For example, we shall apply Newton's laws to objects traveling in circular paths. Also, we shall discuss motion observed from an accelerating frame of reference and motion in a viscous medium. For the most part, this chapter is a series of examples selected to illustrate the application of Newton's laws to a wide variety of circumstances.

6.1 NEWTON'S SECOND LAW APPLIED TO UNIFORM CIRCULAR MOTION

In Section 4.4 we found that a particle moving with uniform speed v in a circular path of radius r experiences an acceleration \mathbf{a}_r that has a magnitude

$$a_r = \frac{v^2}{r}$$

 The acceleration is called the *centripetal acceleration* because \mathbf{a}_r is directed toward the center of the circle. Furthermore, \mathbf{a}_r is *always* perpendicular to \mathbf{v} . (If there were a component of acceleration parallel to \mathbf{v} , the particle's speed would be changing.)

Consider a ball of mass m that is tied to a string of length r and is being whirled at constant speed in a horizontal circular path, as illustrated in Figure 6.1. Its weight is supported by a low-friction table. Why does the ball move in a circle? Because of its inertia, the tendency of the ball is to move in a straight line; however, the string prevents motion along a straight line by exerting on the ball a force that makes it follow the circular path. This force is directed along the string toward the center of the circle, as shown in Figure 6.1. This force can be any one of our familiar forces causing an object to follow a circular path.

If we apply Newton's second law along the radial direction, we find that the value of the net force causing the centripetal acceleration can be evaluated:

Force causing centripetal acceleration

$$\sum F_r = ma_r = m \frac{v^2}{r} \quad (6.1)$$

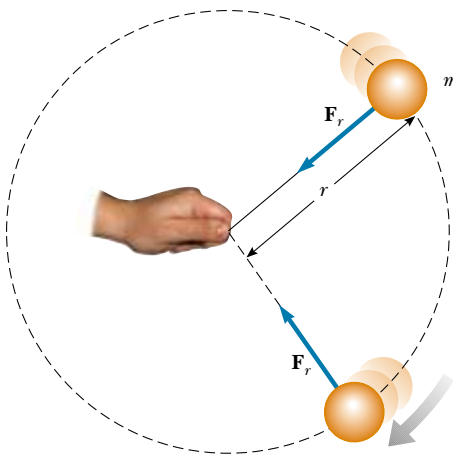


Figure 6.1 Overhead view of a ball moving in a circular path in a horizontal plane. A force \mathbf{F}_r directed toward the center of the circle keeps the ball moving in its circular path.

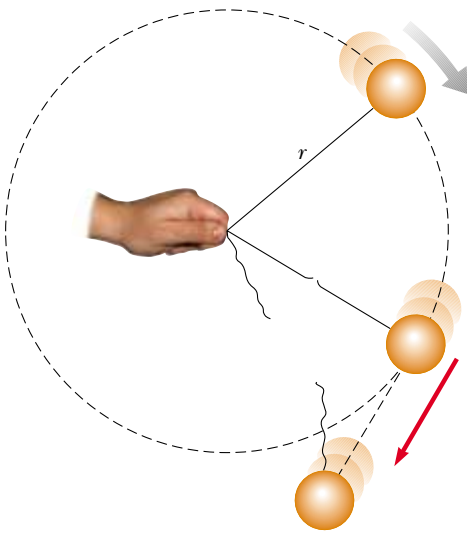
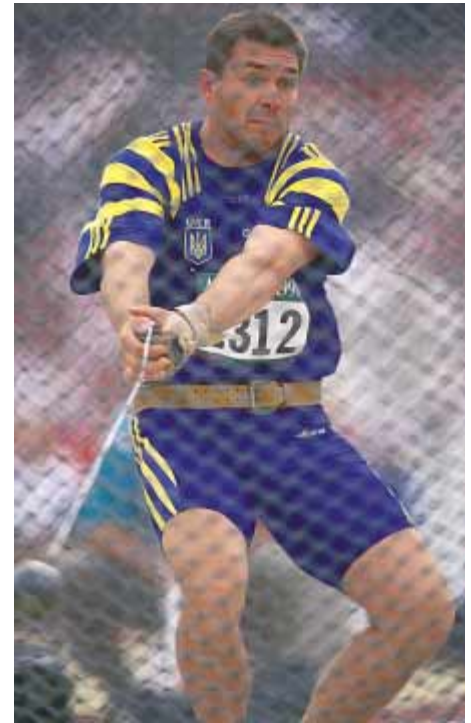


Figure 6.2 When the string breaks, the ball moves in the direction tangent to the circle.

A force causing a centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector. If that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangent to the circle. This idea is illustrated in Figure 6.2 for the ball whirling at the end of a string. If the string breaks at some instant, the ball moves along the straight-line path tangent to the circle at the point where the string broke.

Quick Quiz 6.1

Is it possible for a car to move in a circular path in such a way that it has a tangential acceleration but no centripetal acceleration?



An athlete in the process of throwing the hammer at the 1996 Olympic Games in Atlanta, Georgia. The force exerted by the chain is the force causing the circular motion. Only when the athlete releases the hammer will it move along a straight-line path tangent to the circle.

CONCEPTUAL EXAMPLE 6.1 Forces That Cause Centripetal Acceleration

The force causing centripetal acceleration is sometimes called a *centripetal force*. We are familiar with a variety of forces in nature—friction, gravity, normal forces, tension, and so forth. Should we add *centripetal* force to this list?

Solution No; centripetal force *should not* be added to this list. This is a pitfall for many students. Giving the force causing circular motion a name—centripetal force—leads many students to consider it a new kind of force rather than a new *role* for force. A common mistake in force diagrams is to draw all the usual forces and then to add another vector for the centripetal force. But it is not a separate force—it is simply one of our familiar forces *acting in the role of a force that causes a circular motion*.

Consider some examples. For the motion of the Earth around the Sun, the centripetal force is *gravity*. For an object sitting on a rotating turntable, the centripetal force is *friction*. For a rock whirled on the end of a string, the centripetal force is the force of *tension* in the string. For an amusement-park patron pressed against the inner wall of a rapidly rotating circular room, the centripetal force is the *normal force* exerted by the wall. What's more, the centripetal force could be a combination of two or more forces. For example, as a Ferris-wheel rider passes through the lowest point, the centripetal force on her is the difference between the normal force exerted by the seat and her weight.

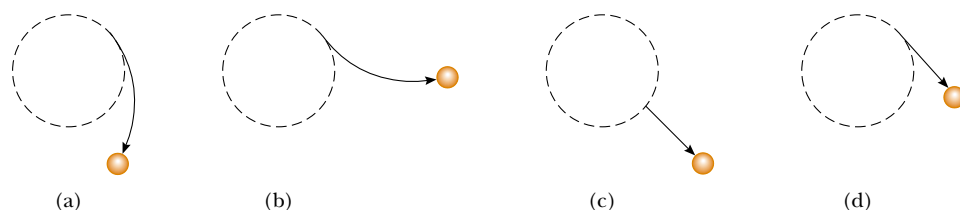


Figure 6.3 A ball that had been moving in a circular path is acted on by various external forces that change its path.

Quick Quiz 6.2

QuickLab

Tie a string to a tennis ball, swing it in a circle, and then, while it is swinging, let go of the string to verify your answer to the last part of Quick Quiz 6.2.

A ball is following the dotted circular path shown in Figure 6.3 under the influence of a force. At a certain instant of time, the force on the ball changes abruptly to a new force, and the ball follows the paths indicated by the solid line with an arrowhead in each of the four parts of the figure. For each part of the figure, describe the magnitude and direction of the force required to make the ball move in the solid path. If the dotted line represents the path of a ball being whirled on the end of a string, which path does the ball follow if the string breaks?

Let us consider some examples of uniform circular motion. In each case, be sure to recognize the external force (or forces) that causes the body to move in its circular path.



EXAMPLE 6.2 How Fast Can It Spin?

A ball of mass 0.500 kg is attached to the end of a cord 1.50 m long. The ball is whirled in a horizontal circle as was shown in Figure 6.1. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed the ball can attain before the cord breaks? Assume that the string remains horizontal during the motion.

Solution It is difficult to know what might be a reasonable value for the answer. Nonetheless, we know that it cannot be too large, say 100 m/s, because a person cannot make a ball move so quickly. It makes sense that the stronger the cord, the faster the ball can twirl before the cord breaks. Also, we expect a more massive ball to break the cord at a lower speed. (Imagine whirling a bowling ball!)

Because the force causing the centripetal acceleration in this case is the force \mathbf{T} exerted by the cord on the ball, Equation 6.1 yields for $\Sigma F_r = ma_r$

$$T = m \frac{v^2}{r}$$

Solving for v , we have

$$v = \sqrt{\frac{Tr}{m}}$$

This shows that v increases with T and decreases with larger m , as we expect to see—for a given v , a large mass requires a large tension and a small mass needs only a small tension. The maximum speed the ball can have corresponds to the maximum tension. Hence, we find

$$\begin{aligned} v_{\max} &= \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{(50.0 \text{ N})(1.50 \text{ m})}{0.500 \text{ kg}}} \\ &= 12.2 \text{ m/s} \end{aligned}$$

Exercise Calculate the tension in the cord if the speed of the ball is 5.00 m/s.

Answer 8.33 N.

EXAMPLE 6.3 The Conical Pendulum

A small object of mass m is suspended from a string of length L . The object revolves with constant speed v in a horizontal circle of radius r , as shown in Figure 6.4. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum*.) Find an expression for v .

Solution Let us choose θ to represent the angle between string and vertical. In the free-body diagram shown in Figure 6.4, the force \mathbf{T} exerted by the string is resolved into a vertical component $T \cos \theta$ and a horizontal component $T \sin \theta$ acting toward the center of revolution. Because the object does

not accelerate in the vertical direction, $\Sigma F_y = ma_y = 0$, and the upward vertical component of \mathbf{T} must balance the downward force of gravity. Therefore,

$$(1) \quad T \cos \theta = mg$$

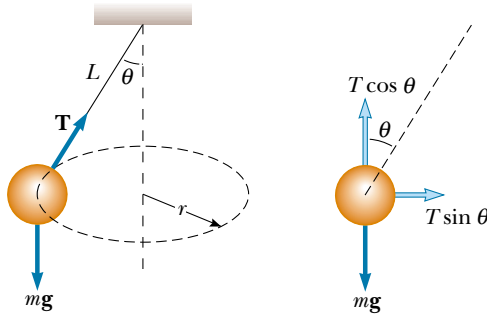


Figure 6.4 The conical pendulum and its free-body diagram.

Because the force providing the centripetal acceleration in this example is the component $T \sin \theta$, we can use Newton's second law and Equation 6.1 to obtain

$$(2) \quad \Sigma F_r = T \sin \theta = ma_r = \frac{mv^2}{r}$$

Dividing (2) by (1) and remembering that $\sin \theta / \cos \theta = \tan \theta$, we eliminate T and find that

$$\begin{aligned} \tan \theta &= \frac{v^2}{rg} \\ v &= \sqrt{rg \tan \theta} \end{aligned}$$

From the geometry in Figure 6.4, we note that $r = L \sin \theta$; therefore,

$$v = \sqrt{Lg \sin \theta \tan \theta}$$

Note that the speed is independent of the mass of the object.



EXAMPLE 6.4 What Is the Maximum Speed of the Car?

A 1500-kg car moving on a flat, horizontal road negotiates a curve, as illustrated in Figure 6.5. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires

and dry pavement is 0.500, find the maximum speed the car can have and still make the turn successfully.

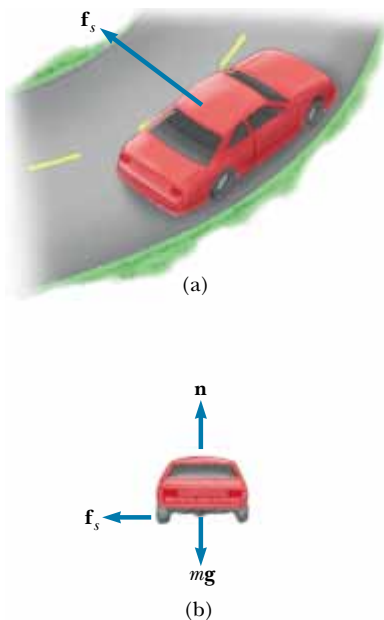


Figure 6.5 (a) The force of static friction directed toward the center of the curve keeps the car moving in a circular path. (b) The free-body diagram for the car.

Solution From experience, we should expect a maximum speed less than 50 m/s. (A convenient mental conversion is that 1 m/s is roughly 2 mi/h.) In this case, the force that enables the car to remain in its circular path is the force of static friction. (Because no slipping occurs at the point of contact between road and tires, the acting force is a force of static friction directed toward the center of the curve. If this force of static friction were zero—for example, if the car were on an icy road—the car would continue in a straight line and slide off the road.) Hence, from Equation 6.1 we have

$$(1) \quad f_s = m \frac{v^2}{r}$$

The maximum speed the car can have around the curve is the speed at which it is on the verge of skidding outward. At this point, the friction force has its maximum value $f_{s,\max} = \mu_s n$. Because the car is on a horizontal road, the magnitude of the normal force equals the weight ($n = mg$) and thus $f_{s,\max} = \mu_s mg$. Substituting this value for f_s into (1), we find that the maximum speed is

$$\begin{aligned} v_{\max} &= \sqrt{\frac{f_{s,\max} r}{m}} = \sqrt{\frac{\mu_s mg r}{m}} = \sqrt{\mu_s g r} \\ &= \sqrt{(0.500)(9.80 \text{ m/s}^2)(35.0 \text{ m})} = 13.1 \text{ m/s} \end{aligned}$$

Note that the maximum speed does not depend on the mass of the car. That is why curved highways do not need multiple speed limit signs to cover the various masses of vehicles using the road.

Exercise On a wet day, the car begins to skid on the curve when its speed reaches 8.00 m/s. What is the coefficient of static friction in this case?

Answer 0.187.

EXAMPLE 6.5 The Banked Exit Ramp

A civil engineer wishes to design a curved exit ramp for a highway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually *banked*; this means the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s (30.0 mi/h) and the radius of the curve is 50.0 m. At what angle should the curve be banked?

Solution On a level (unbanked) road, the force that causes the centripetal acceleration is the force of static friction between car and road, as we saw in the previous example. However, if the road is banked at an angle θ , as shown in Figure 6.6, the normal force \mathbf{n} has a horizontal component

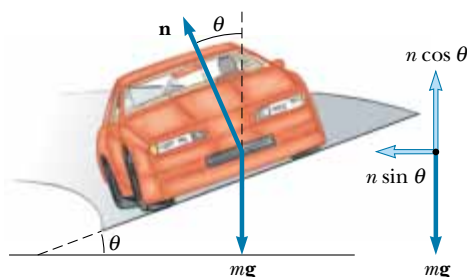


Figure 6.6 Car rounding a curve on a road banked at an angle θ to the horizontal. When friction is neglected, the force that causes the centripetal acceleration and keeps the car moving in its circular path is the horizontal component of the normal force. Note that \mathbf{n} is the sum of the forces exerted by the road on the wheels.

$n \sin \theta$ pointing toward the center of the curve. Because the ramp is to be designed so that the force of static friction is zero, only the component $n \sin \theta$ causes the centripetal acceleration. Hence, Newton's second law written for the radial direction gives

$$(1) \quad \sum F_r = n \sin \theta = \frac{mv^2}{r}$$

The car is in equilibrium in the vertical direction. Thus, from $\sum F_y = 0$, we have

$$(2) \quad n \cos \theta = mg$$

Dividing (1) by (2) gives

$$\tan \theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1} \left[\frac{(13.4 \text{ m/s})^2}{(50.0 \text{ m})(9.80 \text{ m/s}^2)} \right] = 20.1^\circ$$

If a car rounds the curve at a speed less than 13.4 m/s, friction is needed to keep it from sliding down the bank (to the left in Fig. 6.6). A driver who attempts to negotiate the curve at a speed greater than 13.4 m/s has to depend on friction to keep from sliding up the bank (to the right in Fig. 6.6). The banking angle is independent of the mass of the vehicle negotiating the curve.

Exercise Write Newton's second law applied to the radial direction when a frictional force \mathbf{f}_s is directed down the bank, toward the center of the curve.

Answer $n \sin \theta + f_s \cos \theta = \frac{mv^2}{r}$

EXAMPLE 6.6 Satellite Motion

This example treats a satellite moving in a circular orbit around the Earth. To understand this situation, you must know that the gravitational force between spherical objects and small objects that can be modeled as particles having

masses m_1 and m_2 and separated by a distance r is attractive and has a magnitude

$$F_g = G \frac{m_1 m_2}{r^2}$$

where $G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$. This is Newton's law of gravitation, which we study in Chapter 14.

Consider a satellite of mass m moving in a circular orbit around the Earth at a constant speed v and at an altitude h above the Earth's surface, as illustrated in Figure 6.7. Determine the speed of the satellite in terms of G , h , R_E (the radius of the Earth), and M_E (the mass of the Earth).

Solution The only external force acting on the satellite is the force of gravity, which acts toward the center of the Earth

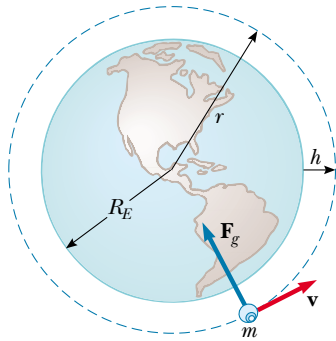


Figure 6.7 A satellite of mass m moving around the Earth at a constant speed v in a circular orbit of radius $r = R_E + h$. The force \mathbf{F}_g acting on the satellite that causes the centripetal acceleration is the gravitational force exerted by the Earth on the satellite.

and keeps the satellite in its circular orbit. Therefore,

$$F_r = F_g = G \frac{M_E m}{r^2}$$

From Newton's second law and Equation 6.1 we obtain

$$G \frac{M_E m}{r^2} = m \frac{v^2}{r}$$

Solving for v and remembering that the distance r from the center of the Earth to the satellite is $r = R_E + h$, we obtain

$$(1) \quad v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{GM_E}{R_E + h}}$$

If the satellite were orbiting a different planet, its velocity would increase with the mass of the planet and decrease as the satellite's distance from the center of the planet increased.

Exercise A satellite is in a circular orbit around the Earth at an altitude of 1 000 km. The radius of the Earth is equal to $6.37 \times 10^6 \text{ m}$, and its mass is $5.98 \times 10^{24} \text{ kg}$. Find the speed of the satellite, and then find the *period*, which is the time it needs to make one complete revolution.

Answer $7.36 \times 10^3 \text{ m/s}$; $6.29 \times 10^3 \text{ s} = 105 \text{ min}$.

EXAMPLE 6.7 Let's Go Loop-the-Loop!

A pilot of mass m in a jet aircraft executes a loop-the-loop, as shown in Figure 6.8a. In this maneuver, the aircraft moves in a vertical circle of radius 2.70 km at a constant speed of 225 m/s. Determine the force exerted by the seat on the pilot (a) at the bottom of the loop and (b) at the top of the loop. Express your answers in terms of the weight of the pilot mg .

Solution We expect the answer for (a) to be greater than that for (b) because at the bottom of the loop the normal and gravitational forces act in opposite directions, whereas at the top of the loop these two forces act in the same direction. It is the vector sum of these two forces that gives the force of constant magnitude that keeps the pilot moving in a circular path. To yield net force vectors with the same magnitude, the normal force at the bottom (where the normal and gravitational forces are in opposite directions) must be greater than that at the top (where the normal and gravitational forces are in the same direction). (a) The free-body diagram for the pilot at the bottom of the loop is shown in Figure 6.8b. The only forces acting on him are the downward force of gravity $\mathbf{F}_g = m\mathbf{g}$ and the upward force \mathbf{n}_{bot} exerted by the seat. Because the net upward force that provides the centripetal ac-

celeration has a magnitude $n_{\text{bot}} - mg$, Newton's second law for the radial direction combined with Equation 6.1 gives

$$\begin{aligned} \sum F_r &= n_{\text{bot}} - mg = m \frac{v^2}{r} \\ n_{\text{bot}} &= mg + m \frac{v^2}{r} = mg \left(1 + \frac{v^2}{rg} \right) \end{aligned}$$

Substituting the values given for the speed and radius gives

$$n_{\text{bot}} = mg \left[1 + \frac{(225 \text{ m/s})^2}{(2.70 \times 10^3 \text{ m})(9.80 \text{ m/s}^2)} \right] = 2.91mg$$

Hence, the magnitude of the force \mathbf{n}_{bot} exerted by the seat on the pilot is *greater* than the weight of the pilot by a factor of 2.91. This means that the pilot experiences an apparent weight that is greater than his true weight by a factor of 2.91.

(b) The free-body diagram for the pilot at the top of the loop is shown in Figure 6.8c. As we noted earlier, both the gravitational force exerted by the Earth and the force \mathbf{n}_{top} exerted by the seat on the pilot act downward, and so the net downward force that provides the centripetal acceleration has

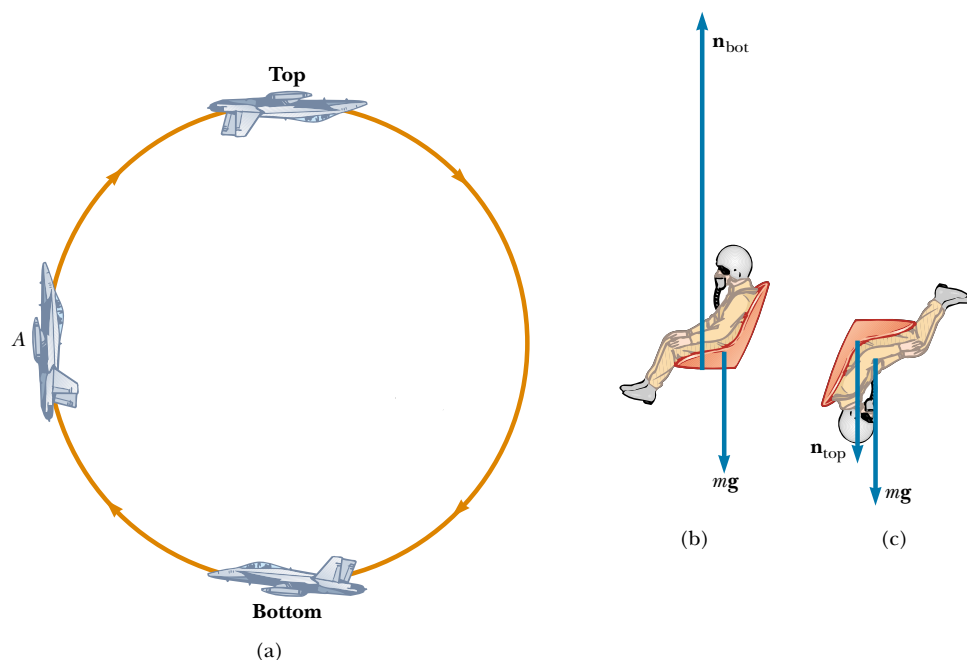


Figure 6.8 (a) An aircraft executes a loop-the-loop maneuver as it moves in a vertical circle at constant speed. (b) Free-body diagram for the pilot at the bottom of the loop. In this position the pilot experiences an apparent weight greater than his true weight. (c) Free-body diagram for the pilot at the top of the loop.

a magnitude $n_{\text{top}} + mg$. Applying Newton's second law yields

$$\sum F_r = n_{\text{top}} + mg = m \frac{v^2}{r}$$

$$n_{\text{top}} = m \frac{v^2}{r} - mg = mg \left(\frac{v^2}{rg} - 1 \right)$$

$$n_{\text{top}} = mg \left[\frac{(225 \text{ m/s})^2}{(2.70 \times 10^3 \text{ m})(9.80 \text{ m/s}^2)} - 1 \right] = 0.913mg$$

In this case, the magnitude of the force exerted by the seat on the pilot is *less* than his true weight by a factor of 0.913, and the pilot feels lighter.

Exercise Determine the magnitude of the radially directed force exerted on the pilot by the seat when the aircraft is at point A in Figure 6.8a, midway up the loop.

Answer $n_A = 1.913mg$ directed to the right.

Quick Quiz 6.3

A bead slides freely along a curved wire at constant speed, as shown in the overhead view of Figure 6.9. At each of the points Ⓐ, Ⓑ, and Ⓒ, draw the vector representing the force that the wire exerts on the bead in order to cause it to follow the path of the wire at that point.

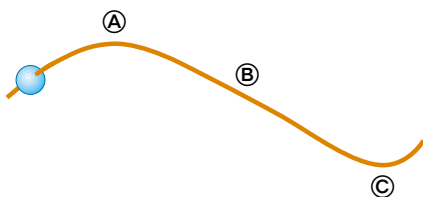


Figure 6.9

QuickLab

Hold a shoe by the end of its lace and spin it in a vertical circle. Can you feel the difference in the tension in the lace when the shoe is at top of the circle compared with when the shoe is at the bottom?

6.2 NONUNIFORM CIRCULAR MOTION

In Chapter 4 we found that if a particle moves with varying speed in a circular path, there is, in addition to the centripetal (radial) component of acceleration, a tangential component having magnitude dv/dt . Therefore, the force acting on the



Some examples of forces acting during circular motion. (*Left*) As these speed skaters round a curve, the force exerted by the ice on their skates provides the centripetal acceleration. (*Right*) Passengers on a “corkscrew” roller coaster. What are the origins of the forces in this example?

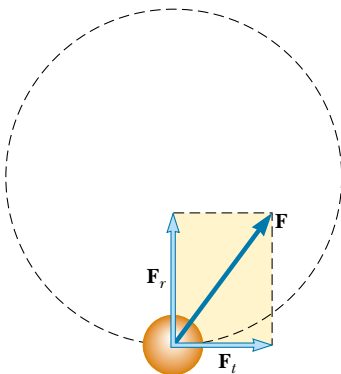


Figure 6.10 When the force acting on a particle moving in a circular path has a tangential component F_t , the particle’s speed changes. The total force exerted on the particle in this case is the vector sum of the radial force and the tangential force. That is, $\mathbf{F} = \mathbf{F}_r + \mathbf{F}_t$.

particle must also have a tangential and a radial component. Because the total acceleration is $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the total force exerted on the particle is $\mathbf{F} = \mathbf{F}_r + \mathbf{F}_t$, as shown in Figure 6.10. The vector \mathbf{F}_r is directed toward the center of the circle and is responsible for the centripetal acceleration. The vector \mathbf{F}_t tangent to the circle is responsible for the tangential acceleration, which represents a change in the speed of the particle with time. The following example demonstrates this type of motion.

EXAMPLE 6.8 Keep Your Eye on the Ball

A small sphere of mass m is attached to the end of a cord of length R and whirls in a *vertical* circle about a fixed point O , as illustrated in Figure 6.11a. Determine the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

Solution Unlike the situation in Example 6.7, the speed is *not* uniform in this example because, at most points along the path, a tangential component of acceleration arises from the gravitational force exerted on the sphere. From the free-body diagram in Figure 6.11b, we see that the only forces acting on

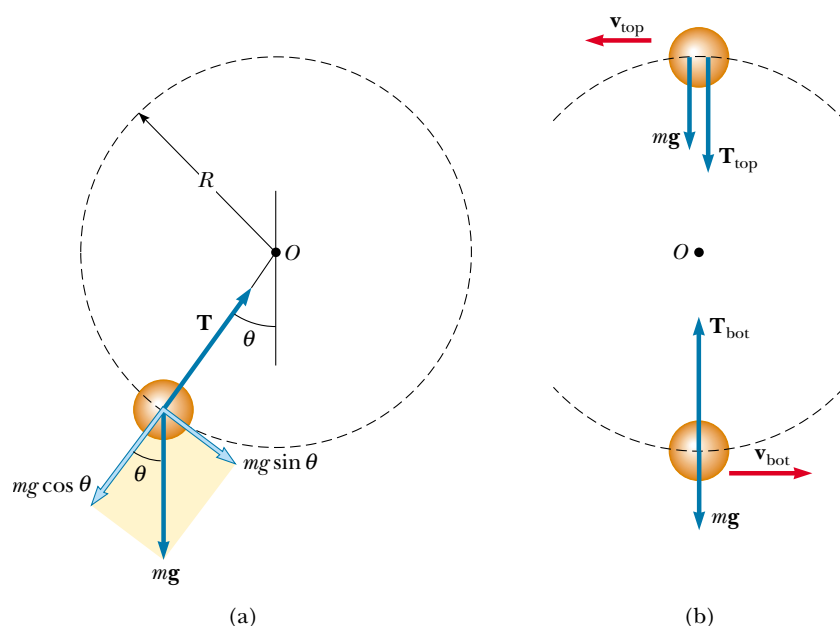


Figure 6.11 (a) Forces acting on a sphere of mass m connected to a cord of length R and rotating in a vertical circle centered at O . (b) Forces acting on the sphere at the top and bottom of the circle. The tension is a maximum at the bottom and a minimum at the top.

the sphere are the gravitational force $\mathbf{F}_g = m\mathbf{g}$ exerted by the Earth and the force \mathbf{T} exerted by the cord. Now we resolve \mathbf{F}_g into a tangential component $mg \sin \theta$ and a radial component $mg \cos \theta$. Applying Newton's second law to the forces acting on the sphere in the tangential direction yields

$$\begin{aligned}\sum F_t &= mg \sin \theta = ma_t \\ a_t &= g \sin \theta\end{aligned}$$

This tangential component of the acceleration causes v to change in time because $a_t = dv/dt$.

Applying Newton's second law to the forces acting on the sphere in the radial direction and noting that both \mathbf{T} and \mathbf{a}_r are directed toward O , we obtain

$$\sum F_r = T - mg \cos \theta = \frac{mv^2}{R}$$

$$T = m \left(\frac{v^2}{R} + g \cos \theta \right)$$

Special Cases At the top of the path, where $\theta = 180^\circ$, we have $\cos 180^\circ = -1$, and the tension equation becomes

$$T_{\text{top}} = m \left(\frac{v_{\text{top}}^2}{R} - g \right)$$

This is the minimum value of T . Note that at this point $a_t = 0$ and therefore the acceleration is purely radial and directed downward.

At the bottom of the path, where $\theta = 0$, we see that, because $\cos 0 = 1$,

$$T_{\text{bot}} = m \left(\frac{v_{\text{bot}}^2}{R} + g \right)$$

This is the maximum value of T . At this point, a_t is again 0 and the acceleration is now purely radial and directed upward.

Exercise At what position of the sphere would the cord most likely break if the average speed were to increase?

Answer At the bottom, where T has its maximum value.

Optional Section

6.3 MOTION IN ACCELERATED FRAMES

When Newton's laws of motion were introduced in Chapter 5, we emphasized that they are valid only when observations are made in an inertial frame of reference. In this section, we analyze how an observer in a noninertial frame of reference (one that is accelerating) applies Newton's second law.

To understand the motion of a system that is noninertial because an object is moving along a curved path, consider a car traveling along a highway at a high speed and approaching a curved exit ramp, as shown in Figure 6.12a. As the car takes the sharp left turn onto the ramp, a person sitting in the passenger seat slides to the right and hits the door. At that point, the force exerted on her by the door keeps her from being ejected from the car. What causes her to move toward the door? A popular, but improper, explanation is that some mysterious force acting from left to right pushes her outward. (This is often called the “centrifugal” force, but we shall not use this term because it often creates confusion.) The passenger invents this **fictitious force** to explain what is going on in her accelerated frame of reference, as shown in Figure 6.12b. (The driver also experiences this effect but holds on to the steering wheel to keep from sliding to the right.)

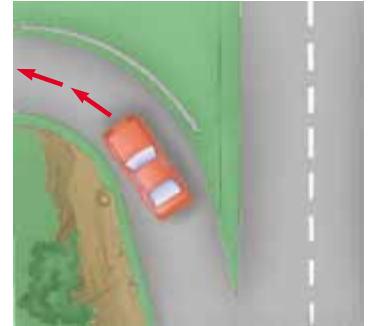
The phenomenon is correctly explained as follows. Before the car enters the ramp, the passenger is moving in a straight-line path. As the car enters the ramp and travels a curved path, the passenger tends to move along the original straight-line path. This is in accordance with Newton’s first law: The natural tendency of a body is to continue moving in a straight line. However, if a sufficiently large force (toward the center of curvature) acts on the passenger, as in Figure 6.12c, she will move in a curved path along with the car. The origin of this force is the force of friction between her and the car seat. If this frictional force is not large enough, she will slide to the right as the car turns to the left under her. Eventually, she encounters the door, which provides a force large enough to enable her to follow the same curved path as the car. She slides toward the door not because of some mysterious outward force but because **the force of friction is not sufficiently great to allow her to travel along the circular path followed by the car.**

In general, if a particle moves with an acceleration \mathbf{a} relative to an observer in an inertial frame, that observer may use Newton’s second law and correctly claim that $\Sigma \mathbf{F} = m\mathbf{a}$. If another observer in an accelerated frame tries to apply Newton’s second law to the motion of the particle, the person must introduce fictitious forces to make Newton’s second law work. These forces “invented” by the observer in the accelerating frame appear to be real. However, we emphasize that **these fictitious forces do not exist when the motion is observed in an inertial frame.** Fictitious forces are used only in an accelerating frame and do not represent “real” forces acting on the particle. (By real forces, we mean the interaction of the particle with its environment.) If the fictitious forces are properly defined in the accelerating frame, the description of motion in this frame is equivalent to the description given by an inertial observer who considers only real forces. Usually, we analyze motions using inertial reference frames, but there are cases in which it is more convenient to use an accelerating frame.

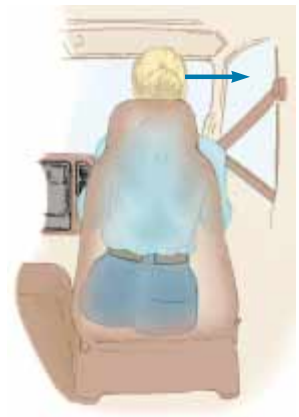
QuickLab

Use a string, a small weight, and a protractor to measure your acceleration as you start sprinting from a standing position.

Fictitious forces



(a)



(b)



(c)

Figure 6.12 (a) A car approaching a curved exit ramp. What causes a front-seat passenger to move toward the right-hand door? (b) From the frame of reference of the passenger, a (fictitious) force pushes her toward the right door. (c) Relative to the reference frame of the Earth, the car seat applies a leftward force to the passenger, causing her to change direction along with the rest of the car.

EXAMPLE 6.9 Fictitious Forces in Linear Motion

A small sphere of mass m is hung by a cord from the ceiling of a boxcar that is accelerating to the right, as shown in Figure 6.13. According to the inertial observer at rest (Fig. 6.13a), the forces on the sphere are the force \mathbf{T} exerted by the cord and the force of gravity. The inertial observer concludes that the acceleration of the sphere is the same as that of the boxcar and that this acceleration is provided by the horizontal component of \mathbf{T} . Also, the vertical component of \mathbf{T} balances the force of gravity. Therefore, she writes Newton's second law as $\Sigma \mathbf{F} = \mathbf{T} + m\mathbf{g} = m\mathbf{a}$, which in component form becomes

$$\text{Inertial observer} \quad \begin{cases} (1) & \Sigma F_x = T \sin \theta = ma \\ (2) & \Sigma F_y = T \cos \theta - mg = 0 \end{cases}$$

Thus, by solving (1) and (2) simultaneously for a , the inertial observer can determine the magnitude of the car's acceleration through the relationship

$$a = g \tan \theta$$

Because the deflection of the cord from the vertical serves as a measure of acceleration, *a simple pendulum can be used as an accelerometer.*

According to the noninertial observer riding in the car (Fig. 6.13b), the cord still makes an angle θ with the vertical; however, to her the sphere is at rest and so its acceleration is zero. Therefore, she introduces a fictitious force to balance the horizontal component of \mathbf{T} and claims that the net force on the sphere is *zero*! In this noninertial frame of reference, Newton's second law in component form yields

$$\text{Noninertial observer} \quad \begin{cases} \Sigma F'_x = T \sin \theta - F_{\text{fictitious}} = 0 \\ \Sigma F'_y = T \cos \theta - mg = 0 \end{cases}$$

If we recognize that $F_{\text{fictitious}} = ma_{\text{inertial}} = ma$, then these expressions are equivalent to (1) and (2); therefore, the noninertial observer obtains the same mathematical results as the inertial observer does. However, the physical interpretation of the deflection of the cord differs in the two frames of reference.

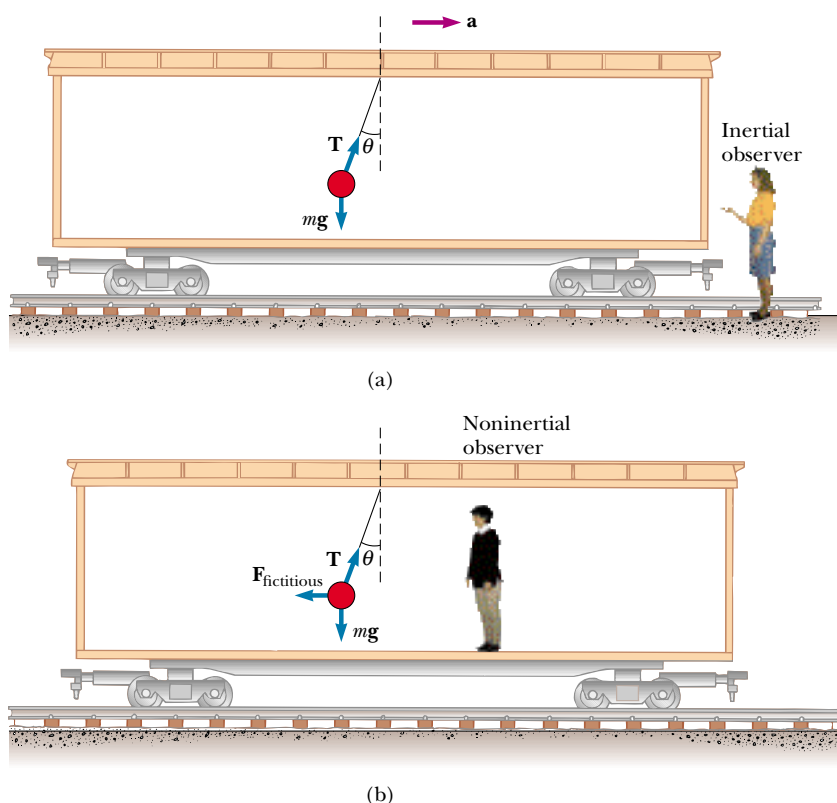


Figure 6.13 A small sphere suspended from the ceiling of a boxcar accelerating to the right is deflected as shown. (a) An inertial observer at rest outside the car claims that the acceleration of the sphere is provided by the horizontal component of \mathbf{T} . (b) A noninertial observer riding in the car says that the net force on the sphere is zero and that the deflection of the cord from the vertical is due to a fictitious force $\mathbf{F}_{\text{fictitious}}$ that balances the horizontal component of \mathbf{T} .

EXAMPLE 6.10 Fictitious Force in a Rotating System

Suppose a block of mass m lying on a horizontal, frictionless turntable is connected to a string attached to the center of the turntable, as shown in Figure 6.14. According to an inertial observer, if the block rotates uniformly, it undergoes an acceleration of magnitude v^2/r , where v is its linear speed. The inertial observer concludes that this centripetal acceleration is provided by the force \mathbf{T} exerted by the string and writes Newton's second law as $T = mv^2/r$.

According to a noninertial observer attached to the turntable, the block is at rest and its acceleration is zero. Therefore, she must introduce a fictitious outward force of magnitude mv^2/r to balance the inward force exerted by the string. According to her, the net force on the block is zero, and she writes Newton's second law as $T - mv^2/r = 0$.

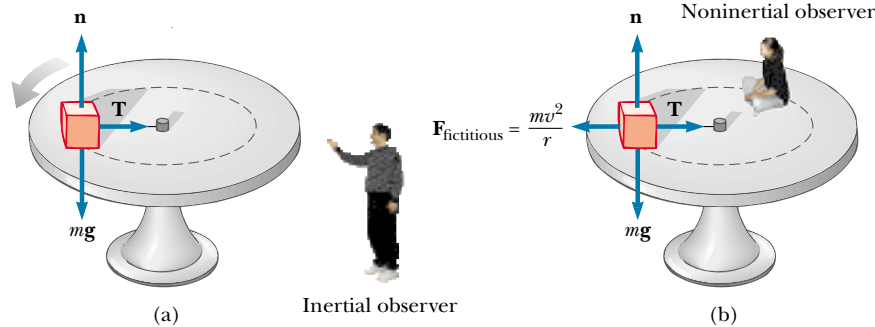


Figure 6.14 A block of mass m connected to a string tied to the center of a rotating turntable. (a) The inertial observer claims that the force causing the circular motion is provided by the force \mathbf{T} exerted by the string on the block. (b) The noninertial observer claims that the block is not accelerating, and therefore she introduces a fictitious force of magnitude mv^2/r that acts outward and balances the force \mathbf{T} .

*Optional Section***6.4 MOTION IN THE PRESENCE OF RESISTIVE FORCES**

In the preceding chapter we described the force of kinetic friction exerted on an object moving on some surface. We completely ignored any interaction between the object and the medium through which it moves. Now let us consider the effect of that medium, which can be either a liquid or a gas. The medium exerts a **resistive force** \mathbf{R} on the object moving through it. Some examples are the air resistance associated with moving vehicles (sometimes called *air drag*) and the viscous forces that act on objects moving through a liquid. The magnitude of \mathbf{R} depends on such factors as the speed of the object, and the direction of \mathbf{R} is always opposite the direction of motion of the object relative to the medium. The magnitude of \mathbf{R} nearly always increases with increasing speed.

The magnitude of the resistive force can depend on speed in a complex way, and here we consider only two situations. In the first situation, we assume the resistive force is proportional to the speed of the moving object; this assumption is valid for objects falling slowly through a liquid and for very small objects, such as dust particles, moving through air. In the second situation, we assume a resistive force that is proportional to the square of the speed of the moving object; large objects, such as a skydiver moving through air in free fall, experience such a force.

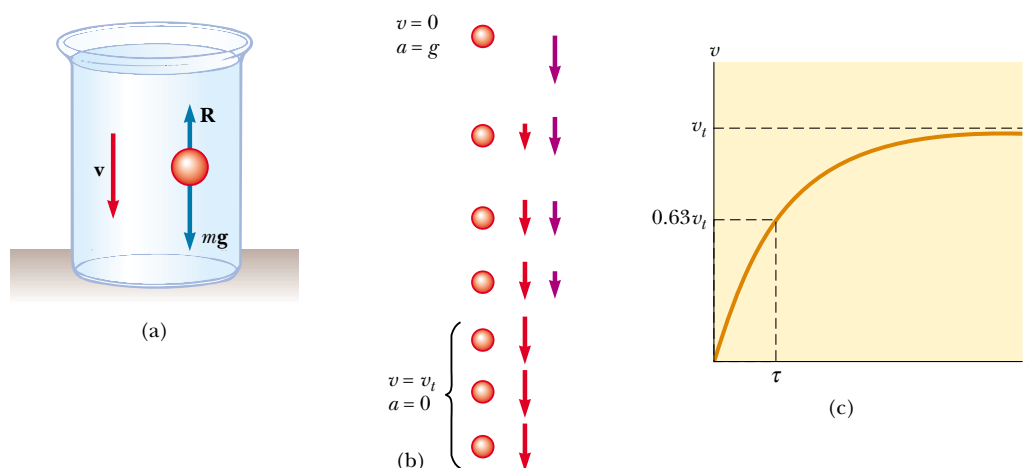


Figure 6.15 (a) A small sphere falling through a liquid. (b) Motion diagram of the sphere as it falls. (c) Speed–time graph for the sphere. The sphere reaches a maximum, or terminal, speed v_t , and the time constant τ is the time it takes to reach $0.63v_t$.

Resistive Force Proportional to Object Speed

If we assume that the resistive force acting on an object moving through a liquid or gas is proportional to the object's speed, then the magnitude of the resistive force can be expressed as

$$R = bv \quad (6.2)$$

where v is the speed of the object and b is a constant whose value depends on the properties of the medium and on the shape and dimensions of the object. If the object is a sphere of radius r , then b is proportional to r .

Consider a small sphere of mass m released from rest in a liquid, as in Figure 6.15a. Assuming that the only forces acting on the sphere are the resistive force bv and the force of gravity F_g , let us describe its motion.¹ Applying Newton's second law to the vertical motion, choosing the downward direction to be positive, and noting that $\Sigma F_y = mg - bv$, we obtain

$$mg - bv = ma = m \frac{dv}{dt} \quad (6.3)$$

where the acceleration dv/dt is downward. Solving this expression for the acceleration gives

$$\frac{dv}{dt} = g - \frac{b}{m} v \quad (6.4)$$

This equation is called a *differential equation*, and the methods of solving it may not be familiar to you as yet. However, note that initially, when $v = 0$, the resistive force $-bv$ is also zero and the acceleration dv/dt is simply g . As t increases, the resistive force increases and the acceleration decreases. Eventually, the acceleration becomes zero when the magnitude of the resistive force equals the sphere's weight. At this point, the sphere reaches its **terminal speed** v_t , and from then on

Terminal speed

¹ There is also a *buoyant force* acting on the submerged object. This force is constant, and its magnitude is equal to the weight of the displaced liquid. This force changes the apparent weight of the sphere by a constant factor, so we will ignore the force here. We discuss buoyant forces in Chapter 15.

it continues to move at this speed with zero acceleration, as shown in Figure 6.15b. We can obtain the terminal speed from Equation 6.3 by setting $a = dv/dt = 0$. This gives

$$mg - bv_t = 0 \quad \text{or} \quad v_t = mg/b$$

The expression for v that satisfies Equation 6.4 with $v = 0$ at $t = 0$ is

$$v = \frac{mg}{b} (1 - e^{-bt/m}) = v_t (1 - e^{-t/\tau}) \quad (6.5)$$

This function is plotted in Figure 6.15c. The **time constant** $\tau = m/b$ (Greek letter tau) is the time it takes the sphere to reach 63.2% ($= 1 - 1/e$) of its terminal speed. This can be seen by noting that when $t = \tau$, Equation 6.5 yields $v = 0.632v_t$.

We can check that Equation 6.5 is a solution to Equation 6.4 by direct differentiation:

$$\frac{dv}{dt} = \frac{d}{dt} \left(\frac{mg}{b} - \frac{mg}{b} e^{-bt/m} \right) = -\frac{mg}{b} \frac{d}{dt} e^{-bt/m} = ge^{-bt/m}$$

(See Appendix Table B.4 for the derivative of e raised to some power.) Substituting into Equation 6.4 both this expression for dv/dt and the expression for v given by Equation 6.5 shows that our solution satisfies the differential equation.



Aerodynamic car. A streamlined body reduces air drag and increases fuel efficiency.

EXAMPLE 6.11 Sphere Falling in Oil

A small sphere of mass 2.00 g is released from rest in a large vessel filled with oil, where it experiences a resistive force proportional to its speed. The sphere reaches a terminal speed of 5.00 cm/s. Determine the time constant τ and the time it takes the sphere to reach 90% of its terminal speed.

Solution Because the terminal speed is given by $v_t = mg/b$, the coefficient b is

$$b = \frac{mg}{v_t} = \frac{(2.00 \text{ g})(980 \text{ cm/s}^2)}{5.00 \text{ cm/s}} = 392 \text{ g/s}$$

Therefore, the time constant τ is

$$\tau = \frac{m}{b} = \frac{2.00 \text{ g}}{392 \text{ g/s}} = 5.10 \times 10^{-3} \text{ s}$$

The speed of the sphere as a function of time is given by Equation 6.5. To find the time t it takes the sphere to reach a speed of $0.900v_t$, we set $v = 0.900v_t$ in Equation 6.5 and solve for t :

$$0.900v_t = v_t(1 - e^{-t/\tau})$$

$$1 - e^{-t/\tau} = 0.900$$

$$e^{-t/\tau} = 0.100$$

$$-\frac{t}{\tau} = \ln(0.100) = -2.30$$

$$t = 2.30\tau = 2.30(5.10 \times 10^{-3} \text{ s}) = 11.7 \times 10^{-3} \text{ s} = 11.7 \text{ ms}$$

Thus, the sphere reaches 90% of its terminal (maximum) speed in a very short time.

Exercise What is the sphere's speed through the oil at $t = 11.7 \text{ ms}$? Compare this value with the speed the sphere would have if it were falling in a vacuum and so were influenced only by gravity.

Answer 4.50 cm/s in oil versus 11.5 cm/s in free fall.

Air Drag at High Speeds

For objects moving at high speeds through air, such as airplanes, sky divers, cars, and baseballs, the resistive force is approximately proportional to the square of the speed. In these situations, the magnitude of the resistive force can be expressed as

$$R = \frac{1}{2}D\rho Av^2 \quad (6.6)$$

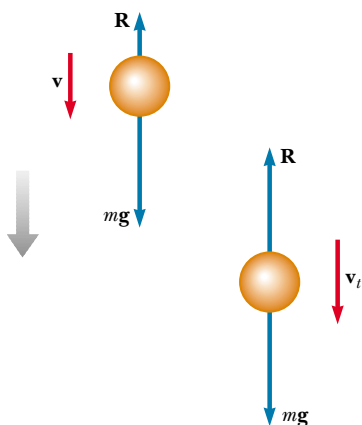


Figure 6.16 An object falling through air experiences a resistive force \mathbf{R} and a gravitational force $\mathbf{F}_g = m\mathbf{g}$. The object reaches terminal speed (on the right) when the net force acting on it is zero, that is, when $\mathbf{R} = -\mathbf{F}_g$ or $R = mg$. Before this occurs, the acceleration varies with speed according to Equation 6.8.

where ρ is the density of air, A is the cross-sectional area of the falling object measured in a plane perpendicular to its motion, and D is a dimensionless empirical quantity called the *drag coefficient*. The drag coefficient has a value of about 0.5 for spherical objects but can have a value as great as 2 for irregularly shaped objects.

Let us analyze the motion of an object in free fall subject to an upward air resistive force of magnitude $R = \frac{1}{2}D\rho A v^2$. Suppose an object of mass m is released from rest. As Figure 6.16 shows, the object experiences two external forces: the downward force of gravity $\mathbf{F}_g = m\mathbf{g}$ and the upward resistive force \mathbf{R} . (There is also an upward buoyant force that we neglect.) Hence, the magnitude of the net force is

$$\Sigma F = mg - \frac{1}{2}D\rho A v^2 \quad (6.7)$$

where we have taken downward to be the positive vertical direction. Substituting $\Sigma F = ma$ into Equation 6.7, we find that the object has a downward acceleration of magnitude

$$a = g - \left(\frac{D\rho A}{2m} \right) v^2 \quad (6.8)$$

We can calculate the terminal speed v_t by using the fact that when the force of gravity is balanced by the resistive force, the net force on the object is zero and therefore its acceleration is zero. Setting $a = 0$ in Equation 6.8 gives

$$g - \left(\frac{D\rho A}{2m} \right) v_t^2 = 0$$

$$v_t = \sqrt{\frac{2mg}{D\rho A}} \quad (6.9)$$

Using this expression, we can determine how the terminal speed depends on the dimensions of the object. Suppose the object is a sphere of radius r . In this case, $A \propto r^2$ (from $A = \pi r^2$) and $m \propto r^3$ (because the mass is proportional to the volume of the sphere, which is $V = \frac{4}{3}\pi r^3$). Therefore, $v_t \propto \sqrt{r}$.

Table 6.1 lists the terminal speeds for several objects falling through air.



The high cost of fuel has prompted many truck owners to install wind deflectors on their cabs to reduce drag.

TABLE 6.1 Terminal Speed for Various Objects Falling Through Air

Object	Mass (kg)	Cross-Sectional Area (m ²)	v_t (m/s)
Sky diver	75	0.70	60
Baseball (radius 3.7 cm)	0.145	4.2×10^{-3}	43
Golf ball (radius 2.1 cm)	0.046	1.4×10^{-3}	44
Hailstone (radius 0.50 cm)	4.8×10^{-4}	7.9×10^{-5}	14
Raindrop (radius 0.20 cm)	3.4×10^{-5}	1.3×10^{-5}	9.0

CONCEPTUAL EXAMPLE 6.12

Consider a sky surfer who jumps from a plane with her feet attached firmly to her surfboard, does some tricks, and then opens her parachute. Describe the forces acting on her during these maneuvers.

Solution When the surfer first steps out of the plane, she has no vertical velocity. The downward force of gravity causes her to accelerate toward the ground. As her downward speed increases, so does the upward resistive force exerted by the air on her body and the board. This upward force reduces their acceleration, and so their speed increases more slowly. Eventually, they are going so fast that the upward resistive force matches the downward force of gravity. Now the net force is zero and they no longer accelerate, but reach their terminal speed. At some point after reaching terminal speed, she opens her parachute, resulting in a drastic increase in the upward resistive force. The net force (and thus the acceleration) is now upward, in the direction opposite the direction of the velocity. This causes the downward velocity to decrease rapidly; this means the resistive force on the chute also decreases. Eventually the upward resistive force and the downward force of gravity balance each other and a much smaller terminal speed is reached, permitting a safe landing.

(Contrary to popular belief, the velocity vector of a sky diver never points upward. You may have seen a videotape in which a sky diver appeared to “rocket” upward once the chute opened. In fact, what happened is that the diver slowed down while the person holding the camera continued falling at high speed.)



A sky surfer takes advantage of the upward force of the air on her board. (

EXAMPLE 6.13 Falling Coffee Filters

The dependence of resistive force on speed is an empirical relationship. In other words, it is based on observation rather than on a theoretical model. A series of stacked filters is dropped, and the terminal speeds are measured. Table 6.2

presents data for these coffee filters as they fall through the air. The time constant τ is small, so that a dropped filter quickly reaches terminal speed. Each filter has a mass of 1.64 g. When the filters are nested together, they stack in

such a way that the front-facing surface area does not increase. Determine the relationship between the resistive force exerted by the air and the speed of the falling filters.

Solution At terminal speed, the upward resistive force balances the downward force of gravity. So, a single filter falling at its terminal speed experiences a resistive force of

$$R = mg = \left(\frac{1.64 \text{ g}}{1000 \text{ g/kg}} \right) (9.80 \text{ m/s}^2) = 0.0161 \text{ N}$$

Two filters nested together experience 0.0322 N of resistive force, and so forth. A graph of the resistive force on the filters as a function of terminal speed is shown in Figure 6.17a. A straight line would not be a good fit, indicating that the resistive force is not proportional to the speed. The curved line is for a second-order polynomial, indicating a proportionality of the resistive force to the square of the speed. This proportionality is more clearly seen in Figure 6.17b, in which the resistive force is plotted as a function of the square of the terminal speed.

TABLE 6.2
Terminal Speed for
Stacked Coffee Filters

Number of Filters	v_t (m/s) ^a
1	1.01
2	1.40
3	1.63
4	2.00
5	2.25
6	2.40
7	2.57
8	2.80
9	3.05
10	3.22

^a All values of v_t are approximate.



Pleated coffee filters can be nested together so that the force of air resistance can be studied.

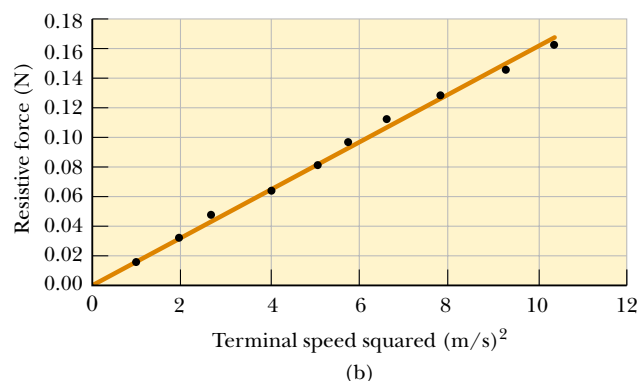
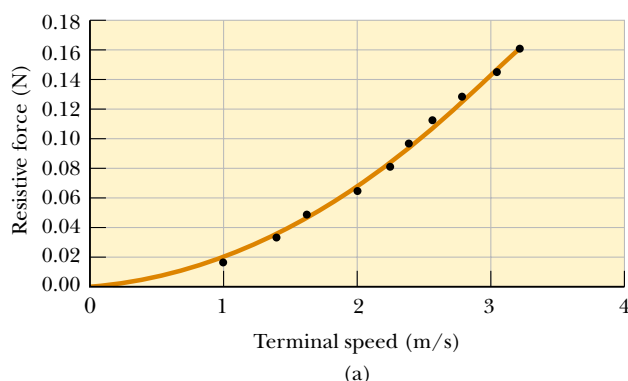


Figure 6.17 (a) Relationship between the resistive force acting on falling coffee filters and their terminal speed. The curved line is a second-order polynomial fit. (b) Graph relating the resistive force to the square of the terminal speed. The fit of the straight line to the data points indicates that the resistive force is proportional to the terminal speed squared. Can you find the proportionality constant?

EXAMPLE 6.14 Resistive Force Exerted on a Baseball

A pitcher hurls a 0.145-kg baseball past a batter at 40.2 m/s (= 90 mi/h). Find the resistive force acting on the ball at this speed.

Solution We do not expect the air to exert a huge force on the ball, and so the resistive force we calculate from Equation 6.6 should not be more than a few newtons. First, we must determine the drag coefficient D . We do this by imagining that we drop the baseball and allow it to reach terminal speed. We solve Equation 6.9 for D and substitute the appropriate values for m , v_t , and A from Table 6.1. Taking the density of air as 1.29 kg/m^3 , we obtain

$$D = \frac{2mg}{v_t^2 \rho A} = \frac{2(0.145 \text{ kg})(9.80 \text{ m/s}^2)}{(43 \text{ m/s})^2 (1.29 \text{ kg/m}^3)(4.2 \times 10^{-3} \text{ m}^2)} = 0.284$$

This number has no dimensions. We have kept an extra digit beyond the two that are significant and will drop it at the end of our calculation.

We can now use this value for D in Equation 6.6 to find the magnitude of the resistive force:

$$\begin{aligned} R &= \frac{1}{2} D \rho A v^2 \\ &= \frac{1}{2} (0.284) (1.29 \text{ kg/m}^3) (4.2 \times 10^{-3} \text{ m}^2) (40.2 \text{ m/s})^2 \\ &= 1.2 \text{ N} \end{aligned}$$

*Optional Section***6.5 NUMERICAL MODELING IN PARTICLE DYNAMICS²**

As we have seen in this and the preceding chapter, the study of the dynamics of a particle focuses on describing the position, velocity, and acceleration as functions of time. Cause-and-effect relationships exist among these quantities: Velocity causes position to change, and acceleration causes velocity to change. Because acceleration is the direct result of applied forces, any analysis of the dynamics of a particle usually begins with an evaluation of the net force being exerted on the particle.

Up till now, we have used what is called the *analytical method* to investigate the position, velocity, and acceleration of a moving particle. Let us review this method briefly before learning about a second way of approaching problems in dynamics. (Because we confine our discussion to one-dimensional motion in this section, boldface notation will not be used for vector quantities.)

If a particle of mass m moves under the influence of a net force ΣF , Newton's second law tells us that the acceleration of the particle is $a = \Sigma F/m$. In general, we apply the analytical method to a dynamics problem using the following procedure:

1. Sum all the forces acting on the particle to get the net force ΣF .
2. Use this net force to determine the acceleration from the relationship $a = \Sigma F/m$.
3. Use this acceleration to determine the velocity from the relationship $dv/dt = a$.
4. Use this velocity to determine the position from the relationship $dx/dt = v$.

The following straightforward example illustrates this method.

EXAMPLE 6.15 An Object Falling in a Vacuum—Analytical Method

Consider a particle falling in a vacuum under the influence of the force of gravity, as shown in Figure 6.18. Use the analytical method to find the acceleration, velocity, and position of the particle.

Solution The only force acting on the particle is the downward force of gravity of magnitude F_g , which is also the net force. Applying Newton's second law, we set the net force acting on the particle equal to the mass of the particle times

² The authors are most grateful to Colonel James Head of the U.S. Air Force Academy for preparing this section. See the *Student Tools CD-ROM* for some assistance with numerical modeling.

its acceleration (taking upward to be the positive y direction):

$$F_g = ma_y = -mg$$

Thus, $a_y = -g$, which means the acceleration is constant. Because $dv_y/dt = a_y$, we see that $dv_y/dt = -g$, which may be integrated to yield

$$v_y(t) = v_{yi} - gt$$

Then, because $v_y = dy/dt$, the position of the particle is obtained from another integration, which yields the well-known result

$$y(t) = y_i + v_{yi}t - \frac{1}{2}gt^2$$

In these expressions, y_i and v_{yi} represent the position and speed of the particle at $t_i = 0$.



Figure 6.18 An object falling in vacuum under the influence of gravity.

The analytical method is straightforward for many physical situations. In the “real world,” however, complications often arise that make analytical solutions difficult and perhaps beyond the mathematical abilities of most students taking introductory physics. For example, the net force acting on a particle may depend on the particle’s position, as in cases where the gravitational acceleration varies with height. Or the force may vary with velocity, as in cases of resistive forces caused by motion through a liquid or gas.

Another complication arises because the expressions relating acceleration, velocity, position, and time are differential equations rather than algebraic ones. Differential equations are usually solved using integral calculus and other special techniques that introductory students may not have mastered.

When such situations arise, scientists often use a procedure called *numerical modeling* to study motion. The simplest numerical model is called the Euler method, after the Swiss mathematician Leonhard Euler (1707–1783).

The Euler Method

In the **Euler method** for solving differential equations, derivatives are approximated as ratios of finite differences. Considering a small increment of time Δt , we can approximate the relationship between a particle’s speed and the magnitude of its acceleration as

$$a(t) \approx \frac{\Delta v}{\Delta t} = \frac{v(t + \Delta t) - v(t)}{\Delta t}$$

Then the speed $v(t + \Delta t)$ of the particle at the end of the time interval Δt is approximately equal to the speed $v(t)$ at the beginning of the time interval plus the magnitude of the acceleration during the interval multiplied by Δt :

$$v(t + \Delta t) \approx v(t) + a(t)\Delta t \quad (6.10)$$

Because the acceleration is a function of time, this estimate of $v(t + \Delta t)$ is accurate only if the time interval Δt is short enough that the change in acceleration during it is very small (as is discussed later). Of course, Equation 6.10 is exact if the acceleration is constant.

The position $x(t + \Delta t)$ of the particle at the end of the interval Δt can be found in the same manner:

$$v(t) \approx \frac{\Delta x}{\Delta t} = \frac{x(t + \Delta t) - x(t)}{\Delta t}$$

$$x(t + \Delta t) \approx x(t) + v(t)\Delta t \quad (6.11)$$

You may be tempted to add the term $\frac{1}{2} a(\Delta t)^2$ to this result to make it look like the familiar kinematics equation, but this term is not included in the Euler method because Δt is assumed to be so small that Δt^2 is nearly zero.

If the acceleration at any instant t is known, the particle's velocity and position at a time $t + \Delta t$ can be calculated from Equations 6.10 and 6.11. The calculation then proceeds in a series of finite steps to determine the velocity and position at any later time. The acceleration is determined from the net force acting on the particle, and this force may depend on position, velocity, or time:

$$a(x, v, t) = \frac{\sum F(x, v, t)}{m} \quad (6.12)$$

It is convenient to set up the numerical solution to this kind of problem by numbering the steps and entering the calculations in a table, a procedure that is illustrated in Table 6.3.

The equations in the table can be entered into a spreadsheet and the calculations performed row by row to determine the velocity, position, and acceleration as functions of time. The calculations can also be carried out by using a program written in either BASIC, C++, or FORTRAN or by using commercially available mathematics packages for personal computers. Many small increments can be taken, and accurate results can usually be obtained with the help of a computer. Graphs of velocity versus time or position versus time can be displayed to help you visualize the motion.

One advantage of the Euler method is that the dynamics is not obscured—the fundamental relationships between acceleration and force, velocity and acceleration, and position and velocity are clearly evident. Indeed, these relationships form the heart of the calculations. There is no need to use advanced mathematics, and the basic physics governs the dynamics.

The Euler method is completely reliable for infinitesimally small time increments, but for practical reasons a finite increment size must be chosen. For the finite difference approximation of Equation 6.10 to be valid, the time increment must be small enough that the acceleration can be approximated as being constant during the increment. We can determine an appropriate size for the time in-

See the spreadsheet file “Baseball with Drag” on the Student Web site (address below) for an example of how this technique can be applied to find the initial speed of the baseball described in Example 6.14. We cannot use our regular approach because our kinematics equations assume constant acceleration. Euler's method provides a way to circumvent this difficulty.

A detailed solution to Problem 41 involving iterative integration appears in the *Student Solutions Manual and Study Guide* and is posted on the Web at <http://www.saunderscollege.com/physics>

TABLE 6.3 The Euler Method for Solving Dynamics Problems

Step	Time	Position	Velocity	Acceleration
0	t_0	x_0	v_0	$a_0 = F(x_0, v_0, t_0)/m$
1	$t_1 = t_0 + \Delta t$	$x_1 = x_0 + v_0 \Delta t$	$v_1 = v_0 + a_0 \Delta t$	$a_1 = F(x_1, v_1, t_1)/m$
2	$t_2 = t_1 + \Delta t$	$x_2 = x_1 + v_1 \Delta t$	$v_2 = v_1 + a_1 \Delta t$	$a_2 = F(x_2, v_2, t_2)/m$
3	$t_3 = t_2 + \Delta t$	$x_3 = x_2 + v_2 \Delta t$	$v_3 = v_2 + a_2 \Delta t$	$a_3 = F(x_3, v_3, t_3)/m$
	\vdots	\vdots	\vdots	\vdots
n	t_n	x_n	v_n	a_n

crement by examining the particular problem being investigated. The criterion for the size of the time increment may need to be changed during the course of the motion. In practice, however, we usually choose a time increment appropriate to the initial conditions and use the same value throughout the calculations.

The size of the time increment influences the accuracy of the result, but unfortunately it is not easy to determine the accuracy of an Euler-method solution without a knowledge of the correct analytical solution. One method of determining the accuracy of the numerical solution is to repeat the calculations with a smaller time increment and compare results. If the two calculations agree to a certain number of significant figures, you can assume that the results are correct to that precision.

SUMMARY

Newton's second law applied to a particle moving in uniform circular motion states that the net force causing the particle to undergo a centripetal acceleration is

$$\sum F_r = ma_r = \frac{mv^2}{r} \quad (6.1)$$

You should be able to use this formula in situations where the force providing the centripetal acceleration could be the force of gravity, a force of friction, a force of string tension, or a normal force.

A particle moving in nonuniform circular motion has both a centripetal component of acceleration and a nonzero tangential component of acceleration. In the case of a particle rotating in a vertical circle, the force of gravity provides the tangential component of acceleration and part or all of the centripetal component of acceleration. Be sure you understand the directions and magnitudes of the velocity and acceleration vectors for nonuniform circular motion.

An observer in a noninertial (accelerating) frame of reference must introduce **fictitious forces** when applying Newton's second law in that frame. If these fictitious forces are properly defined, the description of motion in the noninertial frame is equivalent to that made by an observer in an inertial frame. However, the observers in the two frames do not agree on the causes of the motion. You should be able to distinguish between inertial and noninertial frames and identify the fictitious forces acting in a noninertial frame.


A body moving through a liquid or gas experiences a **resistive force** that is speed-dependent. This resistive force, which opposes the motion, generally increases with speed. The magnitude of the resistive force depends on the shape of the body and on the properties of the medium through which the body is moving. In the limiting case for a falling body, when the magnitude of the resistive force equals the body's weight, the body reaches its **terminal speed**. You should be able to apply Newton's laws to analyze the motion of objects moving under the influence of resistive forces. You may need to apply **Euler's method** if the force depends on velocity, as it does for air drag.

QUESTIONS


1. Because the Earth rotates about its axis and revolves around the Sun, it is a noninertial frame of reference. Assuming the Earth is a uniform sphere, why would the apparent weight of an object be greater at the poles than at the equator?
2. Explain why the Earth bulges at the equator.

3. Why is it that an astronaut in a space capsule orbiting the Earth experiences a feeling of weightlessness?
4. Why does mud fly off a rapidly turning automobile tire?
5. Imagine that you attach a heavy object to one end of a spring and then whirl the spring and object in a horizontal circle (by holding the free end of the spring). Does the spring stretch? If so, why? Discuss this in terms of the force causing the circular motion.
6. It has been suggested that rotating cylinders about 10 mi in length and 5 mi in diameter be placed in space and used as colonies. The purpose of the rotation is to simulate gravity for the inhabitants. Explain this concept for producing an effective gravity.
7. Why does a pilot tend to black out when pulling out of a steep dive?
8. Describe a situation in which a car driver can have a centripetal acceleration but no tangential acceleration.
9. Describe the path of a moving object if its acceleration is constant in magnitude at all times and (a) perpendicular to the velocity; (b) parallel to the velocity.
10. Analyze the motion of a rock falling through water in terms of its speed and acceleration as it falls. Assume that the resistive force acting on the rock increases as the speed increases.
11. Consider a small raindrop and a large raindrop falling through the atmosphere. Compare their terminal speeds. What are their accelerations when they reach terminal speed?


PROBLEMS

1, 2, 3 = straightforward, intermediate, challenging  = full solution available in the *Student Solutions Manual and Study Guide*

WEB = solution posted at <http://www.saunderscollege.com/physics/>  = Computer useful in solving problem  = Interactive Physics

 = paired numerical/symbolic problems

Section 6.1 Newton's Second Law Applied to Uniform Circular Motion

1. A toy car moving at constant speed completes one lap around a circular track (a distance of 200 m) in 25.0 s. (a) What is its average speed? (b) If the mass of the car is 1.50 kg, what is the magnitude of the force that keeps it in a circle?
2. A 55.0-kg ice skater is moving at 4.00 m/s when she grabs the loose end of a rope, the opposite end of which is tied to a pole. She then moves in a circle of radius 0.800 m around the pole. (a) Determine the force exerted by the rope on her arms. (b) Compare this force with her weight.
-  3. A light string can support a stationary hanging load of 25.0 kg before breaking. A 3.00-kg mass attached to the string rotates on a horizontal, frictionless table in a circle of radius 0.800 m. What range of speeds can the mass have before the string breaks?
4. In the Bohr model of the hydrogen atom, the speed of the electron is approximately 2.20×10^6 m/s. Find (a) the force acting on the electron as it revolves in a circular orbit of radius 0.530×10^{-10} m and (b) the centripetal acceleration of the electron.
5. In a cyclotron (one type of particle accelerator), a deuteron (of atomic mass 2.00 u) reaches a final speed of 10.0% of the speed of light while moving in a circular path of radius 0.480 m. The deuteron is maintained in the circular path by a magnetic force. What magnitude of force is required?
6. A satellite of mass 300 kg is in a circular orbit around the Earth at an altitude equal to the Earth's mean radius (see Example 6.6). Find (a) the satellite's orbital speed, (b) the period of its revolution, and (c) the gravitational force acting on it.
7. Whenever two Apollo astronauts were on the surface of the Moon, a third astronaut orbited the Moon. Assume the orbit to be circular and 100 km above the surface of the Moon. If the mass of the Moon is 7.40×10^{22} kg and its radius is 1.70×10^6 m, determine (a) the orbiting astronaut's acceleration, (b) his orbital speed, and (c) the period of the orbit.
8. The speed of the tip of the minute hand on a town clock is 1.75×10^{-3} m/s. (a) What is the speed of the tip of the second hand of the same length? (b) What is the centripetal acceleration of the tip of the second hand?
9. A coin placed 30.0 cm from the center of a rotating, horizontal turntable slips when its speed is 50.0 cm/s. (a) What provides the force in the radial direction when the coin is stationary relative to the turntable? (b) What is the coefficient of static friction between coin and turntable?
10. The cornering performance of an automobile is evaluated on a skid pad, where the maximum speed that a car can maintain around a circular path on a dry, flat surface is measured. The centripetal acceleration, also called the lateral acceleration, is then calculated as a multiple of the free-fall acceleration g . The main factors affecting the performance are the tire characteristics and the suspension system of the car. A Dodge Viper GTS can negotiate a skid pad of radius 61.0 m at 86.5 km/h. Calculate its maximum lateral acceleration.
11. A crate of eggs is located in the middle of the flatbed of a pickup truck as the truck negotiates an unbanked

curve in the road. The curve may be regarded as an arc of a circle of radius 35.0 m. If the coefficient of static friction between crate and truck is 0.600, how fast can the truck be moving without the crate sliding?

12. A car initially traveling eastward turns north by traveling in a circular path at uniform speed as in Figure P6.12. The length of the arc ABC is 235 m, and the car completes the turn in 36.0 s. (a) What is the acceleration when the car is at B located at an angle of 35.0° ? Express your answer in terms of the unit vectors \mathbf{i} and \mathbf{j} . Determine (b) the car's average speed and (c) its average acceleration during the 36.0-s interval.

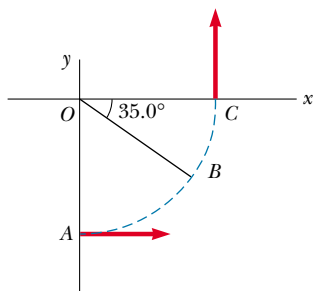


Figure P6.12

13. Consider a conical pendulum with an 80.0-kg bob on a 10.0-m wire making an angle of $\theta = 5.00^\circ$ with the vertical (Fig. P6.13). Determine (a) the horizontal and vertical components of the force exerted by the wire on the pendulum and (b) the radial acceleration of the bob.

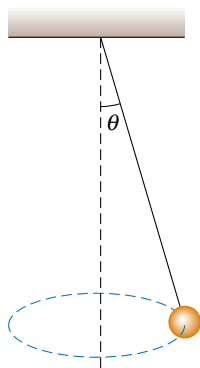


Figure P6.13

Section 6.2 Nonuniform Circular Motion

14. A car traveling on a straight road at 9.00 m/s goes over a hump in the road. The hump may be regarded as an arc of a circle of radius 11.0 m. (a) What is the apparent weight of a 600-N woman in the car as she rides over the

hump? (b) What must be the speed of the car over the hump if she is to experience weightlessness? (That is, if her apparent weight is zero.)



15. Tarzan ($m = 85.0$ kg) tries to cross a river by swinging from a vine. The vine is 10.0 m long, and his speed at the bottom of the swing (as he just clears the water) is 8.00 m/s. Tarzan doesn't know that the vine has a breaking strength of 1 000 N. Does he make it safely across the river?

16. A hawk flies in a horizontal arc of radius 12.0 m at a constant speed of 4.00 m/s. (a) Find its centripetal acceleration. (b) It continues to fly along the same horizontal arc but steadily increases its speed at the rate of 1.20 m/s^2 . Find the acceleration (magnitude and direction) under these conditions.

17. A 40.0-kg child sits in a swing supported by two chains, each 3.00 m long. If the tension in each chain at the lowest point is 350 N, find (a) the child's speed at the lowest point and (b) the force exerted by the seat on the child at the lowest point. (Neglect the mass of the seat.)

18. A child of mass m sits in a swing supported by two chains, each of length R . If the tension in each chain at the lowest point is T , find (a) the child's speed at the lowest point and (b) the force exerted by the seat on the child at the lowest point. (Neglect the mass of the seat.)

19. A pail of water is rotated in a vertical circle of radius 1.00 m. What must be the minimum speed of the pail at the top of the circle if no water is to spill out?

20. A 0.400-kg object is swung in a vertical circular path on a string 0.500 m long. If its speed is 4.00 m/s at the top of the circle, what is the tension in the string there?

21. A roller-coaster car has a mass of 500 kg when fully loaded with passengers (Fig. P6.21). (a) If the car has a speed of 20.0 m/s at point A, what is the force exerted by the track on the car at this point? (b) What is the maximum speed the car can have at B and still remain on the track?

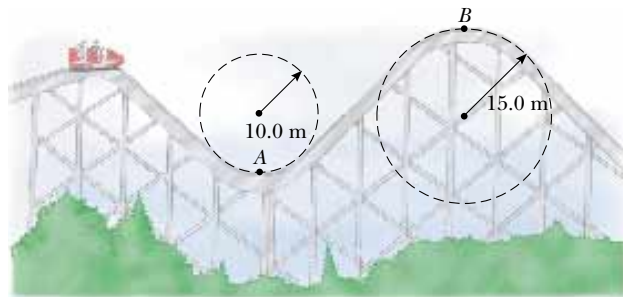


Figure P6.21

22. A roller coaster at the Six Flags Great America amusement park in Gurnee, Illinois, incorporates some of the latest design technology and some basic physics. Each vertical loop, instead of being circular, is shaped like a teardrop (Fig. P6.22). The cars ride on the inside of the loop at the top, and the speeds are high enough to ensure that the cars remain on the track. The biggest loop is 40.0 m high, with a maximum speed of 31.0 m/s (nearly 70 mi/h) at the bottom. Suppose the speed at the top is 13.0 m/s and the corresponding centripetal acceleration is $2g$. (a) What is the radius of the arc of the teardrop at the top? (b) If the total mass of the cars plus people is M , what force does the rail exert on this total mass at the top? (c) Suppose the roller coaster had a loop of radius 20.0 m. If the cars have the same speed, 13.0 m/s at the top, what is the centripetal acceleration at the top? Comment on the normal force at the top in this situation.



Figure P6.22 (Frank Cezus/EPG International)

(Optional)

Section 6.3 Motion in Accelerated Frames

23. A merry-go-round makes one complete revolution in 12.0 s. If a 45.0-kg child sits on the horizontal floor of the merry-go-round 3.00 m from the center, find (a) the child's acceleration and (b) the horizontal force of friction that acts on the child. (c) What minimum coefficient of static friction is necessary to keep the child from slipping?
24. A 5.00-kg mass attached to a spring scale rests on a frictionless, horizontal surface as in Figure P6.24. The spring scale, attached to the front end of a boxcar, reads 18.0 N when the car is in motion. (a) If the spring scale reads zero when the car is at rest, determine the acceleration of the car. (b) What will the spring scale read if the car moves with constant velocity? (c) Describe the forces acting on the mass as observed by someone in the car and by someone at rest outside the car.

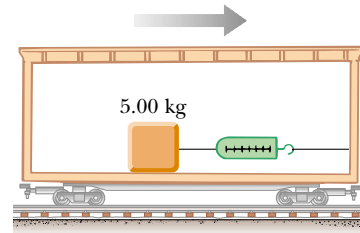


Figure P6.24

25. A 0.500-kg object is suspended from the ceiling of an accelerating boxcar as was seen in Figure 6.13. If $a = 3.00 \text{ m/s}^2$, find (a) the angle that the string makes with the vertical and (b) the tension in the string.
26. The Earth rotates about its axis with a period of 24.0 h. Imagine that the rotational speed can be increased. If an object at the equator is to have zero apparent weight, (a) what must the new period be? (b) By what factor would the speed of the object be increased when the planet is rotating at the higher speed? (Hint: See Problem 53 and note that the apparent weight of the object becomes zero when the normal force exerted on it is zero. Also, the distance traveled during one period is $2\pi R$, where R is the Earth's radius.)
27. A person stands on a scale in an elevator. As the elevator starts, the scale has a constant reading of 591 N. As the elevator later stops, the scale reading is 391 N. Assume the magnitude of the acceleration is the same during starting and stopping, and determine (a) the weight of the person, (b) the person's mass, and (c) the acceleration of the elevator.
28. A child on vacation wakes up. She is lying on her back. The tension in the muscles on both sides of her neck is 55.0 N as she raises her head to look past her toes and out the motel window. Finally, it is not raining! Ten minutes later she is screaming and sliding feet first down a water slide at a constant speed of 5.70 m/s, riding high on the outside wall of a horizontal curve of radius 2.40 m (Fig. P6.28). She raises her head to look forward past her toes; find the tension in the muscles on both sides of her neck.



Figure P6.28

29. A plumb bob does not hang exactly along a line directed to the center of the Earth, because of the Earth's rotation. How much does the plumb bob deviate from a radial line at 35.0° north latitude? Assume that the Earth is spherical.

(Optional)

Section 6.4 Motion in the Presence of Resistive Forces

30. A sky diver of mass 80.0 kg jumps from a slow-moving aircraft and reaches a terminal speed of 50.0 m/s .
 (a) What is the acceleration of the sky diver when her speed is 30.0 m/s ? What is the drag force exerted on the diver when her speed is (b) 50.0 m/s ? (c) 30.0 m/s ?
31. A small piece of Styrofoam packing material is dropped from a height of 2.00 m above the ground. Until it reaches terminal speed, the magnitude of its acceleration is given by $a = g - bv$. After falling 0.500 m , the Styrofoam effectively reaches its terminal speed, and then takes 5.00 s more to reach the ground. (a) What is the value of the constant b ? (b) What is the acceleration at $t = 0$? (c) What is the acceleration when the speed is 0.150 m/s ?
32. (a) Estimate the terminal speed of a wooden sphere (density 0.830 g/cm^3) falling through the air if its radius is 8.00 cm . (b) From what height would a freely falling object reach this speed in the absence of air resistance?
33. Calculate the force required to pull a copper ball of radius 2.00 cm upward through a fluid at the constant speed 9.00 cm/s . Take the drag force to be proportional to the speed, with proportionality constant 0.950 kg/s . Ignore the buoyant force.
34. A fire helicopter carries a 620-kg bucket at the end of a cable 20.0 m long as in Figure P6.34. As the helicopter flies to a fire at a constant speed of 40.0 m/s , the cable makes an angle of 40.0° with respect to the vertical. The bucket presents a cross-sectional area of 3.80 m^2 in a plane perpendicular to the air moving past it. Determine the drag coefficient assuming that the resistive

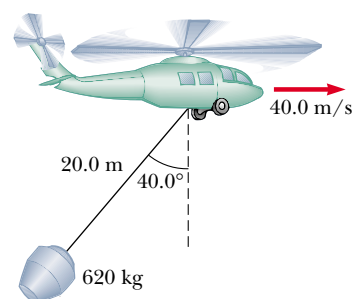


Figure P6.34

force is proportional to the square of the bucket's speed.

35. A small, spherical bead of mass 3.00 g is released from rest at $t = 0$ in a bottle of liquid shampoo. The terminal speed is observed to be $v_t = 2.00\text{ cm/s}$. Find (a) the value of the constant b in Equation 6.4, (b) the time τ the bead takes to reach $0.632v_t$, and (c) the value of the resistive force when the bead reaches terminal speed.
36. The mass of a sports car is $1\,200\text{ kg}$. The shape of the car is such that the aerodynamic drag coefficient is 0.250 and the frontal area is 2.20 m^2 . Neglecting all other sources of friction, calculate the initial acceleration of the car if, after traveling at 100 km/h , it is shifted into neutral and is allowed to coast.
- WEB 37. A motorboat cuts its engine when its speed is 10.0 m/s and coasts to rest. The equation governing the motion of the motorboat during this period is $v = v_i e^{-ct}$, where v is the speed at time t , v_i is the initial speed, and c is a constant. At $t = 20.0\text{ s}$, the speed is 5.00 m/s . (a) Find the constant c . (b) What is the speed at $t = 40.0\text{ s}$? (c) Differentiate the expression for $v(t)$ and thus show that the acceleration of the boat is proportional to the speed at any time.
38. Assume that the resistive force acting on a speed skater is $f = -kmv^2$, where k is a constant and m is the skater's mass. The skater crosses the finish line of a straight-line race with speed v_f and then slows down by coasting on his skates. Show that the skater's speed at any time t after crossing the finish line is $v(t) = v_f / (1 + ktv_f)$.
39. You can feel a force of air drag on your hand if you stretch your arm out of the open window of a speeding car. (Note: Do not get hurt.) What is the order of magnitude of this force? In your solution, state the quantities you measure or estimate and their values.

(Optional)

6.5 Numerical Modeling in Particle Dynamics

40. A 3.00-g leaf is dropped from a height of 2.00 m above the ground. Assume the net downward force exerted on the leaf is $F = mg - bv$, where the drag factor is $b = 0.030\text{ kg/s}$. (a) Calculate the terminal speed of the leaf. (b) Use Euler's method of numerical analysis to find the speed and position of the leaf as functions of

time, from the instant it is released until 99% of terminal speed is reached. (*Hint:* Try $\Delta t = 0.005$ s.)

- 41.** A hailstone of mass 4.80×10^{-4} kg falls through the air and experiences a net force given by

$$F = -mg + Cv^2$$

where $C = 2.50 \times 10^{-5}$ kg/m. (a) Calculate the terminal speed of the hailstone. (b) Use Euler's method of numerical analysis to find the speed and position of the hailstone at 0.2-s intervals, taking the initial speed to be zero. Continue the calculation until the hailstone reaches 99% of terminal speed.

- 42.** A 0.142-kg baseball has a terminal speed of 42.5 m/s (95 mi/h). (a) If a baseball experiences a drag force of magnitude $R = Cv^2$, what is the value of the constant C ? (b) What is the magnitude of the drag force when the speed of the baseball is 36.0 m/s? (c) Use a computer to determine the motion of a baseball thrown vertically upward at an initial speed of 36.0 m/s. What maximum height does the ball reach? How long is it in the air? What is its speed just before it hits the ground?
- 43.** A 50.0-kg parachutist jumps from an airplane and falls with a drag force proportional to the square of the speed $R = Cv^2$. Take $C = 0.200$ kg/m with the parachute closed and $C = 20.0$ kg/m with the chute open. (a) Determine the terminal speed of the parachutist in both configurations, before and after the chute is opened. (b) Set up a numerical analysis of the motion and compute the speed and position as functions of time, assuming the jumper begins the descent at 1 000 m above the ground and is in free fall for 10.0 s before opening the parachute. (*Hint:* When the parachute opens, a sudden large acceleration takes place; a smaller time step may be necessary in this region.)
- 44.** Consider a 10.0-kg projectile launched with an initial speed of 100 m/s, at an angle of 35.0° elevation. The resistive force is $\mathbf{R} = -b\mathbf{v}$, where $b = 10.0$ kg/s. (a) Use a numerical method to determine the horizontal and vertical positions of the projectile as functions of time. (b) What is the range of this projectile? (c) Determine the elevation angle that gives the maximum range for the projectile. (*Hint:* Adjust the elevation angle by trial and error to find the greatest range.)
- 45.** A professional golfer hits a golf ball of mass 46.0 g with her 5-iron, and the ball first strikes the ground 155 m (170 yards) away. The ball experiences a drag force of magnitude $R = Cv^2$ and has a terminal speed of 44.0 m/s. (a) Calculate the drag constant C for the golf ball. (b) Use a numerical method to analyze the trajectory of this shot. If the initial velocity of the ball makes an angle of 31.0° (the loft angle) with the horizontal, what initial speed must the ball have to reach the 155-m distance? (c) If the same golfer hits the ball with her 9-iron (47.0° loft) and it first strikes the ground 119 m away, what is the initial speed of the ball? Discuss the differences in trajectories between the two shots.

ADDITIONAL PROBLEMS

- 46.** An 1 800-kg car passes over a bump in a road that follows the arc of a circle of radius 42.0 m as in Figure P6.46. (a) What force does the road exert on the car as the car passes the highest point of the bump if the car travels at 16.0 m/s? (b) What is the maximum speed the car can have as it passes this highest point before losing contact with the road?
- 47.** A car of mass m passes over a bump in a road that follows the arc of a circle of radius R as in Figure P6.46. (a) What force does the road exert on the car as the car passes the highest point of the bump if the car travels at a speed v ? (b) What is the maximum speed the car can have as it passes this highest point before losing contact with the road?

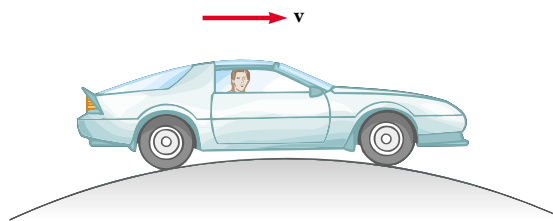


Figure P6.46 Problems 46 and 47.

- 48.** In one model of a hydrogen atom, the electron in orbit around the proton experiences an attractive force of about 8.20×10^{-8} N. If the radius of the orbit is 5.30×10^{-11} m, how many revolutions does the electron make each second? (This number of revolutions per unit time is called the *frequency* of the motion.) See the inside front cover for additional data.
- 49.** A student builds and calibrates an accelerometer, which she uses to determine the speed of her car around a certain unbanked highway curve. The accelerometer is a plumb bob with a protractor that she attaches to the roof of her car. A friend riding in the car with her observes that the plumb bob hangs at an angle of 15.0° from the vertical when the car has a speed of 23.0 m/s. (a) What is the centripetal acceleration of the car rounding the curve? (b) What is the radius of the curve? (c) What is the speed of the car if the plumb bob deflection is 9.00° while the car is rounding the same curve?
- 50.** Suppose the boxcar shown in Figure 6.13 is moving with constant acceleration a up a hill that makes an angle ϕ with the horizontal. If the hanging pendulum makes a constant angle θ with the perpendicular to the ceiling, what is a ?
- 51.** An air puck of mass 0.250 kg is tied to a string and allowed to revolve in a circle of radius 1.00 m on a fric-

tionless horizontal table. The other end of the string passes through a hole in the center of the table, and a mass of 1.00 kg is tied to it (Fig. P6.51). The suspended mass remains in equilibrium while the puck on the tabletop revolves. What are (a) the tension in the string, (b) the force exerted by the string on the puck, and (c) the speed of the puck?

52. An air puck of mass m_1 is tied to a string and allowed to revolve in a circle of radius R on a frictionless horizontal table. The other end of the string passes through a hole in the center of the table, and a mass m_2 is tied to it (Fig. P6.51). The suspended mass remains in equilibrium while the puck on the tabletop revolves. What are (a) the tension in the string? (b) the central force exerted on the puck? (c) the speed of the puck?

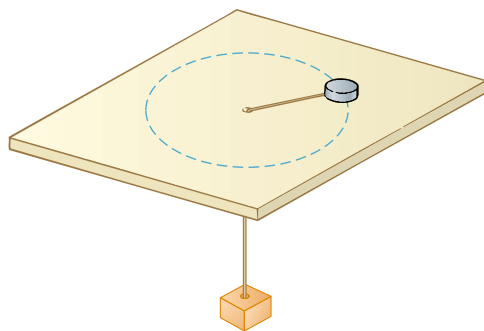


Figure P6.51 Problems 51 and 52.

- WEB 53. Because the Earth rotates about its axis, a point on the equator experiences a centripetal acceleration of 0.0337 m/s^2 , while a point at one of the poles experiences no centripetal acceleration. (a) Show that at the equator the gravitational force acting on an object (the true weight) must exceed the object's apparent weight. (b) What is the apparent weight at the equator and at the poles of a person having a mass of 75.0 kg ? (Assume the Earth is a uniform sphere and take $g = 9.800 \text{ m/s}^2$.)

54. A string under a tension of 50.0 N is used to whirl a rock in a horizontal circle of radius 2.50 m at a speed of 20.4 m/s . The string is pulled in and the speed of the rock increases. When the string is 1.00 m long and the speed of the rock is 51.0 m/s , the string breaks. What is the breaking strength (in newtons) of the string?

55. A child's toy consists of a small wedge that has an acute angle θ (Fig. P6.55). The sloping side of the wedge is frictionless, and a mass m on it remains at constant height if the wedge is spun at a certain constant speed. The wedge is spun by rotating a vertical rod that is firmly attached to the wedge at the bottom end. Show

that, when the mass sits a distance L up along the sloping side, the speed of the mass must be

$$v = (gL \sin \theta)^{1/2}$$

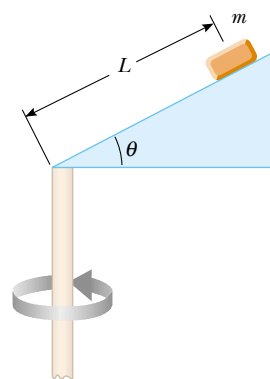


Figure P6.55

56. The pilot of an airplane executes a constant-speed loop-the-loop maneuver. His path is a vertical circle. The speed of the airplane is 300 mi/h , and the radius of the circle is 1200 ft . (a) What is the pilot's apparent weight at the lowest point if his true weight is 160 lb ? (b) What is his apparent weight at the highest point? (c) Describe how the pilot could experience apparent weightlessness if both the radius and the speed can be varied. (Note: His apparent weight is equal to the force that the seat exerts on his body.)
57. For a satellite to move in a stable circular orbit at a constant speed, its centripetal acceleration must be inversely proportional to the square of the radius r of the orbit. (a) Show that the tangential speed of a satellite is proportional to $r^{-1/2}$. (b) Show that the time required to complete one orbit is proportional to $r^{3/2}$.
58. A penny of mass 3.10 g rests on a small 20.0-g block supported by a spinning disk (Fig. P6.58). If the coeffi-

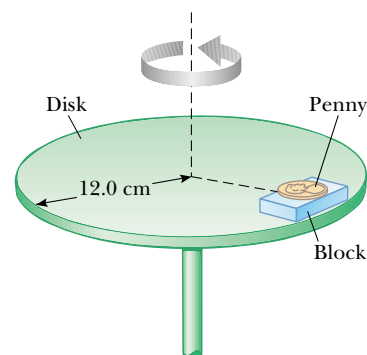


Figure P6.58

cients of friction between block and disk are 0.750 (static) and 0.640 (kinetic) while those for the penny and block are 0.450 (kinetic) and 0.520 (static), what is the maximum rate of rotation (in revolutions per minute) that the disk can have before either the block or the penny starts to slip?

59. Figure P6.59 shows a Ferris wheel that rotates four times each minute and has a diameter of 18.0 m. (a) What is the centripetal acceleration of a rider? What force does the seat exert on a 40.0-kg rider (b) at the lowest point of the ride and (c) at the highest point of the ride? (d) What force (magnitude and direction) does the seat exert on a rider when the rider is halfway between top and bottom?



Figure P6.59 (Color Box/FPG)

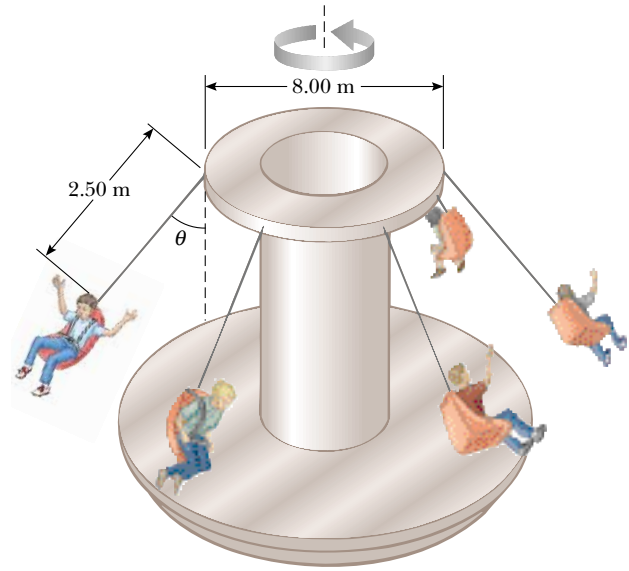


Figure P6.61

63. An amusement park ride consists of a large vertical cylinder that spins about its axis fast enough that any person inside is held up against the wall when the floor drops away (Fig. P6.63). The coefficient of static friction between person and wall is μ_s , and the radius of the cylinder is R . (a) Show that the maximum period of revolution necessary to keep the person from falling is $T = (4\pi^2 R \mu_s / g)^{1/2}$. (b) Obtain a numerical value for T

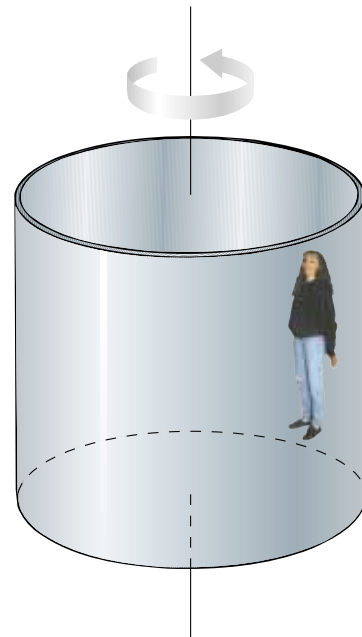


Figure P6.63

60. A space station, in the form of a large wheel 120 m in diameter, rotates to provide an “artificial gravity” of 3.00 m/s^2 for persons situated at the outer rim. Find the rotational frequency of the wheel (in revolutions per minute) that will produce this effect.
61. An amusement park ride consists of a rotating circular platform 8.00 m in diameter from which 10.0-kg seats are suspended at the end of 2.50-m massless chains (Fig. P6.61). When the system rotates, the chains make an angle $\theta = 28.0^\circ$ with the vertical. (a) What is the speed of each seat? (b) Draw a free-body diagram of a 40.0-kg child riding in a seat and find the tension in the chain.
62. A piece of putty is initially located at point A on the rim of a grinding wheel rotating about a horizontal axis. The putty is dislodged from point A when the diameter through A is horizontal. The putty then rises vertically and returns to A the instant the wheel completes one revolution. (a) Find the speed of a point on the rim of the wheel in terms of the acceleration due to gravity and the radius R of the wheel. (b) If the mass of the putty is m , what is the magnitude of the force that held it to the wheel?

if $R = 4.00$ m and $\mu_s = 0.400$. How many revolutions per minute does the cylinder make?

64. *An example of the Coriolis effect.* Suppose air resistance is negligible for a golf ball. A golfer tees off from a location precisely at $\phi_i = 35.0^\circ$ north latitude. He hits the ball due south, with range 285 m. The ball's initial velocity is at 48.0° above the horizontal. (a) For what length of time is the ball in flight? The cup is due south of the golfer's location, and he would have a hole-in-one if the Earth were not rotating. As shown in Figure P6.64, the Earth's rotation makes the tee move in a circle of radius $R_E \cos \phi_i = (6.37 \times 10^6 \text{ m}) \cos 35.0^\circ$, completing one revolution each day. (b) Find the eastward speed of the tee, relative to the stars. The hole is also moving eastward, but it is 285 m farther south and thus at a slightly lower latitude ϕ_f . Because the hole moves eastward in a slightly larger circle, its speed must be greater than that of the tee. (c) By how much does the hole's speed exceed that of the tee? During the time the ball is in flight, it moves both upward and downward, as well as southward with the projectile motion you studied in Chapter 4, but it also moves eastward with the speed you found in part (b). The hole moves to the east at a faster speed, however, pulling ahead of the ball with the relative speed you found in part (c). (d) How far to the west of the hole does the ball land?

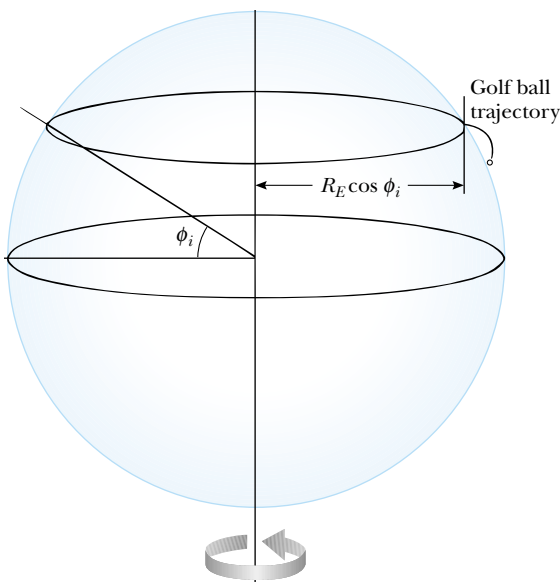


Figure P6.64

65. A curve in a road forms part of a horizontal circle. As a car goes around it at constant speed 14.0 m/s, the total force exerted on the driver has magnitude 130 N. What are the magnitude and direction of the total force exerted on the driver if the speed is 18.0 m/s instead?

66. A car rounds a banked curve as shown in Figure 6.6. The radius of curvature of the road is R , the banking angle is θ , and the coefficient of static friction is μ_s . (a) Determine the range of speeds the car can have without slipping up or down the banked surface. (b) Find the minimum value for μ_s such that the minimum speed is zero. (c) What is the range of speeds possible if $R = 100$ m, $\theta = 10.0^\circ$, and $\mu_s = 0.100$ (slippery conditions)?
67. A single bead can slide with negligible friction on a wire that is bent into a circle of radius 15.0 cm, as in Figure P6.67. The circle is always in a vertical plane and rotates steadily about its vertical diameter with a period of 0.450 s. The position of the bead is described by the angle θ that the radial line from the center of the loop to the bead makes with the vertical. (a) At what angle up from the lowest point can the bead stay motionless relative to the turning circle? (b) Repeat the problem if the period of the circle's rotation is 0.850 s.

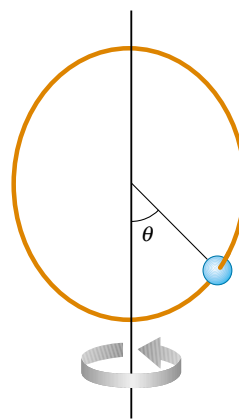


Figure P6.67

68. The expression $F = arv + br^2v^2$ gives the magnitude of the resistive force (in newtons) exerted on a sphere of radius r (in meters) by a stream of air moving at speed v (in meters per second), where a and b are constants with appropriate SI units. Their numerical values are $a = 3.10 \times 10^{-4}$ and $b = 0.870$. Using this formula, find the terminal speed for water droplets falling under their own weight in air, taking the following values for the drop radii: (a) $10.0 \mu\text{m}$, (b) $100 \mu\text{m}$, (c) 1.00 mm. Note that for (a) and (c) you can obtain accurate answers without solving a quadratic equation, by considering which of the two contributions to the air resistance is dominant and ignoring the lesser contribution.
69. A model airplane of mass 0.750 kg flies in a horizontal circle at the end of a 60.0-m control wire, with a speed of 35.0 m/s. Compute the tension in the wire if it makes a constant angle of 20.0° with the horizontal. The forces exerted on the airplane are the pull of the control wire,

its own weight, and aerodynamic lift, which acts at 20.0° inward from the vertical as shown in Figure P6.69.

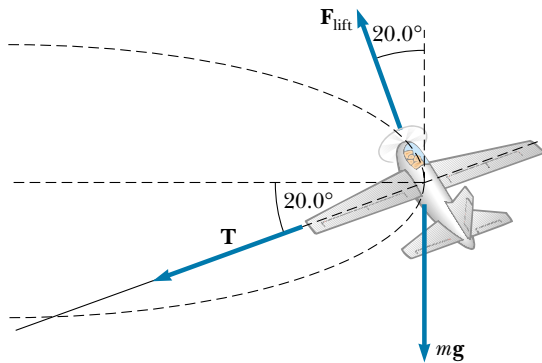


Figure P6.69

70. A 9.00-kg object starting from rest falls through a viscous medium and experiences a resistive force $\mathbf{R} = -b\mathbf{v}$, where \mathbf{v} is the velocity of the object. If the object's speed reaches one-half its terminal speed in 5.54 s, (a) determine the terminal speed. (b) At what time is the speed of the object three-fourths the terminal speed? (c) How far has the object traveled in the first 5.54 s of motion?
71. Members of a skydiving club were given the following data to use in planning their jumps. In the table, d is the distance fallen from rest by a sky diver in a "free-fall

stable spread position" versus the time of fall t . (a) Convert the distances in feet into meters. (b) Graph d (in meters) versus t . (c) Determine the value of the terminal speed v_t by finding the slope of the straight portion of the curve. Use a least-squares fit to determine this slope.

t (s)	d (ft)
1	16
2	62
3	138
4	242
5	366
6	504
7	652
8	808
9	971
10	1 138
11	1 309
12	1 483
13	1 657
14	1 831
15	2 005
16	2 179
17	2 353
18	2 527
19	2 701
20	2 875

ANSWERS TO QUICK QUIZZES

- 6.1 No. The tangential acceleration changes just the speed part of the velocity vector. For the car to move in a circle, the *direction* of its velocity vector must change, and the only way this can happen is for there to be a centripetal acceleration.
- 6.2 (a) The ball travels in a circular path that has a larger radius than the original circular path, and so there must be some external force causing the change in the velocity vector's direction. The external force must not be as strong as the original tension in the string because if it were, the ball would follow the original path. (b) The ball again travels in an arc, implying some kind of external force. As in part (a), the external force is directed toward the center of the new arc and not toward the center of the original circular path. (c) The ball undergoes an abrupt change in velocity—from tangent to the circle to perpendicular to it—and so must have experienced a large force that had one component opposite the ball's velocity (tangent to the circle) and another component radially outward. (d) The ball travels in a straight line tangent to the original path. If there is an external force, it cannot have a component perpendicular to this line because if it did, the path would curve. In

fact, if the string breaks and there is no other force acting on the ball, Newton's first law says the ball will travel along such a tangent line at constant speed.

- 6.3 At Ⓐ the path is along the circumference of the larger circle. Therefore, the wire must be exerting a force on the bead directed toward the center of the circle. Because the speed is constant, there is no tangential force component. At Ⓑ the path is not curved, and so the wire exerts no force on the bead. At Ⓒ the path is again curved, and so the wire is again exerting a force on the bead. This time the force is directed toward the center of the smaller circle. Because the radius of this circle is smaller, the magnitude of the force exerted on the bead is larger here than at Ⓐ.

