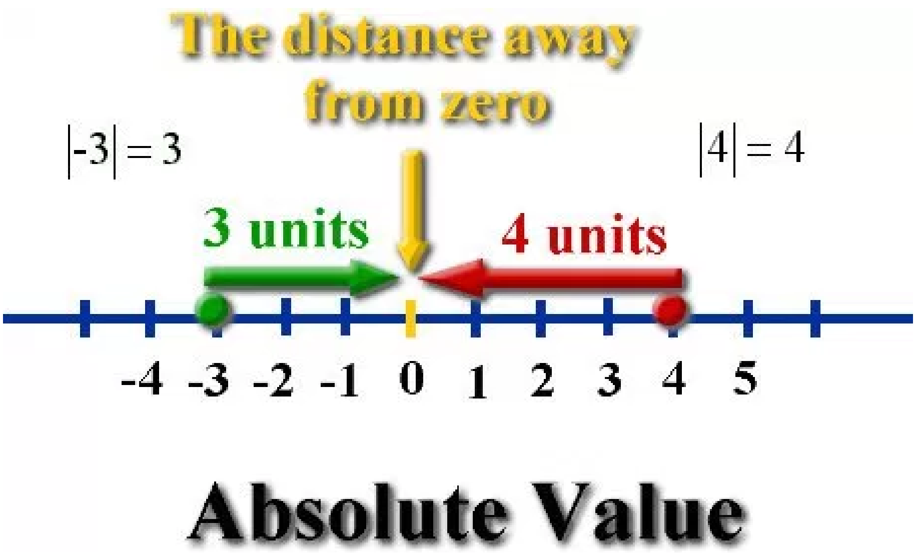
10. Solving Inequalities

Absolute Value

The **absolute value** of a number can be considered as the **distance** between 0 and that number on the real number line.



Remember that distance is always a positive quantity (or zero).
The distance in the diagram above from +4 to 0 is 4 units and the distance from -3 to 0 is 3 units. These units are never negative values.

Using absolute value, we write this as:

$|4| = 4$ and $|-3| = 3$

The rule for computing absolute value is:

$$|a| = a \text{ if } a \geq 0$$

$$|a| = -a \text{ if } a < 0$$

Examples:

$|25| = 25$

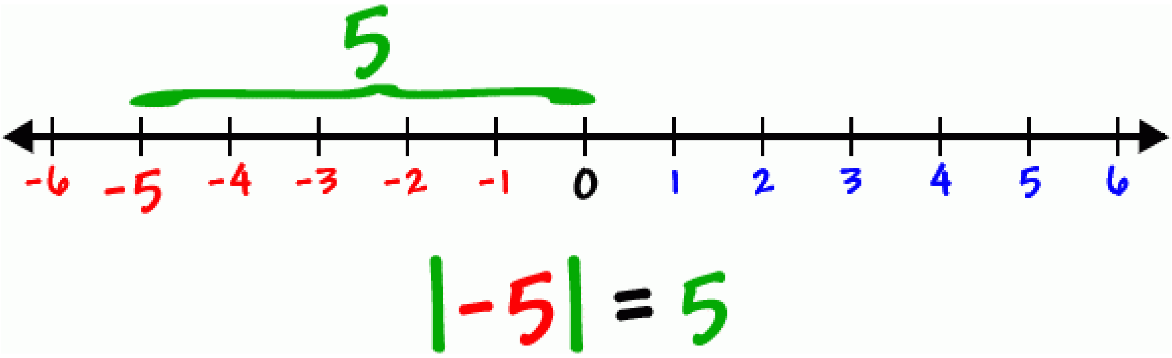
$|-25| = -(-25) = 25$

Absolute Value of an Integer:

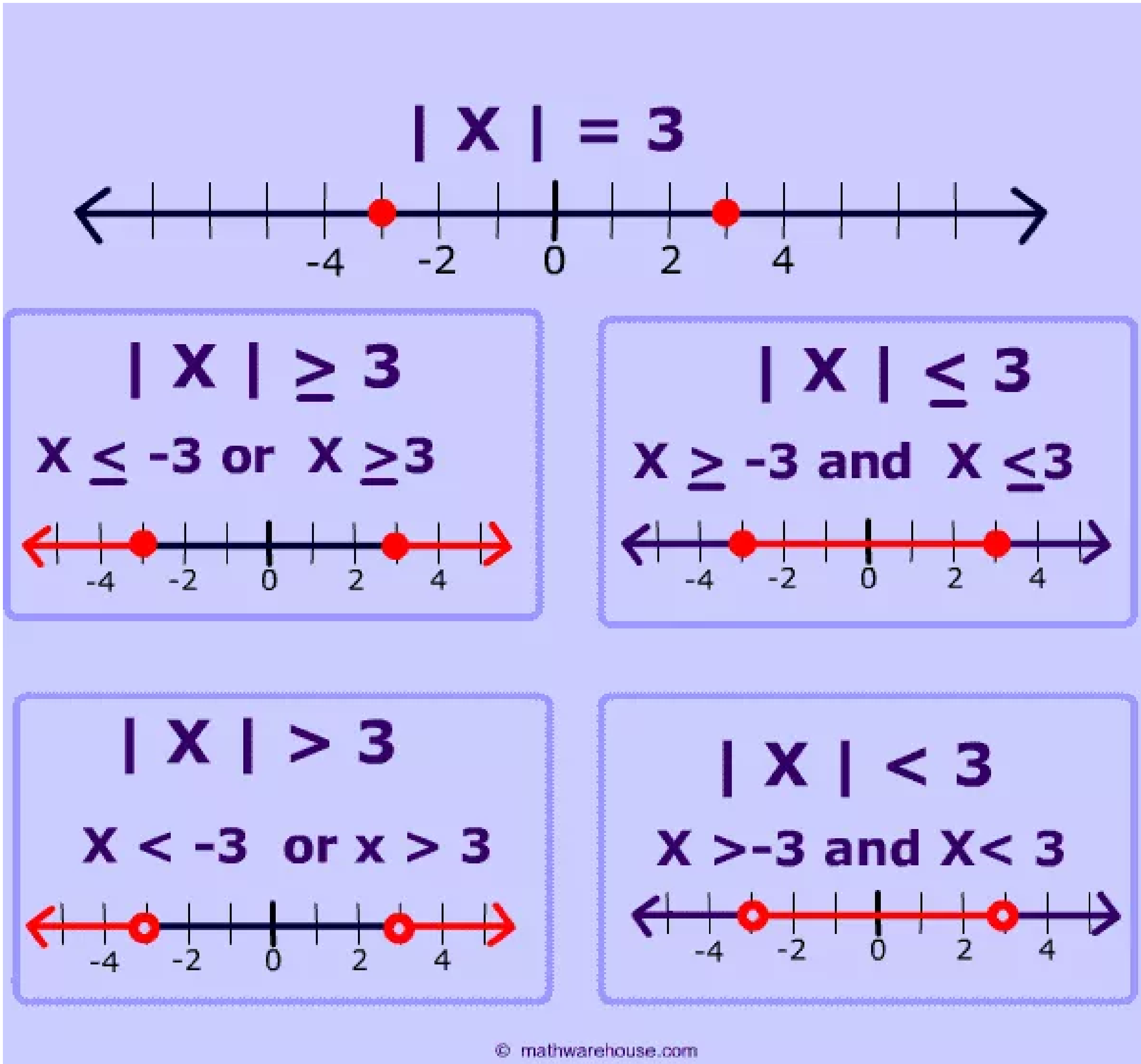
The absolute value of an integer is the numerical value (magnitude) of an integer regardless of its sign (direction). It is denoted by the symbol **? ?**. The absolute value of an integer is either zero or positive. Also, the corresponding positive and negative integers have the same absolute value.

Examples:

The absolute value of -2 is ?-2? = 2.
The absolute value of 5 is ?5? = 5.
The absolute value of 0 is ?0? = 0



Absolute Value Inequality Graph



Absolute Value Equations

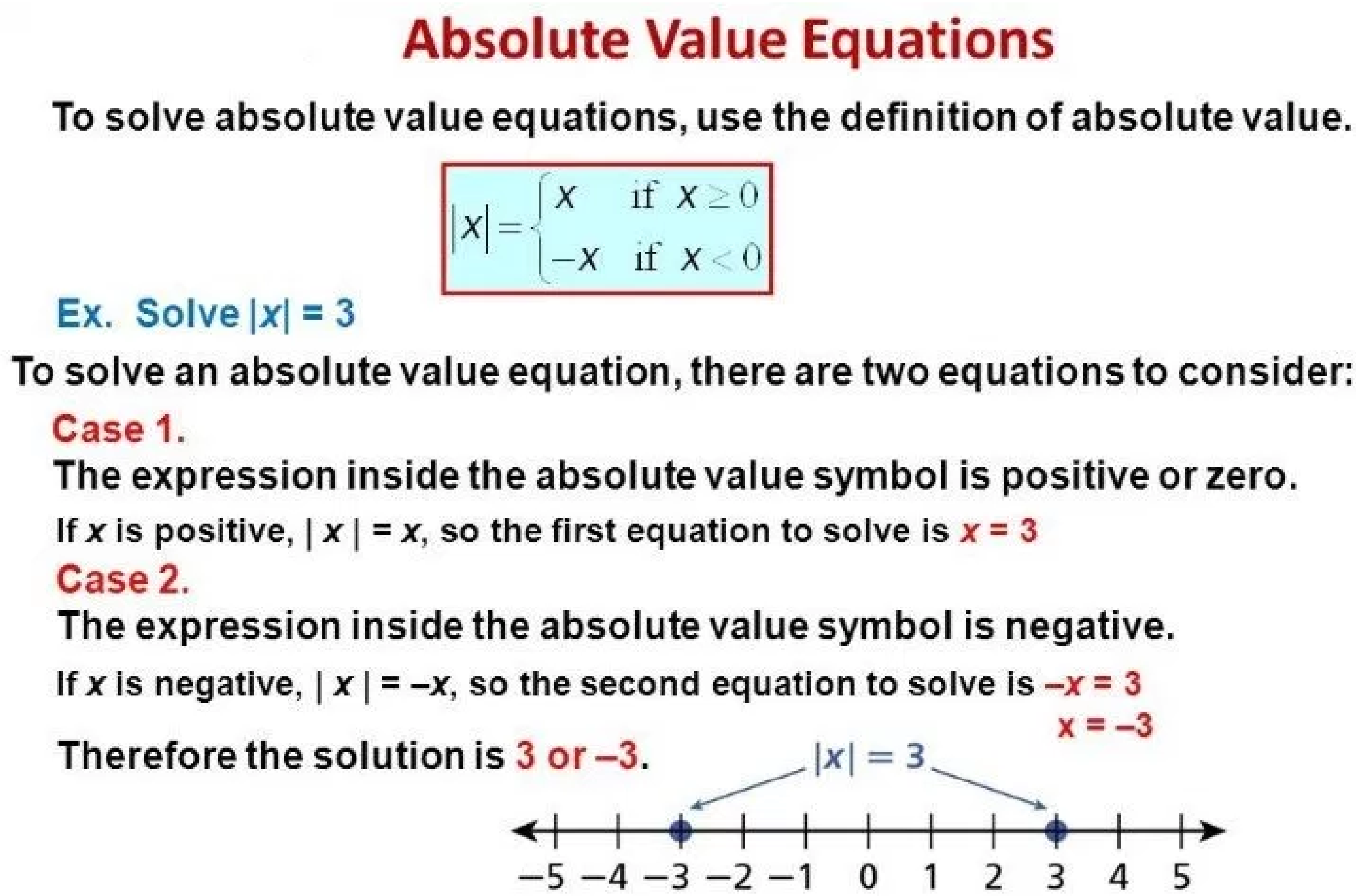
To solve an absolute value equation, isolate the absolute value on one side of the equal sign, and establish two cases:

Case 1:

|a| = b set a = b
Set the expression inside the absolute value symbol equal to the other given expression.

Case 2:

|a| = b set a = − b
Set the expression inside the absolute value symbol equal to the negation of the other given expression.



Always Check your answers.

The two cases create "derived" equations. These derived equations may not always be true equivalents to the original equation. Consequently, the roots of the derived equations MUST BE CHECKED in the original equation so that you do not list extraneous roots as answers.

Case 1: $x - 10 = 6$ $x = 16$	**Case 2:** $x - 10 = -6$ $x = 4$	Answer: $x = 16,\ x = 4$
Check: $\lvert 16 - 10 \rvert = 6$ $\lvert 6 \rvert = 6$ $6 = 6$	*Check:* $\lvert 4 - 10 \rvert = 6$ $\lvert -6 \rvert = 6$ $6 = 6$	The solutions are 16 or 4. On a number line, these value are each 6 units away from 10.

Example 2: (No solution)

As soon as you isolate the absolute value expression, you observe:

There is no need to work out the two cases in this problem. Absolute value is NEVER equal to a negative value. This equation is never true. The answer is the empty set .

Example 3: $|3x+2| = 4x+5$ (Two cases with one solution)

Case 1:	Case 2:	
$\lvert 3x+2\rvert = 4x+5$ $3x+2 = 4x+5$ $2 = x+5$ $-3 = x$	$\lvert 3x+2\rvert = 4x+5$ $3x+2 = -(4x+5)$ $3x+2 = -4x-5$ $7x = -7$ $x = -1$	Answer: $x = -1$
Check: $\lvert 3(-3)+2\rvert = 4(-3)+5$ $\lvert -9+2\rvert = -12+5$ $\lvert -7\rvert = -7$ $7 \neq -7$ *Not an answer!*	*Check:* $\lvert 3(-1)+2\rvert = 4(-1)+5$ $\lvert -3+2\rvert = -4+5$ $\lvert -1\rvert = 1$ $1 = 1$	The check shows that $x = -3$ is NOT a solution to this absolute value equation. There is only ONE answer. $x = -1$. **Always check!!!!**

Example 4: A machine fills Quaker Oatmeal containers with 32 ounces of oatmeal. After the containers are filled, another machine weighs them. If the container's weight differs from the desired 32 ounce weight by more than 0.5 ounces, the container is rejected. Write an equation that can be used to find the heaviest and lightest acceptable weights for the Quaker Oatmeal container. Solve the equation.

Solution: Let x = the weight of the container

$$|x-32| = 0.5$$

Case 1:	Case 2:
$x-32 = 0.5$	$x-32 = -0.5$
$x = 32.5$	$x = 31.5$
Check:	*Check:*
$\lvert 32.5-32\rvert = 0.5$	$\lvert 31.5-32\rvert = 0.5$
$\lvert 0.5\rvert = 0.5$	$\lvert -0.5\rvert = 0.5$
$0.5 = 0.5$	$0.5 = 0.5$

Answer: $x = 31.5$ ounces (lightest)
$x = 32.5$ ounces (heaviest)

When setting up a word problem involving absolute value, remember that absolute value can represent "distance" from a given point.

The difference between the answer (x) and the desired point (32) is placed under the absolute value symbol. This absolute value is then set equal to the desired "distance" (0.5).