Chapter 3: Indefinite Integration

EXERCISE 3.1 [PAGE 102]

Exercise 3.1 | Q 1.1 | Page 102

Integrate the following w.r.t. x : x^3 + x2 – x + 1

SOLUTION

$$\int (x^3 + x^2 - x + 1) dx = \int x3\ dx + \int x^2 dx - \int x dx + \int 1 dx$$

$$= \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x + c.$$

SOLUTION

$$\int (x^3 + x^2 - x + 1) dx = \int x3\ dx + \int x^2 dx - \int x dx + \int 1 dx$$

$$= \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x + c.$$

Exercise 3.1 | Q 1.2 | Page 102

Integrate the following w.r.t. x : $\int x^2 \left(1 - \frac{2}{x}\right)^2 dx$

SOLUTION

$$\int x^2 \left(1 - \frac{2}{x}\right)^2 dx$$

$$= \int x^2 \left(1 - \frac{4}{x} + \frac{4}{x^2}\right) dx$$

$$= \int (x^2 - 4x + 4) dx$$

$= \int x^2 dx - 4\int x dx + 4\int 1 dx$

$= \frac{x^3}{3} - 4\left(\frac{x^2}{2}\right) + 4x + c$

$= \frac{1}{3}x^3 - 2x^2 + 4x + c.$

Exercise 3.1 | Q 1.3 | Page 102

Integrate the following w.r.t. x : $3\sec^2 x - \frac{4}{x} + \frac{1}{x\sqrt{x}} - 7$

SOLUTION

$\int\left(3\sec^2 x - \frac{4}{x} + \frac{1}{x\sqrt{x}} - 7\right)dx$

$= 3\int \sec^2 x\, dx - 4\int \frac{1}{x} dx + \int x^{-\frac{3}{2}} dx - 7\int 1 dx$

$= 3\tan x - 4\log|x| + \frac{x - \frac{3}{2} + 1}{-\frac{3}{2} + 1} - 7x + c$

$= 3\tan x - 4\log|x| - \frac{2}{\sqrt{x}} - 7x + c$

Exercise 3.1 | Q 1.4 | Page 102

Integrate the following w.r.t. x : $2x^3 - 5x + \frac{3}{x} + \frac{4}{x^5}$

SOLUTION

$\int\left(2x^3 - 5x + \frac{3}{x} + \frac{4}{x^5}\right)dx$

$= 2\int x^3 dx - 5\int x dx + 3\int \frac{1}{x} dx + 4\int x^{-5} dx$

$= 2\left(\frac{x^4}{4}\right) - 5\left(\frac{x^2}{2}\right) + 3\log|x| + 4\left(\frac{x}{-4}\right) + c$

$$= \frac{x^4}{2} - \frac{5}{2}x^2 + 3\log|x| - \frac{1}{x^4} + c$$

Exercise 3.1 | Q 1.5 | Page 102

Integrate the following w.r.t. x : $\frac{3x^3 - 2x + 5}{x\sqrt{x}}$

SOLUTION

$$\int \frac{3x^3 - 2x + 5}{x\sqrt{x}} dx$$

$$= \int x^{\frac{-3}{2}}\left(3x^3 - 2x + 5\right)dx$$

$$= \int \left(3x^{\frac{3}{2}} - 2x^{-\frac{1}{2}} + 5x^{-\frac{3}{2}}\right)dx$$

$$= 3\int x^{\frac{3}{2}}dx - 2\int x^{-\frac{1}{2}}dx + 5\int x^{-\frac{3}{2}}dx$$

$$= 3\left(\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right) - 2\left(\frac{x^{\frac{1}{2}+1}}{-\frac{1}{2}+1}\right) + 5\left(\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}\right) + c$$

$$= \frac{6}{5}x^2\sqrt{x^2} - 4\sqrt{x} - \frac{10}{\sqrt{x}} + c.$$

Exercise 3.1 | Q 2.01 | Page 102

Evaluate the following integrals : $\tan^2 x$

SOLUTION

$$\int \tan^2 x dx = \int \left(\sec^2 x - 1\right)dx$$

$$= \int \sec^2 x dx - f1dx$$

= tan x – x + c.

Exercise 3.1 | Q 2.02 | Page 102

Evaluate the following integrals : $\int \frac{\sin 2x}{\cos x} dx$

SOLUTION

$$\int \frac{\sin 2x}{\cos x} dx = \int \frac{2 \sin x \cos x}{\cos x} dx$$

$= 2 \int \sin x dx$

= − 2 cos x + c.

Exercise 3.1 | Q 2.03 | Page 102

Evaluate the following integrals : $\int \frac{\sin x}{\cos^2 x} dx$

SOLUTION

$$\int \frac{\sin x}{\cos^2 x} dx = \int \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right) dx$$

$= \int \sec x \tan x dx$

= sec x + c.

Exercise 3.1 | Q 2.04 | Page 102

Evaluate the following integrals : $\int \frac{\cos 2x}{\sin^2 x} dx$

SOLUTION

$$\int \frac{\cos 2x}{\sin^2 x} dx = \int \frac{(1 - 2 \sin^2 x)}{\sin^2 x} dx$$

$= \int \left(\frac{1}{\sin^2 x} - \frac{2 \sin^2 x}{\sin^2 x}\right) dx$

$= \int \text{cosec}^2 x dx - 2\int dx$

= – cot x – 2x + c.

Exercise 3.1 | Q 2.05 | Page 102

Evaluate the following integrals : $\int \frac{\cos 2x}{\sin^2 x . \cos^2 x} dx$

SOLUTION

$\int \frac{\cos 2x}{\sin^2 x . \cos^2 x} dx$

$= \int \frac{\cos^2 x - \sin^2 x}{\sin^2 x . \cos^2 x} dx$

$= \int \left(\frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}\right) dx$

$= \int \text{cosec}^2 x dx - \int \sec^2 x \, dx$

= – cot x – tan x + c.

Exercise 3.1 | Q 2.06 | Page 102

Evaluate the following integrals : $\int \frac{\sin x}{1 + \sin x} dx$

SOLUTION

$\int \frac{\sin x}{1 + \sin x} dx$

$= \int \frac{\sin x}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x} dx$

$= \int \frac{\sin x - \sin^2 x}{1 - \sin^2 x} dx$

$= \int \frac{\sin x - \sin^2 x}{\cos^2 x} dx$

= $\int\left(\frac{\sin x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}\right)dx$

= $\int\left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right)dx-\int \tan^2 x dx$

= $\int \sec x \tan x dx-\int\left(\sec^2 x-1\right)dx$

= $\int \sec x \tan x dx-\int \sec^2 x dx+\int 1 dx$

= sec x – tan x + x + c.

Exercise 3.1 | Q 2.07 | Page 102

Evaluate the following integrals : $\int \frac{\tan x}{\sec x+\tan x}dx$

SOLUTION

$\int \frac{\tan x}{\sec x+\tan x}dx$

= $\int \frac{\tan x}{\sec x+\tan x}\times\frac{\sec x-\tan x}{\sec - \tan x}dx$

= $\int \frac{\sec x \tan x-\tan^2 x}{\sec^2 x-\tan^2 x}dx$

= $\int \frac{sec \tan x-\left(\sec^2 x-1\right)}{1}dx$

= $\int \sec x \tan x dx-\int \sec^2 x dx+\int 1 dx$

= sec x – tan x + x + c.

Exercise 3.1 | Q 2.08 | Page 102

Evaluate the following integrals : $\int \sqrt{1+\sin 2x} dx$

SOLUTION

$$\int \sqrt{1+\sin 2x}dx$$

$$= \int \sqrt{\cos^2 x + \sin^2 x + 2\sin x \cos x dx}$$

$$= \int \sqrt{(\cos x + \sin x)^2} dx$$

$$= \int (\cos x + \sin x) dx$$

$$= \int \cos x \ dx + \int \sin x \ dx$$

= sin x − cos x + c.

Exercise 3.1 | Q 2.09 | Page 102

Evaluate the following integrals : $\int \sqrt{1-\cos 2x}dx$

SOLUTION

$$\int \sqrt{1-\cos 2x}dx$$

$$= \int \sqrt{2\sin^2 x dx}$$

$$= \sqrt{2} \int \sin x dx$$

$$= -\sqrt{2}\cos x + c.$$

Exercise 3.1 | Q 2.1 | Page 102

Evaluate the following integrals : $\int \sin 4x \cos 3x dx$

SOLUTION

$$\int \sin 4x \cos 3x dx$$

$$= \frac{1}{2}\int \sin 4x \cos 3x dx$$

$$= \frac{1}{2}\int [\sin(4x+3x)+\sin(4x-3x)]dx$$

$$= \frac{1}{2}\int \sin 7x dx + \frac{1}{2}\int \sin x dx$$

$$= \frac{1}{2}\left(\frac{-\cos 7x}{7}\right) - \frac{1}{2}\cos x + c$$

$$= -\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + c.$$

Exercise 3.1 | Q 3.01 | Page 102

Evaluate the following integrals : $\int \frac{x}{x+2}.dx$

SOLUTION

$$\int \frac{x}{x+2}.dx$$

$$= \int \frac{(x+2)-2}{x+2}.dx$$

$$= \int \left(\frac{x+2}{x+2} - \frac{2}{x+2}\right).dx$$

$$= \int 1dx - 2\int \frac{1}{x+2}.dx$$

= x – 2 log |x + 2| + c.

Exercise 3.1 | Q 3.02 | Page 102

Evaluate the following integrals : $\int \frac{4x+3}{2x+1}.dx$

SOLUTION

$$\int \frac{4x+3}{2x+1}.\,dx$$

$$= \int \frac{(2(2x+1)+1)}{2x+1}.\,dx$$

$$= \int \left(\frac{2(2x+1)}{2x+1} + \frac{1}{2x+1}\right).\,dx$$

$$= 2\int 1dx + \int \frac{1}{2x+1}.\,dx$$

$$= 2x + \frac{1}{2}\log|2x+1| + c.$$

Exercise 3.1 | Q 3.03 | Page 102

Evaluate the following integrals : $\int \frac{5x+2}{3x-4}.\,dx$

SOLUTION

$$\int \frac{5x+2}{3x-4}.\,dx$$

$$= \int \frac{\frac{5}{3}\left(3x-4+\frac{20}{3}+2\right)}{3x-4}.\,dx$$

$$= \int \frac{\frac{5}{3}(3x-4)+\frac{26}{3}}{3x-4}.\,dx$$

$$= \int \left[\frac{5}{3} + \frac{\left(\frac{26}{3}\right)}{3x-4}\right].\,dx$$

$$= \frac{5}{3}\int 1\,dx + \frac{26}{3}\int \frac{1}{3x-4}.\,dx$$

$= (5x)(3) + \frac{26}{3} . \frac{1}{3} \log|3x - 4| + c$

$= (5x)(3) + \frac{26}{3} \log|3x - 4| + c.$

Exercise 3.1 | Q 3.03 | Page 102

Evaluate the following integrals : $\int \frac{5x + 2}{3x - 4} . dx$

SOLUTION

$\int \frac{5x + 2}{3x - 4} . dx$

$= \int \frac{\frac{5}{3}\left(3x - 4 + \frac{20}{3} + 2\right)}{3x - 4} . dx$

$= \int \frac{\frac{5}{3}(3x - 4) + \frac{26}{3}}{3x - 4} . dx$

$= \int \left[\frac{5}{3} + \frac{\left(\frac{26}{3}\right)}{3x - 4}\right] . dx$

$= \frac{5}{3} \int 1\ dx + \frac{26}{3} \int \frac{1}{3x - 4} . dx$

$= (5x)(3) + \frac{26}{3} . \frac{1}{3} \log|3x - 4| + c$

$= (5x)(3) + \frac{26}{3} \log|3x - 4| + c.$

Exercise 3.1 | Q 3.04 | Page 102

Evaluate the following integrals : $\int \frac{x - 2}{\sqrt{x + 5}} . dx$

SOLUTION

$$\int \frac{x-2}{\sqrt{x+5}}.dx$$

$$= \int \frac{(x+5)-7}{\sqrt{x+5}}.dx$$

$$= \int \left(\frac{x+5}{\sqrt{x+5}} - \frac{7}{\sqrt{x+5}}\right).dx$$

$$= \int (x+5)^{\frac{1}{2}} dx - 7\int (x+5)^{-\frac{1}{2}} dx$$

$$= \frac{(x+5)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{7(x+5)^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c$$

$$= \frac{1}{3}(x+5)^{\frac{3}{2}} - 14\sqrt{x+5} + c.$$

Exercise 3.1 | Q 3.05 | Page 102

Evaluate the following integrals : $\int \frac{2x-7}{\sqrt{4x-1}}.dx$

SOLUTION

$$\int \frac{2x-7}{\sqrt{4x-1}}.dx$$

$$= \frac{1}{2}\int \frac{2(2x-7)}{\sqrt{4x-1}}.dx$$

$$= \frac{1}{2}\int \frac{(4x-1)-13}{\sqrt{4x-1}}.dx$$

$$= \frac{1}{2}\int \left(\frac{4x-1}{\sqrt{4x-1}} - \frac{13}{\sqrt{4x-1}}\right).dx$$

$$= \frac{1}{2}\int (4x-1)^{\frac{1}{2}}.dx - \frac{13}{2}\int (4x-1)^{-\frac{1}{2}}.dx$$

$= \frac{1}{2}\int \frac{(4x-1)^{\frac{3}{2}}}{(4)\left(\frac{3}{2}\right)} - \frac{13}{2} \cdot \frac{(4x-1)^{\frac{1}{2}}}{(4)\left(\frac{1}{2}\right)} + c$

$= \frac{1}{12}(4x-1)^{\frac{3}{2}} - \frac{13}{4}\sqrt{4x-1} + c.$

Exercise 3.1 | Q 3.06 | Page 102

Evaluate the following integrals : $\int \frac{\sin 4x}{\cos 2x} . dx$

SOLUTION

$\int \frac{\sin 4x}{\cos 2x} . dx$

$= \int \frac{2 \sin 2x \cos 2x}{\cos 2x} . dx$

$= 2\int \sin 2x . dx$

$= 2\left(-\frac{\cos 2x}{2}\right) + c$

= − cos 2x + c.

Exercise 3.1 | Q 3.07 | Page 102

Evaluate the following integrals : $\int \sqrt{1 + \sin 5x} . dx$

SOLUTION

$\int \sqrt{1 + \sin 5x} . dx$

$= \int \sqrt{\sin^2 x + \cos^2 x + 5 \sin x \cos x} . dx$

$= \int \sqrt{(\sin x + \cos x)^2} . dx$

= $\int (\sin x + \cos x).\, dx$

= $\int \sin x\, dx + \int \cos x.\, dx$

= $\left(\frac{2}{5}\sin\ \frac{5x}{2} - \cos\ \frac{5x}{2}\right) + c.$

Exercise 3.1 | Q 3.08 | Page 102

Evaluate the following integrals : $\int \cos^2 x.\, dx$

SOLUTION

Recall the identity cos 2x = 2 $\cos^2$x – 1, which gives

$\cos^2 x = \frac{1 + \cos 2x}{2}$

Therefore, $\int \cos^2 x.\, dx$

= $\frac{1}{2}\int (1 + \cos 2x).\, dx$

= $\frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x.\, dx$

= $\frac{x}{2} + \frac{1}{4}\sin 2x + C.$

Exercise 3.1 | Q 3.09 | Page 102

Evaluate the following integrals : $\int \frac{2}{\sqrt{x} - \sqrt{x+3}}.\, dx$

SOLUTION

$$\int \frac{2}{\sqrt{x} - \sqrt{x+3}} . dx$$

$$= \int \frac{2}{\sqrt{x} - \sqrt{x+3}} \times \frac{\sqrt{x} + \sqrt{x+3}}{\sqrt{x} + \sqrt{x+3}} . dx$$

$$= \int \frac{2\left(\sqrt{x} + \sqrt{x+3}\right)}{x - (x+3)} . dx$$

$$= -\frac{2}{3} \int \left(\sqrt{x} + \sqrt{x+3}\right) . dx$$

$$= -\frac{2}{3} \int x^{\frac{1}{2}} dx - \frac{2}{3} \int (x+3)^{\frac{1}{2}} . dx$$

$$= -\frac{2}{3} . \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{2}{3} . \frac{(x+3)^{\frac{3}{2}}}{\left(\frac{3}{2}\right) + c}$$

$$= -\frac{4}{9}\left[x^{\frac{3}{2}} + (x+3)^{\frac{3}{2}}\right] + c.$$

Exercise 3.1 | Q 3.1 | Page 102

Evaluate the following integrals : $\int \frac{3}{\sqrt{7x-2} - \sqrt{7x-5}} . dx$

SOLUTION

$$\int \frac{3}{\sqrt{7x-2} - \sqrt{7x-5}} . dx$$

$$= \int \frac{3}{\sqrt{7x-2} - \sqrt{7x-5}} \times \frac{\sqrt{7x-2} + \sqrt{7x-5}}{\sqrt{7x-2} + \sqrt{7x-5}} . dx$$

$$= \int \frac{3\left(\sqrt{7x-2} + \sqrt{7x-5}\right)}{(7x-2) - (7x-5)} . dx$$

$$= \int \left(\sqrt{7x-2} + \sqrt{7x-5}\right) . dx$$

$$= \int (7x-2)^{\frac{1}{2}}.dx + \int (7x-5)^{\frac{1}{2}}.dx$$

$$= \frac{(7x-2)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{7} + \frac{(7x-5)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{7} + c$$

$$= \frac{2}{21}(7x-2)^{\frac{3}{2}} + \frac{2}{21}(7x-5)^{\frac{3}{2}} + c.$$

Exercise 3.1 | Q 4 | Page 102

If $f\prime(x) = x - \frac{3}{x^3}, f(1) = \frac{11}{2}$, find f(x)

SOLUTION

By the definition of integral,

f(x) = $\int f\prime(x).dx$

$$= \int \left(x - \frac{3}{x^3}\right).dx$$

$$= \int x\ dx - 3\int x^{-3}.dx$$

$$= \frac{x^2}{2} - \frac{3x^{(-2)}}{(-2)} + c$$

$$= \frac{x^2}{2} + \frac{3}{2x^2} + c \qquad ...(1)$$

f(1) = $\frac{11}{2}$ ...(Given)

$$\therefore \frac{1}{2} + \frac{3}{2} + c = \frac{11}{2}$$

$\therefore$ c = $\frac{7}{2}$

$\therefore$ f(x) = $\frac{x^2}{2} + \frac{3}{2x^2} + \frac{7}{2}$. ...[By (1)]

EXERCISE 3.2 (A) [PAGE 110]

Exercise 3.2 (A) | Q 1.01 | Page 110

Integrate the following functions w.r.t. x : $\frac{(\log x)^n}{x}$

SOLUTION

Let I = $\int \frac{(\log x)^n}{x}. dx$

Put log x = t.

$\therefore \frac{1}{x}. dx = dt$

$\therefore$ I = $\int t^n dt$

= $\frac{t^{n+1}}{n+1} + c$

= $\frac{1}{n+1}. (\log x)^{n+1} + c.$

Exercise 3.2 (A) | Q 1.02 | Page 110

Integrate the following functions w.r.t. x : $\frac{(\sin^{-1} x)^{\frac{3}{2}}}{\sqrt{1 - x^2}}$

SOLUTION

Let I = $\int \frac{(\sin^{-1} x)^{\frac{3}{2}}}{\sqrt{1 - x^2}}. dx$

Put $\sin^{-1}x$ = t.

$\therefore \frac{1}{\sqrt{1 - x^2}}. dx$ = dt

$\therefore$ I = $\int t^{\frac{3}{2}}\, dt$

= $\frac{t^{\frac{5}{2}}}{\frac{5}{2}} + c$

= $\frac{2}{5}\left(\sin^{-1} x\right)^{\frac{5}{2}} + c.$

Exercise 3.2 (A) | Q 1.03 | Page 110

Integrate the following functions w.r.t. x : $\frac{1+x}{x.\sin(x+\log x)}$

SOLUTION

Let I = $\int \frac{1+x}{x.\sin(x+\log x)}.dx$

= $\int \frac{1}{\sin(x+\log x)}.\left(\frac{1+x}{x}\right).dx$

= $\int \frac{1}{\sin(x+\log x)}.\left(\frac{1}{x}+1\right).dx$

Put x + log x = t

$\therefore$ $\left(1+\frac{1}{x}\right).dx$ = dt

$\therefore$ I = $\int \frac{1}{\sin t} dt = \int \text{cosec}\ t\ dt$

= log | cosec t – cot t | + c

= log | cosec (x + log x) – cot (x + logx) |+ c.

Exercise 3.2 (A) | Q 1.04 | Page 110

Integrate the following functions w.r.t. x : $\frac{x.\sec^2(x^2)}{\sqrt{\tan^3(x^2)}}$

SOLUTION

Let I = $\int \frac{x.\sec^2(x^2)}{\sqrt{\tan^3(x^2)}}.dx$

Put tan(x^2) = t

∴ $\sec^2(x^2)$ x 2x dx = dt

$\therefore x.\sec^2(x^2)dx = \frac{dt}{2}$

∴ I = $\int \frac{1}{\sqrt{t^3}}.\frac{dt}{2}$

= $\frac{1}{2}\int t^{-\frac{3}{2}}dt$

= $\frac{1}{2}.\frac{t^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

= $\frac{-1}{\sqrt{t}} + c$

= $\frac{-1}{\sqrt{\tan(x^2)}} + c.$

Exercise 3.2 (A) | Q 1.05 | Page 110

Integrate the following functions w.r.t. x : $\frac{e^{3x}}{e^{3x}+1}$

SOLUTION

Let I = $\int \frac{e^{3x}}{e^{3x}+1}.dx$

Put e^{3x} + 1 = t.

∴ $3e^{3x}$ dx = dt

$\therefore e^{3x}\,dx = \dfrac{dt}{3}$

$\therefore I = \displaystyle\int \frac{1}{t}\cdot\frac{dt}{3}$

$= \dfrac{1}{3}\displaystyle\int \frac{1}{t}dt$

$= \dfrac{1}{3}\log|t| + c$

$= \dfrac{1}{3}\log\left|e^{3x} + 1\right| + c.$

Exercise 3.2 (A) | Q 1.06 | Page 110

Integrate the following functions w.r.t. x : $\dfrac{x^2+2}{(x^2+1)}\cdot a^{x+\tan^{-1}x}$

SOLUTION

Let I = $\displaystyle\int \frac{x^2+2}{(x^2+1)}\cdot a^{x+\tan^{-1}x}\cdot dx$

$= \displaystyle\int a^{x+\tan^{-1}x}\cdot\left(\frac{x^2+2}{x^2+1}\right)\cdot dx$

Put x + $\tan^{-1}$x = t

$\therefore \left(1+\dfrac{1}{1+x^2}\right)\cdot dx$ = dt

$\therefore \left(\dfrac{1+x^2+1}{1+x^2}\right)\cdot dx$ = dt

$\therefore \left(\dfrac{x^2+2}{x^2+1}\right)\cdot dx$ = dt

$\therefore$ I = $\int a^t dt = \frac{a^t}{\log a} + c$

= $\frac{a^{x+\tan^{-1} x}}{\log a} + c.$

Exercise 3.2 (A) | Q 1.07 | Page 110

Integrate the following functions w.r.t. x : $e^x . \frac{\log(\sin e^x)}{\tan(e^x)}$

SOLUTION

Let I = $\int \frac{e^x . \log(\sin e^x)}{\tan(e^x)} . dx$

= $\int \log(\sin e^x) . e^x . \cot(e^x) dx$

Put log (sin e^x) = t

$\therefore$ $\frac{1}{\sin(e^x)} . \cos(e^x) . e^x dx$ = dt

$\therefore$ e^x . cot (e^x) dx = dt

$\therefore$ I = $\int t\ dt = \frac{t^2}{2} + c$

= $\frac{1}{2} [\log(\sin e^x)]^2 + c.$

Exercise 3.2 (A) | Q 1.08 | Page 110

Integrate the following functions w.r.t. x : $\frac{e^{2x} + 1}{e^{2x} - 1}$

SOLUTION

Let I = $\int \frac{e^{2x}+1}{e^{2x}-1}.dx$

= $\int \frac{\left(\frac{e^{2x}+1}{e^{x}}\right)}{\left(\frac{e^{2x}-1}{e^{x}}\right)}.dx$

= $\int \left(\frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\right).dx$

= $\int \frac{\frac{d}{dx}\left(e^{x}-e^{-x}\right)}{e^{x}-e^{-x}}.dx$

= $\log|e^{x} - e^{-x}| + c.$ ...$\left[\because \int \frac{f\prime(x)}{f(x)}.dx = \log|f(x)|+c\right]$

Exercise 3.2 (A) | Q 1.09 | Page 110

Integrate the following functions w.r.t. x : $\sin^4x.\cos^3x$

SOLUTION

Let I = $\int \sin^4 x.\cos^3 x dx$

= $\int \sin^4 x.\cos^2 x.\cos x dx$

= $\int \sin^4 x\left(1-\sin^2 x\right)\cos x dx$

Put sin x = t

∴ cos x dx = dt

∴ I = $\int t^4\left(1-t^2\right)dt$

= $\int \left(t^4-t^6\right)dt$

$= \int t^4 dt - \int t^6 dt$

$= \frac{t^5}{5} - \frac{t^7}{7} + c$

$= \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c.$

Exercise 3.2 (A) | Q 1.1 | Page 110

Integrate the following functions w.r.t. x : $\frac{1}{4x + 5x^{-11}}$

SOLUTION

Let I = $\int \frac{1}{4x + 5x^{-11}} . dx$

$= \int \frac{x_{11}}{x^{11}(4x + 5x^{-11})} . dx$

$= \int \frac{x^{11}}{4x^{12} + 5} . dx$

$= \frac{1}{48} \int \frac{48x^{11}}{4x^{12} + 5} . dx$

$= \frac{1}{48} \int \frac{\frac{d}{dx}(4x^{12} + 5)}{4x^{12} + 5} . dx$

$= \frac{1}{48} \log|4x^{12} + 5| + c$ ...$\left[\because \int \frac{f\prime(x)}{f(x)} dx = \log|f(x)| + c\right]$

Exercise 3.2 (A) | Q 1.11 | Page 110

Integrate the following functions w.r.t. x : $x^9.\sec^2(x^{10})$

SOLUTION

Let I = $\int x^9 . \sec^2(x^{10}) . dx$

Put x^{10} = t

∴ $10x^9 dx$ = dt

∴ $x^9 dx$ = $\frac{1}{10} dt$

∴ I = $\int \sec^2 t . \frac{dt}{10}$

= $\frac{1}{10} \tan t + c$

= $\frac{1}{10} \tan(x^{10}) + c.$

Exercise 3.2 (A) | Q 1.12 | Page 110

Integrate the following functions w.r.t. x : $e^{3\log x}(x^4 + 1)^{-1}$

SOLUTION

Let I = $e^{3\log x}(x^4 + 1)^{-1}$.dx

= $\int \frac{e^{\log x^3}}{x^4 + 1} . dx$

= $\int \frac{x^3}{x^4 + 1} . dx$ ...[∵ $e^{\log N} = N$]

= $\frac{1}{4} \int \frac{4x^3}{x^4 + 1} . dx$

= $\frac{1}{4} \int \frac{\frac{d}{dx}(x^4 + 1)}{x^4 + 1} . dx$

$= \frac{1}{4}\log\left|x^4 + 1\right| + c. \quad ...\left[\because \int \frac{f\prime(x)}{f(x)} dx = \log|f(x)| + c\right]$

Exercise 3.2 (A) | Q 1.13 | Page 110

Integrate the following functions w.r.t. x : $\frac{\sqrt{\tan x}}{\sin x . \cos x}$

SOLUTION

Let I = $\int \frac{\sqrt{\tan x}}{\sin x . \cos x} . dx$

Dividing numerator and denominator by $\cos^2 x$, we get

I = $\int \frac{\left(\frac{\sqrt{\tan x}}{\cos^2 x}\right)}{\left(\frac{\sin x}{\cos x}\right)} . dx$

= $\int \frac{\sqrt{\tan x} . \sec^2 x}{\tan x} . dx$

= $\int \frac{\sec^2 x}{\sqrt{\tan x}} . dx$

Put tan x = t

$\therefore \sec^2 x dx = dt$

$\therefore$ I = $\int \frac{1}{\sqrt{t}} dt$

= $\int t^{-\frac{1}{2}} dt$

= $\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$

= $2\sqrt{t} + c$

= $2\sqrt{\tan x} + c.$

Exercise 3.2 (A) | Q 1.14 | Page 110

Integrate the following functions w.r.t. x : $\dfrac{(x-1)^2}{(x^2+1)^2}$

SOLUTION

Let I = $\displaystyle\int \frac{(x-1)^2}{(x^2+1)^2}.dx$

= $\displaystyle\int \frac{x^2-2x+1}{(x^2+1)^2}.dx$

= $\displaystyle\int \frac{(x^2+1)-2x}{(x^2+1)^2}.dx$

= $\displaystyle\int \left[\frac{x^2+1}{(x^2+1)^2} - \frac{2x}{(x^2+1)^2}\right].dx$

= $\displaystyle\int \frac{1}{x^2+1}dx - \int \frac{2x}{(x^2+1)^2}.dx$

= $I_1 - I_2$ ...(Let)

In I_2, Put $x^2 + 1 = t$

∴ 2x dx = dt

= I = $\displaystyle\int \frac{1}{x^2+1}.dx - \int t^{-2}dt$

= $\tan^{-1}x - \dfrac{t^{-1}}{(-1)} + c$

= $\tan^{-1}x + \dfrac{1}{x^2+1} + c.$

Exercise 3.2 (A) | Q 1.15 | Page 110

Integrate the following functions w.r.t. x : $\dfrac{2\sin x\cos x}{3\cos^2 x + 4\sin^2 x}$

SOLUTION

Let I = $\int \frac{2\sin x\cos x}{3\cos^2 x + 4\sin^2 x}.dx$

Put 3 $\cos^2$x + 4 $\sin^2$x = t

$\therefore$ $\left[3(2\cos x)\frac{d}{dx}(\cos x) + 4(2\sin x)\frac{d}{dx}(\sin x)\right]dx$ = dt

$\therefore$ [– 6 cos x sin x + 8 sin x cos x]dx = dt

$\therefore$ 2 sin x cos x dx = dt

I = $\int \frac{dt}{t}$ = log|t| + c

= log |3 $\cos^2$x + 4 $\sin^2$x| + c.

Exercise 3.2 (A) | Q 1.16 | Page 110

Integrate the following functions w.r.t. x : $\frac{1}{\sqrt{x} + \sqrt{x^3}}$

SOLUTION

Let I = $\int \frac{1}{\sqrt{x} + \sqrt{x^3}}.dx$

= $\int \frac{1}{x^{\frac{1}{2}} + x^{\frac{3}{2}}}.dx$

Put x = t^2

$\therefore$ dx = 2t dt

Also $x^{\frac{1}{2}} = \left(t^2\right)^{\frac{1}{2}}$ = t

and

$x^{\frac{3}{2}} = \left(t^2\right)^{\frac{3}{2}}$ = t^3

$\therefore$ I = $\int \frac{2tdt}{t + t^3}$

= $2\int \frac{\text{tdt}}{t(1+t^2)}$

= $2\int \frac{1}{1+t^2}dt$

= $2\tan^{-1}$ t+ c

= $2\tan^{-1}(\sqrt{x}) + c.$

Exercise 3.2 (A) | Q 1.17 | Page 110

Integrate the following functions w.r.t. x : $\frac{10x^9\ 10^x.\log 10}{10^x + 10^{10}}$

SOLUTION

Let I = $\int \frac{10x^9\ 10^x.\log 10}{10^x + 10^{10}}.dx$

Put $10^x + x^{10}$ = t

$\therefore$ $(10^x.\log 10 + 10x^9).dx$ = dt

$\therefore$ I = $\int \frac{1}{t}dt$ = log | t | + c

= log | $10^x + x^{10}$ | + c.

Exercise 3.2 (A) | Q 1.18 | Page 110

Integrate the following functions w.r.t. x : $\frac{x^n - 1}{\sqrt{1+4x^n}}$

SOLUTION

Let I = $\int \frac{x^n - 1}{\sqrt{1+4x^n}} . dx$

Put x^n = t

∴ nx^{n-1} dx = dt

∴ x^{n-1} dx = $\frac{dt}{n}$

∴ I = $\int \frac{1}{\sqrt{1+4t}} . \frac{dt}{n}$

= $\frac{1}{n} \int (1+4t)^{-\frac{1}{2}} dt$

= $\frac{1}{n} . \frac{(1+4t)^{\frac{1}{2}}}{\frac{1}{2}} \times \frac{1}{4} + c$

= $\frac{1}{2n} . \sqrt{1+4x^n} + c.$

Exercise 3.2 (A) | Q 1.19 | Page 110

Integrate the following functions w.r.t. x : $(2x+1)\sqrt{x+2}$

SOLUTION

Let I = $ff(2x+1)\sqrt{x+2}. dx$

Put x + 2 = t

∴ dx = dt

Also, x = t – 2

∴ 2x + 1 = 2(t – 2) + 1 = 2t – 3

∴ I = $\int (2t-3)\sqrt{t} dt$

$= \int \left(2t^{\frac{3}{2}} - 3t^{\frac{1}{2}}\right) dt$

$= 2 \int t^{\frac{3}{2}} dt - 3 \int t^{\frac{1}{2}} dt$

$= 2. \frac{\frac{t^5}{2}}{\left(\frac{5}{2}\right)} - 3. \frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c$

$= \frac{4}{5}(x+2)^{\frac{5}{2}} - 2(x+2)^{\frac{3}{2}} + c.$

Exercise 3.2 (A) | Q 1.2 | Page 110

Integrate the following functions w.r.t. x : $x^5\sqrt{a^2+x^2}$

SOLUTION

Let I = $\int x^5 \sqrt{a^2+x^2}. dx$

Put, $a^2 + x^2 = t$

$\therefore$ 2 dx = dt

$\therefore$ x dx = $\frac{1}{2} dt$

Also, $x^2 = t - a^2$

I = $\int x^2 . x^2 \sqrt{a^2+x^2} x \, dx$

$= \frac{1}{2} \int \left(t - a^2\right)^2 \sqrt{t} \, dt$

$= \frac{1}{2} \int \left(t^2 - 2a^2 t + a^4\right) \sqrt{t} dt$

$= \frac{1}{2} \int \left(t^{\frac{5}{2}} - 2a^2 t^{\frac{3}{2}} + a^4 t^{\frac{1}{2}}\right) \sqrt{t} dt$

$= \frac{1}{2} \int t^{\frac{5}{2}} dt - a^2 \int t^{\frac{3}{2}} dt + \frac{a^4}{2} \int t^{\frac{1}{2}} dt$

$$= \frac{1}{2} \cdot \frac{t^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} - a^2 \cdot \frac{t^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + \frac{a^4}{2} \cdot \frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right) + c}$$

$$= \frac{1}{7}\left(a^2 + x^{22}\right)^{\frac{7}{2}} - \frac{2a^2}{5}\left(a^2 + x^2\right)^{\frac{5}{2}} + \frac{a^4}{3}\left(a^2 + x^2\right)^{\frac{3}{2}} + c.$$

Exercise 3.2 (A) | Q 1.21 | Page 110

Integrate the following functions w.r.t. x : $(5 - 3x)(2 - 3x)^{-\frac{1}{2}}$

SOLUTION

Let I = $\int (5 - 3x)(2 - 3x)^{-\frac{1}{2}} . dx$

Put 2 – 3x = t

∴ – 3dx = dt

∴ dx = $\frac{-dt}{3}$

Also, x = $\frac{2 - t}{3}$

∴ I = $\int \left[5 - 3\left(\frac{2 - t}{3}\right)\right] t^{-\frac{1}{2}} . \left(\frac{-dt}{3}\right)$

= $-\frac{1}{3}(5 - 2 + t)t^{-\frac{1}{2}} dt$

= $-\frac{1}{3} \int (3 + t)t^{\frac{1}{2}} dt$

= $-\frac{1}{3} \int \left(3t^{-\frac{1}{2}} + t^{\frac{1}{2}}\right) dt$

$$= -\frac{3}{3}\int t^{-\frac{1}{2}}dt - \frac{1}{3}\int t^{\frac{1}{2}}dt$$

$$= -\frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} - \frac{1}{3}.\frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + c$$

$$= -2\sqrt{2-3x} - \frac{2}{9}(2-3x)^{\frac{3}{2}} + c.$$

Exercise 3.2 (A) | Q 1.22 | Page 110

Integrate the following functions w.r.t. x : $\frac{7+4+5x^2}{(2x+3)^{\frac{3}{2}}}$

SOLUTION

Let I = $\int \frac{7+4x+5x^2}{(2x3)^{\frac{3}{2}}}.dx$

$$= \int \frac{5x^2+4x+7}{(2x+3)^{\frac{3}{2}}}.dx$$

Put 2x + 3 = t

∴ 2dx = dt

∴ dx = $\frac{dt}{2}$

Also, x = $\frac{t-3}{2}$

∴ I = $\int \frac{5\left(\frac{t-3}{2}\right)^2 + 4\left(\frac{t-3}{2}\right) + 7}{t^{\frac{3}{2}}}.\frac{dt}{2}$

$$= \frac{1}{2}\int \frac{5\left(\frac{t^2-6t+9}{4}\right) + 2(t-3) + 7}{t^{\frac{3}{2}}}dt$$

$$= \frac{1}{2}\int \frac{5t^2-30t+4+8t-24+28}{4t^{\frac{3}{2}}}dt$$

$= \frac{1}{8}\int \frac{5^2 - 22t + 49}{t^{\frac{3}{2}}} dt$

$= \frac{1}{8}\int \left(5t^{\frac{1}{2}} - 22t^{-\frac{1}{2}} + 49t^{-\frac{3}{2}}\right) dt$

$= \frac{5}{8}\int t^{\frac{1}{2}} dt - \frac{22}{8}\int t^{-\frac{1}{2}} dt + \frac{49}{8}\int t^{-\frac{3}{2}} dt$

$= \frac{5}{8} . \frac{t^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} - \frac{11}{4} . \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + \frac{49}{8} . \frac{t^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)} + c$

$= \frac{5}{12}(x+3)^{\frac{3}{2}} - \frac{11}{2}\sqrt{2x+3} - \frac{49}{4} . \frac{1}{\sqrt{2x+3}} + c.$

Exercise 3.2 (A) | Q 1.23 | Page 110

Integrate the following functions w.r.t. x : $\frac{x^2}{\sqrt{9 - x^6}}$

SOLUTION

Let I = $\int \frac{x^2}{\sqrt{9 - x^6}} . dx$

Put $x^3 = t$

∴ $3x^2$ dx = dt

∴ x^2dx = $\frac{1}{3} dt$

∴ I = $\int \frac{1}{\sqrt{9 - t^2}} . \frac{dt}{3}$

$= \frac{1}{3}\int \frac{dt}{\sqrt{3^2 - t^2}}$

$= \frac{1}{3}\sin^{-1}\left(\frac{t}{3}\right) + c$

$= \frac{1}{3}\sin^{-1}\left(\frac{x^3}{3}\right) + c.$

Exercise 3.2 (A) | Q 1.24 | Page 110

Integrate the following functions w.r.t. x : $\frac{1}{x(x^3-1)}$

SOLUTION

Let I = $\int \frac{1}{x(x^3-1)}.dx$

$= \int \frac{x^{-4}}{x^{-4}x(x^3-1)}.dx$

$= \int \frac{x^{-4}}{1-x^{-3}}.dx$

$= \frac{1}{3}\int \frac{3x^{-4}}{1-x^{-3}}.dx$

$= \frac{1}{3}\int \frac{\frac{d}{dx}\left(1-x^{-3}\right)}{1-x^{-3}}.dx$

$= \frac{1}{3}\log\left|1-x^{-3}\right| + c \quad ...\left[\because \int \frac{f\prime(x)}{f(x)}dx = \log|f(x)| + c\right]$

$= \frac{1}{3}\log\left|1-\frac{1}{x^3}\right| + c$

$= \frac{1}{3}\log\left|\frac{x^3-1}{x^3}\right| + c.$

Alternative Method :

Let I = $\int \frac{1}{x(x^3-1)}.dx$

$= \int \frac{x^2}{x^3(x^3 - 1)} . dx$

Put $x^3 = t$

$\therefore 3x^2 dx = dt$

$\therefore x^2 dx = \frac{dt}{3}$

$\therefore I = \int \frac{1}{t(t - 1)} . \frac{dt}{3}$

$= \frac{1}{3} \int \frac{1}{t(t-1)} dt$

$= \frac{1}{3} \int \frac{t - (t-1)}{t(t-1)} dt$

$= \frac{1}{3} \int \left(\frac{1}{t-1} - \frac{1}{t} \right) dt$

$= \frac{1}{3} \left[\int \frac{1}{t-1} dt - \int \frac{1}{t} dt \right]$

$= \frac{1}{3} [\log|t - 1| - \log|t|] + c$

$= \frac{1}{3} \log \left| \frac{t-1}{t} \right| + c$

$= \frac{1}{3} \log \left| \frac{x^3 - 1}{x^3} \right| + c.$

Exercise 3.2 (A) | Q 1.25 | Page 110

Integrate the following functions w.r.t. x : $\frac{1}{x . \log x . \log(\log x)}$.

SOLUTION

Let I = $\int \frac{1}{x.\log x.\log(\log x)}.dx$

= $\int \frac{1}{\log(\log x)}.\frac{1}{x.\log x}.dx$

Put log(log x) = t

∴ $\frac{1}{\log x}.\frac{1}{x}.dx$ = dt

∴ $\frac{1}{x.\log x}.dx$ = dt

∴ I = $\int \frac{1}{t}dt = \log|t| + c$

= log|log (logx)| + c.

Exercise 3.2 (A) | Q 2.01 | Page 110

Integrate the following functions w.r.t. x : $\frac{\cos 3x - \cos 4x}{\sin 3x + \sin 4x}$

SOLUTION

Let I = $\int \frac{\cos 3x - \cos 4x}{\sin 3x + \sin 4x}.dx$

= $\int \frac{-2\sin\left(\frac{3x+4x}{2}\right)\sin\left(\frac{3x-4x}{2}\right)}{2\sin\left(\frac{3x+4x}{2}\right)\cos\left(\frac{3x-4x}{2}\right)}.dx$

= $\int -\frac{\sin\left(-\frac{x}{2}\right)}{\cos\left(-\frac{x}{2}\right)}.dx$

= $\int \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}.dx$

$= \int \tan\left(\frac{x}{2}\right). dx$

$= \log \frac{\left|\sec\left(\frac{x}{2}\right)\right|}{\left(\frac{1}{2}\right)} + c$

$= 2\log\left|\sec\left(\frac{x}{2}\right)\right| + c.$

Exercise 3.2 (A) | Q 2.02 | Page 110

Integrate the following functions w.r.t. x : $\frac{\cos x}{\sin(x-a)}$

SOLUTION

Let I = $\int \frac{\cos x}{\sin(x-a)}. dx$

$= \int \frac{\cos[(x-a)+a]}{\sin(x-a)}. dx$

$= \int \frac{\cos(x-a)\cos a - \sin(x-a)\sin a}{\sin(x-a)}. dx$

$= \int\left[\frac{\cos(x-a)\cos a}{\sin(x-a)} - \frac{\sin(x-a)\sin a}{\sin(x-a)}\right]. dx$

$= \cos a \int \cot(x-a)dx - \sin a \int 1dx$

= cos a log |sin(x – a)| – x sin a + c.

Exercise 3.2 (A) | Q 2.03 | Page 110

Integrate the following functions w.r.t. x : $\frac{\sin(x-a)}{\cos(x\ +b)}$

SOLUTION

Let I = $\int \frac{\sin(x-a)}{\cos(x+b)}. dx$

$= \int \frac{\sin[(x+b)-(a\ +b)]}{\cos(x+b)}.dx$

$= \int \frac{\sin(x+b)\cos(a+b)-\cos(x+b)\sin(a+b)}{\cos(x+b)}.dx$

$= \int \left[\frac{\sin(x+b)\cos(a+b)}{\cos(x+b)} - \frac{\cos(x+b)\sin(a+b)}{\cos(x+b)}\right].dx$

$= \cos(a+b)\int \tan(x+b)dx - \sin(a+b)\int 1dx$

= cos (a + b) log | sec (x + b) | – x sin (a + b) + c.

Exercise 3.2 (A) | Q 2.04 | Page 110

Integrate the following functions w.r.t. x : $\frac{1}{\sin x.\cos x + 2\cos^2 x}$

SOLUTION

Let I = $\int \frac{1}{\sin x.\cos x + 2\cos^2 x}.dx$

Dividing numerator and denominator of $\cos^2 x$, we get

I = $\int \frac{\left(\frac{1}{\cos^2 x}\right)}{\frac{\sin x}{\cos x} + 2}.dx$

$= \int \frac{\sec^2 x}{\tan x + 2}.dx$

Put tan x = t

∴ $\sec^2 x$ dx = dt

∴ I = $\int \frac{1}{t+2}dt$

= log |t + 2| + c

= log|tan x + 2| + c.

Exercise 3.2 (A) | Q 2.05 | Page 110

Integrate the following functions w.r.t. x : $\frac{\sin x + 2\cos x}{3\sin x + 4\cos x}$

SOLUTION

Let I = $\int \frac{\sin x + 2\cos x}{3\sin x + 4\cos x} . dx$

Put,

Numberator = $\text{A (Denominator)} + \text{B}\left[\frac{d}{dx}\text{(Denominator)}\right]$

∴ sinx + 2cosx = $\text{A}(3\sin x + 4\cos x) + \text{B}\left[\frac{d}{dx}(3\sin x + 4\cos x)\right]$

= A(3 sin x + 4 cos x) + B(3 cos x – 4 sin x)

∴ sin x + 2 cos x = (3A – 4B)sin x + (4A + 3B)cos x

Equaliting the coefficients of sin x and cos x on both the sides, we get

3A – 4B = 1 ...(1)

and

4A + 3B = 2 ...(2)

Multiplying equation (1) bt 3 and equation (2) byy 4, we get

9A – 12B = 3

16A + 12B = 8

On adding, we get

25A = 11

∴ A = $\frac{11}{25}$

∴ from (2), $4\left(\frac{11}{25}\right) + 3\text{B}$ = 2

∴ 3B = $2 - \frac{44}{25} = \frac{6}{25}$

∴ B = $\frac{2}{25}$

$$\therefore \sin x + 2\cos x = \frac{11}{25}(3\sin x + 4\cos x) + \frac{2}{25}(3\cos x - 4\sin x)$$

$$\therefore \text{I} = \int \left[\frac{\frac{11}{25}(3\sin x + 4\cos x) + \frac{2}{25}(3\cos x - 4\sin x)}{3\sin x + 4\cos x}\right].dx$$

$$= \int \left[\frac{11}{25} + \frac{\frac{2}{25}(3\cos x - 4\sin x)}{(3\sin x + 4\cos x)}\right].dx$$

$$= \frac{11}{25}\int 1dx + \frac{2}{25}\int \frac{3\cos x - 4\sin x}{3\sin x + 4\cos x}.dx$$

$$= \frac{11}{25}x + \frac{2}{25}\log|3\sin x + 4\cos x| + c. \quad ...\left[\because \int \frac{f\prime(x)}{f\prime(x)}dx = \log|f(x)| \ + \ c\right]$$

Exercise 3.2 (A) | Q 2.06 | Page 110

Integrate the following functions w.r.t. x : $\frac{1}{2 + 3\tan x}$

SOLUTION

Let I = $\int \frac{1}{2 + 3\tan x}.dx$

$$= \int \frac{1}{2 + 3\left(\frac{\sin x}{\cos x}\right)}.dx$$

$$= \int \frac{\cos x}{2\cos x + 3\sin x}.dx$$

Put,

Numerator = $\text{A (Denominator)} + \text{B}\left[\frac{d}{dx}\text{(Denominator)}\right]$

∴ cos x = $\text{A}(\cos x + 3\sin x) + \text{B}\left[\frac{d}{dx}(2\cos x + 3\sin x)\right]$

= A(2 cos x + 3 sin x) + B(− sin x + 3 cos x)

∴ cos x = (2A + 3B)cos x + (3A – 2B)sin x

Equating the coefficients of cos x sin x on both the sides, we get

2A 3B = 1   ...(1)

and

3A – 2B = 0   ...(2)

Multiplying equation (1) by 22 and equation (2) by 3, we get

4A +6B = 2

9A – 6B = 0

On adding, we get

13A = 2

∴ A = $\dfrac{2}{13}$

∴ from (2), 2B = 3A = $3\left(\dfrac{2}{13}\right) = \dfrac{6}{13}$

∴ B = $\dfrac{3}{13}$

∴ cos x = $\dfrac{2}{13}(2\cos x + 3\sin x) + \dfrac{3}{13}(-2\sin x + \ 3\cos x)$

∴ I = $\int\left[\dfrac{\frac{2}{13}(2\cos x + 3\sin x) + \frac{3}{13}(-2\sin x + 3\cos x)}{2\cos x + 3\sin x}\right].dx$

= $\int\left[\dfrac{2}{13} + \dfrac{\frac{3}{13}(-2\sin x + 3\cos x)}{2\cos x + 3\sin x}\right].dx$

= $\dfrac{2}{13}1dx + \dfrac{3}{13}\int\dfrac{-2\sin x + 3\cos x}{2\cos x + 3\sin x}.dx$

= $\dfrac{2}{13}x + \dfrac{3}{13}\log|2\cos x + 3\sin x| + c.$ ...$\left[\because \int\dfrac{f\prime(x)}{f(x)}dx = \log|f(x)| + c\right]$

Exercise 3.2 (A) | Q 2.07 | Page 110

Integrate the following functions w.r.t. x : $\dfrac{4e^x - 25}{2e^x - 5}$

SOLUTION

Let I = $\int \frac{4e^x - 25}{2e^x - 5} . dx$

Put,

Numerator = $\mathbf{A}$ $(\mathbf{Denominator}) + \mathbf{B}\left[\frac{d}{dx}(\mathbf{Denominator})\right]$

$\therefore$ $4e^x - 25$ = $\mathbf{A}(2e^x - 5) + \mathbf{B}\left[\frac{d}{dx}(2e^x - 5)\right]$

= $A(2e^x - 5) + B(2e^x - 0)$

$\therefore$ $4e^x - 25 = (2A + 2B)e^x - 5A$

Equating the coefficient of e^x and constant on both sides, we get

2A + 2B = 4 ...(1)

and

5A = 25

$\therefore$ A = 5

$\therefore$ from (1),2(5) + 2B = 4

$\therefore$ 2B = – 6

$\therefore$ B = – 3

$\therefore$ $4e^x - 25 = 5(2e^x - 5) - 3(2e^x)$

$\therefore$ I = $\int \left[\frac{5(2e^x - 5) - 3(2e^x)}{2e^x - 5}\right] . dx$

= $\int \left[5 - \frac{3(2e^x)}{2e^x - 5}\right] . dx$

= $5\int 1dx - 3\int \frac{2e^x}{2e^x - 5} . dx$

= $5x - 3\log|2e^x - 5| + c$ ... $\left[\because \int \frac{f\prime(x)}{f(x)} dx = \log|f(x)| + c\right]$

Exercise 3.2 (A) | Q 2.08 | Page 110

Integrate the following functions w.r.t. x : $\dfrac{20+12e^x}{3e^x+4}$

SOLUTION

Let I = $\displaystyle\int \frac{20+12e^x}{3e^x+4}.dx$

Put,

Numerator = $\mathbf{A}$ $\mathbf{(Denominator)} + \mathbf{B}\left[\dfrac{d}{dx}\mathbf{(Denominator)}\right]$

$\therefore$ 20 +12e^x = $\mathbf{A}(3e^x+4)+\mathbf{B}\left[\dfrac{d}{dx}(3e^x+4)\right]$

= A(3e^x + 4) + B(3e^x + 0)

$\therefore$ 20 +12e^x = (2A + 2B)e^x – 5A

Equating the coefficient of e^x and constant on both sides, we get

2A + 2B = 4 ...(1)

and

5A = 25

$\therefore$ A = 5

$\therefore$ from (1),2(5) + 2B = 4

$\therefore$ 2B = – 6

$\therefore$ B = – 3

$\therefore$ 20 +12e^x = 5(3e^x + 4) – 3 (3e^x)

$\therefore$ I = $\displaystyle\int\left[\frac{5(3e^x+5)-3(3e^x)}{3e^x+4}\right].dx$

= $\displaystyle\int\left[5-\frac{3(3e^x)}{3^x+4}\right].dx$

$= 5\int 1dx - 3\int \frac{3e^x}{3e^x + 4} . dx$

= 5x − log|3e^x + 4| + c .

Exercise 3.2 (A) | Q 2.09 | Page 110

Integrate the following functions w.r.t. x : $\frac{3e^{2x} + 5}{4e^{2x} - 5}$

SOLUTION

Let I = $\int \frac{3e^{2x} + 5}{4e^{2x} - 5} . dx$

Put,

Numerator = $\mathbf{A}$ $\mathbf{(Denominator)} + \mathbf{B}\left[\frac{d}{dx}\mathbf{(Denominator)}\right]$

∴ 3e^{2x} + 5 = $\mathbf{A}\left(4e^{2x} - 5\right) + \mathbf{B}\left[\frac{d}{dx}\left(4e^{2x} - 5\right)\right]$

= A(4e^{2x} – 5) + B(4.e^{2x} x 2 – 0)

∴ 3e^{2x} + 5 = (4A + 8B)e^{2x} – 5A

Equating the coeffiecient of e^{2x} and constant on both sides, we get

4A + 8B = 3 ...(1)

and

– 5A = 5

∴ A = – 1

∴ from (1), 4(– 1) + 8B = 3

∴ 8B = 7

∴ B = $\frac{7}{8}$

∴ 3e^{2x} + 5 = $-\left(4e^{2x} - 5\right) + \frac{7}{8}\left(8e^{2x}\right)$

$\therefore$ I = $\int\left[\frac{-\left(4e^{2x}-5\right)+\frac{7}{8}\left(8e^{2x}\right)}{4e^{2x}-5}\right].dx$

= $\int\left[-1+\frac{\frac{7}{8}\left(8e^{2x}\right)}{4e^{2x}-5}\right].dx$

= $\int 1dx+\frac{7}{8}\int\frac{8e^{2x}}{4e^{2x}-5}.dx$

= $-x+\frac{7}{8}\log\left|4e^{2x}-\ 5\right|+c\ \ldots\left[\because\int\frac{f\prime(x)}{f(x)}dx=\log|f(x)|+c\right]$

Exercise 3.2 (A) | Q 2.1 | Page 110

Integrate the following functions w.r.t. x : $\cos^8 x\cot x$

SOLUTION

Let I = $\int\cos^8 x\cot x dx$

= $\int\cos^8 x.\frac{\cos x}{\sin x}.dx$

Put sin x = t

$\therefore$ cos x dx = dt

$\cos^8 x = (\cos^2 x)^4 = (1 - \sin^2 x)^4$

$= (1 - t^2)^4 = 1 - 4t^2 + 6t^4 - 4t^6 + t^8$

I = $\int\frac{1-4t^2+6t^4-4t^6+t^8}{t}dt$

= $\int\left[\frac{1}{t}-4t+6t^3-4t^5+t^7\right]dt$

= $\int\frac{1}{t}dx-4\int tdt+6\int t^3dt-4\int t^5dt+\int t^7dt$

$$= \log|t| - 4\left(\frac{t^2}{2}\right) + 6\left(\frac{t^4}{4}\right) - 4\left(\frac{t^6}{6}\right) + \frac{t^8}{8} + c$$

$$= \log|\sin x| - 2\sin^2 x + \frac{3}{2}\sin^4 x - \frac{2}{3}\sin^6 + \frac{\sin^8 x}{8} + c.$$

Exercise 3.2 (A) | Q 2.11 | Page 110

Integrate the following functions w.r.t. x : $\tan^5 x$

SOLUTION

Let I = $\int \tan^5 x \, dx$

$$= \int \tan^3 x \tan^2 x dx$$

$$= \int \tan^3 x (\sec^2 x - 1) dx$$

$$= \int (\tan^3 x \sec^2 x - \tan^3 x) dx$$

$$= \int (\tan^3 x \sec^2 x - \tan x . \tan^2 x) dx$$

$$= \int [\tan^3 x \sec^2 x - \tan x (\sec^2 x - 1)] dx$$

$$= \int (\tan^3 x \sec^2 x - \tan x \sec^2 x + \tan x) dx$$

$$= \int [(\tan^3 x - \tan x) \sec^2 x + \tan x] dx$$

$$= \int (\tan^3 x - \tan x) \sec^2 x dx + \int \tan x dx$$

= $I_1 + I_2$

In I_1, put tan x = t

∴ $\sec^2 x\, dx = dt$

∴ I = $\int (t^3 - t)dt + \int \tan x dx$

= $\frac{t^4}{4} - \frac{t^2}{2} + \log|\sec x| + c$

= $\frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| + c.$

Exercise 3.2 (A) | Q 2.12 | Page 110

Integrate the following functions w.r.t. x : $\cos^7 x$

SOLUTION

Let I = $\int \cos^7 x dx$

= $\int \cos^6 x . \cos x dx$

= $\int (1 - \sin^2 x)^3 \cos x dx$

Put, sin x = t

∴ cos x dx = dt

I = $\int (1 - t^2)^3 dt$

= $\int (1 - 3t^2 + 3t^4 - t^6)dt$

= $\int 1dt - 3\int t^2 dt + 3\int t^4 dt - \int t^6 dt$

= $t - 3\left(\frac{t^3}{3}\right) + 3\left(\frac{t^5}{3}\right) - \frac{t^7}{7} + c$

= $\sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c.$

Exercise 3.2 (A) | Q 2.13 | Page 110

Integrate the following functions w.r.t. x : tan 3x tan 2x tan x

SOLUTION

Let I = $\int \tan 3x \tan 2x \tan x dx$

Consider tan 3x = tan (2x + x)

= $\frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

∴ tan3x (1 – tan 2x tan x) = tan 2x + tan x

∴ tan 3x – tan 3x tan 2x tan x = tan 2x + tan x

∴ tan 3x – tan 2x – tan x = tan 3x tan 2x tan x

I = $\int (\tan 3x - \tan 2x - \tan x) dx$

= $\int \tan 3x dx - \int \tan 2x dx - \int \tan x dx$

= $\frac{1}{3} \log|\sec 3x| - \frac{1}{2} \log|\sec 2x| - \log|\sec x| + c.$

Exercise 3.2 (A) | Q 2.14 | Page 110

Integrate the following functions w.r.t. x : $\sin^5 x.\cos^8 x$

SOLUTION

Let I = $\int \sin^5 x \cos^8 x dx$

= $\int \sin^4 x \cos^8 x \sin x dx$

= $\int (1 - \cos^2 x)^2 \cos^8 x \sin x dx$

Put cos x = t

∴ – sin x dx = dt

∴ sin x dx = – dt

I = $-\int\left(1-t^{2}\right)^{2} t^{8} dt$

= $-\int\left(1-2 t^{2}+t^{4}\right) t^{8} dt$

= $-\int\left(t^{8}-2 t^{10}+t^{12}\right) dt$

= $-\int t^{8} dt+2 \int t^{10} dt-\int t^{12} dt$

= $-\frac{t^{9}}{9}+2\left(\frac{t^{11}}{11}\right)-\frac{t^{13}}{13}+c$

= $-\frac{1}{9} \cos ^{9} x+\frac{2}{11} \cos ^{11} x-\frac{1}{13} \cos ^{13} x+c.$

Exercise 3.2 (A) | Q 2.15 | Page 110

Integrate the following functions w.r.t. x : $3^{\cos ^{2} x} \sin 2 x$

SOLUTION

let I = $\int 3^{\cos ^{2} x} \sin 2 x d x$

Put $\cos^2 x$ = t

$\therefore$ $\left[2 \cos x \frac{d}{d x}(\cos x)\right] d x$ = dt

$\therefore$ – 2 sin x cos x dx = dt

$\therefore$ sin 2x dx = – dt

I = $-\int 3^{t} dt$

= $-\frac{1}{\log 3} .3^{t}+c$

= $-\frac{1}{\log 3} .3^{\cos ^{2} x}+c.$

Exercise 3.2 (A) | Q 2.16 | Page 110

Integrate the following functions w.r.t. x : $\dfrac{\sin 6x}{\sin 10x \sin 4x}$

SOLUTION

Let I = $\displaystyle\int \frac{\sin 6x}{\sin 10x \sin 4x} . dx$

= $\displaystyle\int \frac{\sin(10x - 4x)}{\sin 10x \sin 4x} . dx$

= $\displaystyle\int \frac{\sin 10x \cos 4x - \cos 10x \sin 4x}{\sin 10x \sin 4x} . dx$

= $\displaystyle\int \left[\frac{\sin 10x \cos 4x}{\sin 10x \sin 4x} - \frac{\cos 10x \sin 4x}{\sin 10x \sin 4x}\right] . dx$

= $\displaystyle\int \cot 4x dx - \int \cot 10x dx$

= $\dfrac{1}{4}\log|\sin 4x| - \dfrac{1}{10}\log|\sin 10x| + c.$

Exercise 3.2 (A) | Q 2.17 | Page 110

Integrate the following functions w.r.t. x : $\dfrac{\sin x \cos^3 x}{1 + \cos^2 x}$

SOLUTION

Let I = $\displaystyle\int \frac{\sin x \cos^3 x}{1 + \cos^2 x} . dx$

Put cos x = t

∴ − sin x dx = dt

∴ sin x dx = − dt

I = $\displaystyle - \int \frac{t^3}{t^2 + 1} dt$

$$= -\int \frac{t\left(t^2+1\right)-t}{t^2+1}dt$$

$$= -\int \left[\frac{t\left(t^2+1\right)}{t^2+1} - \frac{t}{t^2+1}\right]dt$$

$$= -\int tdt + \int \frac{t}{t^2+1}dt$$

$$= -\int tdt + \frac{1}{2}\int \frac{2t}{t^2+1}dt$$

$$= \frac{t^2}{2} + \frac{1}{2}\log\left|t^2+1\right| + c$$

$$\dots \left[\because \frac{d}{dt}\left(t^2+1\right) = 2t \text{ and } \int \frac{f\prime(x)}{f(x)}dx = \log[f(x)] + c\right]$$

$$= -\frac{1}{2}\cos^2 x + \frac{1}{2}\log\left|\cos^2 x + 1\right| + c$$

$$= \frac{1}{2}\left[\log\left|\cos^2 x + 1\right| - \cos^2 x\right] + c.$$

EXERCISE 3.2 (B) [PAGE 123]

Exercise 3.2 (B) | Q 1.01 | Page 123

Evaluate the following : $\int \frac{1}{4x^2-3}.dx$

SOLUTION

$$\text{I} = \int \frac{1}{4x^2-3}.dx$$

$$= \frac{1}{4}\int \frac{1}{x^2-\frac{3}{4}}.dx$$

$$= \frac{1}{4}\int \frac{1}{x^2-\left(\frac{\sqrt{3}}{2}\right)^2}.dx$$

$= \frac{1}{4}\frac{1}{2\left(\frac{\sqrt{3}}{2}\right)}\log\left|\frac{x-\frac{\sqrt{3}}{2}}{x+\frac{\sqrt{3}}{2}}\right| + c$

$= \frac{1}{4\sqrt{3}}\log\left|\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right| + c.$

Exercise 3.2 (B) | Q 1.02 | Page 123

Evaluate the following : $\int \frac{1}{25-9x^2}.dx$

SOLUTION

I = $\int \frac{1}{25-9x^2}.dx$

$= \int \frac{1}{5^2-(3x)^2}.dx$

$= \frac{1}{2(5)}\log\left|\frac{5+3x}{5-3x}\right|.\frac{1}{3} + c$

$= \frac{1}{30}\log\left|\frac{5+3x}{5-3x}\right| + c.$

Alternative Method :

$\int \frac{1}{25-9x^2}.dx$

$= \frac{1}{9}\int \frac{1}{\frac{25}{9}x^2}.dx$

$= \frac{1}{9}\int \frac{1}{\left(\frac{5}{3}\right)^2 - x^2}.dx$

$= \frac{1}{9}\times\frac{1}{2\times\frac{5}{3}}\log\left|\frac{\frac{5}{3}+x}{\frac{5}{3}-x}\right| + c$

$$= \frac{1}{30} \log \left| \frac{5 + 3x}{5 - 3x} \right| + c.$$

Exercise 3.2 (B) | Q 1.03 | Page 123

Evaluate the following : $\int \frac{1}{7 + 2x^2} . dx$

SOLUTION

$$I = \int \frac{1}{7 + 2x^2} . dx$$

$$= \frac{1}{2} \int \frac{1}{\frac{7}{2} + x^2} . dx$$

$$= \frac{1}{2} \int \frac{1}{\left(\sqrt{\frac{7}{2}}\right)^2 + x^2} . dx$$

$$= \frac{1}{2} . \frac{1}{\left(\sqrt{\frac{7}{2}}\right)} \tan^{-1} \left| \frac{x}{\sqrt{\frac{7}{2}}} \right| + c$$

$$= \frac{1}{\sqrt{14}} \tan^{-1} \left| \frac{\sqrt{2}x}{\sqrt{7}} \right| + c.$$

Exercise 3.2 (B) | Q 1.04 | Page 123

Evaluate the following : $\int \frac{1}{\sqrt{3x^2 - 8}} . dx$

SOLUTION

$$\int \frac{1}{\sqrt{3x^2+8}}.dx$$

$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2+\frac{8}{3}}}.dx$$

$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2+\left(\sqrt{\frac{8}{3}}\right)^2}}.dx$$

$$= \frac{1}{\sqrt{3}} \log\left|x+\sqrt{x^2+\left(\sqrt{\frac{8}{3}}\right)^2}\right| + c_1$$

$$= \frac{1}{\sqrt{3}} \log\left|x+\sqrt{x^2+\frac{8}{3}}\right| + c_1$$

$$= \frac{1}{\sqrt{3}} \log\left|\frac{\sqrt{3}x+\sqrt{3x^2+8}}{\sqrt{3}}\right| + c_1$$

$$= \frac{1}{\sqrt{3}} \log\left|\sqrt{3}x+\sqrt{3x^2+8}\right| - \log\sqrt{3} + c_1$$

$$= \frac{1}{\sqrt{3}} \log\left|\sqrt{3}x+\sqrt{3x^2+8}\right| + c, \text{where } c = c_1 - \log\sqrt{3}$$

Alternative Method :

$$\int \frac{1}{\sqrt{3x^2+8}}.dx$$

$$= \int \frac{1}{\sqrt{\left(\sqrt{3}x\right)^2+\left(\sqrt{8}\right)^2}}.dx$$

$$= \frac{\log \mid \sqrt{3}x + \sqrt{\left(\sqrt{3}x\right)^2 + \sqrt{(8)}^2} \mid + c}{\sqrt{3}}$$

$$= \frac{1}{\sqrt{3}} \log\left|\sqrt{3}x + \sqrt{3x^2 + 8}\right| + c.$$

Exercise 3.2 (B) | Q 1.05 | Page 123

Evaluate the following : $\int \frac{1}{\sqrt{11 - 4x^2}} . dx$

SOLUTION

$$\int \frac{1}{\sqrt{11 - 4x^2}} . dx$$

$$= \int \frac{1}{\sqrt{\left(\sqrt{11}\right)^2 - (2x)^2}} . dx$$

$$= \frac{1}{2} \sin^{-1}\left(2\frac{x}{\sqrt{11}}\right) + c.$$

Exercise 3.2 (B) | Q 1.06 | Page 123

Evaluate the following : $\int \frac{1}{\sqrt{2x^2 - 5}} . dx$

SOLUTION

$$\int \frac{1}{\sqrt{2x^2 - 5}} . dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 - \frac{5}{2}}} . dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 - \left(\sqrt{\frac{5}{2}}\right)^2}} . dx$$

$$= \frac{1}{\sqrt{2}} \log\left| x + \sqrt{x^2 - \frac{5}{2}} \right| + c.$$

Exercise 3.2 (B) | Q 1.07 | Page 123

Evaluate the following : $\int \sqrt{\frac{9+x}{9-x}} . dx$

SOLUTION

Let I = $\int \sqrt{\frac{9+x}{9-x}} . dx$

$$= \int \sqrt{\frac{9+x}{9-x} \times \frac{9+x}{9+x}} . dx$$

$$= \int \frac{9+x}{\sqrt{81 - x^2}} . dx$$

$$= \int \frac{9}{\sqrt{81 - x^2}} . dx + \int \frac{x}{\sqrt{81 - x^2}} . dx$$

$$= 9 \int \frac{1}{\sqrt{9^2 - x^2}} . dx + \frac{1}{2} \int \frac{2x}{\sqrt{81 - x^2}} . dx$$

= $I_1 + I_2$ ...(Let)

$I_1 = 9 \int \frac{1}{\sqrt{9^2 - x^2}} . dx$

$$= 9 \sin^{-1}\left(\frac{x}{9}\right) + c_1$$

In I_2, put $81 - x^2 = t$

$\therefore -2x\,dx = dt$

$\therefore 2x\,dx = -dt$

$I_2 = -\frac{1}{2}\int t^{-\frac{1}{2}} dt$

$= -\frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_2$

$= -\sqrt{81 - x^2} + c_2$

$I = 9\sin^{-1}\left(\frac{x}{9}\right) - \sqrt{81 - x^2} + c,$

where $c = c_1 + c_2$.

Exercise 3.2 (B) | Q 1.08 | Page 123

Evaluate the following : $\int \sqrt{\frac{2 + x}{2 - x}} . dx$

SOLUTION

Let $I = \int \sqrt{\frac{2 + x}{2 - x}} . dx$

$= \int \sqrt{\frac{2 + x}{2 - x} \times \frac{2 + x}{2 + x}} . dx$

$= \int \frac{2 + x}{\sqrt{4 - x^2}} . dx$

$= \int \frac{2}{\sqrt{4 - x^2}} . dx + \int \frac{x}{\sqrt{4 - x^2}} . dx$

$= 2\int \frac{1}{\sqrt{2^2 - x^2}} . dx + \frac{1}{2}\int \frac{2x}{\sqrt{4 - x^2}} . dx$

$= I_1 + I_2$ ...(Let)

$I_1 = 2\int \frac{1}{\sqrt{2^2 - x^2}} . dx$

$= 2\sin^{-1}\left(\frac{x}{2}\right) + c_1$

In I_2, put $4 - x^2 = t$

$\therefore - 2x\,dx = dt$

$\therefore 2x\,dx = - dt$

$I_2 = -\frac{1}{2}\int t^{-\frac{1}{2}}dt$

$= -\frac{1}{2}.\frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_2$

$= -\sqrt{4 - x^2} + c_2$

$I = 2\sin^{-1}\left(\frac{x}{2}\right) - \sqrt{4 - x^2} + c.$

Exercise 3.2 (B) | Q 1.09 | Page 123

Evaluate the following : $\int\sqrt{\frac{10 + x}{10 - x}}.dx$

SOLUTION

Let $I = \int\sqrt{\frac{10 + x}{10 - x}}.dx$

$= \int\sqrt{\frac{10 + x}{10 - x}\times\frac{10 + x}{10 + x}}.dx$

$= \int\frac{10 + x}{\sqrt{100 - x^2}}.dx$

$= \int\frac{10}{\sqrt{100 - x^2}}.dx + \int\frac{x}{\sqrt{100 - x^2}}.dx$

$= 10\int\frac{1}{\sqrt{10^2 - x^2}}.dx + \frac{1}{2}\int\frac{2x}{\sqrt{100 - x^2}}.dx$

$= I_1 + I_2$ ...(Let)

$I_1 = 10\int\frac{1}{\sqrt{10^2 - x^2}}.dx$

= $10\sin^{-1}\left(\frac{x}{10}\right) + c_1$

In I_2, put $100 - x^2 = t$

∴ $-2x\,dx = dt$

∴ $2x\,dx = -dt$

$I_2 = -\frac{1}{2}\int t^{-\frac{1}{2}}\,dt$

$= -\frac{1}{2}.\frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_2$

$= -\sqrt{100 - x^2} + c_2$

$I = 10\sin^{-1}\left(\frac{x}{10}\right) - \sqrt{100 - x^2} + c.$

Exercise 3.2 (B) | Q 1.1 | Page 123

Evaluate the following : $\int \frac{1}{x^2 + 8x + 12}.dx$

SOLUTION

$$\int \frac{1}{x^2 + 8x + 12}.dx$$

$$= \int \frac{1}{(x^2 + 8x + 16) - 16 + 12}.dx$$

$$= \int \frac{1}{(x+4)^2 - 2^2}.dx$$

$$= \frac{1}{2(2)}\log\left|\frac{(x+4)-2}{(x+4)+2}\right| + c$$

$$= \frac{1}{4}\log\left|\frac{x+2}{x+6}\right| + c.$$

Exercise 3.2 (B) | Q 1.11 | Page 123

Evaluate the following : $\int \frac{1}{1 + x - x^2}.dx$

SOLUTION

Let I = $\int \frac{1}{1 + x - x^2}.dx$

1 + x – x^2 = 1 – (x^2 – x)

= $1 - \left(x^2 - x + \frac{1}{4}\right) + \frac{1}{4}$

= $\frac{5}{4} - \left(x^2 - x + \frac{1}{4}\right)$

= $\left(\frac{\sqrt{5}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2$

∴ I = $\int \frac{1}{\left(\frac{\sqrt{5}}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2}.dx$

= $\frac{1}{2\left(\frac{\sqrt{5}}{2}\right)} \log\left|\frac{\frac{\sqrt{5}}{2} + \left(x - \frac{1}{2}\right)}{\frac{\sqrt{5}}{2} - \left(x - \frac{1}{2}\right)}\right| + c$

= $\frac{1}{\sqrt{5}} \log\left|\frac{\sqrt{5} - 1 + 2x}{\sqrt{5} + 1 - 2x}\right| + c.$

Exercise 3.2 (B) | Q 1.12 | Page 123

Evaluate the following : $\frac{1}{4x^2 - 20x + 17}$

SOLUTION

$$\int \frac{1}{4x^2 - 20x + 17}.dx$$

$$= \frac{1}{4}\int \frac{1}{x^2 - 5x + \frac{17}{4}}.dx$$

$$= \frac{1}{4}\int \frac{1}{\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} + \frac{17}{4}}.dx$$

$$= \frac{1}{4}\int \frac{1}{\left(x - \frac{5}{2}\right)^2 - \left(\sqrt{2}\right)^2}.dx$$

$$= \frac{1}{4} \times \frac{1}{2\sqrt{2}}\log\left|\frac{x - \frac{5}{2} - \sqrt{2}}{x - \frac{5}{2} + \sqrt{2}}\right| + c$$

$$= \frac{1}{8\sqrt{2}}\log\left|\frac{2x - 5 - 2\sqrt{2}}{2x - 5 + 2\sqrt{2}}\right| + c.$$

Exercise 3.2 (B) | Q 1.13 | Page 123

Evaluate the following : $\int \frac{1}{5 - 4x - 3x^2}.dx$

SOLUTION

Let I = $\int \frac{1}{5 - 4x - 3x^2}.dx$

$5 - 4x - 3x^2 = \left[\frac{5}{3} - \left(x^2 + \frac{4}{3}x\right)\right]$

$= 3\left[\frac{5}{3} - \left(x^2 + \frac{4}{3}x + \frac{4}{9}\right) + \frac{4}{9}\right]$

$= 3\left[\frac{19}{9} - \left(x^2 + \frac{4x}{3} + \frac{4}{9}\right)\right]$

$$= 3\left[\left(\frac{\sqrt{19}}{3}\right)^2 - \left(x + \frac{2}{3}\right)^2\right]$$

$$I = \int \frac{1}{3\left[\left(\frac{\sqrt{19}}{3}\right)^2 - \left(x + \frac{2}{3}\right)^2\right]}.dx$$

$$= \frac{1}{3}\frac{1}{2\left(\frac{\sqrt{19}}{3}\right)}\log\left|\frac{\frac{\sqrt{19}}{3} + \left(x + \frac{2}{3}\right)}{\frac{\sqrt{19}}{3} - \left(x + \frac{2}{3}\right)}\right| + c$$

$$= \frac{1}{2\sqrt{19}}\log\left|\frac{\sqrt{19} + 2 + 3x}{\sqrt{19} - 2 - 3x}\right| + c$$

$$= \frac{1}{2\sqrt{19}}\log\left|\frac{3x + 2 + \sqrt{19}}{-\left(3x + 2 - \sqrt{19}\right)}\right| + c$$

$$= \frac{1}{2\sqrt{19}}\log\left|\frac{3x + 2 + \sqrt{19}}{3x + 2 - \sqrt{19}}\right| + c. \quad ...[\because |-x| = x]$$

Exercise 3.2 (B) | Q 1.14 | Page 123

Evaluate the following : $\int \frac{1}{\sqrt{3x^2 + 5x + 7}}.dx$

SOLUTION

Let I = $\int \frac{1}{\sqrt{3x^2 + 5x + 7}}.dx$

$3x^2 + 5x + 7 = 3\left[x^2 + \frac{5}{3}x + \frac{7}{3}\right]$

$$= 3\left[\left(x^2 + \frac{5x}{3} + \frac{25}{36}\right) + \left(\frac{7}{3} - \frac{25}{36}\right)\right]$$

$$= 3\left[\left(x + \frac{5}{6}\right)^2 + \left(\frac{\sqrt{59}}{6}\right)^2\right]$$

$$\therefore \sqrt{3x^2 + 5x + 7} = \sqrt{3}\sqrt{\left(x + \frac{5}{6}\right)^2 + \left(\frac{\sqrt{59}}{6}\right)^2}$$

$$\therefore \text{I} = \frac{1}{\sqrt{3}}\int \frac{1}{\left(x + \frac{5}{6}\right)^2 + \left(\frac{\sqrt{59}}{6}\right)^2} . dx$$

$$= \frac{1}{\sqrt{3}}\log\left|x + \frac{5}{6} + \sqrt{\left(x + \frac{5}{6}\right)^2 + \left(\frac{\sqrt{59}}{6}\right)^2}\right| + c$$

$$= \frac{1}{\sqrt{3}}\log\left|x + \frac{5}{6} + \sqrt{x^2 + \frac{5x}{3} + \frac{7}{3}}\right| + c.$$

Exercise 3.2 (B) | Q 1.15 | Page 123

Evaluate the following : $\int \frac{1}{\sqrt{x^2 + 8x - 20}} . dx$

SOLUTION

$$\int \frac{1}{\sqrt{x^2 + 8x - 20}} . dx$$

$$= \int \frac{1}{\sqrt{(x^2 + 8x + 16) - 16 - 20}} . dx$$

$$= \int \frac{1}{\sqrt{(x + 4)^2 - (6)^2}} . dx$$

$$= \log\left|(x + 4) + \sqrt{(x - 4)^2 - (6)^2}\right| + c$$

$$= \log\left|(x + 4) + \sqrt{x^2 - 8x - 20}\right| + c.$$

Exercise 3.2 (B) | Q 1.16 | Page 123

Evaluate the following : $\int \frac{1}{\sqrt{8 - 3x + 2x^2}} . dx$

SOLUTION

Let I = $\int \frac{1}{\sqrt{8 - 3x + 2x^2}} . dx$

$8 - 3x + 2x^2 = 8\left[x^2 + \frac{3}{2}x + \frac{2}{2}\right]$

$= 8\left[\left(x^2 + \frac{3x}{2} + \frac{6}{4}\right) + \left(\frac{2}{2} - \frac{6}{4}\right)\right]$

$= 8\left[\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{1}}{4}\right)^2\right]$

$\therefore \sqrt{3x^2 + 5x + 7} = \sqrt{3}\sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{1}}{4}\right)^2}$

$\therefore$ I = $\frac{1}{\sqrt{3}} \int \frac{1}{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{1}}{4}\right)^2} . dx$

$= \frac{1}{\sqrt{2}} \log\left|x - \frac{3}{4} + \sqrt{\left(x + \frac{3}{4}\right)^2 + \left(\frac{\sqrt{1}}{4}\right)^2}\right| + c$

$= \frac{1}{\sqrt{2}} \log\left|x - \frac{3}{4} + \sqrt{x^2 - \frac{3x}{2} + 4}\right| + c.$

Exercise 3.2 (B) | Q 1.17 | Page 123

Evaluate the following : $\int \frac{1}{\sqrt{(x - 3)(x + 2)}} . dx$

SOLUTION

Let I = $\int \frac{1}{\sqrt{(x-3)(x+2)}}.dx$

= $\int \frac{1}{\sqrt{x^2-x-6}}.dx$

= $\int \frac{1}{\sqrt{\left(x^2-x+\frac{1}{4}\right)-\frac{1}{4}-6}}.dx$

= $\int \frac{1}{\sqrt{\left(x-\frac{1}{2}\right)^2-\left(\frac{5}{2}\right)^2}}.dx$

= $\log\left|\left(x-\frac{1}{2}\right)+\sqrt{\left(x-\frac{1}{2}\right)^2-\left(\frac{5}{2}\right)^2}\right|+c$

= $\log\left|\left(x-\frac{1}{2}\right)+\sqrt{x^2-x-6}\right|+c.$

Exercise 3.2 (B) | Q 1.18 | Page 123

Evaluate the following : $\int \frac{1}{4+3\cos^2 x}.dx$

SOLUTION

Let I = $\int \frac{1}{4+3\cos^2 x}.dx$

Dividing both numerator and denominator by $\cos^2 x$, we get

I = $\int \frac{\sec^2 x}{4\sec^2 x+3}.dx$

= $\int \frac{\sec^2 x}{4(1+\tan^2 x)+3}.dx$

$$= \int \frac{\sec^2 x}{4\tan^2 x + 7} . dx$$

Put tan x = t

$\therefore \sec^2 x \, dx = dt$

$$I = \int \frac{dt}{4t^2 + 7}$$

$$= \int \frac{dt}{(2t)^2 + \left(\sqrt{7}\right)^2}$$

$$= \frac{1}{\sqrt{7}} \tan^{-1}\left(\frac{2t}{\sqrt{7}}\right) . \frac{1}{2} + c$$

$$= \frac{1}{2\sqrt{7}} \tan^{-1}\left(\frac{2\tan x}{\sqrt{7}}\right) + c.$$

Exercise 3.2 (B) | Q 1.19 | Page 123

Evaluate the following : $\int \frac{1}{\cos 2x + 3\sin^2 x} . dx$

SOLUTION

Let $I = \int \frac{1}{\cos 2x + 3\sin^2 x} . dx$

$$= \int \frac{1}{1 - 2\sin^2 x + 3\sin^2 x} . dx$$

$$= \int \frac{1}{1 + \sin^2 x} . dx$$

Dividing both numerator and denominator by $\cos^2 x$, we get

$$I = \int \frac{\sec^2 x dx}{\sec^2 x + \tan^2 x}$$

$$= \int \frac{\sec^2 x dx}{1 + \tan^2 x + \tan^2 x}$$

$= \int \frac{\sec^2 x dx}{2\tan^2 x + 1}$

Put tan x = t

∴ $\sec^2 x$ dx = dt

∴ I = $\int \frac{1}{2t^2 + 1} dt$

$= \frac{1}{2} \int \frac{1}{t^2 + \left(\frac{1}{\sqrt{2}}\right)^2} dt$

$= \frac{1}{2} \times \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} \tan^{-1}\left(\frac{t}{\frac{1}{\sqrt{2}}}\right) + c$

$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2} \tan x\right) + c.$

Exercise 3.2 (B) | Q 1.2 | Page 123

Evaluate the following : $\int \frac{\sin x}{\sin 3x} . dx$

SOLUTION

Let I = $\int \frac{\sin x}{\sin 3x} . dx$

$= \int \frac{\sin x}{3 \sin x - 4 \sin^2 x} . dx$

$= \int \frac{1}{3 - 4 \sin^2 x} . dx$

Dividing both numeratpr and denominator by $\cos^2 x$, we get

I = $\int \frac{\sec^2 x}{3\sec^2 x - 4\tan^2 x} . dx$

= $\int \frac{\sec^2 x}{3(1 + \tan^2 x) - 4\tan^2 x} . dx$

= $\int \frac{\sec^2 x}{3 - \tan^2 x} . dx$

Put tan x = t

∴ $\sec^2 x$ dx = dt

I = $\int \frac{dt}{\left(\sqrt{3}\right)^2 - t^2}$

= $\frac{1}{2\sqrt{3}} \log\left|\frac{\sqrt{3} + t}{\sqrt{3} - t}\right| + c$

= $\frac{1}{2\sqrt{3}} \log\left|\frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x}\right| + c.$

Exercise 3.2 (B) | Q 2.1 | Page 123

Integrate the following functions w.r.t. x : $\int \frac{1}{3 + 2\sin x} . dx$

SOLUTION

Let I = $\int \frac{1}{3 + 2\sin x} . dx$

Put $\tan\left(\frac{x}{2}\right) = t$

∴ x = 2 $\tan^{-1}$ t

∴ dx = $\frac{2t}{1 + t^2}$ and $\sin x = \frac{2t}{1 + t^2}$

$$\therefore I = \int \frac{1}{3 + 2\left(\frac{2t}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}$$

$$= \int \frac{1+t^2}{3+3t^2+4t} \cdot \frac{2dt}{1+t^2}$$

$$= 2\int \frac{1}{3t^2+4t+3} dt$$

$$= \frac{2}{3}\int \frac{1}{t^2+\frac{4}{3}t+1} dt$$

$$= \frac{2}{3}\int \frac{1}{\left(t^2+\frac{4}{3}t+\frac{4}{9}\right)-\frac{4}{9}+1} dt$$

$$= \frac{2}{3}\int \frac{1}{\left(t+\frac{2}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2} dt$$

$$= \frac{2}{3} \times \frac{1}{\left(\frac{\sqrt{5}}{3}\right)} \tan^{-1}\left[\frac{t+\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right] + c$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{3t+2}{\sqrt{5}}\right) + c$$

$$= \frac{2}{\sqrt{5}} \tan^{-1}\left[\frac{3\tan\left(\frac{x}{2}\right)+2}{\sqrt{5}}\right] + c.$$

Exercise 3.2 (B) | Q 2.2 | Page 123

Integrate the following functions w.r.t. x : $\int \frac{1}{4-5\cos x} . dx$

SOLUTION

Let I = $\int \frac{1}{4 - 5\cos x} . dx$

Put $\tan\left(\frac{x}{2}\right)$ = t

∴ x = 2 $\tan^{-1}$ t

∴ dx = $\frac{2dt}{1+t^2}$ and $\cos x = \frac{1-t^2}{1+t^2}$

∴ I = $\int \frac{1}{4 - 5\left(\frac{1-t^2}{1+t^2}\right)} . \frac{2dt}{1+t^2}$

= $\int \frac{1+t^2}{4(1+t^2) - 5(1-t^2)} . \frac{2dt}{1+t^2}$

= $\int \frac{2dt}{9t^2 - 1}$

= $\frac{2}{9} \int \frac{1}{t^2 - \frac{1}{9}} dt$

= $\frac{2}{9} \int \frac{1}{t^2 - \left(\frac{1}{3}\right)^2} dt$

= $\frac{2}{9} \times \frac{1}{2 \times \frac{1}{3}} \log\left|\frac{t - \frac{1}{3}}{t + \frac{1}{3}}\right| + c$

= $\frac{1}{3} \log\left|\frac{3\tan\left(\frac{x}{2}\right) - 1}{3\tan\left(\frac{x}{2}\right) + 1}\right| + c.$

Exercise 3.2 (B) | Q 2.3 | Page 123

Integrate the following functions w.r.t. x : $\int \frac{1}{2 + \cos x - \sin x} . dx$

SOLUTION

Let I = $\int \frac{1}{2+\cos x-\sin x}.dx$

Put $\tan\left(\frac{x}{2}\right)$ = t

∴ x 2 $\tan^{-1}$ t

∴ dx = $\frac{2dt}{1+t^2}$ and $\sin x=\frac{2t}{1+t^2}, \cos x=\frac{1-t^2}{1+t^2}$

∴ I = $\int \frac{1}{2+\left(\frac{1-t^2}{1+t^2}\right)-\left(\frac{2t}{1+t^2}\right)}.\frac{2dt}{1+t^2}$

= $\int \frac{1+t^2}{2+2t^2+1-t^2-2t}.\frac{2dt}{1+t^2}$

= $2\int \frac{1}{t^2 \ -2t+3}dt$

= $2\int \frac{1}{(t^2-2t+1)+2}dt$

= $2\int \frac{1}{(t-1)^2+\left(\sqrt{2}\right)^2}.dt$

= $2\times\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t-1}{\sqrt{2}}\right)+c$

= $\sqrt{2}\tan^{-1}\left[\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right]+c.$

Exercise 3.2 (B) | Q 2.4 | Page 123

Integrate the following functions w.r.t. x : $\int \frac{1}{3+2\sin x-\cos x}dx$

SOLUTION

Let I = $\int \frac{1}{3 + 2\sin x - \cos x} dx$

Put $\tan\left(\frac{x}{2}\right)$ = t

∴ x = 2 $\tan^{-1}$ t

∴ dx = $\frac{2}{1+t^2} dt$ and

sinx = $\frac{2t}{1+t^2}$, $\cos x = \frac{1-t^2}{1+t^2}$

∴ I = $\int \frac{1}{3 + 2\left(\frac{2t}{1+t^2}\right) - \left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2}$

= $\int \frac{1+t^2}{3(1+t^2) + 4t - (1-t^2)} \cdot \frac{2dt}{1+t^2}$

= $2\int \frac{dt}{4t^2 + 4t + 2}$

= $2\int \frac{dt}{4t^2 + 4t + 1 + 1}$

= $2\int \frac{dt}{(2t+1)^2 + 1^2}$

= $\frac{2}{2}\tan^{-1}\left(\frac{2t+1}{1}\right) + c$

= $\tan^{-1}\left[2\tan^{-1}\left(\frac{x}{2}\right) + 1\right] + c.$

Exercise 3.2 (B) | Q 2.5 | Page 123

Integrate the following functions w.r.t. x : $\int \frac{1}{3 - 2\cos 2x} . dx$

SOLUTION

Let I = $\int \frac{1}{3 - 2\cos 2x} . dx$

Put tan x = t

∴ x = $\tan^{-1}$ t

∴ dx = $\frac{dt}{1+t^2}$ and $\cos 2x = \frac{1-t^2}{1+t^2}$

∴ I = $\int \frac{1}{3 - 2\left(\frac{1-t^2}{1+t^2}\right)} . \frac{dt}{1+t^2}$

= $\int \frac{1+t^2}{3+3t^2-2+2t^2} . \frac{dt}{1+t^2}$

= $\int \frac{1}{1+5t^2} dt$

= $\frac{1}{5} \int \frac{1}{\left(\frac{1}{\sqrt{5}}\right)^2 + t^2} dt$

= $\frac{1}{5} \times \frac{1}{\left(\frac{1}{\sqrt{5}}\right)} \tan^{-1}\left(\frac{t}{\frac{1}{\sqrt{5}}}\right) + c$

= $\frac{1}{\sqrt{5}} \tan^{-1}\left(\sqrt{5} \tan x\right) + c.$

Exercise 3.2 (B) | Q 2.6 | Page 123

Integrate the following functions w.r.t. x : $\int \frac{1}{2\sin 2x - 3} dx$

SOLUTION

Let I = $\int \frac{1}{2\sin 2x - 3} dx$

Put tan x = t

∴ x = $\tan^{-1}$ t

∴ dx = $\frac{dt}{1+t^2}$ and $\sin 2x = \frac{2t}{1+t^2}$

∴ I = $\int \frac{1}{2\left(\frac{2t}{1+t^2}\right) - 3} \cdot \frac{dt}{1+t^2}$

= $\int \frac{1+t^2}{4t - 3 - 3t^2} \cdot \frac{dt}{1+t^2}$

= $\int \frac{1}{-3t^2 + 4t - 3} dt$

= $\frac{1}{3} \int \frac{1}{t^2 - \frac{4}{3}t + 1} dt$

= $-\frac{1}{3} \int \frac{1}{\left(t^2 - \frac{4}{3}t + \frac{4}{9}\right) - \frac{4}{9} + 1} dt$

= $-\frac{1}{3} \int \frac{1}{\left(t - \frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} dt$

= $-\frac{1}{3} \times \frac{1}{\left(\frac{\sqrt{5}}{3}\right)} \tan^{-1}\left(\frac{t - \frac{2}{3}}{\frac{\sqrt{5}}{3}}\right) + c$

= $-\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{3t-2}{\sqrt{5}}\right) + c$

= $-\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{3\tan x - 2}{\sqrt{5}}\right) + c.$

Exercise 3.2 (B) | Q 2.7 | Page 123

Integrate the following functions w.r.t. x : $\int \frac{1}{3 + 2\sin 2x + 4\cos 2x}.dx$

SOLUTION

Let I = $\int \frac{1}{3 + 2\sin 2x + 4\cos 2x}.dx$

Put tan x = t

∴ x = $\tan^{-1}$ t

∴ dx = $\frac{dt}{1+t^2}$ $\text{ and }\ \sin 2x = \frac{2t}{1+t^2},, \cos 2x = \frac{1-t^2}{1+t^2}$

∴ I = $\int \frac{1}{3 + 2\left(\frac{2t}{1+t^2}\right) + 4\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{dt}{1+t^2}$

= $\int \frac{1+t^2}{3(1+t^2) + 4t + 4(1-t^2)} \cdot \frac{dt}{1+t^2}$

= $\int \frac{1}{7+4t-t^2}dt = \int \frac{1}{7-(t^2-4t+4)+4}dt$

= $\int \frac{1}{\left(\sqrt{11}\right)^2 - (t-2)^2}dt$

= $\frac{1}{2\sqrt{11}}\log\left|\frac{\sqrt{11}+t-2}{\sqrt{11}-t+2}\right| + c$

= $\frac{1}{2\sqrt{11}}\log\left|\frac{\sqrt{11}+\tan x-2}{\sqrt{11}-\tan x+2}\right| + c.$

Exercise 3.2 (B) | Q 2.8 | Page 123

Integrate the following functions w.r.t. x : $\int \frac{1}{\cos x - \sin x}.dx$

SOLUTION

Let I = $\int \frac{1}{\cos x - \sin x} . dx$

Dividing each term by $\sqrt{1^2 + (-1)^2} = \sqrt{2}$, we get

I = $\frac{1}{\sqrt{2}} \int \frac{1}{\cos x . \frac{1}{\sqrt{2}} - \sin x . \frac{1}{\sqrt{2}}} . dx$

= $\frac{1}{\sqrt{2}} \int \frac{1}{\cos x . \cos \frac{\pi}{4} - \sin x . \sin \frac{\pi}{4}} . dx$

= $\frac{1}{\sqrt{2}} \int \frac{1}{\cos\left(x + \frac{\pi}{4}\right)} . dx$

= $\frac{1}{\sqrt{2}} \int \sec\left(x + \frac{\pi}{4}\right) . dx$

= $\frac{1}{\sqrt{2}} \log\left|\sec\left(x + \frac{\pi}{4}\right) + \tan\left(x + \frac{\pi}{4}\right)\right| + c.$

Exercise 3.2 (B) | Q 2.9 | Page 123

Integrate the following functions w.r.t. x : $\int \frac{1}{\cos x - \sqrt{3} \sin x} . dx$

SOLUTION

Let I = $\int \frac{1}{\cos x - \sqrt{3} \sin x} . dx$

Dividing each term by $\sqrt{1^2 + (-1)^2} = \sqrt{3}$, we get

I = $\frac{1}{2} \int \frac{1}{\cos x . \frac{1}{\sqrt{3}} - \sin x . \frac{1}{\sqrt{3}}} . dx$

= $\frac{1}{2} \int \frac{1}{\cos x . \cos \frac{\pi}{3} - \sin x . \sin \frac{\pi}{3}} . dx$

$= \frac{1}{2}\int \frac{1}{\cos\left(x+\frac{\pi}{3}\right)}.dx$

$= \frac{1}{2}\int \sec\left(x+\frac{\pi}{3}\right).dx$

$= \frac{1}{2}\log\left|\sec\left(x+\frac{\pi}{3}\right)+\tan\left(x+\frac{\pi}{3}\right)\right|+c.$

EXERCISE 3.2 (C) [PAGE 128]

Exercise 3.2 (C) | Q 1.1 | Page 128

Evaluate the following integrals : $\int \frac{3x+4}{x^2+6x+5}.dx$

SOLUTION

Let I = $\int \frac{3x+4}{x^2+6x+5}.dx$

Let 3x + 4 = $\mathbf{A}\left[\frac{d}{dx}\left(x^2+6x+5\right)\right]+\mathbf{B}$

= A(2x + 6) + B

∴ 3x + 4 = 2Ax + (6A + B)

Comparing the coefficient of x and constant on both sides, we get

2A = 3 and 6A + B = 4

∴ $\mathbf{A}=\frac{3}{2}$ and $6\left(\frac{3}{2}\right)+\mathbf{B}$ = 4

∴ B = – 5

∴ 3x + 4 = $\frac{3}{2}(2x+6)-5$

∴ I = $\int \frac{\frac{3}{2}(2x+6)-5}{x^2+6x+5}.dx$

$= \frac{3}{2}\int \frac{2x+6}{x^2+6x+5}.dx - 5\int \frac{1}{x^2+6x+5}.dx$

$= \frac{3}{2}I_1 - 5I_2$

I_1 is of the type $\int \frac{f\prime(x)}{f(x)}.dx = \log|f(x)| + c$

$\therefore I_1 = \log\left|x^2+6x+5\right| + c_1$

$I_2 = \int \frac{1}{x^2+6x+5}.dx$

$= \int \frac{1}{(x^2+6x+9)-4}.dx$

$= \int \frac{1}{(x+3)^2-2^2}.dx$

$= \frac{1}{2\times 2}\log\left|\frac{x+3-2}{x+3+2}\right| + c_2$

$= \frac{1}{4}\log\left|\frac{x+1}{x+5}\right| + c_2$

$\therefore I = \frac{3}{2}\log\left|x^2+6x+\ 5\right| - \frac{5}{4}\log\left|\frac{x+1}{x+5}\right| + c$, where c = c + c_2.

Exercise 3.2 (C) | Q 1.2 | Page 128

Evaluate the following integrals : $\int \frac{2x+1}{x^2+4x-5}.dx$

SOLUTION

Let I = $\int \frac{2x+1}{x^2+4x-5}.dx$

Let 2x + 1 = $\mathbf{A}\left[\frac{d}{dx}(x^2+4x-5)\right]+\mathbf{B}$

= A(2x + 1) + B

∴ 2x + 1 = 2Ax + (4A + B)

Comparing the coefficient of x and constant on both sides, we get

4A = 2 and 4A + B = 4

$\therefore \mathbf{A}=\frac{3}{2}$ and $6\left(\frac{3}{2}\right)+\mathbf{B}$ = 4

∴ B = – 5

∴ 2x + 1 = $\frac{3}{2}(2x+1)-5$

∴ I = $\int \frac{\frac{3}{2}(2x+1)-5}{x^2+6x+5}.dx$

= $\frac{3}{2}\int \frac{2x+1}{x^2+4x-5}.dx-5\int \frac{1}{x^2+4x+5}.dx$

= $\frac{3}{2}\mathrm{I}_1-5\mathrm{I}_2$

I_1 is of the type $\int \frac{f\prime(x)}{f(x)}.dx=\log|f(x)|+c$

$\therefore \mathrm{I}_1=\log\left|x^2+4x-5\right|+c_1$

I_2 = $\int \frac{1}{x^2+4x-5}.dx$

= $\int \frac{1}{(x^2+4x-9)-4}.dx$

$= \int \frac{1}{(x+3)^2 - 2^2} . dx$

$= \log\left|\frac{x+3-2}{x+3+2}\right| + c_2$

$= \log\left|\frac{x-1}{x+5}\right| + c_2$

$\therefore$ I = $\log|x^2 + 4x - 5| - \frac{1}{2}\log\left|\frac{x-1}{x+5}\right| + c,$

Exercise 3.2 (C) | Q 1.3 | Page 128

Evaluate the following integrals : $\int \frac{2x+3}{2x^2+3x-1} . dx$

SOLUTION

Let I = $\int \frac{2x+3}{2x^2+3x-1} . dx$

Let 2x + 3 = $\mathrm{A}\left[\frac{d}{dx}\left(2x^2+3x-1\right)\right] + \mathrm{B}$

= A(4x + 3) + B

$\therefore$ 2x + 3 = 4Ax + (3A + B)

Comapring the coefficientof x and constant on both sides, we get

4A = 2 and 3A + B = 3

$\therefore$ A = $\frac{1}{2}$ and $3\left(\frac{1}{2}\right) + \mathrm{B}$ = 3

$\therefore$ B = $\frac{3}{2}$

$\therefore$ 2x + 3 = $\frac{1}{2}(4x+3) + \frac{3}{2}$

$\therefore$ I = $\int \frac{\frac{1}{2}(4x+3) + \frac{3}{2}}{2x^2+3x-1} . dx$

$= \frac{1}{2}\int \frac{4x+3}{2x^2+3x-1}.dx + \frac{3}{2}\int \frac{1}{2x^2+3x-1}.dx$

$= \frac{1}{2}\mathrm{I}_1 + \frac{3}{2}\mathrm{I}_2$

I_1 is of the type $\int \frac{f\prime(x)}{f(x)}dx = \log|f(x)| + c$

$\therefore I_1 = \log|2x^2 + 3x - 1| + c_1$

$I_2 = \int \frac{1}{2x^2+3x-1}.dx$

$= \frac{1}{2}\int \frac{1}{x^2+\frac{3}{2}x-\frac{1}{2}}.dx$

$= \frac{1}{2}\int \frac{1}{\left(x^2+\frac{3}{2}x+\frac{9}{16}\right)-\frac{9}{16}-\frac{1}{2}}.dx$

$= \frac{1}{2}\int \frac{1}{\left(x+\frac{3}{4}\right)^2-\left(\frac{\sqrt{17}}{4}\right)^2}.dx$

$= \frac{1}{2}\times\frac{1}{2\times\frac{\sqrt{17}}{4}}\log\left|\frac{x+\frac{3}{4}-\frac{\sqrt{17}}{4}}{x+\frac{3}{4}+\frac{\sqrt{17}}{4}}\right| + c_2$

$= \frac{1}{\sqrt{17}}\log\left|\frac{4x+3-\sqrt{17}}{4x+3+\sqrt{17}}\right| + c_2$

$\therefore I = \frac{1}{2}\log\left|2x^2+3x-1\right| + \frac{3}{2\sqrt{17}}\log\left|\frac{4x+3-\sqrt{17}}{4x+3+\sqrt{17}}\right| + c$, where $c = c + c^2$.

Exercise 3.2 (C) | Q 1.4 | Page 128

Evaluate the following integrals : $\int \frac{3x+4}{\sqrt{2x^2+2x+1}}.dx$

SOLUTION

Let I = $\int \frac{3x+4}{\sqrt{2x^2+2x+1}}.dx$

Let 3x + 4 = $\mathbf{A}\left[\frac{d}{dx}(2x^2+2x+1)\right.+\mathbf{B}$

= A(4x + 2) + B

∴ 3x + 4 = 4Ax + (2A + B)

Comapring the coefficient of x and the constant on both the sides, we get

4A = 3 and 2A + B = 4

∴ A = $\frac{3}{4}$ and $2\left(\frac{3}{4}\right)+\mathbf{B}$ = 4

∴ B = $\frac{5}{2}$

∴ 3x + 4 = $(3)(4)(4x+2)+\frac{5}{2}$

∴ I = $\int \frac{\frac{3}{4}(4x+2)+\frac{5}{2}}{\sqrt{2x^2+2x+1}}.dx$

= $\frac{3}{4}\int \frac{4x+2}{\sqrt{2x^2+2x+1}}.dx+\frac{5}{2}\int \frac{1}{\sqrt{2x^2+2x+1}}.dx$

= $\frac{3}{4}\mathrm{I}_1+\frac{5}{2}\mathrm{I}_2$

In I_1, put $2x^2 + 2x + 1 = t$

∴ (4x + 2)dx = dt

∴ I1 = $\int \frac{1}{\sqrt{t}}dt$

= $\int t^{-\frac{1}{2}}dt$

$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1$

$= 2\sqrt{2x^2 + 2x + 1} + c$

$I_2 = \frac{5}{\sqrt{2}} \int \frac{1}{\sqrt{x^2 + x + \frac{1}{2}}} . dx$

$= \frac{5}{\sqrt{2}} \int \frac{1}{\sqrt{\left(x^2 + x + \frac{1}{4}\right) + \frac{1}{4}}} . dx$

$= \frac{5}{\sqrt{2}} \int \frac{1}{\sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}} . dx$

$= \frac{5}{\sqrt{2}} \log\left|\left(x + \frac{1}{2}\right) + \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}\right| + c_2$

$= \frac{5}{\sqrt{2}} \log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + \frac{1}{2}}\right| + c_2$

$\therefore I = \frac{3}{2}\sqrt{2x^2 + 2x + 1} + \frac{5}{2\sqrt{2}} \log\left|\left(x + \frac{1}{2}\right) + \sqrt{x^2 + x + \frac{1}{2}}\right| + c$, where $c = c_1 + c_2$.

Exercise 3.2 (C) | Q 1.5 | Page 128

x + Evaluate the following integrals : $\int \frac{7x + 3}{\sqrt{3 + 2x - x^2}} . dx$

SOLUTION

Let I $= \int \frac{7x + 3}{\sqrt{3 + 2x - x^2}} . dx$

Let 7x + 3 = $\mathbf{A}\left[\frac{d}{dx}\left(3 + 2x - x^2\right)\right] + \mathbf{B}$

= A(2 – 2x) + B

∴ 7x + 3 = 2Ax + (2A + B)

Comparing the coefficient of x and constant on both the sides, we get

$-2A = 7$ and $2A + B = 3$

$\therefore A = \dfrac{-7}{2}$ and $2\left(-\dfrac{7}{2}\right) + B = 3$

$\therefore B = 10$

$\therefore 7x + 3 = \dfrac{-7}{2}(2 - 2x) + 10$

$$\therefore I = \int \frac{\frac{-7}{2}(2-2x)+10}{\sqrt{3+2x-x^2}}.dx$$

$$= \frac{-7}{2}\int \frac{(2-2x)}{\sqrt{3+2x-x^2}}.dx + 10\int \frac{1}{\sqrt{3+2x-x^2}}x$$

$$= \frac{-7}{2}I_1 + 10I_2$$

In I_1, put $3 + 2x - x^2 = t$

$\therefore (2 - 2x)dx = dt$

$$\therefore I_1 = \int \frac{1}{\sqrt{t}}dt$$

$$= \int t^{-\frac{1}{2}}dt$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1$$

$$= 2\sqrt{3+2x-x^2} + c_1$$

$$I_2 = \int \frac{1}{\sqrt{3-(x^2-2x+1)+1}}.dx$$

$$= \int \frac{1}{\sqrt{(2)^2-(x-1)^2}}.dx$$

$= \sin^{-1}\left(\frac{x-1}{2}\right) + c_2$

$\therefore$ I = $-7\sqrt{3+2x-x^2} + 10\sin^{-1}\left(\frac{x-1}{2}\right) + c$, where c = $c_1 + c_2$.

$= \frac{5}{\sqrt{2}}\int \frac{1}{\sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}}.dx$

$= \frac{5}{\sqrt{2}}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}\right| + c_2$

$= \frac{5}{\sqrt{2}}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+\frac{1}{2}}\right| + c_2$

$\therefore$ I = $\frac{3}{2}\sqrt{2x^2+2x+1} + \frac{5}{2\sqrt{2}}\log\left|\left(x+\frac{1}{2}\right) + \sqrt{x^2+x+\frac{1}{2}}\right| + c$, where c = $c_1 + c_2$.

Exercise 3.2 (C) | Q 1.6 | Page 128

Evaluate the following integrals : $\int\sqrt{\frac{x-7}{x-9}}.dx$

SOLUTION

Let I = $\int\sqrt{\frac{x-7}{x-9}}.dx$

$= \int\sqrt{\frac{x-7}{x-9}\times\frac{x-7}{x-7}}.dx$

$= \int\sqrt{\frac{(x-7)^2}{x^2-16x+63}}.dx$

$= \int\frac{x-7}{\sqrt{x^2-16x+63}}.dx$

Let x – 7 = $\mathrm{A}\left[\frac{d}{dx}\left(x^2-16x+63\right)\right] + \mathrm{B}$

= A(2x – 16) + B

= 2Ax + (B – 16A)

Comparing the coefficient of x and constant term on both sides, we get

2A = 1

∴ A = $\frac{1}{2}$ and

B – 16A = – 7

∴ $\mathbf{B-16\left(\frac{1}{2}\right)}$ = – 7

∴ B = 1

∴ x – 7 = $\frac{1}{2}(2x-16)+1$

∴ I = $\int \frac{\frac{1}{2}(2x-16)+1}{\sqrt{x^2-16x+63}}.dx$

= $\frac{1}{2}\int \frac{2x-16}{\sqrt{x^2-16x+63}}.dx+\int \frac{1}{\sqrt{x^2-16x+63}}.dx$

= $\frac{1}{2}I_1+I_2$

In I_1, put x^2 – 16x + 63 = t

∴ (2x – 16)dx = dt

∴ I_1 = $\frac{1}{2}\int \frac{1}{\sqrt{t}}dt$

= $\frac{1}{2}\int t^{-\frac{1}{2}}dt$

$= \frac{1}{2} \frac{t^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_1$

$= \sqrt{x^2 - 16x + 63} + c_1$

$I_2 = \int \frac{1}{\sqrt{x^2 - 16x + 63}}.dx$

$= \int \frac{1}{\sqrt{(x-8)^2 - 1^2}}.dx$

$= \log\left|x - 8 + \sqrt{(x-8)^2 - 1^2}\right| + c_2$

$= \log\left|x \ - 8 + \sqrt{x^2 - 16x + 63}\right| + c_2$

$\therefore$ I = $\sqrt{x^2 - 16x + 63} + \log\left|x - 8 + \sqrt{x^2 - 16x + 63}\right| + c$,, where c = c_1 + c_2.

Exercise 3.2 (C) | Q 1.7 | Page 128

Evaluate the following integrals : $\int \sqrt{\frac{9-x}{x}}.dx$

SOLUTION

Let I = $\int \sqrt{\frac{9-x}{x}}.dx$

$= \int \sqrt{\frac{9-x}{x} . \frac{9-x}{9-x}}.dx$

$= \int \frac{9-x}{\sqrt{9x - x^2}}.dx$

Let 9 – x = $\mathbf{A}\left[\frac{d}{dx}\left(9x - x^2\right)\right] + \mathbf{B}$

= A(9 – 2x) + B

∴ 9 – x = (9A + B) – 2Ax

Comparing the coefficient of x and constant on both the sides, we get

$-2A = -1$ and $9A + B = 9$

$\therefore \mathbf{A = \frac{1}{2}}$ **and** $\mathbf{9\left(\frac{1}{2}\right) + B} = 9$

$\therefore B = \frac{9}{2}$

$\therefore 9 - x = \frac{1}{2}(9 - 2x) + \frac{9}{2}$

$\therefore I = \int \frac{\frac{1}{2}(9-2x) + \frac{9}{2}}{\sqrt{9x - x^2}} . dx$

$= \frac{1}{2} \int \frac{9 - 2x}{\sqrt{9x - x^2}} . dx + \frac{9}{2} \int \frac{1}{\sqrt{9x - x^2}} . dx$

$= \frac{1}{2} I_1 + \frac{9}{2} I_2$

In I_1, put $9x - x^2 = t$

$\therefore (9 - 2x)dx = dt$

$\therefore I_1 = \int \frac{1}{\sqrt{t}} dt$

$= \int t^{-\frac{1}{2}} dt$

$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1$

$= 2\sqrt{9x - x^2} + c_1$

$I_2 = \int \frac{1}{\sqrt{\frac{81}{4} - \left(x^2 - 9x + \frac{81}{4}\right)}} . dx$

$= \int \frac{1}{\sqrt{\left(\frac{9}{2}\right)^2 - \left(x - \frac{9}{2}\right)^2}} . dx$

$= \sin^{-1}\left(\frac{x - \frac{9}{2}}{\frac{9}{2}}\right) + c_2$

$= = \sin^{-1}\left(\frac{2x - 9}{9}\right) + c_2$

∴ I = $\sqrt{9x - x^2} + \frac{9}{2}\sin^{-1}\left(\frac{2x - 9}{9}\right) + c$, where c = $c_1 + c_2$.

Exercise 3.2 (C) | Q 1.8 | Page 128

Evaluate the following integrals : $\int \frac{3\cos x}{4\sin^2 x + 4\sin x - 1} . dx$

SOLUTION

Let I = $\int \frac{3\cos x}{4\sin^2 x + 4\sin x - 1} . dx$

Put sin x = t

∴ cosx dx = dt

∴ I = $3\int \frac{dt}{4t^2 + 4t - 1}$

$= 3\int \frac{dt}{(4t^2 + 4t + 1) - 2}$

$= 3\int \frac{dt}{(2t + 1)^2 - \left(\sqrt{2}\right)^2}$

$= \frac{3}{2\left(2\sqrt{2}\right)}\log\left|\frac{2t + 1 - \sqrt{2}}{2t + 1 + \sqrt{2}}\right| + c$

$= \frac{3}{2\left(2\sqrt{2}\right)} \log\left|\frac{2t+1-\sqrt{2}}{2t+1+\sqrt{2}}\right| + c$

$= \frac{3}{4\sqrt{2}} \log\left|\frac{2\sin x+1-\sqrt{2}}{2\sin x+1+\sqrt{2}}\right| + c.$

Exercise 3.2 (C) | Q 1.9 | Page 128

Evaluate the following integrals : $\int\sqrt{\frac{e^{3x}-e^{2x}}{e^x+1}}.dx$

SOLUTION

Let I = $\int\sqrt{\frac{e^{3x}-e^{2x}}{e^x+1}}.dx$

$= \int\sqrt{\frac{e^{2x}(e^x-1)}{e^x+1}}.dx$

$= \int e^x\sqrt{\frac{e^x-1}{e^x+1}}.dx$

Put $e^x = t$

$\therefore e^x\, dx = dt$

$\therefore$ I = $\int\sqrt{\frac{t-1}{t+1}}dt$

$= \int\sqrt{\frac{t-1}{t+1}\times\frac{t-1}{t+1}}dt$

$= \int\sqrt{\frac{(t-1)^2}{t^2-1}}dt$

$= \int\frac{t-1}{\sqrt{t^2-1}}dt$

$= \frac{1}{2}\int \frac{2t}{\sqrt{t^2-1}}dt - \int \frac{1}{\sqrt{t^2-1}}dt$

$= I_1 - I_2$

In I_1, put $t^2 - 1 = \theta$

$\therefore 2t\, dt = d\theta$

$\therefore I_1 = \frac{1}{2}\int \frac{d\theta}{\sqrt{\theta}}$

$= \frac{1}{2}\int \theta^{-\frac{1}{2}} d\theta$

$= \frac{1}{2}\frac{\theta^{\frac{1}{2}}}{\left(\frac{1}{2}\right)} + c_1$

$= \sqrt{\theta} + c_1$

$= \sqrt{t^2-1} + c_1$

$= \sqrt{e^{2x}-1} + c_1$

$I_2 = \int \frac{1}{\sqrt{t^2-1}}dt$

$= \log\left|t + \sqrt{t^2-1}\right| + c_2$

$= \log\left|e^x + \sqrt{e^{2x}-1}\right| + c_2$

$\therefore I = \sqrt{e^{2x}-1} - \log|e^x + \sqrt{e^{2x}-1} + c$, where $c = c_1 + c_2$.

EXERCISE 3.3 [PAGES 137 - 138]

Exercise 3.3 | Q 1.01 | Page 137

Evaluate the following : $\int x^2 . \log x . dx$

SOLUTION

Let I = $\int x^2 . \log x . dx$

= $\int \log x . x^2 . dx$

= $(\log x) \int x^2 . dx - \int \left[\left\{\frac{d}{dx}(\log x) \int x^2 . dx\right\}\right] . dx$

= $(\log x) . \frac{x^3}{3} - \int \frac{1}{x} . \frac{x^3}{3} . dx$

= $\frac{x^3}{3} \log x - \frac{1}{3} \int x^2 . dx$

= $\frac{x^3}{3} \log x - \frac{1}{3}\left(\frac{x^3}{3}\right) + c$

= $\frac{x^3}{9}(3 . \log x - 1) + c.$

Exercise 3.3 | Q 1.02 | Page 137

Evaluate the following : $\int x^2 \sin 3x \, dx$

SOLUTION

Let I = $\int x^2 \sin 3x \, dx$

= $x^2 \int \sin 3x dx - \int \left[\left\{\frac{d}{dx}(x^2) \int \sin 3x dx\right\}\right] . dx$

= $x^2 \left(\frac{-\cos 3x}{3}\right) - \int 2x \left(\frac{-\cos 3x}{3}\right) . dx$

= $\frac{x^2}{3} \cos 3x + \frac{2}{3} \int x \cos 3x dx$

$$= \frac{x^2}{3}\cos 3x + \frac{2}{3}\left[x\int \cos 3x dx - \int\left\{\frac{d}{dx}(x)\int \cos 3x dx\right\}. dx\right]$$

$$= \frac{x^2}{3}\cos 3x + \frac{2}{3}\left[\frac{x\sin 3x}{3} - \int 1. \frac{\sin 3x}{3}. dx\right]$$

$$= -\frac{x^2}{3}\cos 3x + \frac{2}{9}x\sin 3x - \frac{2}{9}\int \sin 3x dx$$

$$= -\frac{x^2}{3}\cos 3x + \frac{2}{9}x\sin 3x - \frac{2}{9}\int\left(\frac{-\cos 3x}{3}\right) + c$$

$$= -\frac{x^2}{3}\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + c.$$

Exercise 3.3 | Q 1.03 | Page 137

Evaluate the following : $\int x\tan^{-1} x. dx$

SOLUTION

Let I = $\int x\tan^{-1} x. dx$

$$= \int (\tan^{-1} x). dx$$

$$= (\tan^{-1} x)\int x. dx - \int\left[\left\{\frac{d}{dx}(\tan^{-1} x)\int x. dx\right\}\right]. dx$$

$$= (\tan^{-1} x)\left(\frac{x^2}{2}\right) - \int\left(\frac{1}{1+x^2}\right)\left(\frac{x^2}{2}\right). dx$$

$$= \frac{x^2\tan^{-1}}{2} - \frac{1}{2}\int\frac{x^2}{x^2+1}. dx$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\frac{(x^2+1)-1}{x^2+1}. dx$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\left[\int\left(1 - \frac{1}{x^2+1}\right). dx\right]$$

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\left[\int 1.\,dx - \int \frac{1}{x^2+1}.\,dx\right]$

$= \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}(x - \tan^{-1}x) + c.$

Exercise 3.3 | Q 1.04 | Page 137

Evaluate the following : $\int x^2 \tan^{-1} x.\,dx$

SOLUTION

Let I = $\int x^2 \tan^{-1} x.\,dx$

$= \int (\tan^{-1} x).\,x^2 dx$

$= (\tan^{-1} x)\int x^2.\,dx - \int\left[\left\{\frac{d}{dx}(\tan^{-1} x)\int x^2.\,dx\right\}\right].\,dx$

$= (\tan^{-1} x)\left(\frac{x^3}{3}\right) - \int\left(\frac{1}{1+x^2}\right)\left(\frac{x^3}{3}\right).\,dx$

$= x\frac{3}{3}\tan^{-1}x - \frac{1}{3}\frac{x(x^2+1)-x}{x^2+1}.\,dx$

$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\left[\int\left\{x - \frac{x}{x^2+1}\right\}.\,dx\right]$

$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\left[\int x.\,dx - \frac{1}{2}\int \frac{2x}{x^2+1}.\,dx\right]$

$= \frac{x^3}{3}\tan^{-1}x - \frac{1}{3}\left[\frac{x^2}{2} - \frac{1}{2}\log|x^2+1|\right] + c$

...$\left[\because \frac{d}{dx}(x^2+1) = 2x \text{ and } \int \frac{f\prime(x)}{f(x)}dx = \log|f(x)| + c\right]$

$= \frac{x^3}{3}\tan^{-1}x - \frac{x^2}{6} + \frac{1}{6}\log\left|x^2+1\right| + c.$

Exercise 3.3 | Q 1.05 | Page 137

Evaluate the following : $\int x^3 . \tan^{-1} x . dx$

SOLUTION

Let I = $\int x^3 . \tan^{-1} x . dx$

$= \int (\tan^{-1} x) . x^3 dx$

$= (\tan^{-1} x) \int x^3 . dx - \int \left[\left\{\frac{d}{dx}(\tan^{-1} x) \int x^3 . dx\right\}\right] . dx$

$= (\tan^{-1} x)\left(\frac{x^4}{4}\right) - \int \left(\frac{1}{1+x^2}\right)\frac{x^4}{4} . dx$

$= \frac{x^4}{4}\tan^{-1}x - \frac{1}{4}\frac{(x^4-1)+1}{x^2+1}$

$= \frac{x^4}{4}\tan^{-1}x - \frac{1}{4}\int \frac{(x^2-1)(x^2+1)+1}{x^2+1} . dx$

$= \frac{x^4}{4}\tan^{-1}x - \frac{1}{4}\int \left[x^2 - 1 + \frac{1}{x^2+1}\right] . dx$

$= \frac{x^4}{4}\tan^{-1}x - \frac{1}{4}\int \left[\int x^2 . dx - \int 1 . dx + \int \frac{1}{x^2+1} . dx\right]$

$= \frac{x^4}{4}\tan^{-1}x - \frac{1}{4}\left[\frac{x^3}{3} - x + \tan^{-1}x\right] + c$

$= \frac{x^4}{4}\tan^{-1}x - \tan^{-1}\ \frac{x}{4} - \frac{x^3}{12} - \frac{x}{4} + c$

$= \frac{1}{4}(\tan^{-1} x)(x^4 - 1) - \frac{x}{12}(x^2 - 3) + c.$

Exercise 3.3 | Q 1.06 | Page 137

Evaluate the following : $\int (\log x)2.\, dx$

SOLUTION

Let I = $\int (\log x)^2.\, dx$

Put log x = t

∴ x = e^t

∴ dx = e^t dt

∴ I = $\int t^2 e^t dt$

= $t^2 \int e^t dt - \int \left[\frac{d}{dx}(t^2) \int e^t - dt\right] dt$

= $t^2 e^t - \int 2te^t dt$

= $t^2 e^t - 2\left[t \int e^t dt - \int \left\{\frac{d}{dt}(t) \int e^t dt\right\} dt\right]$

= $t^2 e^t - 2\left[te^t - \int 1.\, e^t dt\right]$

= $t^2 e^t - 2te^t + 2e^t + c$

= $e^t\left[t^2 - 2t + 2\right] + c$

= x[(log x)^2 – 2(log x) + 2] + c.

Alternative Method :

Let I = $\int (\log x)^2.\, dx$

= $\int (\log x)^2.\, 1dx$

= $(\log x)^2 \int 1.\, dx - \int \left[\frac{d}{dx}(\log x)^2.\int 1.\, dx\right].\, dx$

= $(\log x)^2 . x - \int 2\log x . \frac{d}{dx}(\log x) . x dx$

= $x(\log x)^2 - \int 2\log x \times \frac{1}{x} \times x . dx$

= $x(\log x)^2 - 2\int (\log x) .1 dx$

= $x(\log x)2 - 2\left[(\log x)\int 1 . dx - \int\left\{\frac{d}{dx}(\log x)\int 1 . dx\right\} . dx\right]$

= $x(\log x)^2 - 2\left[(\log x)x - \int \frac{1}{x} \times x . dx\right.$

= $x(\log x) - 2x(\log x) + 2\int 1 . dx$

= $x(\log x)^2 - 2x(\log x) + 2x + c$

= $x\left[(\log x)^2 - 2(\log x) + 2\right] + c.$

Exercise 3.3 | Q 1.07 | Page 137

Evaluate the following : $\int \sec^3 x . dx$

SOLUTION

Let I = $\int \sec^3 x . dx$

= $\int \sec x \sec^2 x . dx$

= $\sec x \int \sec^2 x . dx - \int\left[\frac{d}{dx}(\sec x)\int \sec^2 x . dx\right] . dx$

= $\sec x \tan x - \int (\sec x \tan x)(\tan x) . dx$

= $\sec x \tan x - \int \sec x \tan^2 x . dx$

$= \sec x \tan x - \int \sec x \left(\sec^2 x - 1\right). dx$

$= \sec x \tan x - \int \sec^3 x. dx + \int \sec x. dx$

∴ I = sec x tan x – I + log |sec x + tanx|

∴ 2I = sec x tan x + log |sec x + tan x|

∴ I = $\frac{1}{2}[\sec x \tan x + \log|\sec x + \tan|] + c.$

Exercise 3.3 | Q 1.08 | Page 137

Evaluate the following : $\int x. \sin^2 x. dx$

SOLUTION

$\int x. \sin^2 x. dx$

$= \int x\left(\frac{1-\cos 2x}{2}\right). dx$

$= \frac{1}{2}\int (x - x\cos 2x). dx$

$= \frac{1}{2}\int x. dx - \frac{1}{2}\int x\cos 2x. dx$

$= \frac{1}{2}.\frac{x^2}{2} - \frac{1}{2}\left[x\int \cos 2x. dx - \int\left\{\frac{d}{dx}(x)\int \cos 2x. dx\right\}. dx\right]$

$= \frac{x^2}{4} - \frac{1}{2}\left[x. \frac{\sin 2x}{2} - \int 1. \frac{\sin 2x}{2}. dx\right]$

$= \frac{x^2}{4} - \frac{1}{2}x. \sin 2x + \frac{1}{4}\sin 2x. dx$

$$= \frac{x^2}{4} - \frac{1}{4}x.\sin 2x + \frac{1}{4}.\frac{(-\cos 2x)}{2} + c$$

$$= \frac{x^2}{4} - \frac{1}{4}x.\sin 2x - \frac{1}{8}\cos 2x + c$$

Exercise 3.3 | Q 1.09 | Page 137

Evaluate the following : $\int x^3.\log x.dx$

SOLUTION

Let I = $\int x^3.\log x.dx$

$$= \int \log x.x^3.dx$$

$$= (\log x)\int x^3.dx - \int\left[\left\{\frac{d}{dx}(\log x)\int x^3.dx\right\}\right].dx$$

$$= (\log x).\frac{x^4}{4} - \int \frac{1}{x}.\frac{x^4}{4}.dx$$

$$= \frac{x^4}{4}\log x - \frac{1}{4}\int x^3.dx$$

$$= \frac{x^4}{4}\log x - \frac{1}{4}\left(\frac{x^4}{4}\right) + c$$

$$= \frac{x^4}{4}\log x - \frac{x^4}{16} + c.$$

Exercise 3.3 | Q 1.1 | Page 137

Evaluate the following : $\int e^{2x}.\cos 3x.dx$

SOLUTION

Let I = $\int e^{2x}.\cos 3x.\,dx$

= $e^{2x}\int \cos 3x.\,dx - \int\left[\frac{d}{dx}\left(e^{2x}\right)\int \cos 3x.\,dx\right].\,dx$

= $e^{2x}.\frac{\sin 3x}{3} - \int e^{2x}\times 2\times\frac{\sin 3x}{3}.\,dx$

= $\frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x.\,dx$

= $\frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left[e^{2x}\int \sin 3x.\,dx\right]$

= $\frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left[e^{2x}.\left(\frac{-\cos 3x}{3}\right) - \int e^{2x}\times 2\times\left(\frac{-\cos 3x}{3}\right).\,dx\right]$

= $\frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}\int e^{2x}\cos 3x.\,dx$

∴ I = $\frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}\mathrm{I}$

∴ $\left(1+\frac{4}{9}\right)\mathrm{I} = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x$

∴ $\frac{13}{9}\mathrm{I} = \frac{e^{2x}}{9}(3\sin 3x + 2\cos 3x)$

∴ I = $\frac{e^{2x}}{13}(2\cos 3x + 3\sin 3x) + c.$

Exercise 3.3 | Q 1.11 | Page 137

Evaluate the following : $\int x.\sin^2 x.\,dx$

SOLUTION

Let I = $\int x . \sin^2 x . dx$

= $\int (\sin^{-1} x) . x dx$

= $(\sin^{-1} x) \int x . dx - \int \left[\left\{ \frac{d}{dx} (\sin^{-1} x) \int x . dx \right\} \right] . dx$

= $(\sin^{-1} x) \left(\frac{x^2}{2} \right) - \int \left(\frac{1}{\sqrt{1 - x^2}} \right) \left(\frac{x^2}{2} \right) . dx$

= $\frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \frac{-x^2}{\sqrt{1 - x^2}} . dx$

= $\frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \frac{(1 - x^2) - 1}{\sqrt{1 - x^2}} . dx$

= $\frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \left[\sqrt{1 - x^2} - \frac{1}{\sqrt{1 - x^2}} \right] . dx$

= $\frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \int \sqrt{1 - x^2} . dx - \frac{1}{2} \int \frac{1}{\sqrt{1 - x^2}} . dx$

= $\frac{x^2}{2} \sin^{-1} x + \frac{1}{2} \left[\frac{x}{2} \sqrt{1 - x^2} + \frac{1}{2} \sin^{-1} x \right] - \frac{1}{2} \sin -1 x + c$

= $\frac{x^2}{2} \sin^{-1} x + \frac{1}{4} x \sqrt{1 - x^2} - \frac{1}{4} \sin^{-1} x + c.$

Exercise 3.3 | Q 1.12 | Page 137

Evaluate the following : $\int x^2 \cdot \cos^{-1} x \cdot dx$

SOLUTION

Let I = $\int x^2 . \cos^{-1} x \cdot dx$

= $\int (\cos^{-1} x) \cdot x^2 dx$

= $(\cos^{-1} x) \int x^2 \cdot dx - \int \frac{d}{dx}(\cos^{-1} x) \int x^2 \cdot dx \Big] \cdot dx$

= $(\cos^{-1} x)\left(\frac{x^3}{3}\right) - \int \left(\frac{-1}{\sqrt{1-x^2}}\right)\left(\frac{x^3}{3}\right) \cdot dx$

= $\frac{x^3}{3} \cos^{-1} x + \frac{1}{3} \int \frac{x^2 . x}{\sqrt{1-x^2}} \cdot dx$

In $\int \frac{x^3}{\sqrt{1-x^2}} \cdot dx$, put 1 – x^2 = t

∴ – 2x.dx= dt

∴ x.dx = $-\frac{1}{2} dt$

Also, x^2 = 1 – t

∴ I = $\frac{x^3}{3} \cos^{-1} x + \frac{1}{3} \int \frac{(1-t)}{\sqrt{t}} \left(-\frac{1}{2}\right) \cdot dt$

= $\frac{x^3}{3} \cos^{-1} x - \frac{1}{6} \int \left(\frac{1}{\sqrt{t}} - \sqrt{t}\right) \cdot dt$

= $\frac{x^3}{3} \cos^{-1} x - \frac{1}{6} \int t^{-\frac{1}{2}} dt + \frac{1}{6} \int t^{\frac{1}{2}} \cdot dt$

= $\frac{x^3}{3} \cos^{-1} x - \frac{1}{6} \left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\right) + \frac{1}{6} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$

= $\frac{x^3}{3} \cos^{-1} x - \frac{1}{3} \sqrt{1-x^2} + \frac{1}{9} (1-x^2)^{\frac{3}{2}} + c.$

Exercise 3.3 | Q 1.13 | Page 137

Evaluate the following : $\int \frac{\log(\log x)}{x}.dx$

SOLUTION

Let I = $\int \frac{\log(\log x)}{x}.dx$

= $\int \log(\log x).\frac{1}{x}dx$

Put log = t

$\therefore \frac{1}{x}.dx = dt$

∴ I = $\int \log t dt$

= $\int (\log t).1dt$

= $(\log t)\int 1dt - \int\left[\frac{d}{d}(\log t)\int 1dt\right]dt$

= $(\log t)t - \int \frac{1}{t} + tdt$

= $t\log t - \int 1dt$

= t logt t – t + c

= t(log t – 1) + c

= (log x).[log(log x) – 1] + c.

Exercise 3.3 | Q 1.14 | Page 137

Evaluate the following : $\int \frac{t.\sin^{-1}t}{\sqrt{1-t^2}}.dt$

SOLUTION

Let I = $\int \frac{t.\sin^{-1} t}{\sqrt{1-t^2}}.dt$

= $\int t.\sin^{-1} t.\frac{1}{\sqrt{1-t^2}}.dt$

Put $\sin^{-1}$ t = θ

$\therefore \frac{1}{\sqrt{1-t^2}}.dt$ = dθ

and

t = sin θ

∴ I = $\int (\sin\theta).\theta d\theta$

= $\int \theta \sin\theta d\theta$

= θ $\int \sin\theta d\theta - \int \left[\frac{d}{d\theta}(\theta)\int \sin\theta d\theta\right] d\theta$

= θ$(-\cos\theta) - \int 1.(-\cos\theta)d\theta$

= $-\theta\cos\theta + \int \cos\theta\, d\theta$

= – θ cos θ + sin θ + c

= $-\theta.\sqrt{1-\sin^2\theta} + \sin\theta + c$

= $-\sin^{-1} t.\sqrt{1-t^2} + t + c$

= $-\sqrt{1-t^2}.\sin^{-1} t + t + c.$

Exercise 3.3 | Q 1.15 | Page 137

Evaluate the following : $\int \cos\sqrt{x}.dx$

SOLUTION

Let I = $\int \cos\sqrt{x}.\, dx$

Put $\sqrt{x} = t$

∴ x = t^2

∴ dx = 2t .dt

∴ I = $\int (\cos t) 2t.\, dt$

= $\int 2t \cos t.\, dt$

= $2t \int \cos.\, dt - \int \left[\frac{d}{dt}(2t)\int \cos t.\, dt\right].\, dt$

= $2t \sin t - \int 2 \sin t.\, dt$

= 2t sin t + 2 cos t + c

= $2\left[\sqrt{x}\sin\sqrt{x} + \cos\sqrt{x}\right] + c.$

Exercise 3.3 | Q 1.16 | Page 137

Evaluate the following : $\int \sin\theta.\log(\cos\theta).\, d\theta$

SOLUTION

Let I = $\int \sin\theta.\log(\cos\theta).\, d\theta$

= $\int \log(\cos\theta).\sin\theta\, d\theta$

Put cos θ = t

∴ – sin θ dθ = dt

∴ sin θ dθ = – dt

$\therefore$ I = $\int \log t.(-dt)$

= $-\int (\log t).1dt$

= $-\left[(\log t)\int 1dt - \int\left\{\frac{d}{dt}(\log t)\int 1dt\right\}dt\right]$

= $-\left[(\log t)t - \int \frac{1}{t}.tdt\right]$

= $-t\log t + \int 1dt$

= – t log t + t + c

= – cos θ . log (cos θ) + cos θ + c

= – cos θ [log (cos θ) – 1] + c.

Exercise 3.3 | Q 1.17 | Page 137

Evaluate the following : $\int x.\cos^3 x.dx$

SOLUTION

cos 3x = 4 $\cos^3$x – 3cos x

$\therefore$ cos 3x + 3 cos x = 4 $\cos^3$x

$\therefore \int \cos^3 x = \frac{1}{4}\cos 3x + \frac{3}{4}\cos x$

$\therefore \int \cos^3 x.dx = \frac{1}{4}\int \cos 3x.dx + \frac{3}{4}\int \cos x.dx$

= $\frac{1}{4}\left(\frac{\sin 3x}{3}\right) + \frac{3}{4}\sin x$

= $\frac{\sin 3x}{12} + \frac{3\sin x}{4}$ ...(1)

Let I = $\int x\cos^3 x.dx$

Let I = $\int x\cos^3 x.\,dx$

= $x\int \cos^3 x.\,dx - \int\left[\left\{\frac{d}{dx}(x)\int \cos^3 x.\,dx\right\}\right].\,dx$

= $x\left[\frac{\sin 3x}{12}+\frac{3\sin x}{4}\right] - \int 1.\left(\frac{\sin 3x}{12}+\frac{3\sin x}{4}\right).\,dx$ ...[By (1)]

= $\frac{x\sin 3x}{12}+\frac{3x\sin x}{4}-\frac{1}{12}\int \sin 3x.\,dx-\frac{3}{4}\int \sin x.\,dx$

= $\frac{x\sin 3x}{12}+\frac{3x\sin x}{4}-\frac{1}{12}\left(\frac{-\cos 3x}{3}\right)-\frac{3}{4}(-\cos x)+c$

Exercise 3.3 | Q 1.18 | Page 137

Evaluate the following : $\int \frac{\sin(\log x)^2}{x}.\log.x.\,dx$

SOLUTION

Let I = $\int \frac{\sin(\log x)^2}{x}.\log.x.\,dx$

Put $(\log x)^2$ = t

$\therefore 2\log x.\frac{1}{x}.\,dx$ = dt

$\therefore \frac{1}{x}\log x.\,dx = \frac{1}{2}dt$

$\therefore$ I = $\frac{1}{2}\int \sin t.\,dt$

= $-\frac{1}{2}\cos t + c$

= $-\frac{1}{2}\cos\left[(\log x)^2\right]+c.$

Exercise 3.3 | Q 1.19 | Page 137

Evaluate the following : $\int \frac{\log x}{x} . dx$

SOLUTION

Let I = $\int \frac{\log x}{x} . dx$

Put log x = t $\therefore \frac{1}{x} . dx$ = dt

$\therefore$ I = $\int t . dt$

= $\frac{1}{2} t^2 + c$

= $\frac{1}{2} (\log x)^2 + c$

Exercise 3.3 | Q 1.2 | Page 137

Evaluate the following : $\int x . \sin 2x . \cos 5x . dx$

SOLUTION

Let I = $\int x . \sin 2x . \cos 5x . dx$

sin 2x cos 5x = $\frac{1}{2} [2 \sin 2x \cos 5x]$

= $\frac{1}{2} [\sin(2x + 5x) + \sin(2x - 5x)]$

= $\frac{1}{2} [\sin 7x - \sin 3x]$

$\therefore \int \sin 2x \cos s5x . dx = \frac{1}{2} \left[\int \sin 7x . . dx - \int \sin 3x . dx \right]$

= $\frac{1}{2} \left(\frac{-\cos 7x}{7} \right) - \frac{1}{2} \left(\frac{-\cos 3x}{3} \right)$

$= -\frac{1}{14}\cos 7x + \frac{1}{6}\cos 3x$ ...(1)

I = $\int x \sin 2x \cos 5x . dx$

$= x \int \sin 2x \cos 5x . dx - \int \left[\frac{d}{dx}(x) \int \sin 2x \cos 5x . dx\right] . dx$

$= x\left[-\frac{1}{14}\cos 7x + \frac{1}{6}\cos 3x\right] - \int 1 . \left(-\frac{1}{14}\cos 7x + \frac{1}{6}\cos 3x\right) . dx$ ...[By (1)]

$= -\frac{x}{14}\cos 7x + \frac{x}{6}\cos 3x + \frac{1}{14}\int \cos 7x . dx - \frac{1}{6}\int \cos 3x . dx$

$= -\frac{x}{14}\cos 7x + \frac{x}{6}\cos 3x + \frac{1}{14}\left(\frac{\sin 7x}{7}\right) - \frac{1}{6}\left(\frac{\sin 3x}{3}\right) + c$

$= -\frac{x}{14}\cos 7x + \frac{x}{6}\cos 3x + \frac{\sin 7x}{98} - \frac{\sin 3x}{18} + c.$

Exercise 3.3 | Q 1.21 | Page 137

Evaluate the following : $\int \cos\left(\sqrt[3]{x}\right) . dx$

SOLUTION

Let I = $\int \cos\left(\sqrt[3]{x}\right) . dx$

Put $\sqrt[3]{x}$ = t

∴ x = t^3

∴ dx = $3t^2$.dt

∴ I = $\int 3t^2 \cos t . dt$

$= 3t^2 \int \cos t . dt - \int \left[\frac{d}{dt}(3t)^2 \int \cos t . dt\right] . dt$

$= 3t^2 \sin t - \int 6t \sin t . dt$

$$= 3t^2 \sin t - \left[6t \sin t.\, dt - \int \left\{\frac{d}{dt}(6t) \int \sin t.\, dt\right\}.\, dt\right]$$

$$= 3t^2 \sin t - \left[6t(-\cos t) - \int 6(-\cos t).\, dt\right]$$

= $3t^2$ sin t + 6t cos t – 6 sin t + c

= $3(t^2 - 2)$ sin t + 6t cos t + c

$$= 3\left(x^{\frac{2}{3}} - 2\right) \sin\left(\sqrt[3]{x}\right) + 6\sqrt[3]{x} \cos\left(\sqrt[3]{x}\right) + c.$$

Exercise 3.3 | Q 2.01 | Page 138

Integrate the following functions w.r.t. x : $e^{2x}.\sin 3x$

SOLUTION

Let I = $\int e^{2x}.\sin 3x$`

$$= e^{2x}.\sin 3x - \int \left[\frac{d}{dx}(e^{2x}) \int \sin 3x.\, dx\right].\, dx$$

$$= e^{2x}.\frac{\sin 3x}{3} - \int e^{2x} \times 2 \times \frac{\sin 3x}{3}.\, dx$$

$$= \frac{1}{3}e^{2x} \sin 3x - \frac{2}{3} \int e^{2x} \sin 3x.\, dx$$

$$= \frac{1}{3}e^{2x} \sin 3x - \frac{2}{3}\left[e^{2x} \int \sin 3x.\, dx\right]$$

$$= \frac{1}{3}e^{2x} \sin 3x - \frac{2}{3}\left[e^{2x}.\left(\frac{-\cos 3x}{3}\right) - \int e^{2x} \times 2 \times \left(\frac{-\sin 3x}{3}\right).\, dx\right]$$

$$= \frac{1}{3}e^{2x} \sin 3x - \frac{2}{13}e^{2x} \cos 3x - \frac{4}{13} \int e^{2x} \cos 3x.\, dx$$

$$\therefore I = \frac{1}{3}e^{2x} \sin 3x - \frac{2}{13}e^{2x} \cos 3x - \frac{4}{13}I$$

$$\therefore \left(1+\frac{4}{13}\right)\mathrm{I}=\frac{1}{3}e^{2x}\sin 3x-\frac{2}{13}e^{2x}\cos 3x$$

$$\therefore \frac{13}{13}\mathrm{I}=\frac{e^{2x}}{13}(2\sin 3x-3\cos 3x)$$

$\therefore$ I = $\dfrac{e^{2x}}{13}(2\sin 3x-3\cos 3x)+c.$

Exercise 3.3 | Q 2.02 | Page 138

Integrate the following functions w.r.t. x : $e^{-x}\cos 2x$

SOLUTION

Let I = $\displaystyle\int e^{-x}\cos 2x.\,dx$

$= e^{-x}\displaystyle\int \cos 2x.\,dx-\int\left[\frac{d}{dx}(e^{2x})\int \sin 2x.\,dx\right].\,dx$

$= e^{-x}.\,\dfrac{\cos 2x}{3}-\displaystyle\int e^{-x}\times 2\times\frac{\sin 2x}{3}.\,dx$

$= \dfrac{1}{3}e^{-x}\cos 2x-\dfrac{2}{3}\displaystyle\int e^{-x}\sin 2x.\,dx$

$= \dfrac{1}{3}e^{-x}\cos 2x-\dfrac{2}{3}\left[e^{-x}\displaystyle\int \sin 2x.\,dx\right]$

$= \dfrac{1}{3}e^{-x}\cos 2x-\dfrac{2}{3}\left[e^{-x}.\left(\dfrac{-\cos 2x}{3}\right)-\displaystyle\int e^{-x}\times 2\times\left(\frac{-\cos 2x}{3}\right).\,dx\right]$

$= \dfrac{1}{3}e^{-x}\cos 2x+\dfrac{2}{5}e^{-x}\cos 2x-\dfrac{2}{5}\displaystyle\int e^{-x}\sin 2x.\,dx$

$\therefore$ I = $\dfrac{1}{3}e^{-x}\cos 2x+\dfrac{2}{5}e^{-x}\cos 2x-\dfrac{3}{5}\mathrm{I}$

$$\therefore \left(1+\frac{4}{5}\right)\mathrm{I}=\frac{1}{3}e^{-x}\cos 2x+\frac{2}{5}e^{-x}\sin 2x$$

$$\therefore \frac{e^{-x}}{5}\mathrm{I}=\frac{e^{-x}}{5}(2\cos 2x+2\sin 2x)$$

$\therefore$ I = $\dfrac{e^{-x}}{5}(2\cos 2x + 2\sin 2x) + c.$

Exercise 3.3 | Q 2.03 | Page 138

Integrate the following functions w.r.t. x : sin (log x)

SOLUTION

Le I = $\int \sin(\log x)x.\,dx$

Put log x= t

$\therefore$ x = e^t

$\therefore$ dx = e^t.dt

$\therefore$ I = $\int \sin t \times e^4.\,t$

= $\int e^t \sin t.\,dt$

= $e^t \int \sin t.\,dt - \int \left[\dfrac{d}{dt}(e^t) \int \sin t.\,dt\right].\,dt$

= $e^t(-\cos t) - \int e^t(-cot).\,dt$

= $-e^t \cos t + \int e^t \cos t.\,dt$

= $-e^t \cos t + e^t \int \cos.\,dt - \int \left[\dfrac{d}{dt}(e^t) \int \cos.\,dt\right].\,dt$

= $-e^t \cos t + e^t \sin t - \int e^t \sin t.\,dt$

∴ I = – e^t cos t + e^t sin t – I

∴ 2I = e^t (sin t – cos t)

∴ I = $\frac{e^t}{2}(\sin t - \cos t) + c$

= $\frac{x}{2}[\sin(\log x) - \cos(\log x)] + c.$

Exercise 3.3 | Q 2.04 | Page 138

Integrate the following functions w.r.t. x : $\sqrt{5x^2 + 3}$

SOLUTION

Let I = $\int \sqrt{5x^2 + 3}.\,dx$

= $\sqrt{5}\int \sqrt{x^2 + \frac{3}{5}}.\,dx$

= $\sqrt{5}\left[\frac{x}{2}\sqrt{x^2 + \frac{3}{5}} + \frac{\left(\frac{3}{5}\right)}{2}\log\left|x + \sqrt{x^2 + \frac{3}{5}}\right|\right] + c$

= $\frac{\sqrt{5}}{2}\left[x\sqrt{x^2 + \frac{3}{5}} + \frac{3}{5}\log\left|x + \sqrt{x^2 + \frac{3}{5}}\right|\right] + c.$

Exercise 3.3 | Q 2.05 | Page 138

Integrate the following functions w.r.t. x : $x^2.\,\sqrt{a^2 - x^6}$

SOLUTION

Let I = $\int x^2 . \sqrt{a^2 - x^6} . dx$

Put $x^3 = t$

$\therefore 3x^2.dx = dt$

$\therefore x^2 dx = \frac{1}{3} . dt$

$\therefore I = \int \sqrt{a^2 - t^2} . \frac{dt}{3} = \frac{1}{3} \int \sqrt{a^2 - t^2} . dt$

$= \frac{1}{3} \left[\frac{t}{2} \sqrt{a^2 - t^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{t}{a}\right) \right] + c$

$= \frac{1}{6} \left[x^3 \sqrt{a^2 - x^6} + a^2 \sin^{-1}\left(\frac{x^3}{a}\right) \right] + c.$

Exercise 3.3 | Q 2.06 | Page 138

Integrate the following functions w.r.t. x : $\sqrt{(x-3)(7-x)}$

SOLUTION

Let I = $\int \sqrt{(x-3)(7-x)} . dx$

$= \int \sqrt{-x^2 + 10x - 21} . dx$

$= \int \sqrt{-(x^2 - 10x + 21)} . dx$

$= \int \sqrt{4 - (x^2 - 10x + 25)} . dx$

$= \int \sqrt{2^2 - (x-5)^2}$

= $\left(\frac{x-5}{2}\right)\sqrt{2^2-(x-5)^2}+\frac{2^2}{2}\sin^{-1}\left(\frac{x-5}{2}\right)+c$

= $\left(\frac{x-5}{2}\right)\sqrt{(x-3)(7-x)}+2\sin^{-1}\left(\frac{x-5}{2}\right)+c.$

Exercise 3.3 | Q 2.07 | Page 138

Integrate the following functions w.r.t. x : $\sqrt{4^x(4^x+4)}$

SOLUTION

Let I = $\int\sqrt{4^x(4^x+4)}.\,dx$

= $\int 2^x\sqrt{(2^x)^2+2^2}.\,dx$

Put 2^x = t

∴ 2^x log 2 dx = dt

∴ 2^x dx = $\frac{1}{\log 2}.\,dt$

∴ I = $\int\sqrt{t^2+2^2}.\,\frac{dt}{\log 2}$

= $\frac{1}{\log 2}\int\sqrt{t^2+2^2}.\,dt$

= $\frac{1}{\log 2}\left[\frac{t}{2}\sqrt{t^2+2^2}+\frac{2^2}{2}\log\left|t+\sqrt{t^2+2^2}\right|\right]+c$

= $\frac{1}{\log 2}\left[\frac{2^x}{2}\sqrt{4^x+4}+2\log\left|2^x+\sqrt{4^x+4}\right|\right]+c$

Exercise 3.3 | Q 2.08 | Page 138

Integrate the following functions w.r.t. x : $(x+1)\sqrt{2x^2+3}$

SOLUTION

Let I = $\int (x+1)\sqrt{2x^2+3}$

Let x + 1 = $\mathbf{A}\left[\frac{d}{dx}(2x^2+3)\right]+\mathbf{B}$

= A (4x) + B

= 4Ax + B

Comparing the coefficients of and constant on both sides, we get

4A = 1, B = 1

∴ A = $\frac{1}{4}$, B = 1

∴ x + 1 = $\frac{1}{4}(4x)+1$

∴ I = $\int\left[\frac{1}{4}(4x)+1\right]\sqrt{2x^2+3}.\,dx$

= $\frac{1}{4}\int 4x\sqrt{2x^2+3}.\,dx+\int\sqrt{2x^2+3}.\,dx.$

= $I_1 + I^2$

In I_1 = put $2x^2 + 3 = t$

∴ 4x.dx = dt

∴ $I_1 = \frac{1}{4}\int t^{12}.\,dt$

= $\frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+c_1$

= $\frac{1}{6}(2x^2+3)^{\frac{3}{2}}+c_1$

$I_2 = \int\sqrt{2x^2+3}.\,dx$

$I_2 = \int \sqrt{2x^2 + 3}.\,dx$

$= \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}}.\,dx$

$= \sqrt{2}\left[\frac{x}{2}\sqrt{x^2 + \frac{3}{2}} + \frac{\left(\frac{3}{2}\right)}{2}\log\left|x + \sqrt{x^2 + \frac{3}{2}}\right|\right] + c_2$

$= \sqrt{2}\left[\frac{x}{2}\sqrt{x^2 + \frac{3}{2}} + \frac{3}{4}\log\left|x + \sqrt{x^2 + \frac{3}{2}}\right|\right] + c_2$

$\therefore I = \frac{1}{6}\left(2x^2 + 3\right)^{\frac{3}{2}} + \sqrt{2}\left[\frac{x}{2}\sqrt{x^2 + \frac{3}{2}} + \frac{3}{4}\log\left|x + \sqrt{x^2 + \frac{3}{2}}\right|\right] + c$, where $c = c_1 + c_2$.

Exercise 3.3 | Q 2.09 | Page 138

Integrate the following functions w.r.t. x : $x\sqrt{5 - 4x - x^2}$

SOLUTION

Let $I = \int x\sqrt{5 - 4x - x^2}.\,dx$

Let $x = A\left[\frac{d}{dx}\left(5 - 4x - x^2\right)\right] + B$

$= A[-4 - 2x] + B$

$= -2Ax + (B - 4A)$

Comparing the coefficients of x and the constant term on both the sides, we get

$-2A = 1,\ B - 4A = 0$

$\therefore\ A = -\frac{1}{2}, B = 4A = 4\left(-\frac{1}{2}\right) = -2$

$\therefore x = -\frac{1}{2}(-4 - 2x) - 2$

$\therefore$ I = $\int \left[-\frac{1}{2}(-4-2x)-2\right]\sqrt{5-4x-x^2}.\,dx$

= $-\frac{1}{2}\int(-4-2x)\sqrt{5-4x-x^2}.\,dx - 2\int\sqrt{5-4x-x^2}.\,dx$

= I_1 - I_2

In I_1, put 5 - 4x - x2 = t

$\therefore$ (– 4 – 2x).dx = dt

$\therefore$ I_1 = $\frac{1}{2}\int t^{\frac{1}{2}}.\,dt$

= $-\frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)+c_1$

= $-\frac{1}{3}\left(5-4x-x^2\right)^{\frac{3}{2}}+c_1$

I_2 = $2\int\sqrt{5-4x-x^2}.\,dx$

= $2\int\sqrt{5-(x^2+4x)}.\,dx$

= $2\int\sqrt{9-(x^2+4x+4)}.\,dx$

= $2\int\sqrt{3^2-(x+2)^2}.\,dx$

= $2\left[\left(\frac{x+2}{2}\right)\sqrt{3^2-(x+2)^2}+\frac{3^2}{2}\sin^{-1}\left(\frac{x+2}{3}\right)\right]+c_2$

= $(x+2)\sqrt{5-4x-x^2}+9\sin^{-1}\left(\frac{x+2}{3}\right)+c_2$

$\therefore$ I = $-\frac{1}{3}\left(5-4x-x^2\right)^{\frac{3}{2}}-(x+2)\sqrt{5-4x-x^2}-9\sin^{-1}\left(\frac{x+2}{3}\right)+c$, where c = c_1 + c_2 .

Exercise 3.3 | Q 2.1 | Page 138

Integrate the following functions w.r.t. x : $\sec^2 x.\sqrt{\tan^2 x+\tan x-7}$

SOLUTION

Let I = $\int \sec^2 x.\sqrt{\tan^2 x+\tan x-7}$

Put tan x = t

∴ $\sec^2$x.dx = dt

∴ I = $\int \sqrt{t^2+t-7}.dt$

= $\int \sqrt{t^2+t+\frac{1}{4}-\frac{29}{4}}.dt$

= $\int \sqrt{\left(t+\frac{1}{2}\right)^2-\left(\frac{\sqrt{29}}{2}\right)^2}.dt$

= $\left(\frac{t+\frac{1}{2}}{2}\right)\sqrt{\left(t+\frac{1}{2}\right)^2-\frac{29}{4}}-\frac{\left(\frac{29}{4}\right)}{2}\log\left|\left(t+\frac{1}{2}\right)+\sqrt{\left(t+\frac{1}{2}\right)^2-\frac{29}{4}}\right|+c$

= $\frac{(2t+1)}{4}\sqrt{t^2+t-7}-\frac{29}{8}\log\left|\left(t+\frac{1}{2}\right)+\sqrt{t^2+t-7}\right|+c$

= $\left(\frac{2\tan x+1}{4}\right)\sqrt{\tan^2 x+\tan x-7}-\frac{29}{8}\log\left|\left(\tan x+\frac{1}{2}\right)+\sqrt{\tan^2 x+\tan x-7}\right|+c.$

Exercise 3.3 | Q 2.11 | Page 138

Integrate the following functions w.r.t. x : $\sqrt{x^2+2x+5}$

SOLUTION

Let I = $\int \sqrt{x^2+2x+5}.\, dx$

= $\int \sqrt{x^2+2x+1+4} dx$

= $\int \sqrt{(x+1)^2+2^2}.\, dx$

= $\left(\frac{x+1}{2}\right) \int \sqrt{(x+1)^2+2^2} + \frac{2^2}{2} \log \left|(x+1)+\sqrt{(x+1)^2+2^2}\right| + c$

= $\left(\frac{x+1}{2}\right) \sqrt{x^2+2x+5} + 2 \log \left|(x+1)+\sqrt{x^2+2x+5}\right| + c.$

Exercise 3.3 | Q 2.12 | Page 138

Integrate the following functions w.r.t. x : $\sqrt{2x^2+3x+4}$

SOLUTION

Let I = $\int \sqrt{2x^2+3x+4}.\, dx$

= $\sqrt{2} \int \sqrt{x^2+\frac{3}{2}x+2}.\, dx$

= $\sqrt{2} \int \sqrt{\left(x^2+\frac{3}{2}x+\frac{9}{16}\right)-\frac{9}{16}+2}.\, dx$

= $\sqrt{2} \int \sqrt{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{23}}{4}\right)^2}.\, dx$

= $\sqrt{2}\left[\frac{\left(x+\frac{3}{4}\right)}{2}\sqrt{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{23}}{4}\right)^2} + \frac{\left(\frac{23}{16}\right)}{2}\log\left|\left(x+\frac{3}{4}\right)+\sqrt{\left(x+\frac{3}{4}\right)^2+\left(\frac{\sqrt{23}}{4}\right)^2}\right|\right] + c$

= $s\sqrt{2}\left[\left(\frac{4x+3}{8}\right)\sqrt{x^2+\frac{3}{2}x+2} + \frac{23}{32}\log\left|\left(x+\frac{3}{4}\right)+\sqrt{x^2+\frac{3}{2}x+2}\right|\right] + c.$

Exercise 3.3 | Q 3.1 | Page 138

Integrate the following functions w.r.t. x : [2 + cot x – cosec^{2}x]e^x

SOLUTION

Let I = $\int e^x\left[2+\cot x-\operatorname{cosec}^2 x\right].dx$

Put f(x) = 2 + cot x

∴ f'(x) = $\dfrac{d}{dx}(2+\cot x)$

= $\dfrac{d}{dx}(2)+\dfrac{d}{dx}(\cot x)$

= 0 – cosec^{2}x

= – cosec^{2}x

∴ I = $\int e^x[f(x)+f\prime(x)].dx$

= e^x f(x) + c

= e^x (2 + cot x) + c.

Exercise 3.3 | Q 3.2 | Page 138

Integrate the following functions w.r.t. x : $\left(\dfrac{1+\sin x}{1+\cos x}\right).e^x$

SOLUTION

Let I = $\int e^x\left(\dfrac{1+\sin x}{1+\cos x}\right).dx$

= $\int e^x\left[\dfrac{1+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right].dx$

= $\int e^x\left[\dfrac{1}{2\cos^2\frac{x}{2}}+\dfrac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right].dx$

= $\int e^x\left[\frac{1}{2}\sec^2\ \frac{x}{2} + \tan\left(\frac{x}{2}\right)\right].dx$

Put f(x) = $\tan\left(\frac{x}{2}\right)$

∴ f'(x) = $\frac{d}{dx}\left[\tan\ \frac{x}{2}\right]$

= $\sec^2\ \frac{x}{2}.\frac{1}{2}$

= $\frac{1}{2}\sec^2\ \frac{x}{2}$

∴ I = $\int e^x[f(x) + f\prime(x)].dx$

= e^X f(x) + c

= $e^x.\tan\left(\frac{x}{2}\right) + c.$

Exercise 3.3 | Q 3.3 | Page 138

Integrate the following functions w.r.t. x : $e^x.\left(\frac{1}{x} - \frac{1}{x^2}\right)$

SOLUTION

Let I = $\int e^x.\left(\frac{1}{x} - \frac{1}{x^2}\right).dx$

Let f(x) = $\frac{1}{x}$

∴ f'(x) = $-\frac{1}{x^2}$

∴ I = $\int e^x[f(x) + f\prime(x)].dx$

= e^X f(x) + c

= $e^x.\frac{1}{x} + c.$

Exercise 3.3 | Q 3.4 | Page 138

Integrate the following functions w.r.t. x : $\left[\frac{x}{(x+1)^2}\right].e^x$

SOLUTION

Let I = $\int e^x\left[\frac{x}{(x+1)^2}\right].dx$

= $\int e^x\left[\frac{(x+1)-1}{(x+1)^2}\right].dx$

= $\int e^x\left[\frac{1}{x+1}-\frac{1}{(x+1)^2}\right].dx$

Let f(x) = $\frac{1}{x+1}$

= $(x+1)^{-1}$

∴ f'(x) = $\frac{d}{dx}(x+1)^{-1}$

= $-(x+1)^{-2}\frac{d}{dx}(x+1)$

= $\frac{-1}{(x+1)^2}\times 1$

= $\frac{-1}{(x+1)^2}$

∴ I = $\int e^x[f(x)+f\prime(x)].dx$

= e^x.f(x) + c

= $\frac{e^x}{x+1}+c.$

Exercise 3.3 | Q 3.5 | Page 138

Integrate the following functions w.r.t. x : $\frac{e^x}{x}\left[x(\log x)^2 + 2(\log x)\right]$

SOLUTION

Let I = $\int \frac{e^x}{x}\left[x(\log x)^2 + 2\log x\right].dx$

= $\int e^x\left[(\log x)^2 + \frac{2\log x}{x}\right].dx$

Put f(x) = $(\log x)^2$

∴ f'(x) = $\frac{d}{dx}(\log x)^2$

= $2(\log x).\frac{d}{dx}(\log x)$

= $\frac{2\log x}{x}$

∴ I = $\int e^x[f(x) + f\prime(x)].dx$

= e^x . f(x) + c

= e^x . $(\log x)^2$ + c.

Exercise 3.3 | Q 3.6 | Page 138

Integrate the following functions w.r.t. x : $e^{5x}.\left[\frac{5x.\log x + 1}{x}\right]$

SOLUTION

Let I = $\int e^{5x}\left[\frac{5x.\log x + 1}{x}\right].dx$

= $\int e^{5x}\left[5\log x + \frac{1}{x}\right].dx$

Put 5x = t

∴ 5.dx = dt

∴ dx = $\frac{1}{5}.dt$

Also, x = $\frac{t}{5}$

∴ I = $\frac{1}{5}\int e^t\left[5\log\left(\frac{t}{5}\right)+\frac{5}{t}\right].dt$

Let f(t) = $5\log\left(\frac{t}{5}\right)$

= 5 log t – 5 log 5

∴ f'(t) = $\frac{d}{dt}[5\log t-5\log 5]$

= $\frac{5}{t}-0$

= $\frac{5}{t}$

∴ I = $\frac{1}{5}\int e^t[f(t)+f\prime(t)].dt$

= $\frac{1}{5}e^t f(t)+c$

= $\frac{1}{5}e^t.5\log\left(\frac{t}{5}\right)+c$

= e^{5x} log x + c.

Exercise 3.3 | Q 3.7 | Page 138

Integrate the following functions w.r.t. x : $e^{\sin^{-1}x}.\left[\frac{x+\sqrt{1-x^2}}{\sqrt{1-x^2}}\right]$

SOLUTION

Let I = $\int e^{\sin^{-1} x}\left[\frac{x+\sqrt{1-x^2}}{\sqrt{1-x^2}}\right].dx$

= $\int e^{\sin^{-1} x}\left[x+\sqrt{1-x^2}\right].\frac{1}{\sqrt{1-x^2}}.dx$

Put $\sin^{-1} x = t$

$\therefore \frac{1}{\sqrt{1-x^2}}.dx = dt$

and x = sin t

$\therefore$ I = $\int e^t\left[\sin t+\sqrt{1-\sin^2 t}.dt\right.$

= $\int e^t\left[\sin t+\sqrt{\cos^2 t}.dt\right.$

= $\int e^t(\sin t+\cos t).dt$

Let f(t) = sint

$\therefore$ f'(t) = cost

$\therefore$ I = $\int e^t[f(t)+f\prime(t)].dt$

= e^t . f(t) + c

= e^t . sin t + c

= $e^{\sin^- 1_x}.x+c.$

Exercise 3.3 | Q 3.8 | Page 138

Integrate the following functions w.r.t. x : $\log(1+x)^{(1+x)}$

SOLUTION

Let I = $\int \log(1+x)^{(1+x)} . dx$

= $\int (1+x)\log(1+x). dx$

= $\int [\log(1+x)](1+x). dx$

= $\left[\log(1+x)\int (1+x). dx - \int \left[\frac{d}{dt}\{\log(1+x)\}\int (1+x). dx\right]. dx\right.$

= $[\log(1+x)]\left[\frac{(1+x)^2}{2}\right] - \int \frac{1}{x+1} . \frac{(x+1)^2}{2} . dx$

= $\frac{(x+1)^2}{2} . \log(1+x) - \frac{1}{2}\int (x+1). dx$

= $\frac{(x+1)^2}{2} . \log(1+x) - \frac{1}{2} . \frac{(x+1)^2}{2} + c$

= $\frac{(x+1)^2}{2}\left[\log(1+x) - \frac{1}{2}\right] + c.$

Exercise 3.3 | Q 3.9 | Page 138

Integrate the following functions w.r.t. x : cosec (log x)[1 – cot (log x)]

SOLUTION

Let I = $\int \text{cosec}(\log x)[1 - \cot(\log x)]. dx$

Put log x = t

∴ e^t

∴ dx = e^t .dt

$\therefore$ I = $\int \text{cosec}t(1 - \cot t).\, e^t dt$

= $\int e^t[\text{cosec}\, t - \text{cosec}\, t \cot t].\, dt$

= $\int e^t\left[\text{cosec}\, t + \frac{d}{dt}(\text{cosec}\, t)\right].\, dt$

= $e^t\text{cosec}t + c$ ...$\left[\because \int e^t[f(t) + f\prime(t)].\, dt = e^t f(t) + c\right]$

= x . cosec (log x) + c.

EXERCISE 3.4 [PAGES 144 - 145]

Exercise 3.4 | Q 1.01 | Page 144

Integrate the following w.r.t. x : $\frac{x^2 + 2}{(x - 1)(x + 2)(x + 3)}$

SOLUTION

Let I = $\int \frac{x^2 + 2}{(x - 1)(x + 2)(x + 3)}.\, dx$

Let $\frac{x^2 + 2}{(x - 1)(x + 2)(x + 3)}$

= $\frac{A}{x - 1} + \frac{B}{x + 2} + \frac{C}{x + 3}$

$\therefore x^2 + 2 = A(x + 2)(x + 3) + B(x - 1)(x + 3) + C(x - 1)(x + 2)$

Put x – 1 = 0, i.e. x = 1, we get

1 + 2 = A(3)(4) + B(0)(4) + C(0)(3)

$\therefore$ 3 = 12A

$\therefore$ A = $\frac{1}{4}$

Put x + 2 = 0, i.e. x = – 2, we get

4 + 2 = A(0)(1) + B(– 3)(1) + C(– 3)(0)

∴ 6 = – 3B

∴ B = – 2

Put x + 3 = 0, i.e. x = – 3we get

9 + 2 = A(– 1)(0) + B(– 4)(0) + C(– 4)(– 1)

∴ 11 = 4C

∴ C = $\frac{11}{4}$

∴ $\frac{x^2+2}{(x-1)(x+2)(x+3)} = \frac{\left(\frac{1}{4}\right)}{x-1} + \frac{-2}{x+2} + \frac{\left(\frac{11}{4}\right)}{x+3}$

∴ I = $\int\left[\frac{\left(\frac{1}{4}\right)}{x-1} + \frac{-2}{x+2} + \frac{\left(\frac{11}{4}\right)}{x+3}\right].dx$

= $\frac{1}{4}\int\frac{1}{x-1}.dx - 2\int\frac{1}{x+2}.dx + \frac{11}{4}\int\frac{1}{x+3}.dx$

= $\frac{1}{4}\log|x-1| - 2\log|x+2| + \frac{11}{4}\log|x+3| + c.$

Exercise 3.4 | Q 1.02 | Page 144

Integrate the following w.r.t. x : $\frac{x^2}{(x^2+1)(x^2-2)(x^2+3)}$

SOLUTION

Let I = $\int \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} . dx$

Consider, $\frac{x^2}{(x^2+1)(x^2-2)(x^2+3)}$

For finding partial fractions only, put x^2 = t.

$\therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} = \frac{t}{(t-1)(t-2)(t+3)}$

= $\frac{A}{t+1} + \frac{B}{t-2} + \frac{C}{t+3}$ ...(Say)

∴ t = A(t – 2)(t + 3) + B(t + 1)(t + 3) + C(t + 1)(t –2)

Put t + 1 = 0, i.e. t = – 1, we get

–1 = A(– 3)(2) + B(0)(2) + C(0)(– 3)

∴ – 1 = – 6A

∴ A = $\frac{1}{6}$

Put t – 2 = 0, i.e. t = 2, we get

2 = A(0)(5) + B(3)(5) + C(3)(0)

∴ 2 = 15B

∴ B = $\frac{2}{15}$

Put t + 3 = 0, i.e. t = – 3, we get

– 3 = A(– 5)(0) + B(– 2)(0) + C(– 2)(– 5)

–3 = 10C

∴ C = $-\frac{3}{10}$

$$\therefore \frac{t}{(t+1)(t-2)(t+3)} = \frac{\left(\frac{1}{6}\right)}{t+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}$$

$$\therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} = \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}$$

$$\therefore \text{I} = \int\left[\frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}\right].dx$$

$$= \frac{1}{6}\int\frac{1}{1+x^2}.dx + \frac{2}{15}\int\frac{1}{x^2-\left(\sqrt{2}\right)^2}.dx - \frac{3}{10}\int\frac{1}{x^2+\left(\sqrt{3}\right)^2}.dx$$

$$= \frac{1}{6}\tan^{-1}x + \frac{2}{15}\times\frac{1}{2\sqrt{2}}\log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| - \frac{3}{10}\times\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

$$= \frac{1}{6}\tan^{-1}x + \frac{1}{15\sqrt{2}}\log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| - \frac{\sqrt{3}}{10}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c.$$

Exercise 3.4 | Q 1.03 | Page 144

Integrate the following w.r.t. x : $\frac{12x+3}{6x^2+13x-63}$

SOLUTION

Let I = $\int\frac{12x+3}{6x^2+13x-63}.dx$

Let $\frac{12x+3}{6x^2+13x-63}$

$= \frac{12x+3}{(2x+9)(3x-7)}$

$= \frac{A}{2x+9} + \frac{B}{3x-7}$

∴ 12 + 3 = A(3x - 7) + B(2x + 9)

Put 2x + 9 = 0, i.e. x = $\frac{-9}{2}$, we get

$$12\left(\frac{-9}{2}\right) + 3 = \mathrm{A}\left(\frac{-27}{2} - 7\right) + \mathrm{B}(0)$$

∴ − 51 = $\frac{-41}{2}\mathrm{A}$

∴ A = $\frac{102}{41}$

Put 3x − 7 = 0, i.x = $\frac{7}{3}$, we get

$$12\left(\frac{7}{3}\right) + 3 = \mathrm{A}(0) + \mathrm{B}\left(\frac{14}{3} + 9\right)$$

∴ 31 = $\frac{41}{3}\mathrm{B}$

∴ B = $\frac{93}{41}$

∴ $\frac{12x+3}{6x^2+13x-63}\frac{12x+3}{6x^2+13x-63} = \frac{\left(\frac{102}{41}\right)}{2x+9} + \frac{\left(\frac{93}{41}\right)}{3x-7}$

∴ I = $\int\left[\frac{\left(\frac{102}{41}\right)}{2x+9} + \frac{\left(\frac{93}{41}\right)}{3x-7}\right].dx$

= $\frac{102}{41}\int\frac{1}{2x+9}.dx + \frac{93}{41}\int\frac{1}{3x-7}.dx$

= $\frac{102}{41}.\frac{\log|2x+9|}{2} + \frac{93}{41}.\frac{\log|3x-7|}{3} + c$

= $\frac{51}{41}\log|2x+9| + \frac{31}{41}\log|3x-7| + c.$

Exercise 3.4 | Q 1.04 | Page 145

Integrate the following w.r.t. x : $\frac{2x}{4-3x-x^2}$

SOLUTION

Let I = $\displaystyle\int \frac{2x}{4-3x-x^2}.dx$

Let $\displaystyle\frac{2x}{4-3x-x^2}$

= $\displaystyle\frac{2x}{(4+x)(1-x)}$

= $\displaystyle\frac{\text{A}}{4+x}+\frac{\text{B}}{1-x}$

∴ 2x = A(1 – x) + B(4 + x)

Put 4 + x = 0, i.e. x = – 4, we get

– 8 = A(5) + B(0)

∴ A = $-\displaystyle\frac{8}{5}$

Put 1 – x = 0, i.e x =1, we

2 = A(0) + B(5)

∴ B = $\displaystyle\frac{2}{5}$

∴ $\displaystyle\frac{2x}{4-3x-x^2}\ \frac{\left(-\frac{8}{5}\right)}{4+x}+\frac{\left(\frac{2}{5}\right)}{1-x}$

∴ I = $\displaystyle\int\left[\frac{\frac{-8}{5}}{4+x}+\frac{\left(\frac{2}{5}\right)}{1-x}\right].dx$

= $\displaystyle-\frac{8}{5}\ \frac{1}{4+x}.dx+\frac{2}{5}\int\frac{1}{1-x}.dx$

= $\displaystyle-\frac{8}{5}\log|4+x|+\frac{2}{5}.\frac{\log|1-x|}{-1}+c$

= $\displaystyle-\frac{8}{5}\log|4+x|-\frac{2}{5}\log|1-x|+c.$

Exercise 3.4 | Q 1.05 | Page 145

Integrate the following w.r.t. x : $\frac{x^2+x-1}{x^2+x-6}$

SOLUTION

Let I = $\int \frac{x^2+x-1}{x^2+x-6}.dx$

= $\int \frac{(x^2+x-6)+5}{x^2+x-6}.dx$

= $\int \left[1+\frac{5}{x^2+x-6}\right].dx$

= $\int 1dx + 5\int \frac{1}{x^2+x-6}.dx$

Let $\frac{1}{x^2+x-6}$

= $\frac{1}{(x+3)(x-2)}$

= $\frac{A}{x+3}+\frac{B}{x-2}$

∴ 1 = A(x – 2) + B(x + 3)

Put x 3 = 0, i.e. x = –3, we get

1 = A(– 5) + B(0)

∴ A = $\frac{-1}{5}$

Put x – 2 = 0, i.e. x = 2, we get

1 = A(0) + B(5)

∴ B = $\frac{1}{5}$

$$\therefore \frac{1}{x^2+x-6} = \frac{\left(-\frac{1}{5}\right)}{x+3} + \frac{\left(\frac{1}{5}\right)}{x-2}$$

$$\therefore \text{I} = \int 1dx + 5\int\left[\frac{\left(-\frac{1}{5}\right)}{x+3} + \frac{\left(\frac{1}{5}\right)}{x-2}\right].dx$$

$$= \int 1dx - \int \frac{1}{x+3}.dx + \int \frac{1}{x-2}.dx$$

= x – log|x + 3| + log|x – 2| + c

$$= x + \log\left|\frac{x-2}{x+3}\right| + c.$$

Exercise 3.4 | Q 1.06 | Page 145

Integrate the following w.r.t. x : $\frac{6x^3+5x^2-7}{3x^2-2x-1}$

SOLUTION

Let I = $\int \frac{6x^3+5x^2-7}{3x^2-2x-1}.dx$

$$3x^2-2x-1)\overline{6x^3+5x^2-\ \ 7}(2x+3$$

$$6x^3-4x^2-2x$$

$$\underline{-\quad +\quad +}$$

$$9x^2+2x-7$$

$$9x^2-6x-3$$

$$\underline{-\quad +\quad +}$$

8x – 4

$$\therefore \text{I} = \int\left[(2x+3) + \frac{8x-4}{3x^2-2x-1}\right].dx$$

$$= \int 2x+3+\int\frac{8x-4}{(x-1)(3x+1)}.dx$$

Let $\frac{8x-4}{(x-1)(3x+1)}$

= $\frac{\mathrm{A}}{x-1}+\frac{\mathrm{B}}{3x+1}$

∴ 8x – 4 = A(3x + 1) + B(x – 1)

Put x – 1 = 0, i.e. x = 1, we get

8 – 4 = A(4) + B(0)

∴ A = 1

Put 3x + 1 = 0, i.e. x = $-\frac{1}{3}$, we get

$8\left(-\frac{1}{3}\right)-4=\mathrm{A}(0)+\mathrm{B}\left(-\frac{4}{3}\right)$

∴ $\frac{-8-12}{3}=-\frac{4\mathrm{B}}{3}$

∴ B = 5

∴ $\frac{8x-4}{(x-1)(3x+1)}=\frac{1}{x-1}+\frac{5}{3x+1}$

∴ I = $2\int x dx+3\int 1 dx+\int\left[\frac{1}{x-1}+\frac{5}{3x+1}\right].dx$

= $2\left(\frac{x^2}{2}\right)+3x+\int\frac{1}{x-1}dx+5\int\frac{1}{3x+1}.dx$

= $x^2+3x+\log x-1\Big|+\frac{5}{3}\log\Big|3x+1+c.$

Exercise 3.4 | Q 1.07 | Page 145

Integrate the following w.r.t. x : $\frac{12x^2-2x-9}{(4x^2-1)(x+3)}$

SOLUTION

Let I = $\int \frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} . dx$

Let $\frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} = \frac{A}{4x^2 - 1} + \frac{B}{x + 3}$

∴ $12x^2 - 2x - 9 = A(x + 3) + B(4x^2 - 1)$

Put $4x^2 - 1 = 0$, i.e. $x^2 = \frac{1}{4}$, i.e $x = \frac{1}{2}$ we get

$$12 \times \left(\frac{1}{2}\right)^2 - 2 \times \left(\frac{1}{2}\right) - 9 = A\left(\frac{7}{2}\right) + B(0)$$

$$\therefore -7 = \frac{7A}{2}$$

∴ A = -2

Put x + 3 = 0, i.e. x = −3, we get

$$12(-3)^2 - 2(-3) - 9 = A(0) + B\left(4\left(3^2\right) - 1\right)$$

∴ 105 = 35B

∴ B = 3

$$\frac{12x^2 - 2x - 9}{(4x^2 - 1)(x + 3)} = \frac{-2}{4x^2 - 1} + \frac{3}{x + 3}$$

∴ I = $\int \left[\frac{-2}{4x^2 - 1} + \frac{3}{x + 3}\right] . dx$

= $(-2) \int \frac{1}{(2x)^2 - 1} . dx + 3 \int \frac{1}{x + 3} . dx$

= $\frac{1}{2} \log\left|\frac{2x + 1}{2x - 1}\right| + 3 \log|x + 3| + c.$

Exercise 3.4 | Q 1.08 | Page 145

Integrate the following w.r.t. x : $\frac{1}{x(x^5 + 1)}$

SOLUTION

Let I = $\int \frac{1}{x(x^5+1)}.dx$

= $\int \frac{x^4}{x^5(x^5+1)}.dx$

Put x^5 = t.

Then $5x^4$ dx = dt

∴ x^4 dx= $\frac{dt}{5}$

∴ I = $\int \frac{1}{t(t+1)}.\frac{dt}{5}$

= $\frac{1}{5}\int \frac{(t+1)-t}{t(t+1)}.dt$

= $\frac{1}{5}\int\left(\frac{1}{t}-\frac{1}{t+1}\right).dt$

= $\frac{1}{5}\left[\int \frac{1}{t}dt-\int \frac{1}{t+1}dt\right]$

= $\frac{1}{5}[\log|t|-\log|t+1|]+c$

= $\frac{1}{5}\log\left|\frac{t}{t+1}\right|+c$

= $\frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right|+c.$

Exercise 3.4 | Q 1.09 | Page 145

Integrate the following w.r.t. x: $\frac{2x^2-1}{x^4+9x^2+20}$

SOLUTION

Let I = $\int \frac{2x^2 - 1}{x^4 + 9x^2 + 20} . dx$

Consider, $\frac{2x^2 - 1}{x^4 + 9x^2 + 20}$

For finding partial fractions only, put x^2 = t.

$\therefore \frac{2x^2 - 1}{x^4 + 9x^2 + 20} = \frac{t}{(t-1)(t-2)(t+3)}$

$= \frac{A}{t+1} + \frac{B}{t-2} + \frac{C}{t+3}$ ...(Say)

∴ t = A(t – 2)(t + 3) + B(t + 1)(t + 3) + C(t + 1)(t –2)

Put t + 1 = 0, i.e. t = – 1, we get

–1 = A(– 3)(2) + B(0)(2) + C(0)(– 3)

∴ – 1 = – 6A

∴ A = $\frac{1}{6}$

Put t – 2 = 0, i.e. t = 2, we get

2 = A(0)(5) + B(3)(5) + C(3)(0)

∴ 2 = 15B

∴ 2 = 15B

∴ B = $\frac{2}{15}$

Put t + 3 = 0, i.e. t = – 3, we get

– 3 = A(– 5)(0) + B(– 2)(0) + C(– 2)(– 5)

–3 = 10C

∴ C = $-\frac{3}{10}$

$$\therefore \frac{t}{(t+1)(t-2)(t+3)} = \frac{\left(\frac{1}{6}\right)}{t+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}$$

$$\therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} = \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}$$

$$\therefore \text{I} = \int\left[\frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}\right].dx$$

$$= \frac{1}{6}\int\frac{1}{1+x^2}.dx + \frac{2}{15}\int\frac{1}{x^2-\left(\sqrt{2}\right)^2}.dx - \frac{3}{10}\int\frac{1}{x^2+\left(\sqrt{3}\right)^2}.dx$$

$$= \frac{1}{6}\tan^{-1}x + \frac{2}{15}\times\frac{1}{2\sqrt{2}}\log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| - \frac{3}{10}\times\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

$$= \frac{11}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) - \frac{9}{2}\tan^{-1}\left(\frac{x}{2}\right) + c.$$

Exercise 3.4 | Q 1.1 | Page 145

Integrate the following w.r.t. x: $\frac{x^2+3}{(x^2-1)(x^2-2)}$

SOLUTION

Let I = $\int \frac{x^2+3}{(x^2-1)(x^2-2)} . dx$

Consider, $\frac{x^2+3}{(x^2-1)(x^2-2)}$

For finding partial fractions only, put x^2 = t.

$\therefore \frac{x^2+3}{(x^2-1)(x^2-2)} = \frac{t}{(t+1)(t-2)}$

= $\frac{A}{t+1} + \frac{B}{t-2}$ ...(Say)

∴ t = A(t – 2)(t + 3) + B(t + 1)(t + 3) + C(t + 1)(t –2)

Put t + 1 = 0, i.e. t = – 1, we get

–1 = A(– 3)(2) + B(0)(2) + C(0)(– 3)

∴ – 1 = – 6A

∴ A = $\frac{1}{6}$

Put t – 2 = 0, i.e. t = 2, we get

2 = A(0)(5) + B(3)(5) + C(3)(0)

∴ 2 = 15B

∴ B = $\frac{2}{15}$

Put t + 3 = 0, i.e. t = – 3, we get

– 3 = A(– 5)(0) + B(– 2)(0) + C(– 2)(– 5)

–3 = 10C

∴ C = $-\frac{3}{10}$

$$\therefore \frac{t}{(t+1)(t-2)(t+3)} = \frac{\left(\frac{1}{6}\right)}{t+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}$$

$$\therefore \frac{x^2}{(x^2+1)(x^2-2)(x^2+3)} = \frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}$$

$$\therefore \text{I} = \int \left[\frac{\left(\frac{1}{6}\right)}{x^2+1} + \frac{\left(\frac{2}{15}\right)}{x^2-2} + \frac{\left(\frac{-3}{10}\right)}{x^2+3}\right].dx$$

$$= \frac{1}{6}\int \frac{1}{1+x^2}.dx + \frac{2}{15}\int \frac{1}{x^2-\left(\sqrt{2}\right)^2}.dx - \frac{3}{10}\int \frac{1}{x^2+\left(\sqrt{3}\right)^2}.dx$$

$$= \frac{1}{6}\tan^{-1}x + \frac{2}{15} \times \frac{1}{2\sqrt{2}}\log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| - \frac{3}{10} \times \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

$$= 2\log\left|\frac{x+1}{x-1}\right| + \frac{5}{2\sqrt{2}}\log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + c.$$

Exercise 3.4 | Q 1.11 | Page 145

Integrate the following w.r.t. x : $\frac{2x}{(2+x^2)(3+x^2)}$

SOLUTION

Let I = $\int \frac{2x}{(2+x^2)(3+x^2)} \cdot dx$

Put x^2 = t

∴ 2x dx = dt

$$\therefore \text{I} = \int \frac{1}{(2+t)(3+t)} \cdot dt$$

$$= \int \frac{(3+t)-(2+t)}{(2+t)(3+t)} \cdot dt$$

$$= \int \left[\frac{1}{2+t} - \frac{1}{3+t}\right] \cdot dt$$

= log|2 + t| − log|3 + t| + c

$= \log\left|\frac{2+t}{3+t}\right| + c$

$= \log\left|\frac{2+x^2}{3+x^2}\right| + c.$

Exercise 3.4 | Q 1.12 | Page 145

Integrate the following w.r.t. x : $\frac{2^x}{4^x - 3 \cdot 2^x - 4}$

SOLUTION

Let I = $\int \frac{2^x}{4^x - 3 \cdot 2^x - 4} \cdot dx$

$= \int \frac{2^x}{(2^x)^2 - 3 \cdot 2^x - 4}$

Put $2^x = t$

∴ $2^x \log 2\ dx = dt$

∴ $2^x\ dx = \frac{1}{\log 2} \cdot dt$

∴ I = $\frac{1}{\log 2} \int \frac{dt}{t^2 - 3t - 4}$

$= \frac{1}{\log 2} \int \frac{1}{(t+1)(t-4)} \cdot dt$

$= \frac{1}{5\log 2} \int \frac{(t+1)-(t-4)}{(t-4)(t-4)} \cdot dt$ ...[Note this step.]

$= \frac{1}{5\log 2} \int \left[\frac{1}{t-4} - \frac{1}{t+1}\right] \cdot dt$

$= \frac{1}{5\log 2} \left[\int \frac{1}{t-4} \cdot dt - \int \frac{1}{t+1} \cdot dt\right]$

= $\dfrac{1}{5\log 2}[\log|t-4| - \log|t+1|] + c$

= $\dfrac{1}{5\log 2}\log\left|\dfrac{2^x - 4}{2^x + 1}\right| + c.$

Exercise 3.4 | Q 1.13 | Page 145

Integrate the following w.r.t. x : $\dfrac{3x-2}{(x+1)^2(x+3)}$

SOLUTION

Let I = $\displaystyle\int \frac{3x-2}{(x+1)^2(x+3)} \cdot dx$

Let $\dfrac{3x-2}{(x+1)^2(x+3)} = \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2} + \dfrac{C}{x+3}$

∴ $3x - 2 = A(x + 1)(x + 3) + B(x + 3) + C(x + 1)^2$

Put x + 1 = 0, i.e. x = – 1, we get

– 3 – 2 = A(0)(2) + B(2) + C(0)

∴ – 5 = 2B

∴ B = $-\dfrac{5}{2}$

Put x + 3 = 0, i.e. x - – 3, we get

– 9 – 2 = A(– 2)(0) + B(0) + C(– 2)2

∴ – 11 = 4C

∴ C = $-\dfrac{11}{4}$

Put x = 0, we get

– 2 = A(1)(3) + B(3) + C(1)

∴ – 2 = 3A + 3B + C

$\therefore -2 = 3\mathrm{A} - \dfrac{15}{2} - \dfrac{11}{4}$

$\therefore 3\mathrm{A} = -2 + \dfrac{15}{2} + \dfrac{11}{4}$

$= \dfrac{-8+30+11}{4}$

$\therefore \mathrm{A} = \dfrac{11}{4}$

$$\therefore \frac{3x-2}{(x+1)^2(x+3)} = \frac{\left(\frac{11}{4}\right)}{x+1} + \frac{\left(-\frac{5}{4}\right)}{(x+1)^2} + \frac{\left(-\frac{11}{4}\right)}{x+3}$$

$$\therefore \mathrm{I} = \int\left[\frac{\left(\frac{11}{4}\right)}{x+1} + \frac{\left(\frac{-5}{2}\right)}{(x+1)^2} + \frac{\left(\frac{-11}{4}\right)}{x+3}\right]$$

$$= \frac{11}{4}\int\frac{1}{x+1}\cdot dx - \frac{5}{2}\int(x+1)^{-2}\cdot dx - \frac{11}{4}\int\frac{1}{x+3}\cdot dx$$

$$= \frac{11}{4}\log|x+1| - \frac{5}{2}\cdot\frac{(x+1)^{-1}}{-1}\cdot\frac{1}{1} - \frac{11}{4}\log|x+3| + c$$

$$= \frac{11}{4}\log\left|\frac{x+1}{x+3}\right| + \frac{5}{2(x+1)} + c.$$

$$= 2\log\left|\frac{x+1}{x-1}\right| + \frac{5}{2\sqrt{2}}\log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + c.$$

Exercise 3.4 | Q 1.14 | Page 145

Integrate the following w.r.t. x : $\dfrac{5x^2+20x+6}{x^3+2x^2+x}$

SOLUTION

Let I = $\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} \cdot dx$

= $\int \frac{5x^2 + 20x + 6}{x(x^2 + 2x + 1)} \cdot dx$

= $\int \frac{5x^2 + 20x + 6}{x(x+1)^2} \cdot dx$

Let $\frac{5x^2 + 20x + 6}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

∴ $5x^2 + 20x + 6 = A(x + 1)^2 + Bx(x + 1) + Cx$

Put x = 0, we get

0 + 0 + 6 = A(1) + B(0)(1) + C(0)

∴ A = 6

Put x + 1 = 0, i x = – 1, we get

5(1) + 20(– 1) + 6 = A(0) + B(– 1)(0) + C(– 1)

∴ – 9 = – C

∴ C = 9

Put x = 1, we get

5(1) + 20(1) + 6 = A(4) + B(1)(2) + C(1)

But A = 6 and C = 9

∴ 31 = 24 + 2B + 9

∴ B = – 1

∴ $\frac{5x^2 + 20x + 6}{x(x+1)^2} = \frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2}$

∴ I = $\int \left[\frac{6}{x} - \frac{1}{x+1} + \frac{9}{(x+1)^2}\right] \cdot dx$

$$= 6\int \frac{1}{x}\cdot dx - \int \frac{1}{x+1}\cdot dx + 9\int (x+1)^{-2}\cdot dx$$

$$= 6\log|x| - \log|x+1| + 9\cdot\frac{(x+1)^{-1}}{-1} + c$$

$$= \log\left|x^6\right| - \log|x+1| - \frac{9}{(x+1)} + c$$

$$= \log\left|\frac{x^6}{x+1}\right| - \frac{9}{(x+1)} + c.$$

Exercise 3.4 | Q 1.15 | Page 145

Integrate the following w.r.t. x : $\frac{1}{x(1+4x^3+3x^6)}$

SOLUTION

Let I = $\int \frac{1}{x(1+4x^3+3x^6)}\cdot dx$

$$= \int \frac{x^2}{x^3(1+4x^3+3x^6)}\cdot dx$$

Put $x^3 = t$

∴ $3x^2\ dx = dt$

∴ $x^2dx\ (1)(3)\cdot dt$

∴ I = $\frac{1}{3}\int \frac{1}{t(1+4t+3t^2)}\cdot dt$

$$= \frac{1}{3}\int \frac{1}{t(t+1)(3t+1)}\cdot dt$$

Let $\frac{1}{t(t+1)(3t+1)} = \frac{A}{t} + \frac{B}{t+1} + \frac{C}{2t+1}$

∴ 1 = A(t + 1)(3t + 1) + Bt (3t + 1) + Ct (t + 1)

Put t = 0, we get

1 = A(1) + B(0) + C(0)

∴ A = 1

Put t + 1 = 0, i.e. t = – 1 we get

1 = A(0) + B(– 1)(– 2) + C(0)

∴ B = $\frac{1}{2}$

Put 3t + 1 = 0, i.e. t = $-\frac{1}{3}$, we get

1 = $\mathrm{A}(0)+\mathrm{B}(0)+\mathrm{C}\left(-\frac{1}{3}\right)\left(\frac{2}{3}\right)$

∴ C = $-\frac{9}{2}$

∴ $\frac{1}{t(t+1)(3t+1)}=\frac{1}{t}+\frac{\left(\frac{1}{2}\right)}{t+1}+\frac{\left(-\frac{9}{2}\right)}{3t+1}$

∴ I = $\frac{1}{3}\int\left[\frac{1}{t}+\frac{\left(\frac{1}{2}\right)}{t+1}+\frac{\left(-\frac{9}{2}\right)}{3t+1}\right]\cdot dt$

= $\frac{1}{3}\left[\int\frac{1}{t}\cdot dt+\frac{1}{2}\int\frac{1}{t+1}\cdot dt-\frac{9}{2}\int\frac{1}{3t+1}\cdot dt\right]$

= $(1)(3)\left[\log\left|+\frac{1}{2}\log\right|t+1\left|-\frac{9}{2}\cdot\frac{1}{3}\log\right|3t+1\mid\right]+c$

= $\frac{1}{3}\log|x^3|+\frac{1}{2}\log|x^3+1|-\frac{3}{2}\log|3x^3+1|+c$

= $\log|x|+\frac{1}{2}\log|x^3+1|-\frac{3}{2}\log|3x^3+1|+c.$

Exercise 3.4 | Q 1.16 | Page 145

Integrate the following w.r.t. x : $\frac{1}{x^3-1}$

SOLUTION

Let I = $\int \frac{1}{x^3-1} \cdot dx$

= $\int \frac{1}{(x-1)(x^2+x+1)} \cdot dx$

Let $\frac{1}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1}$

∴ 1 = A(x^2 + x + 1) + (Bx + C)(x - 1)

Put x – 1 = 0 i.e x = 1, we get

1 = A(3) + (B + C)(0)

∴ A = $\frac{1}{3}$

Put x = 0, we get

1 = A(1) + C(– 1)

∴ C = A – 1 = $-\frac{2}{3}$

Comparing the coefficients of x^2 on both the sides, we get

0 = A + B

∴ B = – A = $-\frac{1}{3}$

$$\therefore \frac{1}{(x-1)(x^2+x+1)} = \frac{\left(\frac{1}{3}\right)}{x-1} + \frac{\left(-\frac{1}{3}x - \frac{2}{3}\right)}{x^2+x+1}$$

$$= \frac{1}{3}\left[\frac{1}{x-1} - \frac{x+2}{x^2+x+1}\right]$$

Let x + 2 = $p\left[\frac{d}{dx}\left(x^2+x+1\right)\right] + q$

Comapring coefficient of x and the constant term on both the sides, we get

2p = 1 i.e. p = $\frac{1}{2}$ and $p+q$ = 2

$\therefore$ q = 2 – p = $2-\dfrac{1}{2}=\dfrac{3}{2}$

$\therefore$ x + 2 = $\dfrac{1}{2}(2x+1)+\dfrac{3}{2}$

$$\therefore \frac{1}{(x+1)(x^2+x+1)}=\frac{1}{3}\left[\frac{1}{x-1}-\frac{\frac{1}{2}(2x+1)+\frac{3}{2}}{(x^2+x+1)}\right]$$

$$=\frac{1}{3}\left[\frac{1}{x-1}-\frac{1}{2}\left(\frac{2x+1}{x^2+x+1}\right)-\frac{\left(\frac{3}{2}\right)}{x^2+x+1}\right]$$

$$\therefore I=\frac{1}{3}\int\left[\frac{1}{x-1}-\frac{1}{2}\left(\frac{2x+1}{x^2+x+1}\right)-\frac{\left(\frac{3}{2}\right)}{x^2+x+1}\right]\cdot dx$$

$$=\frac{1}{3}\int\frac{1}{x-1}\cdot dx-\frac{1}{6}\int\frac{2x+1}{x^2+x+1}\cdot dx-\frac{1}{2}\int\frac{1}{x^2+x+\frac{1}{4}+\frac{3}{4}}\cdot dx$$

$$=\frac{1}{3}\log|x-1|-\frac{1}{6}\int\frac{\frac{d}{dx}(x^2+x+1)}{x^2+x+1}\cdot dx-\frac{1}{2}\int\frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\cdot dx$$

$$=\frac{1}{3}\log|x-1|-\frac{1}{6}\log\left|x^2+x+1\right|-\frac{1}{2}\frac{1}{\left(\frac{\sqrt{3}}{2}\right)}\tan^{-1}\left[\frac{\left(x+\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)}\right]+c$$

$$=\frac{1}{3}\log|x-1|-\frac{1}{6}\log\left|x^2+x+1\right|-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c.$$

Exercise 3.4 | Q 1.17 | Page 145

Integrate the following w.r.t. x : $\dfrac{(3\sin-2)\cdot\cos x}{5-4\sin x-\cos^2 x}$

SOLUTION

Let I = $\int \frac{(3\sin-2)\cdot\cos x}{5-4\sin x-\cos^2 x}\cdot dx$

= $\int \frac{(3\sin x-2)\cdot\cos x}{5-\left(1-\sin^2 x\right)-4\sin x}\cdot dx$

= $\int \frac{(3\sin x-2)\cdot\cos x}{5-1+\sin^2 x-4\sin x}\cdot dx$

= $\int \frac{(3\sin x-2)\cdot\cos x}{\sin^2 x-4\sin x+4}\cdot dx$

Put sin x = t

∴ cos x dx = dt

∴ I = $\int \frac{3t-2}{t^2-4t+4}\cdot dt$

= $\int \frac{3t-2}{(t-2)^2}\cdot dt$

Let $\frac{3t-2}{(t-2)^2}=\frac{A}{t-2}+\frac{B}{(t-2)^2}$

∴ 3t – 2 = A(t – 2) + B

Put t – 2 = 0, i.e. t = 2, we get

4 = A(0) + B

∴ B = 4

Put t = 0, we get

– 2 = A(– 2) + B

∴ –2 = –2A + 4

∴ 2A = 6

∴ A = 3

∴ $\frac{3t-2}{(t-2)^2}=\frac{3}{t-2}+\frac{4}{(t-2)^2}$

$$\therefore I = \int \left[\frac{3}{t-2} + \frac{4}{(t-2)^2}\right] \cdot dt$$

$$= 3\int \frac{1}{t-2} \cdot dt + 4\int (t-2)^{-2} \cdot dt$$

$$= 3\log|t-2| + 4 \cdot \frac{(t-2)^{-1}}{-1} \cdot \frac{1}{1} + c$$

$$= 3\log|t-2| - \frac{4}{(t-2)} + c$$

$$= 3\log|\sin x - 2| - \frac{4}{(\sin x - 2)} + c.$$

Exercise 3.4 | Q 1.18 | Page 145

Integrate the following w.r.t. x : $\dfrac{1}{\sin x + \sin 2x}$

SOLUTION

Let $I = \displaystyle\int \frac{1}{\sin x + \sin 2x} \cdot dx$

$$= \int \frac{1}{\sin x + 2\sin x \cos x} \cdot dx$$

$$= \int \frac{dx}{\sin x(1 + 2\cos x)}$$

$$= \int \frac{\sin x \cdot dx}{\sin^2 x(1 + 2\cos x)}$$

$$= \int \frac{\sin \cdot dx}{(1 - \cos^2 x)(1 + 2\cos x)}$$

$$= \int \frac{\sin \cdot dx}{(1 - \cos x)(1 + \cos x)(1 + 2\cos x)}$$

Put cos x = t

$\therefore$ − sinx . dx = dt

$\therefore$ sinx .dx = − dt

$\therefore$ I = $\int \frac{-dt}{(1-t)(1+t)(1+2t)}$

= $-\int \frac{dt}{(1-t)(1+t)(1+2t)}$

Let $\frac{1}{(1-t)(1+t)(1+2t)} = \frac{\mathbf{A}}{1-t} + \frac{\mathbf{B}}{1+t} + \frac{\mathbf{C}}{1+2t}$

$\therefore$ 1 = A(1 + t)(1 + 2t) + B(1 − t)(1 + 2t) + C(1 − t)(1 + t)

Putting 1 − t = 0, i.e. t = 1, we get

1 = A(2)(3) + B(0)(3) + C(0)(2)

$\therefore$ A = $\frac{1}{6}$

Putting 1 − t = 0, i.e. t = − 1, we get

1 = A(0)(− 1) + B(2)(− 1) + C(2)(0)

$\therefore$ B = $-\frac{1}{2}$

Putting 1 + 2t = 0, i.e. t = $-\frac{1}{2}$, we get

1 = $\mathbf{A}(0) + \mathbf{B}(0) + \mathbf{C}\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$

$\therefore$ C = $\frac{4}{3}$

$\therefore \frac{1}{(1-t)(1+t)(1+2t)} = \frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t}$

$$\therefore \text{I} = \int \left[\frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t}\right] \cdot dt$$

$$= -\frac{1}{6}\int \frac{1}{1-t} \cdot dt + \frac{1}{2}\int \frac{1}{1+t} \cdot dt - \frac{4}{3}\int \frac{1}{1+2t} \cdot dt$$

$$= -\frac{1}{6} \cdot \frac{\log|1-t|}{-1} + \frac{1}{2}\log|1+t| - \frac{4}{3} \cdot \frac{\log|1+2t|}{2} + c$$

$$= -\frac{1}{6}\log|1-\cos x| + \frac{1}{2}\log|1+\cos x| - \frac{2}{3}\log|1+2\cos x| + c$$

$$= \frac{1}{2}\log|\cos x + 1| + \frac{1}{6}\log|\cos x - 1| - \frac{2}{3}\log|2\cos x + 1| + c.$$

Exercise 3.4 | Q 1.19 | Page 145

Integrate the following w.r.t. x : $\dfrac{1}{2\sin x + \sin 2x}$

SOLUTION

$$\text{Let I} = \int \frac{1}{2\sin x + \sin 2x} \cdot dx$$

$$= \int \frac{1}{2\sin x + 2\sin x\cos x} \cdot dx$$

$$= \int \frac{1}{2\sin x(1+\cos x)} \cdot dx$$

$$= \int \frac{1}{2\sin^2 x(1+\cos x)} \cdot dx$$

$$= \int \frac{\sin x}{2(1-\cos^2 x)(1+\cos x)}$$

$$= \int \frac{\sin \cdot dx}{2(1-\cos x)(1+\cos x)(1+\cos x)}$$

$$= \int \frac{\sin \cdot dx}{2(1-\cos x)(1+\cos x)^2}$$

Put cos x = t

∴ – sinx .dx = dt

∴ sinx .dx = – dt

∴ I = $-\frac{1}{2}\int \frac{1}{(1-t)(1+t)^2} \cdot dt$

= $\frac{1}{2}\int \frac{1}{(t-1)(t+1)^2} \cdot dt$

Let $\frac{1}{(t-1)(t+1)^2} = \frac{A}{t-1} + \frac{B}{t+1} + \frac{C}{(t+1)^2}$

∴ 1 = A(t + 1)^2 + B(t – 1)(t + 1) + C(t – 1)

Put t + 1 = 0, i.e., t = 1, we get

∴ 1 = A(0) + B(0) + C(– 2)

∴ C = $-\frac{1}{2}$

Put t – 1 = 0, i.e., t = 1, we get

∴ 1 = A(4) + B(0) + C(0)

∴ A = $\frac{1}{4}$

Comparing coedfficients of t^2 on both the sides , we get

0 = A + B

∴ B = – A = $-\frac{1}{4}$

∴ $\frac{1}{(t-1)(t+1)^2} = \frac{\left(\frac{1}{4}\right)}{t-1} + \frac{\left(-\frac{1}{4}\right)}{t+1} + \frac{\left(-\frac{1}{2}\right)}{(t+1)^2}$

∴ I = $\frac{1}{2}\int \left[\frac{\left(\frac{1}{4}\right)}{t-1} + \frac{\left(-\frac{1}{4}\right)}{t+1} + \frac{\left(-\frac{1}{2}\right)}{(t+1)^2}\right] \cdot dt$

$= \frac{1}{8}\int \frac{1}{t-1}\cdot dt - \frac{1}{8}\int 91\frac{)}{t+1}\cdot dt - \frac{1}{4}\int \frac{1}{(t-1)^2}\cdot dt$

$= \frac{1}{8}\log|t-1| - \frac{1}{8}\log|t+1| - \frac{1}{4}\frac{(t+1)^{-1}}{(-1)} + c$

$= \frac{1}{8}\log\,|\,\frac{t-1}{t+1} + \frac{1}{4}\cdot\frac{1}{t+1} + c$

$= \frac{1}{8}\log\left|\frac{\cos x - 1}{\cos x + 1}\right| + \frac{1}{4(\cos x + 1)} + c.$

Exercise 3.4 | Q 1.2 | Page 145

Integrate the following w.r.t. x : $\frac{1}{\sin 2x + \cos x}$

SOLUTION

Let I = $\int \frac{1}{\sin 2x + \cos x}\cdot dx$

$= \int \frac{1}{\sin x + \sin 2x\cos x}\cdot dx$

$= \int \frac{dx}{\sin x(1 + 2\cos x)}$

$= \int \frac{\sin x\cdot dx}{\sin^2 x(1 + 2\cos x)}$

$= \int \frac{\sin\cdot dx}{(1 - \cos^2 x)(1 + 2\cos x)}$

$= \int \frac{\sin\cdot dx}{(1 - \cos x)(1 + \sin 2\times)(1 + \cos x)}$

Put cos x = t

∴ – sinx . dx = dt

∴ sinx .dx = − dt

$\therefore$ I = $\int \frac{-dt}{(1-t)(1+t)(1+2t)}$

= $-\int \frac{dt}{(1-t)(1+t)(1+2t)}$

Let $\frac{1}{(1-t)(1+t)(1+2t)} = \frac{A}{1-\ t} + \frac{B}{1+t} + \frac{C}{1+2t}$

∴ 1 = A(1 + t)(1 + 2t) + B(1 − t)(1 + 2t) + C(1 − t)(1 + t)

Putting 1 − t = 0, i.e. t = 1, we get

1 = A(2)(3) + B(0)(3) + C(0)(2)

∴ A = $\frac{1}{6}$

Putting 1 − t = 0, i.e. t = − 1, we get

1 = A(0)(− 1) + B(2)(− 1) + C(2)(0)

∴ B = $-\frac{1}{2}$

Putting 1 + 2t = 0, i.e. t = $-\frac{1}{2}$, we get

1 = $A(0) + B(0) + C\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$

∴ C = $\frac{4}{3}$

$$\therefore \frac{1}{(1-t)(1+t)(1+2t)} = \frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t}$$

$$\therefore \text{I} = \int\left[\frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t}\right]\cdot dt$$

$$= -\frac{1}{6}\int\frac{1}{1-t}\cdot dt + \frac{1}{2}\int\frac{1}{1+t}\cdot dt - \frac{4}{3}\int\frac{1}{1+2t}\cdot dt$$

$$= -\frac{1}{6}\cdot\frac{\log|1-t|}{-1} + \frac{1}{2}\log|1+t| - \frac{4}{3}\cdot\frac{\log|1+2t|}{2} + c$$

$$= -\frac{1}{6}\log|\sin x + 1| + \frac{1}{2}\log|\sin x - 1| - \frac{2}{3}\log|\sin x1 + 2| + c$$

$$= -\frac{1}{6}\log|1 - \sin x| - \frac{1}{2}\log|1 + \sin x| + \frac{2}{3}\log|1 + 2\sin x| + c.$$

Exercise 3.4 | Q 1.21 | Page 145

Integrate the following w.r.t. x : $\frac{1}{\sin x\cdot(3+2\cos x)}$

SOLUTION

Let I = $\frac{1}{\sin x\cdot(3+2\cos x)}\cdot dx$

$$= \int\frac{\sin x}{\sin^2 x\cdot(3+2\cos x)}\cdot dx$$

$$= \int\frac{\sin x}{(1-\cos^2 x)(3+2\cos x)}\cdot dx$$

$$= \int\frac{\sin x}{(1-\cos x)(1+\cos x)(3+2\cos x)}\cdot dx$$

Put cos x = t

∴ – sinx.dx = dt

∴ sinx.dx = – dt

$\therefore$ I = $\int \frac{1}{(1-t)(1+t)(3+2t)} \cdot (-dt)$

= $\int \frac{-1}{(1-t)(1+t)(3+2t)} \cdot dt$

Let $\frac{-1}{(1-t)(1+t)(3+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{3+2t}$

$\therefore -1 = A(1 + t)(3 + 2t) + B(1 - t)(3 + 2t) + C(1 - t)(1 + t)$

Put 1 – t = 0, i.e. t = 1, we get

$-1 = A(2)(5) + B(0)(5) + C(0)(2)$

$\therefore -1 = 10A$

$\therefore A = \frac{-1}{10}$

Put 1 + t = 0, i.e. t = – 1, we get

$-1 = A(0)(1) + B(2)(1) + C(2)(0)$

$\therefore -1 = 2B$

$\therefore B = -\frac{1}{2}$

Put 3 + 2t = 0, i.e. t = $-\frac{3}{2}$, we get

$-1 = A\left(-\frac{1}{2}\right)(0) + B\left(\frac{5}{2}\right)(0) + C\left(\frac{5}{2}\right)\left(-\frac{1}{2}\right)$

$\therefore -1 = -\frac{5}{4}C$

$\therefore C = \frac{4}{5}$

$\therefore \frac{-1}{(1-t)(1+t)(3+2t)} = \frac{\left(\frac{-1}{10}\right)}{1-t} + \frac{\left(-\frac{1}{2}\right)}{1+t} + \frac{\left(\frac{4}{5}\right)}{3+2t}$

$$\therefore \text{I} = \int \left[\frac{\left(\frac{-1}{10}\right)}{1-t} + \frac{\left(-\frac{1}{2}\right)}{1+t} + \frac{\left(\frac{4}{5}\right)}{3+2t} \right] \cdot dt$$

$$= -\frac{1}{10} \int \frac{1}{1-t} \cdot dt - \frac{1}{2} \int \frac{1}{1+t} \cdot dt + \frac{4}{5} \int \frac{1}{3+2t} \cdot dt$$

$$= -\frac{1}{10} \frac{\log|1-t|}{-1} - \frac{1}{2}\log|1+t| + \frac{4}{5} \frac{\log|3+2t|}{2} + c$$

$$= \frac{1}{10}\log|1-\cos x| - \frac{1}{2}\log|1+\cos x| + \frac{2}{5}\log|3+2\cos| + c.$$

Exercise 3.4 | Q 1.22

Integrate the following w.r.t. x : $\frac{5 \cdot e^x}{(e^x+1)(e^{2x}+9)}$

SOLUTION

Let I = $\int \frac{5 \cdot e^x}{(e^x+1)(e^{2x}+9)} \cdot dx$

Put $e^x = t$

$\therefore e^x.dx = dt$

$\therefore \text{I} = 5 \int \frac{1}{(t+1)(t^2+9)} \cdot dt$

Let $\frac{1}{(t+1)(t^2+9)} = \frac{\text{A}}{t+1} + \frac{\text{Bt}+\text{C}}{t^2+9}$

$\therefore 1 = A(t^2 + 9) + (Bt + C)(t + 1)$

Put t + 1 = 0, i.e. t = – 1, we get

1 = A(1 + 9) + C(0)

$\therefore A = \frac{1}{10}$

Put t = 0, we get

1 = A(9) + C(1)

∴ C = 1 − 9A = $1-\frac{9}{10}=\frac{1}{10}$

Comparing coefficients of t2 on both the sides, we get

0 = A + B

∴ B = − A = $-\frac{1}{10}$

$$\therefore \frac{1}{(t+1)(t^2+9)}=\frac{\left(\frac{1}{10}\right)}{t+1}+\frac{\left(-\frac{1}{10}t+\frac{1}{10}\right)}{t^2+9}$$

$$\therefore I = 5\int\left[\frac{\left(\frac{1}{10}\right)}{t+1}+\frac{\left(-\frac{1}{10}t+\frac{1}{10}\right)}{t^2+9}\right]\cdot dt$$

$$= \frac{1}{2}\int\frac{1}{t+1}\cdot dt-\frac{1}{2}\int\frac{t}{t^2+9}\cdot dt+\frac{1}{2}\int\frac{t}{t^2+9}\cdot dt$$

$$= \frac{1}{2}\log|t+1|-\frac{1}{4}\int\frac{2t}{t^2+9}\cdot dt+\frac{1}{2}.(1).(3)\tan^{-1}\left(\frac{t}{3}\right)$$

$$= \frac{1}{2}\log|t+1|-\frac{1}{4}\int\frac{\frac{d}{dt}\left(t^2+9\right)}{t^2+9}\cdot dt+\frac{1}{6}\tan^{-1}\left(\frac{t}{3}\right)$$

$$= \frac{1}{2}\log|t+1|-\frac{1}{4}\log\left|t^2+9\right|+\frac{1}{6}\tan^{-1}\left(\frac{t}{3}\right)+c$$

$$= \frac{1}{2}\log|e^x+1|-\frac{1}{4}\log\left|e^{2x}+9\right|+\frac{1}{6}\tan^{-1}\left(\frac{e^x}{3}\right)+c.$$

Exercise 3.4 | Q 1.23 | Page 145

Integrate the following w.r.t. x : $\frac{2\log x+3}{x(3\log x+2)\left[(\log x)^2+1\right]}$

SOLUTION

Let I = $\int \frac{2\log x + 3}{x(3\log x + 2)\left[(\log x)^2 + 1\right]} \cdot dx$

Put log x = t

∴ $\frac{1}{x} \cdot dx$ = dt

∴ I = $\int \frac{2t + 3}{(3t + 2)(t^2 + 1)} \cdot dt$

Let $\frac{2t + 3}{(3t + 2)(t^2 + 1)} = \frac{A}{3t + 2} + \frac{Bt + C}{t^2 + 1}$

∴ 2t + 3 = A(t^2 + 1) + (Bt + C)(3t + 2)

Put 3t + 2 = 0 i,e, t = $-\frac{2}{3}$, we get

$$2\left(\frac{-2}{3}\right) + 3 = A\left(\frac{4}{9} + 1\right) + \left(\frac{-2}{3}B + C\right)(0)$$

∴ $\frac{5}{3} = A\left(\frac{13}{9}\right)$

∴ A = $\frac{15}{13}$

Put t = 0, we get

3 = A(1) + C(2) = $\frac{15}{13} + 2C$

∴ 2C = $3 - \frac{15}{13} = \frac{24}{13}$

∴ C = $\frac{12}{13}$

Comparing coefficient of t^2 on both the sides, we get

0 = A + 3B

$\therefore$ B = $-\dfrac{A}{3} = -\dfrac{5}{13}$

$$\therefore \frac{2t+3}{(3t+2)(t^2+1)} = \frac{\left(\frac{15}{13}\right)}{3t+2} + \frac{\left(-\frac{5}{13}t + \frac{2}{13}\right)}{t^2+1}$$

$$\therefore I = \int\left[\frac{\left(\frac{15}{13}\right)}{3t+2} + \frac{\left(-\frac{5}{13}t + \frac{12}{3}\right)}{t^2+1}\right]\cdot dt$$

$$= \frac{15}{13}\int\frac{1}{3t+2}\cdot dt - \frac{5}{26}\int\frac{2t}{t\cdot^2+1}\cdot dt + \frac{12}{13}\int\frac{1}{t^2+1}\cdot dt$$

$$= \frac{15}{13}\cdot\frac{1}{3}\log|3t+2| - \frac{5}{26}\log\left|t^2+1\right| + \frac{12}{13}\tan^{-1}(t) + c$$

$$\ldots\left[\because \frac{d}{dt}\left(t^2+1\right) = 2t \text{ and } \int\frac{f\prime(x)}{f(x)}dt = \log|f(t)| + c\right]$$

$$= \frac{5}{13}\log|3\log x + 2| - \frac{5}{26}\log\left|(\log x)^2 + 1\right| + \frac{12}{13}\tan^{-1}(\log x) + c.$$

MISCELLANEOUS EXERCISE 3 [PAGES 148 - 150]

Miscellaneous Exercise 3 | Q 1.01 | Page 148

Choose the correct option from the given alternatives :

$$\int\frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}\cdot dx =$$

$\dfrac{1}{2}\sqrt{x+1}+c$

$\dfrac{2}{3}(x+1)^{\frac{3}{2}}+c$

$\sqrt{x+1}+c$

$2(x-1)^{\frac{3}{2}}+c$

SOLUTION

$$\frac{2}{3}(x+1)^{\frac{3}{2}}+c$$

Miscellaneous Exercise 3 | Q 1.02 | Page 148

Choose the correct options from the given alternatives :

$\int \frac{1}{x+x^5} \cdot dx$ = f(x) + c, then $\int \frac{x^4}{x+x^5} \cdot dx$ =

log x – f(x) + c

f(x) + log x + c

f(x) – log x + c

$\frac{1}{5}x^5 f(x)+c$

SOLUTION

log x – f(x) + c

[Hint: $\int \frac{x^4}{x+x^5} \cdot dx = \int \frac{(x^4+1)-1}{x(x^4+1)} \cdot dx$

= $\int \left(\frac{1}{x} - \frac{1}{x+x^5}\right) \cdot dx$

= log x – f(x) + c].

Miscellaneous Exercise 3 | Q 1.03 | Page 148

Choose the correct options from the given alternatives :

$\int \frac{\log(3x)}{x\log(9x)} \cdot dx$ =

log (3x) – log (9x) + c·

log (x) – (log 3) · log (log 9x) + c

log 9 – (log x) · log (log 3x) + c

log (x) + (log 3) · log (log 9x) + c

SOLUTION

log (x) – (log 3) · log (log 9x) + c

[Hint : $\int \frac{\log 3x}{x \log(x)} \cdot dx = \int \frac{\log\left(\frac{9x}{3}\right)}{x \log(9x)} \cdot dx$

= $\int \frac{\log(9x) - \log 3}{x \log(9x)} \cdot dx$

= $\int \left[\frac{1}{x} - \frac{\log 3}{x \log(9x)}\right] \cdot dx$

= $\int \frac{1}{x} \cdot dx - (\log 3) \int \frac{\left(\frac{1}{x}\right)}{\log(9x)} \cdot dx$

= log (x) – (log 3) · log (log 9x) + c].

Miscellaneous Exercise 3 | Q 1.04 | Page 148

Choose the correct options from the given alternatives :

$\int \frac{\sin^m x}{\cos^{m+2} x} \cdot dx$ =

$\frac{\tan^{m+1} x}{m+1} + c$

(m + 2)tan^{m+1} x + c

$\frac{\tan^m x}{m} + c$

(m + 1)tan^{m+1} x + c

SOLUTION

$\frac{\tan^{m+1} x}{m+1} + c$

Miscellaneous Exercise 3 | Q 1.05 | Page 148

Choose the correct options from the given alternatives :

$$\int \tan\left(\sin^{-1} x\right) \cdot dx =$$

$\left(1-x^2\right)^{-\frac{1}{2}} + c$

$\left(1-x^2\right)^{\frac{1}{2}} + c$

$\dfrac{\tan^m x}{\sqrt{1-x^2}} + c$

$-\sqrt{1-x^2} + c$

SOLUTION

$-\sqrt{1-x^2} + c$

$$\left[\text{Hint} : \sin^{-1} x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right].$$

Miscellaneous Exercise 3 | Q 1.06 | Page 148

Choose the correct options from the given alternatives :

$$\int \frac{x - \sin x}{1 - \cos x} \cdot dx =$$

$x \cot\left(\frac{x}{2}\right) + c$

$-x \cot\left(\frac{x}{2}\right) + c$

$\cot\left(\frac{x}{2}\right) + c$

$x \tan\left(\frac{x}{2}\right) + c$

SOLUTION

$-x\cot\left(\frac{x}{2}\right)+c$

[Hint : $\int\frac{x-\sin x}{1-\cos x}\cdot dx=\int\frac{x-2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{2\sin^2\left(\frac{x}{2}\right)}\cdot dx$

= $\frac{1}{2}\int x\text{cosec}^2\left(\frac{x}{2}\right)\cdot dx-\int\cot\left(\frac{x}{2}\right)\cdot dx$

= $\frac{1}{2}\left[x\int\text{cosec}^2\left(\frac{x}{2}\right)\cdot dx-\int\left[\frac{d}{dx}(x)\int\text{cosec}^2\left(\frac{x}{2}\right)^{dx}\right]\cdot dx-\int\cot\left(\frac{x}{2}\right)\cdot dx\right.$

= $\frac{1}{2}\left[x\left\{\frac{-\cot\left(\frac{x}{2}\right)}{\left(\frac{1}{2}\right)}\right\}-\int 1\cdot\frac{-\cot\left(\frac{x}{2}\right)}{\left(\frac{1}{2}\right)}\cdot dx-\int\cot\left(\frac{x}{2}\right)\cdot dx\right.$

= $x\cot\left(\frac{x}{2}\right)+\int\cot\left(\frac{x}{2}\right)\cdot dx-\int\cot\left(\frac{x}{2}\right)\cdot dx$

= $-x\cot\left(\frac{x}{2}\right)+c$].

Miscellaneous Exercise 3 | Q 1.07 | Page 148

Choose the correct options from the given alternatives :

If f(x) = $\frac{\sin^{-1}x}{\sqrt{1-x^2}}, g(x)=e^{\sin^{-1}x}$, then $\int f(x)\cdot g(x)\cdot dx$ =

- $e^{\sin^{-1}x}\cdot\left(\sin^{-1}x-1\right)+c$
- $e^{\sin^{-1}x}\cdot\left(1-\sin^{-1}x\right)+c$
- $e^{\sin^{-1}x}\cdot\left(\sin^{-1}x+1\right)+c$
- $-e^{\sin^{-1}x}\cdot\left(\sin^{-1}x+1\right)+c$

SOLUTION

$e^{\sin^{-1}x}\cdot\left(\sin^{-1}x-1\right)+c$

Miscellaneous Exercise 3 | Q 1.08 | Page 148

Choose the correct options from the given alternatives :

If $\int \tan^3 x \cdot \sec^3 x \cdot dx = \left(\frac{1}{m}\right) \sec^m x - \left(\frac{1}{n}\right) \sec^n x + c, \text{then} (m, n)$ =

- **(5, 3)**
- (3, 5)
- $\left(\frac{1}{5}, \frac{1}{3}\right)$
- (4, 4)

SOLUTION

(5, 3)

[Hint : $\int \tan^3 x . \sec^3 x \cdot dx$

= $\int \sec^2 x \cdot \tan^2 x \cdot \sec x \tan x \cdot dx$

= $\int \sec^2 x \left(\sec^2 x - 1\right) \sec x \tan x \cdot dx$

Put sec x = t].

Miscellaneous Exercise 3 | Q 1.09 | Page 149

Choose the correct options from the given alternatives :

$\int \frac{1}{\cos x - \cos^2 x} \cdot dx$ =

- $\log(\text{cosec} x - \cot x) + \tan\left(\frac{x}{2}\right) + c$
- sin 2x – cos x + c
- $\log(\sec x + \tan x) - \cot\left(\frac{x}{2}\right) + c$
- cos 2x – sin x + c

SOLUTION

$\log(\sec x + \tan x) - \cot\left(\frac{x}{2}\right) + c$

[Hint : $\int \frac{1}{\cos x - \cos^2 x} \cdot dx$

= $\int \frac{1}{\cos x(1 - \cos x)} \cdot dx$

= $\int \frac{(1 - \cos x) + \cos x}{\cos x(1 - \cos x)} \cdot dx$

= $\int \left(\frac{1}{\cos x} + \frac{1}{1 - \cos x}\right) \cdot dx$

= $\int \left[\sec x + \frac{1}{2}\text{cosec}^2\left(\frac{x}{2}\right)\right] \cdot dx$

= $\log|\sec x + \tan x| \frac{1}{2} \frac{\left(-\frac{\cot x}{2}\right)}{\frac{1}{2}} + c$

= $\log|\sec x + \tan x| - \cot\left(\frac{x}{2}\right) + c$].

Miscellaneous Exercise 3 | Q 1.1 | Page 149

Choose the correct options from the given alternatives :

$\int \frac{\sqrt{\cot x}}{\sin x \cdot \cos x} \cdot dx$ =

$2\sqrt{\cot x} + c$

$-2\sqrt{\cot x} + c$

$\frac{1}{2}\sqrt{\cot x} + c$

$\sqrt{\cot x} + c$

SOLUTION

$-2\sqrt{\cot x} + c$

Miscellaneous Exercise 3 | Q 1.11 | Page 149

Choose the correct options from the given alternatives :

$$\int \frac{e^x(x-1)}{x^2} \cdot dx =$$

- $\frac{e^x}{x} + c$
- $\frac{e^x}{x^2} + c$
- $\left(x - \frac{1}{x}\right)e^x + c$
- $xe^{-x} + c$

SOLUTION

$$\frac{e^x}{x} + c$$

Miscellaneous Exercise 3 | Q 1.12 | Page 149

Choose the correct options from the given alternatives :

$$\int \sin(\log x) \cdot dx =$$

- $\frac{x}{2}[\sin(\log x) - \cos(\log x)] + c$
- $\frac{x}{2}[\sin(\log x) + \cos(\log x)] + c$
- $\frac{x}{2}[\cos(\log x) - \sin(\log x)] + c$
- $\frac{x}{4}[\cos(\log x) - \sin(\log x)] + c$

SOLUTION

$$\frac{x}{2}[\sin(\log x) - \cos(\log x)] + c$$

Miscellaneous Exercise 3 | Q 1.13 | Page 149

Choose the correct options from the given alternatives :

$$\int f x^x (1 + \log x) \cdot dx$$

$\frac{1}{2}(1 + \log x)^2 + c$

$x^{2x} + c$

$x^x \log x + c$

$x^x + c$

SOLUTION

$x^x + c$

[Hint : $\frac{d}{dx}(x^x)$ = x^x (1 + log x)].

Miscellaneous Exercise 3 | Q 1.14 | Page 149

Choose the correct options from the given alternatives :

$$\int \cos -\frac{3}{7} x \cdot \sin -\frac{11}{7} x \cdot dx =$$

$\log\left(\sin^{-\frac{4}{7}} x\right) + c$

$\frac{4}{7} \tan^{\frac{4}{7}} x + c$

$-\frac{7}{4} \tan^{-\frac{4}{7}} x + c$

$\log\left(\cos^{\frac{3}{7}} x\right) + c$

SOLUTION

$-\frac{7}{4} \tan^{-\frac{4}{7}} x + c$

[Hint : $\int \cos^{-\frac{3}{7}} x \sin^{-\frac{11}{7}} x \cdot dx$

$= \int \frac{\sin^{-\frac{11}{7}} x}{\cos^{-\frac{11}{7}} x \cdot \cos^2 x} \cdot dx$

$= \int \tan^{-\frac{11}{7}} x \sec^2 x \cdot dx$

Put tan x = t].

Miscellaneous Exercise 3 | Q 1.15 | Page 149

Choose the correct options from the given alternatives :

$2 \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} \cdot dx =$

- **sin 2x + c**
- cos 2x + c
- tan 2x + c
- 2 sin 2x + c

SOLUTION

sin 2x + c

Miscellaneous Exercise 3 | Q 1.16 | Page 149

Choose the correct options from the given alternatives :

$\int \frac{dx}{\cos x \sqrt{\sin^2 x - \cos^2 x}} \cdot dx =$

- $\log\left(\tan x - \sqrt{\tan^2 x - 1}\right) + c$
- $\sin^{-1}$ (tan x) + c
- 1 + $\sin^{-1}$ (cot x) + c
- $\log\left(\tan x + \sqrt{\tan^2 x - 1}\right) + c$

SOLUTION

$$\log\left(\tan x+\sqrt{\tan^2 x-1}\right)+c$$

[Hint : $\int \dfrac{dx}{\cos x\sqrt{\sin^2 x-\cos^2 x}}$

= $\int \dfrac{\sec 2x \cdot dx}{\sqrt{\tan 2x - 1}}$ ...[Dividing by $\cos^2 x$]

Put tan x = t].

Miscellaneous Exercise 3 | Q 1.17 | Page 150

Choose the correct options from the given alternatives :

$\int \dfrac{\log x}{(\log ex)^2} \cdot dx$ =

- $\dfrac{x}{1+\log x}+c$
- x(1 + log x) + c
- $\dfrac{x}{1+\log x}+c$
- $\dfrac{x}{1-\log x}+c$

SOLUTION

$$\frac{x}{1+\log x}+c$$

Miscellaneous Exercise 3 | Q 1.18 | Page 150

Choose the correct options from the given alternatives :

$\int [\sin(\log x)+\cos(\log x)] \cdot dx$ =

x cos (log x) + c

sin (log x) + c

cos (log x) + c

x sin (log x) + c

SOLUTION

x sin (log x) + c

Miscellaneous Exercise 3 | Q 1.19 | Page 150

Choose the correct options from the given alternatives :

$\int \frac{\cos 2x - 1}{\cos 2x + 1} \cdot dx$ =

tan x – x + c

x + tan x + c

x – tan x + c

– x – cot x + c

SOLUTION

x – tan x + c

[Hint : $\int \frac{\cos 2x - 1}{\cos 2x + 1} \cdot dx$

= $\int \frac{-(1 - \cos 2x)}{1 + \cos^2 x} \cdot dx$

= $\int \frac{-2\sin^2 x}{2\cos^2 x} \cdot dx$

= $\int (\sec^2 x - 1) \cdot dx$

= – tan x + x + c.

Miscellaneous Exercise 3 | Q 1.2 | Page 150

Choose the correct options from the given alternatives :

$\int \frac{e^{2x} + e^{-2}x}{e^x} \cdot dx$ =

$e^x - \frac{1}{3e^{3x}} + c$

$e^x + \frac{1}{3e^{3x}} + c$

$e^{-x} + \frac{1}{3e^{3x}} + c$

$e^{-x} - \frac{1}{3e^{3x}} + c$

SOLUTION

$e^x - \frac{1}{3e^{3x}} + c$

[Hint : $\int \frac{e^{2x} + e^{-2}x}{e^x} \cdot dx$

= $\int e^x \cdot dx + \int e^{-3x} \cdot dx$

= $e^x + \frac{e^{-3x}}{(-3)} + c$

= $e^x - \frac{1}{3e^{3x}} + c$].

Miscellaneous Exercise 3 | Q 2.1 | Page 150

Integrate the following with respect to the respective variable : $(x-2)^2\sqrt{x}$

SOLUTION

Let I = $\int (x-2)^2 \sqrt{x} \cdot dx$

= $\int \left(x^2 - 4x + 4\right) \sqrt{x} \cdot dx$

= $\int \left(x^{\frac{5}{2}} - 4x^{\frac{3}{2}} + 4x^{\frac{1}{2}}\right) \cdot dx$

= $\int x^{\frac{5}{2}} \cdot dx - 4 \int x^{\frac{3}{2}} \cdot dx + 4 \int x^{\frac{1}{2}} \cdot dx$

= $\frac{x^{\frac{7}{2}}}{\left(\frac{7}{2}\right)} - 4 \frac{x^{\frac{5}{2}}}{\left(\frac{5}{2}\right)} + 4 \frac{x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}$

= $\frac{2}{7} x^{\frac{7}{2}} - 8x^{\frac{5}{2}} + \frac{8}{3} x^{\frac{3}{2}} + c.$

Miscellaneous Exercise 3 | Q 2.2 | Page 150

integrate the following with respect to the respective variable : $\frac{x^2}{x+1}$

SOLUTION

Let I = $\int \frac{x^7}{x+1} \cdot dx$

= $\int \frac{\left(x^7+1\right)-1}{x+1} \cdot dx$

= $\int \frac{(x+1)\left(x^6 - x^5 + x^4 - x^3 + x^2 - x + 1\right) - 1}{x+1} \cdot dx$

= $\int \left[x^6 - x^5 + x^4 - x^3 + x^2 - x + 1 - \frac{1}{x+1}\right] \cdot dx$

= $\int x^6 \cdot dx - \int x^5 \cdot dx + \int x^4 \cdot dx - \int x^3 \cdot dx + \int x^2 \cdot dx - \int x \cdot dx + \int 1 dx - \int \frac{1}{x+1} \cdot dx$

= $\frac{x^7}{7} \frac{x^6}{6} + \frac{x^5}{5} - \frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x - \log|x+1| + c.$

Miscellaneous Exercise 3 | Q 2.3 | Page 150

Integrate the following with respect to the respective variable : $(6x+5)^{\frac{3}{2}}$

SOLUTION

$$\int (6x+5)^{\frac{3}{2}\cdot dx}$$

$$= \frac{(6x+5)^{\frac{3}{2}}}{6 \times \frac{5}{2}} + c$$

$$= \frac{1}{15}(6x+5)^{\frac{5}{2}} + c.$$

Miscellaneous Exercise 3 | Q 2.4 | Page 150

Integrate the following with respect to the respective variable : $\frac{t^3}{(t+1)^2}$

SOLUTION

Let I = $\int \frac{t^2}{(t+1)^2} \cdot dt$

$$= \int \frac{(t^3+1)-1}{(t+1)^2} \cdot dt$$

$$= \int \frac{(t+1)(t^2-t+1)-1}{(t+1)^2} \cdot dt$$

$$= \int \left[\frac{t^2-t+1}{t+1} - \frac{1}{(t+1^2)}\right] \cdot dt$$

$$= \int \left[\frac{(t+1)(t-2)+3}{t+1} - \frac{1}{(t+1)^2}\right] \cdot dt$$

$$= \int \left[t - 2 + \frac{3}{t+1} - \frac{1}{(t+1)^2}\right] \cdot dt$$

$$= \int t \cdot dt - 2\int 1 \cdot dt + 3\int \frac{1}{t+1} \cdot dt - \int \frac{1}{(t+1)^2} \cdot dt$$

$$= \frac{t^2}{2} - 2t + 3|\log|t+1\,| - \frac{(t+1)-1}{(-1)} + c$$

$$= \frac{t^2}{2} - 2t + 3\log|t+1| + \frac{1}{t+1} + c.$$

Miscellaneous Exercise 3 | Q 2.5 | Page 150

Integrate the following with respect to the respective variable : $\dfrac{3 - 2\sin x}{\cos^2 x}$

SOLUTION

Let I = $\displaystyle\int \frac{3 - 2\sin x}{\cos^2 x} \cdot dx$

$$= \int \left(\frac{3}{\cos^2 x} - \frac{2\sin x}{\cos^2 x}\right) \cdot dx$$

$$= 3\int \sec^2 x \cdot dx - 2\int \sec x \tan x \cdot dx$$

= 3 tan x – 2 sec x + c.

Miscellaneous Exercise 3 | Q 2.6 | Page 150

Integrate the following with respect to the respective variable : $\dfrac{\sin^6\theta + \cos^6\theta}{\sin^2\theta \cdot \cos^2\theta}$

SOLUTION

$$\int \frac{\sin^6\theta + \cos^6\theta}{\sin^2\theta\cdot\cos^2\theta}$$

$$= \int\left[\frac{\left(\sin^2\theta+\cos^2\theta\right)^3 - 3\sin^2\theta\cdot\cos^2\theta\left(\sin^2\theta+\cos^2\theta\right)}{\sin^2\theta\cdot\cos^2\theta}\right]\cdot d\theta$$ ...[$\because a^3 + b^3 = (a + b)^3 - 3ab(a + b)$]

$$= \int\left[\frac{(1)^3 - 3\sin^2\theta\cdot\cos^2\theta(1)}{\sin^2\theta\cdot\cos^2\theta}\right]\cdot d\theta$$

$$= \int\left[\frac{1}{\sin^2\theta\cdot\cos^2\theta} - 3\right]\cdot d\theta$$

$$= \int\left[\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cdot\cos^2\theta} - 3\right]\cdot d\theta$$

$$= \int\left(\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} - 3\right)\cdot d\theta$$

$$= \int\left(\sec^2\theta + \operatorname{cosec}^2\theta - 3\right)\cdot d\theta$$

$$= \int \sec^2\theta\cdot d\theta + \int \operatorname{cosec}^2\theta\cdot d\theta - 3\int 1\cdot d\theta$$

= $\tan\theta - \cot\theta - 3\theta + c$.

Miscellaneous Exercise 3 | Q 2.7 | Page 150

Integrate the following with respect to the respective variable : cos 3x cos 2x cos x

SOLUTION

Let I = $\int \cos 3x \cos 2x \cos x \cdot dx$

Consider cos 3x cos 2x cos x = $\frac{1}{2}\cos 3x[2\cos 2x\cos x]$

$$= \frac{1}{2}\cos 3x[\cos(2x + x) + \cos(2x - x)]$$

$$= \frac{1}{2}\left[\cos^2 3x + \cos 3x\cos x\right]$$

$$= \frac{1}{4}\left[2\cos^2 3x + 2\cos 3x\cos x\right]$$

$= \frac{1}{4}[1 + \cos 6x + \cos(3x + x) + \cos(3x - x)]$

$= \frac{1}{4}[1 + \cos 6x + \cos 4x + \cos 2x]$

$\therefore$ I = $\frac{1}{4} \int [1 + \cos 6x + \cos 4x + \cos 2x] \cdot dx$

$= \frac{1}{4} \int 1 \cdot dx + \frac{1}{4} \int \cos 6x \cdot dx + \frac{1}{4} \int \cos 4x \cdot dx + \frac{1}{4} \int \cos 2x \cdot dx$

$= \frac{x}{4} + \frac{1}{4}\left(\frac{\sin 6x}{6}\right) + \frac{1}{4}\left(\frac{\sin 4x}{4}\right) + \frac{1}{4}\left(\frac{\sin 2x}{2}\right) + c$

$= \frac{1}{48}[12x + 2\sin 6x + 3\sin 4x + 6\sin 2x] + c.$

Miscellaneous Exercise 3 | Q 2.8 | Page 150

Integrate the following with respect to the respective variable : $\frac{\cos 7x - \cos 8x}{1 + 2\cos 5x}$

SOLUTION

$\int \frac{\cos 7x - \cos 8x}{1 + 2\cos 5x} \cdot dx$

$= \int \frac{\sin 5x(\cos 7x - \cos 8x)}{\sin 5x(1 + 2\cos 5x)} \cdot dx$

$= \int \frac{\sin 5x(\cos 7x - \cos 8x)}{\sin 5x + 2\sin 5x \cos 5x} \cdot dx$

$= \int \frac{\sin 5x(\cos 7x - \cos 8x)}{\sin 5x + \sin 10x} \cdot dx$

$= \int \frac{2\sin\left(5\frac{x}{2}\right) \cdot \cos\left(\frac{5x}{2}\right) \times 2\sin\left(\frac{7x+8x}{2}\right) \cdot \sin\left(\frac{8x-7x}{2}\right)}{2\sin\left(\frac{10x+5x}{2}\right) \cdot \cos\left(\frac{10x-5x}{2}\right)} \cdot dx$

$= \int \frac{2\sin\left(\frac{5x}{2}\right) \cdot \cos\left(\frac{5x}{2}\right) \times 2\sin\left(\frac{15x}{2}\right) \cdot \sin\left(\frac{x}{2}\right)}{2\sin\left(\frac{15x}{2}\right) \cdot \cos\left(\frac{5x}{2}\right)} \cdot dx$

$= \int 2\sin\left(\frac{5x}{2}\right)\cdot\sin\left(\frac{x}{2}\right)\cdot dx$

$= \int\left[\cos\left(\frac{5x}{2}-\frac{x}{2}\right)-\cos\left(\frac{5x}{2}+\frac{x}{2}\right)\right]\cdot dx$

$= \int(\cos 2x - \cos 3x)\cdot dx$

$= \int \cos 2x\cdot dx - \int \cos 3\cdot dx$

$= \frac{\sin 2x}{2} - \frac{\sin 3x}{3} + c.$

Miscellaneous Exercise 3 | Q 2.9 | Page 150

Integrate the following with respect to the respective variable : $\cot^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

SOLUTION

Let I = $\int \cot^{-1}\left(\frac{1+\sin x}{\cos x}\right)\cdot dx$

$\frac{1+\sin x}{\cos x} = \frac{1+\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}$

$= \frac{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cdot\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}$

$= \cot\left(\frac{\pi}{6}-\frac{x}{2}\right)$

$\therefore$ I = $\int \cot^{-1}\left[\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]\cdot dx$

$= \int\left(\frac{\pi}{4}-\frac{x}{2}\right)\cdot dx$

$= \frac{\pi}{4}\int 1\cdot dx - \frac{1}{2}\int x\cdot dx$

$= \frac{\pi}{4} \cdot x - \frac{1}{2} \cdot \frac{x^2}{2} + c$

$= \frac{\pi}{4} x - \frac{1}{4} x^2 + c.$

Miscellaneous Exercise 3 | Q 3.01 | Page 150

Integrate the following w.r.t. x: $\frac{(1 + \log x)^2}{x}$

SOLUTION

Let I = $\int \frac{(1 + \log x)^2}{x} \cdot dx$

Put 1 + log x = t

$\therefore \frac{1}{x} \cdot dx$ = dt

$\therefore$ I = $\int t^3 \cdot dt = \frac{1}{4} t^4 + c$

$= \frac{1}{4} (1 + \log x)^4 + c.$

Miscellaneous Exercise 3 | Q 3.02 | Page 150

Integrate the following w.r.t.x : $\cot^{-1} (1 - x + x^2)$

SOLUTION

Let I = $\int \cot^{-1}(1-x+x^2)\cdot dx$

= $\int \tan^{-1}\left(\frac{1}{1-x+x^2}\right)\cdot dx$

= $\int \tan^{-1}\left[\frac{x+(1-x)}{1-x(1-x)}\right]$

= $\int \left[\tan^{-1}x+\tan^{-1}(1-x)\right]\cdot dx$

= $\int \tan^{-1}x\cdot dx+\int \tan^{-1}(1-x)\cdot dx$

∴ I = $I_1 + I_2$ ...(1)

I_1 = $\int \tan^{-1}x\cdot dx=\int (\tan^{-1}x)1\cdot dx$

= $(\tan^{-1}x)\cdot\int 1dx-\left[\frac{d}{dx}(\tan^{-1}x)\cdot\int 1dx\right]\cdot dx$

= $(\tan^{-1}x)x-\int \frac{1}{1+x^2}\cdot x\cdot dx$

= $x\tan^{-1}x-\frac{1}{2}\int \frac{2x}{1+x^2}\cdot dx$

∴ I_1 = $x\tan^{-1}x-\frac{1}{2}\log|1+x^2|+c_1$

...$\left[\because \frac{d}{dx}(1+x^2)=2x \text{ and } \int \frac{f\prime(x)}{f(x)}dx=\log|f(x)|+c\right]$

I_2 = $\int \tan^{-1}(1-x)\cdot dx$

= $\int \tan^{-1}(1-x)\Big]\cdot 1dx$

= $\left[\tan^{-1}(1-x)\right]\cdot\int 1dx-\int\left\{\frac{d}{dx}\left[\tan^{-1}(1-x)\right]\cdot\int 1dx\right\}\cdot dx$

= $\left[\tan^{-1}(1-x)\right]\cdot x-\int\frac{1}{1+(1-x)^2}\cdot(-1)\cdot xdx$

= $x\tan^{-1}(1-x)+\int\frac{x}{1+1-2x+x^2}\cdot dx$

= $x\tan^{-1}(1-x)+\int\frac{x}{2-2x+x^2}\cdot dx$

Let x = $\mathbf{A}\left[\frac{d}{dx}(2-2x+x^2)\right]+\mathbf{B}$

∴ x = A(– 2 + 2x) + B = 2Ax + (–2A + B)

Comparing the coefficient of x and constant on both the sides, we get

1 = 2A and 0 = – 2A + B

∴ A = $\frac{1}{2}$ and $0=-2\left(\frac{1}{2}\right)+\mathbf{B}$

∴ B = 1

∴ x = $\frac{1}{2}(-2+2x)+1$

∴ I_2= $x\tan^{-1}(1-x)+\int\frac{\frac{1}{2}(-2+2x)+1}{2-2x+x^2}\cdot dx$

= $x\tan^{-1}(1-x)+\frac{1}{2}\frac{-2+2x}{2-2x+x^2}\cdot dx+\int\frac{1}{2-2x+x^2}\cdot dx$

= $x\tan^{-1}(1-x)+\frac{1}{2}\log\left|2-2x+x^2\right|+\int\frac{1}{1+(1-2x+x^2)}\cdot dx$

= $x\tan^{-1}(1-x)+\frac{1}{2}\log\left|x^2-2x+2\right|+\int\frac{1}{1+(1-\ x^2)}\cdot dx$

= $x\tan^{-1}(1-x)+\frac{1}{2}\log\left|x^2-2x+2\right|+\frac{1}{1}\frac{\tan-1(1-x)}{-1}+c_2$

= $x\tan^{-1}(1-x)+\frac{1}{2}\log\left|x^2-2x+2\right|-\tan^{-1}(1-x)+c_2$

= $(x-1)\tan^{-1}(1-x)+\frac{1}{2}\log\left|x^2-2x+2\right|+c_2$

∴ I2 = $-(1-x)\tan^{-1}(1-x)+\frac{1}{2}\log\left|x^2-2x+2\right|+c_2$ ...(3)

From (1),(2) and (3), we get

I = $x\tan^{-1}x-\frac{1}{2}\log\left|1+x^2\right|+c_1-(1-x)\tan^{-1}(1-x)+\frac{1}{2}\log\left|x^2-2x+2\right|+c_2$

= $x\tan^{-1}x-\frac{1}{2}\log\left|1+x^2\right|-(1-x)\tan^{-1}(1-x)+\frac{1}{2}\left|x^2-2x+2\right|+c$, where c = c_1 + c_2.

Miscellaneous Exercise 3 | Q 3.03 | Page 150

Integrate the following w.r.t.x : $\frac{1}{x\sin^2(\log x)}$

SOLUTION

Let I = $\int\frac{1}{x\sin^2(\log x)}\cdot dx$

Put log x = t

∴ $\frac{1}{x}\cdot dx$ = dt

∴ I = $\int\frac{1}{\sin^2 t}\cdot dt$

= $\int \text{cosec}^2 dt$

= – cot t + c

= cot (log x) + c.

Miscellaneous Exercise 3 | Q 3.04 | Page 150

Integrate the following w.r.t.x : $\sqrt{x}\sec\left(x^{\frac{3}{2}}\right)\cdot\tan\left(x^{\frac{3}{2}}\right)$

SOLUTION

Let I = $\int\sqrt{x}\sec\left(x^{\frac{3}{2}}\right)\cdot\tan\left(x^{\frac{3}{2}}\right)$

Put $x^{\frac{3}{2}}$ = t

$\therefore\ \frac{3}{2}\sqrt{x}\cdot dx$ = dt

$\therefore\ \sqrt{x}\cdot dx = \frac{2}{3}\cdot dt$

$\therefore$ I = $\frac{2}{3}\int\sec t\tan t\cdot dt$

= $\frac{2}{3}\sec t + c$

= $\frac{2}{3}\sec\left(x^{\frac{3}{2}}\right) + c.$

Miscellaneous Exercise 3 | Q 3.05 | Page 150

Integrate the following w.r.t.x : $\log(1+\cos x) - x\tan\left(\frac{x}{2}\right)$

SOLUTION

Let I = $\int\left[\log(1+\cos x)-x\tan\left(\frac{x}{2}\right)\right]\cdot dx$

= $\int\Big[\log(1+\cos. x)\cdot 1dx-\int x\tan\left(\frac{x}{2}\right)\cdot dx$

= $[\log(1+\cos x)]\cdot\int 1dx-\int\left\{\frac{d}{dx}[\log(1+\cos x)]\cdot\int 1dx\right\}\cdot dx-x\tan\left(\frac{x}{2}\right)\cdot dx$

= $[\log(1+\cos x)]\cdot(x)-\int\frac{1}{1+\cos x}\cdot(0-\sin x)\cdot xdx-\int x\tan\left(\frac{x}{2}\right)\cdot dx$

= $x\cdot\log(1+\cos x)+\int x\cdot\frac{\sin x}{1+\cos x}\cdot dx-\int x\tan\left(\frac{x}{2}\right)\cdot dx+c$

= $x\cdot\log(1+\cos x)+\int x\cdot\frac{2\sin\left(\frac{x}{2}\right)\cdot\cos\left(\frac{x}{2}\right)}{2\cos^2\left(\frac{x}{2}\right)\cdot dx-\int x\tan\left(\frac{x}{2}\right)\cdot dx+c}$

= $x\log(1+\cos x)+\int x\cdot\tan\left(\frac{x}{2}\right)\cdot dx-\int x\tan\left(\frac{x}{2}\right)\cdot dx+c$

= x·log(1 + cosx) + c.

Miscellaneous Exercise 3 | Q 3.06 | Page 150

Integrate the following w.r.t.x : $\frac{x^2}{\sqrt{1-x^6}}$

SOLUTION

Let I = $\int\frac{x^2}{\sqrt{1-x^6}}\cdot dx$

Put $x^3 = t$

∴ $3x^2dx = dt$

∴ $x^2dx = \frac{1}{3}\cdot dt$

$\therefore$ I = $\frac{1}{3}\int \frac{1}{\sqrt{1-t^2}} \cdot dt$

= $\frac{1}{3}\sin^{-1}(t) + c$

= $\frac{1}{3}\sin^{-1}(x^3) + c.$

Miscellaneous Exercise 3 | Q 3.07 | Page 150

Integrate the following w.r.t.x : $\frac{1}{(1-\cos 4x)(3-\cot 2x)}$

SOLUTION

Let I = $\int \frac{1}{(1-\cos 4x)(3-\cot 2x)} \cdot dx$

= $\int \frac{1}{2\sin^2 2x(3-\cot 2x)} \cdot dx$

= $\frac{1}{2}\int \frac{\text{cosec}^2 x}{3-\cot 2x} \cdot dx$

Put 3 – cot 2x = t

$\therefore$ 2 $\text{cosec}^2 2x \cdot dx$ = dt

$\therefore$ $\text{cosec}^2 2x \cdot dx$ = $\frac{1}{2} \cdot dt$

$\therefore$ I = $\frac{1}{4}\int \frac{1}{t} \cdot dt$

= $\frac{1}{4}\log|t| + c$

= $\frac{1}{4}\log|3-\cot 2x| + c.$

Miscellaneous Exercise 3 | Q 3.08 | Page 150

Integrate the following w.r.t.x : log (log x)+(log x)$^{-2}$

SOLUTION

Let I = $\int \left[\log(\log x) + (\log x)^{-2}\right] \cdot dx$

= $\int \left[\log(\log x) + \dfrac{1}{(\log x)^2}\right] \cdot dx$

Put log x = t

∴ x = e^t

∴ x = $e^t \cdot dt$

∴ I = $\int \left(\log t + \dfrac{1}{t^2}\right) e^t \cdot dt$

= $\int e^t \left(\log t + \dfrac{1}{t} - \dfrac{1}{t} + \dfrac{1}{t^2}\right) \cdot dt$

= $\int \left[e^t \left(\log t \dfrac{1}{t}\right) + e^t \left(-\dfrac{1}{t} + \dfrac{1}{t^2}\right)\right] \cdot dt$

= $\int e^t \left(\log t + \dfrac{1}{t}\right) \cdot dt - \int e^t \left(\dfrac{1}{t} - \dfrac{1}{t^2}\right) \cdot dt$

= $I_1 - I_2$

In I_1, Put f(t) = log t. Then f'(t) = $\left(\dfrac{1}{t}\right)$

∴ I_1 = $\int e^t [f(t) + f\prime(t)] \cdot dt$

= $e^t f(t)$

= $e^t \log t$

In I_2, Put g(t) = $\left(\dfrac{1}{t}\right)$. Then g'(t) = $-\left(\dfrac{1}{t^2}\right)$

$\therefore I_2 = \int e^t[g(t) + g\prime(t)] \cdot dt$

$= e^t g(t)$

$= e^t \cdot \left(\frac{1}{t}\right)$

$\therefore I = e^t \log t - \frac{e^t}{t} + c$

$= x \log(\log x) - \frac{x}{\log x} + c.$

Miscellaneous Exercise 3 | Q 3.09 | Page 150

Integrate the following w.r.t.x : $\frac{1}{2\cos x + 3\sin x}$

SOLUTION

Let I = $\int \frac{1}{2\cos x + 3\sin x} \cdot dx$

= $\int \frac{1}{3\sin x + 2\cos x} \cdot dx$

Dividing numerator and denominator by

$\sqrt{3^2 + 2^2} = \sqrt{13}$, we get

I = $\int \frac{\left(\frac{1}{\sqrt{3}}\right)}{\frac{3}{\sqrt{13}}\sin x + \frac{2}{\sqrt{13}}\cos x} \cdot dx$

Since, $\left(\frac{3}{\sqrt{13}}\right)^2 + \left(\frac{2}{\sqrt{13}}\right)^2 = \frac{9}{13} + \frac{4}{13}$ = 1,

we take $\frac{3}{\sqrt{13}} = \cos\infty, \frac{2}{\sqrt{13}} = \sin\infty$

so that $\infty = \frac{2}{3}$ and $\infty = \tan^{-1}\left(\frac{2}{3}\right)$

$\therefore$ I = $\dfrac{1}{\sqrt{13}} \displaystyle\int \frac{1}{\sin x + \cos \infty + \cos x \sin \infty} \cdot dx$

= $\dfrac{1}{\sqrt{13}} \displaystyle\int \frac{1}{\sin(x+\infty)} \cdot dx$

= $\dfrac{1}{\sqrt{13}} \displaystyle\int \cos ec(x+\infty) \cdot dx$

= $\dfrac{1}{\sqrt{13}} \log|\tan| \tan\left(\dfrac{x+\infty}{2}\right) | + c$

= $\dfrac{1}{\sqrt{13}} \log\left|\tan\left(\dfrac{x+\tan^{-1}\ \frac{2}{3}}{2}\right)\right| + c.$

Alternative Method

Let I = $\displaystyle\int \frac{1}{2\cos x + 3\sin x} \cdot dx$

Put $\tan\left(\dfrac{x}{2}\right)$ = t

$\therefore$ $\dfrac{x}{2} = \tan^{-1} t$

$\therefore$ x = 2tan^{-1} t

$\therefore$ dx = $\dfrac{2}{1+t^2} \cdot dt$

and

sin x = $\dfrac{2t}{1+t^2}$

and

cos x = $\dfrac{1-t^2}{1+t^2}$

∴ I = $\displaystyle\int \frac{1}{2\left(\frac{1-t^2}{1+t^2}\right)+3\left(\frac{2t}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$

= $\displaystyle\int \frac{1+t^2}{2-2t^2+6t}\cdot\frac{2dt}{1+t^2}$

= $\displaystyle\int \frac{1}{1-t^2+3t}\cdot dt$

= $\displaystyle\int \frac{1}{1-\left(t^2-3t+\frac{9}{4}\right)+\frac{9}{4}}\cdot dt$

= $\displaystyle\int \frac{1}{\left(\frac{\sqrt{13}}{2}\right)^2-\left(t-\frac{3}{2}\right)^2}\cdot dt$

= $\displaystyle\frac{1}{2\times\frac{\sqrt{13}}{2}}\log\left|\frac{\frac{\sqrt{13}}{2}+t-\frac{3}{2}}{\frac{\sqrt{13}}{2}-t+\frac{3}{2}}\right|+c$

= $\displaystyle\frac{1}{\sqrt{13}}\log\left|\frac{\sqrt{13}+2t-3}{\sqrt{13}-2t+3}\right|+c$

= $\displaystyle\frac{1}{\sqrt{13}}\log\left|\frac{\sqrt{13}+2\tan\left(\frac{x}{2}\right)-3}{\sqrt{13}-2\tan\left(\frac{x}{2}\right)-3}\right|+c.$

Miscellaneous Exercise 3 | Q 3.1 | Page 150

Integrate the following w.r.t.x : $\dfrac{1}{x^3\sqrt{x^2-1}}$

SOLUTION

Let I = $\int \frac{1}{x^3\sqrt{x^2-1}} \cdot dx$

Put x = secθ

∴ dx secθ tanθ dθ

∴ I = $\int \frac{\sec\theta\tan\theta d\theta}{\sec 3\theta\sqrt{\sec^2\theta - 1}}$

= $\int \frac{\sec\theta\tan\theta d\theta}{\sec^3\theta\sqrt{\tan^2\theta}} \cdot d\theta$

∴ I = $\int \cos^2\theta \cdot d\theta$

= $\frac{1}{2}\int (1+\cos 2\theta) \cdot d\theta$

= $\frac{1}{2}\int d\theta + \frac{1}{2}\int \cos 2\theta \cdot d\theta$

= $\frac{\theta}{2} + \frac{1}{2}\left(\frac{\sin 2\theta}{2}\right) + c$ ...(1)

∴ x = sec θ

∴ θ = $\sec^{-1}x$

sin2θ = 2 sinθ cos θ

= $2\sqrt{1-\cos^2\theta} \cdot \cos\theta$

= $2\sqrt{1-\frac{1}{x^2}}\left(\frac{1}{x}\right)$ ...$\left[\because \sec\theta = x \Rightarrow \cos\theta = \frac{1}{x}\right]$

= $\frac{2\sqrt{x^2-1}}{x^2}$

∴ from (1), we have

I = $\frac{1}{2}\sec^{-1}x+\frac{1}{2}\frac{\sqrt{x^2-1}}{x^2}+c.$

Miscellaneous Exercise 3 | Q 3.11 | Page 150

Integrate the following w.r.t.x : $\frac{3x+1}{\sqrt{-2x^2+x+3}}$

SOLUTION

Let I = $\int\frac{3x+1}{\sqrt{-2x^2+x+3}}.dx$

Let 3x + 1 = $\mathbf{A}\left[\frac{d}{dx}\left(-2x^2+x+3\right)\right]+\mathbf{B}$

= A(2 – 2x) + B

∴ 3x + 1 = 2Ax + (2A + B)

Comparing the coefficient of x and constant on both the sides, we get

– 2A = 7 and 2A + B = 3

∴ A = $\frac{-7}{2}$ and $2\left(-\frac{7}{2}\right)+\mathbf{B}$ = 3

∴ B = 10

∴ 7x + 3 = $\frac{-7}{2}(2-2x)+10$

∴ I = $\int\frac{\frac{-7}{2}(2-2x)+10}{\sqrt{3+2x-x^2}}.dx$

= $\frac{-7}{2}\int\frac{(2-2x)}{\sqrt{3+2x-x^2}}.dx+10\int\frac{1}{\sqrt{3+2x-x^2}}x$

= $\frac{-7}{2}\mathrm{I}_1+10\mathrm{I}_2$

In I_1, put $3 + 2x - x^2 = t$

$\therefore (2 - 2x)dx = dt$

$\therefore I_1 = \int \frac{1}{\sqrt{t}} dt$

$= \int t^{-\frac{1}{2}} dt$

$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c_1$

$= 2\sqrt{3 + 2x - x^2} + c_1$

$I_2 = \int \frac{1}{\sqrt{3 - (x^2 - 2x + 1) + 1}} . dx$

$= \int \frac{1}{\sqrt{(2)^2 - (x - 1)^2}} . dx$

$= \sin^{-1}\left(\frac{x - 1}{2}\right) + c_2$

$-\frac{3}{2}\sqrt{-2x^2 + x + 3} + \frac{7}{4\sqrt{2}} \sin^{-1}\left(\frac{4x - 1}{5}\right) + c.$

Miscellaneous Exercise 3 | Q 3.12 | Page 150

Integrate the following w.r.t.x : $\log (x^2 + 1)$

SOLUTION

Let I = $\int \log\left(x^2+1\right) \cdot dx$

= $\int \left[\log\left(x^2+1\right)\right] \cdot 1dx$

= $\left[\log\left(x^2+1\right)\right] \int 1dx - \int \left[\frac{d}{dx}\left\{\log\left(x^2+1\right)\right\} \int 1dx\right] \cdot dx$

= $\left[\log\left(x^2+1\right)\right] \cdot x - \int \frac{1}{x^2+1} \cdot dx\left(x^2+1\right) \cdot xdx$

= $x\log\left(x^2+1\right) - \int \frac{2x^2}{x^2+1} \cdot dx$

= $x\log\left(x^2+1\right) - \int \frac{2x^2+2-2}{x^2+1} \cdot dx$

= $x\log\left(x^2+1\right) - \int \left[\frac{2\left(x^2+1\right)}{x^2+1} - \frac{2}{x^2+1}\right] \cdot dx$

= $x\log\left(x^2+1\right) - \int \left[2\int 1dx - 2\int \frac{1}{x^2+1} \cdot dx\right]$

= x log (x^2 + 1) – 2x + 2 $\tan^{-1}$ x + c.

Miscellaneous Exercise 3 | Q 3.13 | Page 150

Integrate the following w.r.t.x : e^{2x} sin x cos x

SOLUTION

Let I = $\int e^{2x} \cdot \sin x \cos x \cdot dx$

= $\frac{1}{2} \int e(2x) \cdot 2 \sin x \cos x dx$

= $\frac{1}{2} \int e^{2x} \cdot \sin 2x \cdot dx$ ...(1)

= $\frac{1}{2}\left[e^{2x} \int \sin 2x \cdot dx - \int\left\{\frac{d}{dx}(e^{2x}) \int \sin 2x \cdot dx\right\} \cdot dx\right]$

= $\frac{1}{2}\left[e(2x)\left(\frac{-\cos 2x}{2}\right) - \int e^{2x} \times 2 \times \left(\frac{-\cos 2x}{2}\right) \cdot dx\right]$

= $-\frac{1}{4} e^{2x} \cos 2x + \frac{1}{2} \int e^{2x} \cos 2x \cdot dx$

= $-\frac{1}{4} e^{2x} \cos 2x + \frac{1}{2}\left[e^{2x} \int \cos 2x \cdot dx - \int\left\{\frac{d}{dx}(e^{2x}) \int \cos 2x \cdot dx\right\} \cdot dx\right]$

= $\frac{1}{4} e^{2x} \cos 2x + \frac{1}{2}\left[e^{2x}. \frac{\sin 2x}{2} - \int e^{2x} \times 2 \times \frac{\sin 2x}{2} \cdot dx\right]$

= $-\frac{1}{4} e^{2x} \cos 2x + \frac{1}{4} e^{2x} \sin 2x - \frac{1}{2} \int e^{2x} \sin 2x \cdot dx$

∴ I = $-\frac{1}{4} e^{2x} \cos 2x + \frac{1}{4} e^{2x} \sin 2x - \mathrm{I}$ ..[By (1)]

∴ 2I = $-\frac{1}{4} e^{2x} \cos 2x + \frac{1}{4} e^{2x} \sin 2x$

∴ I = $\frac{e^{2x}}{8}(\sin 2x - \cos 2x) + c.$

Miscellaneous Exercise 3 | Q 3.14 | Page 150

Integrate the following w.r.t.x : $\frac{x^2}{(x-1)(3x-1)(3x-2)}$

SOLUTION

Let I = $\int \frac{x^2}{(x-1)(3x-1)(3x-2)} . dx$

Let $\frac{x^2}{(x-1)(3x-1)(3x-2)}$

= $\frac{A}{x-1} + \frac{B}{3x-1} + \frac{C}{3x-2}$

∴ x^2 = A(3x -1)(3x - 2) + B(x – 1)(3x - 2) + C(x – 1)(3x -1)

Put x – 1 = 0, i.e. x = 1, we get

∴ x^2 = A(2)(1) + B(0)(1) + C(0)(2)

∴ 2 = 4A

∴ A = $\frac{1}{2}$

Put x + 2 = 0, i.e. x = – 2, we get

2 + 2 = A(0)(1) + B(– 3)(1) + C(– 3)(0)

∴ 6 = – 3B

∴ B = – 2

Put x + 3 = 0, i.e. x = – 3we get

9 + 2 = A(– 1)(0) + B(– 4)(0) + C(– 4)(– 1)

∴ 11 = 4C

∴ C = $\frac{11}{4}$

∴ $\frac{x^2+2}{(3x-1)(x-1)(3x-2)} = \frac{\left(\frac{1}{4}\right)}{3x-1} + \frac{-2}{x-1} + \frac{\left(\frac{11}{4}\right)}{3x-2}$

∴ I = $\int \left[\frac{\left(\frac{1}{4}\right)}{3x-1} + \frac{-2}{x-\ 1} + \frac{\left(\frac{11}{4}\right)}{3x-2}\right] . dx$

$= \frac{1}{18}\int \frac{1}{3x-1}.dx - 2\int \frac{1}{x-1}.dx + \frac{4}{9}\int \frac{1}{3x-2}.dx$

$= \frac{1}{18}\log|3x-1| + \frac{1}{2}\log|x-1| - \frac{4}{9}\log|3x-2| + c.$

Miscellaneous Exercise 3 | Q 3.15 | Page 150

Integrate the following w.r.t.x : $\frac{1}{\sin x + \sin 2x}$

SOLUTION

Let I = $\int \frac{1}{\sin x + \sin 2x} \cdot dx$

$= \int \frac{1}{\sin x + 2\sin x\cos x} \cdot dx$

$= \int \frac{dx}{\sin x(1 + 2\cos x)}$

$= \int \frac{\sin x \cdot dx}{\sin^2 x(1 + 2\cos x)}$

$= \int \frac{\sin \cdot dx}{(1 - \cos^2 x)(1 + 2\cos x)}$

$= \int \frac{\sin \cdot dx}{(1 - \cos x)(1 + \cos x)(1 + 2\cos x)}$

Put cos x = t

∴ − sinx . dx = dt

∴ sinx .dx = − dt

∴ I = $\int \frac{-dt}{(1-t)(1+t)(1+2t)}$

$= -\int \frac{dt}{(1-t)(1+t)(1+2t)}$

Let $\dfrac{1}{(1-t)(1+t)(1+2t)} = \dfrac{A}{1-\ t} + \dfrac{B}{1+t} + \dfrac{C}{1+2t}$

∴ 1 = A(1 + t)(1 + 2t) + B(1 – t)(1 + 2t) + C(1 – t)(1 + t)

Putting 1 – t = 0, i.e. t = 1, we get

1 = A(2)(3) + B(0)(3) + C(0)(2)

∴ A = $\dfrac{1}{6}$

Putting 1 – t = 0, i.e. t = – 1, we get

1 = A(0)(– 1) + B(2)(– 1) + C(2)(0)

∴ B = $-\dfrac{1}{2}$

Putting 1 + 2t = 0, i.e. t = $-\dfrac{1}{2}$, we get

1 = $A(0) + B(0) + C\left(\dfrac{3}{2}\right)\left(\dfrac{1}{2}\right)$

∴ C = $\dfrac{4}{3}$

$$\therefore \frac{1}{(1-t)(1+t)(1+2t)} = \frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t}$$

$$\therefore I = \int\left[\frac{\left(\frac{1}{6}\right)}{1-t} + \frac{\left(\frac{-1}{2}\right)}{1+t} + \frac{\left(\frac{4}{3}\right)}{1+2t}\right]\cdot dt$$

$$= \frac{1}{6}\int\frac{1}{1-t}\cdot dt + \frac{1}{2}\int\frac{1}{1+t}\cdot dt - \frac{4}{3}\int\frac{1}{1+2t}\cdot dt$$

$$= \frac{1}{6}\cdot\frac{\log|1-t|}{-1} + \frac{1}{2}\log|1+t| - \frac{4}{3}\cdot\frac{\log|1+2t|}{2} + c$$

$$= \frac{1}{6}\log|1-\cos x| + \frac{1}{2}\log|1+\cos x| - \frac{2}{3}\log|1+2\cos x| + c.$$

Miscellaneous Exercise 3 | Q 3.16 | Page 150

Integrate the following w.r.t.x : $\sec^2 x\sqrt{7+2\tan x-\tan^2 x}$

SOLUTION

Let I = $\int \sec^2 x\sqrt{7 + 2\tan x - \tan^2 x} \cdot dx$

Put tan x = t

∴ $\sec^2 x \cdot dx = dt$

∴ I = $\int \sqrt{7 + 2t - t^2} \cdot dt$

= $\int \sqrt{7 - (t^2 - 2t)} \cdot dt$

= $\int \sqrt{8 - (t^2 - 2t + 1)} \cdot dt$

= $\int \sqrt{\left(2\sqrt{2}\right)^2 - (t - 1)^2} \cdot dt$

= $\left(\frac{t-1}{2}\right)\sqrt{\left(2\sqrt{2}\right)^2 - (t-1)^2} + \frac{\left(2\sqrt{2}\right)^2}{2}\sin^{-1}\left(\frac{t-1}{2\sqrt{2}}\right) + c$

= $\left(\frac{t-1}{2}\right)\sqrt{7 + 2t - t^2} + 4\sin^{-1}\left(\frac{t-1}{2\sqrt{2}}\right) + c$

= $\left(\frac{\tan x - 1}{2}\right)\sqrt{7 + 2\tan x - \tan^2 x} + 4\sin^{-1}\left(\frac{\tan x - 1}{2\sqrt{2}}\right) + c.$

Miscellaneous Exercise 3 | Q 3.17 | Page 150

Integrate the following w.r.t.x : $\frac{x+5}{x^3 + 3x^2 - x - 3}$

SOLUTION

Let I = $\int \frac{x+5}{x^3+3x^2-x-3} \cdot dx$

= $\int \frac{x+5}{x^2(x+3)-(x+3)} \cdot dx$

= $\int \frac{x+5}{(x+3)(x^2-1)}$

= $\int \frac{x+5}{(x+3)(x-1)(x+1)} \cdot dx$

∴ $x^2 + 2 = A(x + 2)(x + 3) + B(x – 1)(x + 3) + C(x – 1)(x + 2)$

Put x – 1 = 0, i.e. x = 1, we get

1 + 2 = A(3)(4) + B(0)(4) + C(0)(3)

∴ 3 = 12A

∴ A = $\frac{1}{4}$

Put x + 2 = 0, i.e. x = – 2, we get

4 + 2 = A(0)(1) + B(– 3)(1) + C(– 3)(0)

∴ 6 = – 3B

∴ B = – 2

Put x + 3 = 0, i.e. x = – 3we get

9 + 2 = A(– 1)(0) + B(– 4)(0) + C(– 4)(– 1)

∴ 11 = 4C

∴ C = $\frac{11}{4}$

$$\therefore \frac{x^2+2}{(x-1)(x+2)(x+3)} = \frac{\left(\frac{1}{4}\right)}{x-1} + \frac{-2}{x+1} + \frac{\left(\frac{11}{4}\right)}{x+3}$$

$$\therefore \text{I} = \int \left[\frac{\left(\frac{1}{4}\right)}{x-1} + \frac{-2}{x+1} + \frac{\left(\frac{11}{4}\right)}{x+3}\right].dx$$

$$= \frac{1}{4}\int \frac{1}{x-1}.dx - 2\int \frac{1}{x+1}.dx + \frac{11}{4}\int \frac{1}{x+3}.dx$$

$$\frac{3}{4}\log|x-1| - \log|x+1| + \frac{1}{4}\log|x+3| + c.$$

Miscellaneous Exercise 3 | Q 3.18 | Page 150

Integrate the following w.r.t. x : $\frac{1}{x(x^5+1)}$

SOLUTION

Let I = $\int \frac{1}{x(x^5+1)}.dx$

= $\int \frac{x^4}{x^5(x^5+1)}.dx$

Put x^5 = t.

Then $5x^4$ dx = dt

∴ x^4 dx= $\frac{dt}{5}$

∴ I = $\int \frac{1}{t(t+1)}.\frac{dt}{5}$

= $\frac{1}{5}\int \frac{(t+1)-t}{t(t+1)}.dt$

= $\frac{1}{5}\int \left(\frac{1}{t} - \frac{1}{t+1}\right).dt$

$= \frac{1}{5}\left[\int \frac{1}{t}dt - \int \frac{1}{t+1}dt\right]$

$= \frac{1}{5}[\log|t| - \log|t+1|] + c$

$= \frac{1}{5}\log\left|\frac{t}{t+1}\right| + c$

$= \frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + c.$

Miscellaneous Exercise 3 | Q 3.19 | Page 150

Integrate the following w.r.t.x : $\frac{\sqrt{\tan x}}{\sin x \cdot \cos x}$

SOLUTION

Let I = $\int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x} \cdot dx$

Dividing numerator and denominator by $\cos^2 x$, we get

I = $\int \frac{\left(\frac{\sqrt{\tan x}}{\cos^2}\right)}{\left(\frac{\sin x}{\cos x}\right)} \cdot dx$

$= \int \frac{\sqrt{\tan x} \cdot \sec^2 x}{\tan x} \cdot dx$

$= \int \frac{\sec^2 x}{\sqrt{\tan x}} \cdot dx$

Put tan x = t

∴ $\sec^2 x \cdot dx = dt$

$\therefore$ I = $\int \frac{1}{\sqrt{t}} \cdot dt$

= $\int t^{-\frac{1}{2}} \cdot dt$

= $\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$

= $2\sqrt{t} + c$

= $2\sqrt{\tan x} + c.$

Miscellaneous Exercise 3 | Q 3.2 | Page 150

Integrate the following w.r.t.x : $\sec^4 x \operatorname{cosec}^2 x$

SOLUTION

Let I = $\int \sec^4 x \operatorname{cosec}^2 x \cdot dx$

= $\int \sec^4 x \operatorname{cosec}^2 x \cdot \sec^2 x \cdot dx$

Put tan x = t

$\therefore \sec^2 x \cdot dx = d$

Also, $\sec^2 x \operatorname{cosec}^2 x = (1 + \tan^2 x)(1 + \cot^2 x)$

= $(1 + t^2)\left(1 + \frac{1}{t^2}\right)$

= $(1 + t^2)\left(\frac{t^2 + 1}{t^2}\right)$

= $\frac{t^4 + 2t^2 + 1}{t^2}$

$$= t^2 + 2 + \frac{1}{t^2}$$

$$\therefore \text{I} = \int \left(t^2 + 2 + \frac{1}{t^2} \right) \cdot dt$$

$$= \int t^2 \cdot dt + 2 \int \cdot dt + \int \frac{1}{t^2} \cdot dt$$

$$= \frac{t^3}{3} + 2t + \frac{t^{-1}}{(-1)} + c$$

$$= \frac{1}{3} \tan^3 x + 2 \tan x - \frac{1}{\tan x} + c$$

$$= \frac{1}{3 \cot^3 x} + \frac{2}{\cot x} - \cot x + c.$$