Chapter 6

# Definite Integral

## Contents



J. Bernoulli

*Newton introduced the basic notion of inverse function called the anti-derivative (integral) or the inverse method of tangents.*

*During* 1684-86 A.D. **Leibnitz** *published an article in the Acta Eruditorum which he called calculas summatorius, since it was connected with the summation of a number of infinitely small areas, whose sum, he indicated by the symbol* $\int$. *In* 1696 A.D. *he followed a suggestion made by* **J.Bernoulli** *and changed this article to calculus integrali. This corresponded to Newton's inverse method of tangents.*

*Both* **Newton** *and* **Leibnitz** *adopted quite independent lines of approach which were radically different. However, respective theories accomplished results that were practically identical. Leibnitz used the notion of definite integral and what is quite certain is that he first clearly appreciated tie up between the anti-derivative and the definite integral. The discovery that differentiation and integration are inverse operations belongs to Newton and Leibnitz.*

# Definite Integral

## 6.1 Definition

Let $\phi(x)$ be the primitive or anti-derivative of a function $f(x)$ defined on $[a, b]$ *i.e.*, $\frac{d}{dx}[\phi(x)] = f(x)$. Then the definite integral of $f(x)$ over $[a, b]$ is denoted by $\int_a^b f(x)dx$ and is defined as $[\phi(b) - \phi(a)]$ *i.e.*, $\int_a^b f(x)dx = \phi(b) - \phi(a)$. This is also called Newton Leibnitz formula.

The numbers $a$ and $b$ are called the limits of integration, '$a$' is called the lower limit and '$b$' the upper limit. The interval $[a, b]$ is called the interval of integration. The interval $[a, b]$ is also known as range of integration.

***Important Tips***

☞ *In the above definition it does not matter which anti-derivative is used to evaluate the definite integral, because if* $\int f(x)dx = \phi(x) + c$, *then*

$$\int_a^b f(x)dx = [\phi(x) + c]_a^b = (\phi(b) + c) - (\phi(a) + c) = \phi(b) - \phi(a).$$

*In other words, to evaluate the definite integral there is no need to keep the constant of integration.*

☞ *Every definite integral has a unique value.*

**Example: 1** $\int_{-1}^{3}\left[\tan^{-1}\frac{x}{x^2+1} + \tan^{-1}\frac{x^2+1}{x}\right]dx =$ **[Karnataka CET 2000]**

(a) $\pi$ (b) $2\pi$ (c) $3\pi$ (d) None of these

**Solution:** (b) $I = \int_{-1}^{3}\left[\tan^{-1}\frac{x}{x^2+1} + \cot^{-1}\frac{x}{x^2+1}\right]dx$

$\Rightarrow \quad I = \int_{-1}^{3}\frac{\pi}{2}dx \Rightarrow I = \frac{\pi}{2}[x]_{-1}^{3} = \frac{\pi}{2}[3+1] = 2\pi$ .

**Example: 2** $\int_0^{\pi}\sin^2 x\,dx$ is equal to **[MP PET 1999]**

(a) $\pi$ (b) $\pi/2$ (c) 0 (d) None of these

**Solution:** (b) $I = \frac{1}{2}\int_0^{\pi}2\sin^2 x\,dx = \frac{1}{2}\int_0^{\pi}[1 - \cos 2x]dx$

$\Rightarrow I = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi} \Rightarrow I = \frac{1}{2}[\pi] = \frac{\pi}{2}$ .

## 6.2 Definite Integral as the Limit of a Sum

Let $f(x)$ be a single valued continuous function defined in the interval $a \le x \le b$, where $a$ and $b$ are both finite. Let this interval be divided into $n$ equal sub-intervals, each of width $h$ by inserting ($n$ – 1) points $a + h, a + 2h, a + 3h......a + (n-1)h$ between $a$ and $b$. Then $nh = b - a$ .

Now, we form the sum $hf(a)+hf(a+h)+hf(a+2h)+........+hf(a+rh)+......+hf[a+(n-1)h]$

$$= h[f(a)+f(a+h)+f(a+2h)+.....+f(a+rh)+....+f\{a+(n-1)h\}]$$

$$= h\sum_{r=0}^{n-1} f(a+rh)$$

where, $a+nh=b \Rightarrow nh=b-a$

The $\lim_{h\to 0} h\sum_{r=0}^{n-1} f(a+rh)$, if it exists is called the **definite integral** of $f(x)$ with respect to $x$ between the limits $a$ and $b$ and we denote it by the symbol $\int_a^b f(x)dx$ .

Thus, $\int_a^b f(x)dx = \lim_{h\to 0} h[f(a)+f(a+h)+f(a+2h)+......+f\{a+(n-1)h\}] \Rightarrow \int_a^b f(x)dx = \lim_{h\to 0} h\sum_{r=0}^{n-1} f(a+rh)$

where, $nh=b-a$, $a$ and $b$ being the limits of integration.

The process of evaluating a definite integral by using the above definition is called integration from the first principle or integration as the limit of a sum.

### *Important Tips*

☞ *In finding the above sum, we have taken the left end points of the subintervals. We can take the right end points of the sub-intervals throughout.*

*Then we have,* $\int_a^b f(x)dx = \lim_{h\to 0} h[f(a+h)+f(a+2h)+.....+f(a+nh)]$, $\Rightarrow \int_a^b f(x)dx = h\sum_{r=1}^{n} f(a+rh)$

*where,* $nh = b - a$.

☞ $\int_\alpha^\beta \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}(\beta>\alpha)=\pi$

☞ $\int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)}\ dx = \frac{\pi}{8}(\beta-\alpha)^2$

☞ $\int_a^b \sqrt{\frac{x-a}{b-x}}\ dx = \frac{\pi}{2}(b-a)$

☞ $\int_a^b f(x)\ dx = \frac{1}{n}\int_{na}^{nb} f(x)\ dx$

☞ $\int_{a-c}^{b-c} f(x+c)\ dx = \int_a^b f(x)\ dx$ or $\int_{a+c}^{b+c} f(x-c)\ dx = \int_a^b f(x)\ dx$

### Some useful results for evaluation of definite integrals as limit for sums

(i) $1+2+3+.......+n = \frac{n(n+1)}{2}$

(ii) $1^2+2^2+3^2+........+n^2 = \frac{n(n+1)(2n+1)}{6}$

(iii) $1^3+2^3+3^3+.......+n^3 = \left[\frac{n(n+1)}{2}\right]^2$

(iv) $a+ar+ar^2+........+ar^{n-1} = \frac{a(r^n-1)}{r-1}, r\neq 1, r>1$

(v) $a+ar+ar^2+..............+ar^{n-1} = \frac{a(1-r^n)}{1-r}$, $r\neq 1$, $r<1$

(vi) $\sin a+\sin(a+h)+........+\sin[a+(n-1)h] = \sum_{r=0}^{n-1}[\sin(a+nh)] = \frac{\sin\left\{a+\left(\frac{n-1}{2}\right)h\right\}\sin\left\{\frac{nh}{2}\right\}}{\sin\left(\frac{h}{2}\right)}$

(vii) $\cos a+\cos(a+h)+\cos(a+2h)+.....+\cos[a+(n-1)h]=\sum_{r=0}^{n-1}[\cos(a+nh)]=\dfrac{\cos\left\{a+\left(\frac{n-1}{2}\right)h\right\}\sin\left\{\frac{nh}{2}\right\}}{\sin\left(\frac{h}{2}\right)}$

(viii) $1-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\dfrac{1}{4^2}+\dfrac{1}{5^2}-\dfrac{1}{6^2}+..................\infty=\dfrac{\pi^2}{12}$ (ix) $1+\dfrac{1}{2^2}+\dfrac{1}{3^2}+\dfrac{1}{4^2}+\dfrac{1}{5^2}+\dfrac{1}{6^2}...............\infty=\dfrac{\pi^2}{6}$

(x) $1+\dfrac{1}{3^2}+\dfrac{1}{5^2}+..................\infty=\dfrac{\pi^2}{8}$ (xi) $\dfrac{1}{2^2}+\dfrac{1}{4^2}+\dfrac{1}{6^2}+..................\infty=\dfrac{\pi^2}{24}$

(xii) $\cos\theta=\dfrac{e^{i\theta}+e^{-i\theta}}{2}$ and $\sin\theta=\dfrac{e^{i\theta}-e^{-i\theta}}{2}$ (xiii) $\cosh\theta=\dfrac{e^{\theta}+e^{-\theta}}{2}$ and $\sinh\theta=\dfrac{e^{\theta}-e^{-\theta}}{2}$

## 6.3 Evaluation of Definite Integral by Substitution

When the variable in a definite integral is changed, the substitutions in terms of new variable should be effected at three places.

(i) In the integrand (ii) In the differential say, $dx$ (iii) In the limits

For example, if we put $\phi(x)=t$ in the integral $\int_a^b f\{\phi(x)\}\phi'(x)dx$, then $\int_a^b f\{\phi(x)\}\phi'(x)dx=\int_{\phi(a)}^{\phi(b)} f(t)\,dt$.

### *Important Tips*

☞ $\int_0^{\pi}\dfrac{dx}{1+\sin x}=2$

☞ $\int_0^{\pi/2}\dfrac{dx}{\sin x+\cos x}=\sqrt{2}\log\left(\sqrt{2}+1\right)$

☞ $\int_0^{\pi/2}\log(\tan x)dx=0$

☞ $\int_0^a\dfrac{dx}{1+e^{f(x)}}=\dfrac{a}{2}$, where $f(a-x)=-f(x)$

☞ $\int_0^a\dfrac{dx}{\sqrt{a^2-x^2}}=\dfrac{\pi}{2}$

☞ $\int_0^a\dfrac{dx}{x^2+a^2}=\dfrac{\pi}{2a}$

☞ $\int_0^a\sqrt{a^2-x^2}\,dx=\dfrac{\pi a^2}{4}$

**Example: 3** If $h(a)=h(b)$, then $\int_a^b[f(g[h(x)])]^{-1}f'(g[h(x)])g'[h(x)]h'(x)dx$ is equal to **[MP PET 2001]**

(a) 0 (b) $f(a)-f(b)$ (c) $f[g(a)]-f[g(b)]$ (d) None of these

**Solution:** (a) Put $f(g[h(x)])=t \Rightarrow f'(g[h(x)])g'[h(x)]h'(x)dx=dt$

$\therefore \int_{f(g[h(a)])}^{f(g[h(b)])}t^{-1}dt=[\log(t)]_{f(g[h(a)])}^{f(g[h(b)])}=0$ $[\because h(a)=h(b)]$

**Example: 4** The value of the integral $\int_{e^{-1}}^{e^2}\left|\dfrac{\log_e x}{x}\right|dx$ is **[IIT 2000]**

(a) 3/2 (b) 5/2 (c) 3 (d) 5

**Solution:** (b) Put $\log_e x=t\Rightarrow e^t=x$

$\therefore dx=e^t dt$

and limits are adjusted as –1 to 2

$\therefore I=\int_{-1}^{2}\left|\dfrac{t}{e^t}\right|e^t dt=\int_{-1}^{2}|t|\,dt \Rightarrow I=\int_{-1}^{0}-t\,dt+\int_0^2 t\,dt \Rightarrow I=\left[\dfrac{-t^2}{2}\right]_{-1}^{0}+\left[\dfrac{t^2}{2}\right]_0^2 \Rightarrow I=5/2$

**Example: 5** $\int_0^{\pi/2} \frac{dx}{1+\sin x}$ equals **[MNR 1983; Rajasthan PET 1990; Kurukshetra CEE 1997]**

(a) 0 (b) 1 (c) –1 (d) 2

**Solution:** (b) $I = \int_0^{\pi/2} \frac{dx}{\sin^2 x/2 + \cos^2 x/2 + 2\sin x/2 \cos x/2}$

$I = \int_0^{\pi/2} \frac{dx}{(\sin x/2 + \cos x/2)^2} = \int_0^{\pi/2} \frac{\sec^2 x/2}{(1+\tan x/2)^2} dx$

Put $(1+\tan x/2) = t \Rightarrow \frac{1}{2}\sec^2 x/2\, dx = dt$

$\therefore\ I = 2\int_1^2 \frac{dt}{t^2} = -2\left[\frac{1}{t}\right]_1^2 = -2\left[\frac{1}{2} - \frac{1}{1}\right] = 1$

## 6.4 Properties of Definite Integral

(1) $\int_a^b f(x)dx = \int_a^b f(t)dt$ *i.e.*, The value of a definite integral remains unchanged if its variable is replaced by any other symbol.

**Example: 6** $\int_3^6 \frac{1}{x+1} dx$ is equal to

(a) $[\log(x+1)]_3^6$ (b) $[\log(t+1]_3^6$ (c) Both (a) and (b) (d) None of these

**Solution:** (c) $I = \int_3^6 \frac{1}{x+1} dx = [\log(x+1)]_3^6$ , $I = \int_3^6 \frac{1}{t+1} dt = [\log(t+1)]_3^6$

(2) $\int_a^b f(x)dx = -\int_b^a f(x)dx$ *i.e.*, by the interchange in the limits of definite integral, the sign of the integral is changed.

**Example: 7** Suppose *f* is such that *f*(–*x*) = –*f*(*x*) for every real *x* and $\int_0^1 f(x)\,dx = 5$ , then $\int_{-1}^0 f(t)\,dt =$ **[MP PET 2000]**

(a) 10 (b) 5 (c) 0 (d) –5

**Solution:** (d) Given, $\int_0^1 f(x)dx = 5$

Put $x = -t \Rightarrow dx = -dt$

$\therefore\ I = -\int_0^{-1} f(-t)dt = -\int_{-1}^0 f(t)dt \Rightarrow I = -5$

(3) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ , (where $a < c < b$)

or $\int_a^b f(x)dx = \int_a^{c_1} f(x)dx + \int_{c_1}^{c_2} f(x)dx + ..... + \int_{c_n}^b f(x)dx$; (where $a < c_1 < c_2 < ........c_n < b$ )

Generally this property is used when the integrand has two or more rules in the integration interval.

***Important Tips***

☞ $\int_a^b (|x-a| + |x-b|)\, dx = (b-a)^2$

**Note** : ❑ Property (3) is useful when $f(x)$ is not continuous in [$a$, $b$] because we can break up the integral into several integrals at the points of discontinuity so that the function is continuous in the sub-intervals.

❑ The expression for $f(x)$ changes at the end points of each of the sub-interval. You might at times be puzzled about the end points as limits of integration. You may not be sure for $x = 0$ as the limit of the first integral or the next one. In fact, it is immaterial, as the area of the line is always zero. Therefore, even if you write $\int_{-1}^{0} (1-2x)\,dx,$ whereas 0 is not included in its domain it is deemed to be understood that you are approaching $x = 0$ from the left in the first integral and from right in the second integral. Similarly for $x = 1$.

**Example: 8** $\int_{-2}^{2} |1-x^2|\,dx$ is equal to **[IIT 1989; BIT Ranchi 1996; Kurukshetra CEE 1998]**

(a) 2 (b) 4 (c) 6 (d) 8

**Solution:** (b) $I = \int_{-2}^{2} |1-x^2|\,dx = \int_{-2}^{-1} |1-x^2|\,dx + \int_{-1}^{1} |1-x^2|\,dx + \int_{1}^{2} |1-x^2|\,dx$

$\Rightarrow I = -\int_{-2}^{-1} (1-x^2)dx + \int_{-1}^{1} (1-x^2)dx - \int_{1}^{2} (1-x^2)dx \Rightarrow I = \frac{4}{3} + \frac{4}{3} + \frac{4}{3} = 4$ .

**Example: 9** $\int_{0}^{1.5} [x^2]dx$ , where [.] denotes the greatest integer function, equals **[DCE 2000, 2001; IIT 1988; AMU 1998]**

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$ (c) $1+\sqrt{2}$ (d) $\sqrt{2}-1$

**Solution:** (b) $I = \int_{0}^{1.5} [x^2]dx = \int_{0}^{1} [x^2]dx + \int_{1}^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{1.5} [x^2]dx \Rightarrow I = 0 + \int_{1}^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{1.5} 2dx = \sqrt{2} - 1 + 3 - 2\sqrt{2} \Rightarrow I = 2 - \sqrt{2}$

**Example: 10** If $f(x) = \begin{cases} e^{\cos x}.\sin x, & |x| \le 2 \\ 2, & \text{otherwise} \end{cases}$, then $\int_{-2}^{3} f(x)dx =$ **[IIT 2000]**

(a) 0 (b) 1 (c) 2 (d) 3

**Solution:** (c) $|x| \le 2 \Rightarrow -2 \le x \le 2$ and $f(x) = e^{\cos x}\sin x$ is an odd function.

$\therefore\ I = \int_{-2}^{3} f(x)dx = \int_{-2}^{2} f(x)dx + \int_{2}^{3} f(x)dx \Rightarrow I = 0 + \int_{2}^{3} 2dx = [2x]_2^3 = 2$ $\left[\because \int_{-a}^{a} f(x) = 0 \text{ if } f(x) \text{ is odd and in } (2, 3)\ f(x) \text{ is } 2\right]$

(4) $\int_{0}^{a} f(x)dx = \int_{0}^{a} f(a-x)dx$ : This property can be used only when lower limit is zero. It is generally used for those complicated integrals whose denominators are unchanged when $x$ is replaced by $(a - x)$.

Following integrals can be obtained with the help of above property.

(i) $\int_{0}^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x}dx = \int_{0}^{\pi/2} \frac{\cos^n x}{\cos^n x + \sin^n x}dx = \frac{\pi}{4}$

(ii) $\int_{0}^{\pi/2} \frac{\tan^n x}{1+\tan^n x}dx = \int_{0}^{\pi/2} \frac{\cot^n x}{1+\cot^n x}dx = \frac{\pi}{4}$

(iii) $\int_{0}^{\pi/2} \frac{1}{1+\tan^n x}dx = \int_{0}^{\pi/2} \frac{1}{1+\cot^n x}dx = \frac{\pi}{4}$

(iv) $\int_{0}^{\pi/2} \frac{\sec^n x}{\sec^n x + \mathrm{cosec}^n x}dx = \int_{0}^{\pi/2} \frac{\mathrm{cosec}^n x}{\mathrm{cosec}^n x + \sec^n x}dx = \frac{\pi}{4}$

(v) $\int_{0}^{\pi/2} f(\sin 2x)\sin x dx = \int_{0}^{\pi/2} f(\sin 2x)\cos x dx$

(vi) $\int_{0}^{\pi/2} f(\sin x)dx = \int_{0}^{\pi/2} f(\cos x)dx$

(vii) $\int_{0}^{\pi/2} f(\tan x)dx = \int_{0}^{\pi/2} f(\cot x)dx$

(viii) $\int_{0}^{1} f(\log x)dx = \int_{0}^{1} f[\log(1-x)]dx$

(ix) $\int_0^{\pi/2} \log \tan x dx = \int_0^{\pi/2} \log \cot x dx$ (x) $\int_0^{\pi/4} \log(1+\tan x)dx = \frac{\pi}{8}\log 2$

(xi) $\int_0^{\pi/2} \log \sin x dx = \int_0^{\pi/2} \log \cos x dx = \frac{-\pi}{2}\log 2 = \frac{\pi}{2}\log\frac{1}{2}$

(xii) $\int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x}dx = \int_0^{\pi/2} \frac{a\sec x + b\,\mathrm{cosec}\, x}{\sec x + \mathrm{cosec}\, x}dx = \int_0^{\pi/2} \frac{a\tan x + b\cot x}{\tan x + \cot x}dx = \frac{\pi}{4}(a+b)$

**Example: 11** $\int_0^{\pi} e^{\sin^2 x}\cos^3 x dx =$

**OR**

For any integer $n$, $\int_0^{\pi} e^{\sin^2 x}\cos^3(2n+1)x dx =$ **[MP PET 2002; MNR 1992, 98]**

(a) –1 (b) 0 (c) 1 (d) None of these

**Solution:** (b) Let, $f_1(x) = \cos^3 x = -f(\pi - x)$

and $f_2(x) = \cos^3(2n+1)x = -f(\pi - x)$

$\therefore$ $I = 0$.

**Example: 12** $\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)}dx$ is equal to **[IIT 1988; Kurukshetra CEE 1999; Karnataka CET 2000]**

(a) $a$ (b) $a/2$ (c) $2a$ (d) 0

**Solution:** (a) $I = \int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)}dx = \int_0^{2a} \frac{f(2a-x)}{f(2a-x)+f(x)}dx$

$2I = \int_0^{2a} \frac{f(x)+f(2a-x)}{f(x)+f(2a-x)}dx = \int_0^{2a} dx = [x]_0^{2a} = 2a$

$\therefore I = a$.

**Example: 13** $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is equal to **[MNR 1989, 90; Rajasthan PET 1991, 95]**

(a) $\pi/4$ (b) $\infty$ (c) –1 (d) 1

**Solution:** (a) We know, $\int_0^{\pi/2} \frac{\tan^n x dx}{1+\tan^n x} = \frac{\pi}{4}$ for any value of $n$

$\therefore$ $I = \pi/4$.

(5) $\int_{-a}^{a} f(x)dx = \int_0^a f(x)+f(-x)\,dx$.

In special case : $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, \text{ if } f(x) \text{ is even function or } f(-x)=f(x) \\ 0 \quad \text{, if } f(x) \text{ is odd is odd function or } f(-x)=-f(x) \end{cases}$

This property is generally used when integrand is either even or odd function of $x$.

**Example: 14** The integral $\int_{-1/2}^{1/2}\left([x]+\ln\left(\frac{1+x}{1-x}\right)\right)dx$ equal to

(a) $\frac{-1}{2}$ (b) 0 (c) 1 (d) $2\ln\left(\frac{1}{2}\right)$

**Solution:** (a) $\log\left(\frac{1+x}{1-x}\right)$ is an odd function of $x$ as $f(-x) = -f(x)$

$I = \int_{-1/2}^{1/2}[x]dx + 0 \Rightarrow I = \int_{-1/2}^{0}[x]dx + \int_0^{1/2}[x]dx \Rightarrow I = \int_{-1/2}^{0} -1dx + 0 \Rightarrow -[x]_{-1/2}^{0} = \frac{-1}{2}$.

**Example: 15** The value of the integral $\int_{-1}^{1} \log\left[x+\sqrt{x^2+1}\right]dx$ is **[MP PET 2001]**

(a) 0 (b) log 2 (c) log 1/2 (d) None of these

**Solution:** (a) $f(x)=\log[x+\sqrt{x^2+1}]$ is a odd function *i.e.* $f(-x)=-f(x) \Rightarrow f(x)+f(-x)=0 \Rightarrow I=0$.

**Example: 16** The value of $\int_{-\pi}^{\pi}(1-x^2)\sin x\cos^2 x dx$ is **[Roorkee 1992; MNR 1998]**

(a) 0 (b) 1 (c) 2 (d) 3

**Solution:** (a) Let, $f_1(x)=(1-x^2)$, $f_2(x)=\sin x$ and $f_3(x)=\cos^2 x$

Now, $f_1(x)=f_1(-x)$, $f_2(x)=-f_2(-x)$ and $f_3(x)=f_3(-x)$

$\therefore\ I=\int_{-\pi}^{\pi} f(x)dx=\int_{-\pi}^{\pi}[f_1(x).f_2(x).f_3(x)]dx=-\int_{-\pi}^{\pi}[f_1(-x).f_2(-x).f_3(-x)]dx$

$\therefore\ I=0$

(6) $\int_0^{2a} f(x)dx=\int_0^{a} f(x)dx+\int_0^{a} f(2a-x)\,dx$

In particular, $\int_0^{2a} f(x)\,dx=\begin{cases}0, & \text{if } f(2a-x)=-f(x)\\ 2\int_0^a f(x)\,dx, & \text{if } f(2a-x)=f(x)\end{cases}$

It is generally used to make half the upper limit.

**Example: 17** If $n$ is any integer, then $\int_0^{\pi} e^{\cos^2 x}\cos^3(2n+1)x\,dx$ is equal to **[IIT 1985; Rajasthan PET 1995]**

(a) $x$ (b) 1 (c) 0 (d) None of these

**Solution:** (c) $I=\int_0^{\pi} e^{\cos^2(\pi-x)}.\cos^3(2n+1)(\pi-x)\,dx$

$\Rightarrow I=-\int_0^{\pi} e^{\cos^2 x}.\cos^3(2n+1)x\,dx \Rightarrow I=-I$

$\Rightarrow 2I=0 \Rightarrow I=0$.

**Example: 18** If $I_1=\int_0^{3\pi} f(\cos^2 x)dx$ and $I_2=\int_0^{\pi} f(\cos^2 x)dx$ then

(a) $I_1=I_2$ (b) $I_1=2I_2$ (c) $I_1=3I_2$ (d) $I_1=4I_2$

**Solution:** (c) $f(\cos^2 x)=f(\cos^2(3\pi-x))$

$\therefore\ I_1=3\int_0^{\pi} f(\cos^2 x)dx \Rightarrow I_1=3I_2$

(7) $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$

**Note:** ❑ $\int_a^b \frac{f(x)dx}{f(x)+f(a+b-x)}=\frac{1}{2}(b-a)$

**Example: 19** $\int_{\pi/4}^{3\pi/4}\frac{dx}{1+\cos x}$ is equal to **[IIT 1999]**

(a) 2 (b) –2 (c) 1/2 (d) –1/2

**Solution:** (a) $I=\int_{\pi/4}^{3\pi/4}\frac{1}{1-\cos x}dx$ $\left[\because \cos\left(\frac{\pi}{4}+\frac{3\pi}{4}-x\right)=-\cos x\right]$

$\therefore\ 2I=\int_{\pi/4}^{3\pi/4}\frac{2}{1-\cos^2 x}dx$

$\Rightarrow 2I=2\int_{\pi/4}^{3\pi/4}\text{cosec}^2 x dx \Rightarrow 2I=-2[\cot x]_{\pi/4}^{3\pi/4}=4 \Rightarrow I=2.$

**Example: 20** The value of $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$ is **[IIT 1994; Kurukshetra CEE 1998]**

(a) 1 (b) 0 (c) –1 (d) 1/2

**Solution:** (d) $I = \int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$

Put $x = 2+3-t \Rightarrow dx = -dt$

$\therefore \quad I = \int_3^2 \frac{\sqrt{5-t}}{\sqrt{5-t}+\sqrt{t}}(-dt) = \int_2^3 \frac{\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x}}dx$ and $2I = \int_2^3 \frac{\sqrt{x}+\sqrt{5-x}}{\sqrt{5-x}+\sqrt{x}}\,dx = \int_2^3 1\,dx$

$\Rightarrow \quad 2I = [x]_2^2 = 1 \Rightarrow I = 1/2$

**Example: 21** If $f(a+b-x) = f(x)$ then $\int_a^b x\,f(x)dx$ is equal to **[Kurukshetra CEE 1993; AIEEE 2003]**

(a) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (b) $\frac{a+b}{2}\int_a^b f(x)dx$ (c) $\frac{b-a}{2}\int_a^b f(x)dx$ (d) None of these

**Solution:** (b) $I = \int_a^b x\,f(x)dx$ and $I = \int_a^b (a+b-x)f(a+b-x)dx$

$\Rightarrow I = \int_a^b (a+b-x)f(x)dx \Rightarrow I = \int_a^b (a+b)f(x)dx - \int_a^b x\,f(x)dx \Rightarrow 2I = \left[\int_a^b f(x)dx\right](a+b) \Rightarrow I = \frac{a+b}{2}\int_a^b f(x)dx$

(8) $\int_0^a x\,f(x)dx = \frac{1}{2}a\int_0^a f(x)dx$ if $f(a-x) = f(x)$

**Example: 22** If $\int_0^\pi x\,f(\sin x)dx = k\int_0^\pi f(\sin x)dx$, then the value of $k$ will be **[IIT 1982]**

(a) $\pi$ (b) $\pi/2$ (c) $\pi/4$ (d) 1

**Solution:** (b) Given, $\int_0^\pi x\,f(\sin x)dx = k\int_0^\pi f(\sin x)dx$

$\Rightarrow \int_0^\pi (\pi-x)f(\sin(\pi-x))dx = k\int_0^\pi f(\sin(\pi-x))dx \Rightarrow \pi\int_0^\pi f(\sin x)dx - \int_0^\pi x\,f(\sin x)dx = k\int_0^\pi f(\sin x)dx$

$\Rightarrow \pi\int_0^\pi f(\sin x)dx - 2k\int_0^\pi f(\sin x)dx = 0 \Rightarrow (\pi-2k)\int_0^\pi f(\sin x)dx = 0$

$\therefore\ \pi - 2k = 0 \Rightarrow k = \pi/2$ .

(9) If $f(x)$ is a periodic function with period $T$, then $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$ ,

**Deduction :** If $f(x)$ is a periodic function with period T and $a \in R^+$, then $\int_{nT}^{a+nT} f(x)dx = \int_0^a f(x)dx$

(10) (i) If $f(x)$ is a periodic function with period $T$, then

$$\int_a^{a+nT} f(x)\,dx = n\int_0^T f(x)\,dx \qquad \text{where } n \in I$$

(a) In particular, if $a = 0$

$$\int_0^{nT} f(x)\,dx = n\int_0^T f(x)\,dx, \qquad \text{where } n \in I$$

(b) If $n = 1$, $\int_0^{a+T} f(x)\,dx = \int_0^T f(x)\,dx,$

(i) $\int_{mT}^{nT} f(x)\,dx = (n-m)\int_0^T f(x)\,dx,$ where $n,\ m \in I$

(ii) $\int_{a+nT}^{b+nT} f(x)\,dx = \int_a^b f(x)\,dx,$ where $n\,\varepsilon\,I$

(11) If $f(x)$ is a periodic function with period *T*, then $\int_a^{a+T} f(x)$ is independent of *a*.

(12) $\int_a^b f(x)\,dx = (b-a)\int_0^1 f((b-a)x+a)\,dx$

(13) If $f(t)$ is an odd function, then $\phi(x) = \int_a^x f(t)\,dt$ is an even function

(14) If $f(x)$ is an even function, then $\phi(x) = \int_0^x f(t)\,dt$ is on odd function.

**Note** : ❑ If $f(t)$ is an even function, then for a non zero '*a*', $\int_0^x f(t)\,dt$ is not necessarily an odd function. It will be odd function if $\int_0^a f(t)\,dt = 0$

**Example: 23** For $n > 0, \int_0^{2\pi} \frac{x\sin^{2n}x}{\sin^{2n}x + \cos^{2n}x}dx$ is equal to **[IIT 1996]**

(a) $\pi^2$ (b) $2\pi^2$ (c) $3\pi^2$ (d) $4\pi^2$

**Solution:** (a) $I = \int_0^{2\pi} \frac{x\sin^{2n}x\,dx}{\sin^{2n}x+\cos^{2n}x}$ and $I = \int_0^{2\pi} \frac{(2\pi-x)\sin^{2n}(2\pi-x)dx}{\sin^{2n}(2\pi-x)+\cos^{2n}(2\pi-x)}$ $\left[\because \int_0^a f(x) = \int_0^a f(a-x)\right]$

$\therefore\ 2I = 2\pi\int_0^{2\pi} \frac{\sin^{2n}\pi}{\sin^{2n}x+\cos^{2n}x}dx \Rightarrow I = \pi\int_0^{2\pi} \frac{\sin^{2n}x}{\sin^{2n}x+\cos^{2n}x}dx$

using $\int_0^{nT} f(x) = n\int_0^T f(x)dx$

$\therefore\ I = 4\pi\int_0^{\pi/2} \frac{\sin^{2n}x}{\sin^{2n}x+\cos^{2n}x}dx \Rightarrow I = 4\pi(\pi/4) = \pi^2$.

**Example: 24** If $f(x)$ is a continuous periodic function with period *T*, then the integral $I = \int_a^{a+T} f(x)dx$ is

(a) Equal to $2a$ (b) Equal to $3a$ (c) Independent of *a* (d) None of these

**Solution:** (c) Consider the function $g(a) = \int_a^{a+T} f(x)dx = \int_a^0 f(x)dx + \int_0^T f(x)dx + \int_T^{a+T} f(x)dx$

Putting $x - T = y$ in last integral, we get $\int_T^{a+T} f(x)dx = \int_0^a f(y+T)dy = \int_0^a f(y)dy$

$\Rightarrow g(a) = \int_a^0 f(x)dx + \int_0^T f(x)dx + \int_0^a f(x)dx = \int_0^T f(x)dx$

Hence $g(a)$ is independent of *a*.

## *Important Tips*

- *Every continuous function defined on [a, b] is integrable over [a, b].*
- *Every monotonic function defined on [a, b] is integrable over [a, b].*
- *If f(x) is a continuous function defined on [a, b], then there exists* $c \in (a,b)$ *such that* $\int_a^b f(x)dx = f(c).(b-a)$.

  *The number* $f(c) = \frac{1}{(b-a)}\int_a^b f(x)dx$ *is called the mean value of the function* $f(x)$ *on the interval [a, b].*
- *If f is continuous on [a, b], then the integral function g defined by g(x) =* $\int_a^x f(t)dt$ *for* $x \in [a,b]$ *is derivable on [a, b] and* $g'(x) = f(x)$ *for all* $x \in [a,b]$.

☞ *If m and M are the smallest and greatest values of a function f(x) on an interval [a, b], then* $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$

☞ *If the function* $\varphi(x)$ *and* $\psi(x)$*, are defined on [a, b] and differentiable at a point* $x \varepsilon (a,b)$ *and f(t) is continuous for* $\phi(a) \le t \le \psi(b)$*, then* $\left(\int_{\varphi(x)}^{\psi(x)} f(t)dt\right) = f(\psi(x))\psi'(x) - f(\varphi(x))\varphi'(x)$

$$\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|\, dx$$

☞ *If* $f^2(x)$ *and* $g^2(x)$ *are integrable on [a, b], then* $\left|\int_a^b f(x)g(x)dx\right| \le \left(\int_a^b f^2(x)dx\right)^{1/2}\left(\int_a^b g^2(x)dx\right)^{1/2}$

☞ ***Change of variables*** *: If the function f(x) is continuous on [a, b] and the function* $x = \varphi(t)$ *is continuously differentiable on the interval* $[t_1, t_2]$ *and* $a = \varphi(t_1), b = \varphi(t_2)$*, then* $\int_a^b f(x)dx = \int_{t_1}^{t_2} f(\varphi(t))\varphi'(t)dt$.

☞ *Let a function* $f(x,\alpha)$ *be continuous for* $a \le x \le b$ *and* $c \le \alpha \le d$*. Then for any* $\alpha \in [c,d]$*, if* $I(\alpha) = \int_a^b f(x,\alpha)dx$*, then* $I'(\alpha) = \int_a^b f'(x,\alpha)dx$,

*Where* $I'(\alpha)$ *is the derivative of* $I(\alpha)$ *w.r.t.* $\alpha$ *and* $f'(x,\alpha)$ *is the derivative of* $f(x,\alpha)$ *w.r.t.* $\alpha$*, keeping x constant.*

☞ *For a given function* $f(x)$ *continuous on [a, b] if you are able to find two continuous function* $f_1(x)$ *and* $f_2(x)$ *on [a, b] such that* $f_1(x) \le f(x) \le f_2(x)\ \forall x \in [a,b]$*, then* $\int_a^b f_1(x)dx \le \int_a^b f(x)dx \le \int_a^b f_2(x)dx$

## 6.5 Summation of Series by Integration

We know that $\int_a^b f(x)dx = \lim_{n\to\infty} h\sum_{r=1}^{n} f(a+rh)$, where $nh = b-a$

Now, put $a = 0$, $b = 1$, $\therefore\ nh = 1$ or $h = \frac{1}{n}$. Hence $\int_0^1 f(x)dx = \lim_{n\to\infty}\frac{1}{n}\sum f\left(\frac{r}{n}\right)$

**Note** : ❑ Express the given series in the form $\Sigma \frac{1}{n} f\left(\frac{r}{h}\right)$. Replace $\frac{r}{n}$ by $x$, $\frac{1}{n}$ by $dx$ and the limit of the sum is $\int_0^1 f(x)dx$.

**Example: 25** If $S_n = \frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + ........ + \frac{1}{n+\sqrt{n^2}}$ then $\lim_{n\to\infty} S_n$ is equal to **[Roorkee 2000]**

(a) log 2 (b) 2 log 2 (c) 3 log 2 (d) 4 log 2

**Solution:** (b) $\sum \lim_{n\to\infty}\frac{1}{r+\sqrt{rn}} = \sum \lim_{n\to\infty}\frac{1}{n\left[\frac{r}{n}+\sqrt{\frac{r}{n}}\right]}$

$$\therefore \lim_{n\to\infty} S_n = \int_0^1 \frac{1}{\sqrt{x}(1+\sqrt{x})}dx$$

$$= 2[\log(1+\sqrt{x})]_0^1 = 2\log 2$$

**Example: 26** $\lim_{n\to\infty}\frac{(n!)^{1/n}}{n}$ or $\lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{1/n}$ is equal to **[WB JEE 1989; Kurukshetra CEE 1998]**

(a) $e$ (b) $e^{-1}$ (c) 1 (d) None of these

**Solution:** (b) Let $A=\lim_{n\to\infty}\frac{(n!)^{1/n}}{n}$

$\Rightarrow \log A=\lim_{n\to\infty}\log\left(\frac{1.2.3......n}{n^n}\right)^{1/n} \Rightarrow \log A=\lim_{n\to\infty}\log\left(\frac{1}{n}.\frac{2}{n}.\frac{3}{n}....\frac{n}{n}\right)^{1/n} \Rightarrow \log A=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\left[\log\left(\frac{r}{n}\right)\right]$

$\Rightarrow \log A=\int_0^1\log x dx = [x\log x - x]_0^1 \Rightarrow \log A=-1 \Rightarrow A=e^{-1}$

## 6.6 Gamma Function

If $m$ and $n$ are non-negative integers, then $\int_0^{\pi/2}\sin^m x\cos^n x dx=\frac{\Gamma\left(\frac{m+1}{2}\right)\Gamma\left(\frac{n+1}{2}\right)}{2\Gamma\left(\frac{m+n+2}{2}\right)}$

where $\Gamma(n)$ is called gamma function which satisfied the following properties

$$\Gamma(n+1)=n\Gamma(n)=n! \quad i.e. \quad \Gamma(1)=1 \text{ and } \Gamma(1/2)=\sqrt{\pi}$$

In place of gamma function, we can also use the following formula :

$$\int_0^{\pi/2}\sin^m x\cos^n x dx=\frac{(m-1)(m-3).....(2 \text{ or } 1)(n-1)(n-3).....(2 \text{ or } 1)}{(m+n)(m+n-2)....(2 \text{ or } 1)}$$

It is important to note that we multiply by ($\pi$/2); when both $m$ and $n$ are even.

**Example: 27** The value of $\int_0^{\pi/2}\sin^4 x\cos^6 x dx$ = **[Rajasthan PET 1999]**

(a) $3\pi/312$ (b) $5\pi/512$ (c) $3\pi/512$ (d) $5\pi/312$

**Solution:** (c) $I=\frac{(4-1).(4-3).(6-1).(6-3).(6-5)}{(4+6)(4+6-2)(4+6-4)(4+6-6)(4+6-8)}.\frac{\pi}{2}=\frac{3.1.5.3.1}{10.8.6.4.2.}.\frac{\pi}{2}=\frac{3\pi}{512}$

## 6.7 Reduction formulae Definite Integration

(1) $\int_0^\infty e^{-ax}\sin bx dx=\frac{b}{a^2+b^2}$ (2) $\int_0^\infty e^{-ax}\cos bx dx=\frac{a}{a^2+b^2}$ (3) $\int_0^\infty e^{-ax}x^n dx=\frac{n!}{a^n+1}$

**Example: 28** If $I_n=\int_0^\infty e^{-x}x^{n-1}dx$, then $\int_0^\infty e^{-\lambda x}x^{n-1}dx$ is equal to

(a) $\lambda I_n$ (b) $\frac{1}{\lambda}I_n$ (c) $\frac{I_n}{\lambda^n}$ (d) $\lambda^n I_n$

**Solution:** (c) Put, $\lambda x=t$, $\lambda dx=dt$, we get,

$\int_0^\infty e^{-\lambda x}x^{n-1}dx=\frac{1}{\lambda^n}\int_0^\infty e^{-t}t^{n-1}dt=\frac{1}{\lambda^n}\int_0^\infty e^{-x}x^{n-1}dx=\frac{I_n}{\lambda^n}$

## 6.8 Walli's Formula

$$\int_0^{\pi/2}\sin^n x dx=\int_0^{\pi/2}\cos^n x dx=\begin{cases}\frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}......\frac{2}{3}, & \text{when } n \text{ is odd}\\ \frac{n-1}{n}.\frac{n-3}{n-2}.\frac{n-5}{n-4}.......\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}, & \text{when } n \text{ is even}\end{cases}$$

$\int_0^{\pi/2} \sin^m x \cos^n dx = \frac{(m-1)(m-3)..........(n-1)(n-3)........}{(m+n)(m+n-2)}$ [If *m*, *n* are both odd +*ve* integers or one odd +*ve* integer]

$= \frac{(m-1)(m-3)............(n-1)(n-3)}{(m+n)(m+n-2)} \cdot \frac{\pi}{2}$ [If *m*, *n* are both +*ve* integers]

**Example: 29** $\int_0^{\pi/2} \sin^7 x dx$ has value **[BIT Ranchi 1999]**

(a) $\frac{37}{184}$ (b) $\frac{17}{45}$ (c) $\frac{16}{35}$ (d) $\frac{16}{45}$

**Solution:** (c) Using Walli's formula, $\Rightarrow I = \frac{7-1}{7} \cdot \frac{7-3}{7-2} \cdot \frac{7-5}{7-4} = \frac{6.4.2}{7.5.3} = \frac{16}{35}$

## 6.9 Leibnitz's Rule

(1) If *f*(*x*) is continuous and *u*(*x*), *v*(*x*) are differentiable functions in the interval [*a*, *b*], then,

$$\frac{d}{dx}\int_{u(x)}^{v(x)} f(t)dt = f\{v(x)\}\frac{d}{dx}\{v(x)\} - f\{u(x)\}\frac{d}{dx}\{u(x)\}$$

(2) If the function $\phi(x)$ and $\psi(x)$ are defined on [*a*,*b*] and differentiable at a point $x \in (a,b)$, and $f(x,t)$ is continuous, then, $\frac{d}{dx}\left[\int_{\phi(x)}^{\psi(x)} f(x,t)dt\right] = \int_{\phi(x)}^{\psi(x)} \frac{d}{dx} f(x,t)dt + \left\{\frac{d\,\psi(x)}{dx}\right\} f(x,\psi(x)) - \left\{\frac{d\phi(x)}{dx}\right\} f(x,\phi(x))$

**Example: 30** Let $f(x) = \int_1^x \sqrt{2-t^2}\, dt$. Then the real roots of the equation $x^2 - f'(x) = 0$ are **[IIT 2002]**

(a) $\pm 1$ (b) $\pm\frac{1}{\sqrt{2}}$ (c) $\pm\frac{1}{2}$ (d) 0 and 1

**Solution:** (a) $f(x) = \int_1^x \sqrt{2-t^2}dt \Rightarrow f'(x) = \sqrt{2-x^2}.1 - \sqrt{2-1}.0 = \sqrt{2-x^2}$

$\therefore x^2 = f'(x) = \sqrt{2-x^2} \Rightarrow x^4 + x^2 - 2 = 0 \Rightarrow (x^2+2)(x^2-1) = 0$

$\therefore x = \pm 1$ (only real).

**Example: 31** Let $f:(0,\infty) \to R$ and $f(x) = \int_0^x f(t)dt$. If $f(x^2) = x^2(1+x)$, then *f* (4) equals **[IIT 2001]**

(a) 5/4 (b) 7 (c) 4 (d) 2

**Solution:** (c) By definition of *f*(*x*) we have $f(x^2) = \int_0^{x^2} f(t)dt = x^2 + x^3$ (given)

Differentiate both sides, $f(x^2).2x + 0 = 2x + 3x^2$

Put, $x = 2 \Rightarrow 4f(4) = 16 \Rightarrow f(4) = 4$

## 6.10 Integrals with Infinite Limits (Improper Integral)

A definite integral $\int_a^b f(x)dx$ is called an improper integral, if

The range of integration is finite and the integrand is unbounded and/or the range of integration is infinite and the integrand is bounded.

*e.g.*, The integral $\int_0^1 \frac{1}{x^2}dx$ is an improper integral, because the integrand is unbounded on [0, 1]. Infact, $\frac{1}{x^2} \to \infty$ as $x \to 0$. The integral $\int_0^\infty \frac{1}{1+x^2}dx$ is an improper integral, because the range of integration is not finite.

There are following two kinds of improper definite integrals:

(1) **Improper integral of first kind :** A definite integral $\int_a^b f(x)dx$ is called an improper integral of first kind if the range of integration is not finite (*i.e.*, either $a \to \infty$ or $b \to \infty$ or $a \to \infty$ and $b \to \infty$) and the integrand *f*(*x*) is bounded on [*a*, *b*].

$$\int_1^\infty \frac{1}{x^2}dx, \int_0^\infty \frac{1}{1+x^2}dx, \int_{-\infty}^\infty \frac{1}{1+x^2}dx, \int_1^\infty \frac{3x}{(1+2x)^3}dx \text{ are improper integrals of first kind.}$$

### *Important Tips*

☞ *In an improper integral of first kind, the interval of integration is one of the following types* $[a, \infty)$, $(-\infty, b]$, $(-\infty, \infty)$.

☞ *The improper integral* $\int_a^\infty f(x)dx$ *is said to be convergent, if* $\lim_{k\to\infty}\int_a^k f(x)dx$ *exists finitely and this limit is called the value of the improper integral. If* $\lim_{k\to\infty}\int_a^k f(x)dx$ *is either* $+\infty$ *or* $-\infty$, *then the integral is said to be divergent.*

☞ *The improper integral* $\int_{-\infty}^\infty f(x)dx$ *is said to be convergent, if both the limits on the right-hand side exist finitely and are independent of each other. The improper integral* $\int_{-\infty}^\infty f(x)dx$ *is said to be divergent if the right hand side is* $+\infty$ *or* $-\infty$

(2) **Improper integral of second kind :** A definite integral $\int_a^b f(x)dx$ is called an improper integral of second kind if the range of integration [*a*, *b*] is finite and the integrand is unbounded at one or more points of [*a*, *b*].

If $\int_a^b f(x)dx$ is an improper integral of second kind, then *a*, *b* are finite real numbers and there exists at least one point $c \in [a, b]$ such that $f(x) \to +\infty$ or $f(x) \to -\infty$ as $x \to c$ *i.e.*, *f*(*x*) has at least one point of finite discontinuity in [*a*, *b*].

For example :

(i) The integral $\int_1^3 \frac{1}{x-2}dx$, is an improper integral of second kind, because $\lim_{x\to 2}\left(\frac{1}{x-2}\right) = \infty$.

(ii) The integral $\int_0^1 \log x dx$; is an improper integral of second kind, because $\log x \to \infty$ as $x \to 0$.

(iii) The integral $\int_0^{2\pi} \frac{1}{1+\cos x}dx$, is an improper integral of second kind since integrand $\frac{1}{1+\cos x}$ becomes infinite at $x = \pi \in [0, 2\pi]$.

(iv) $\int_0^1 \frac{\sin x}{x}dx$, is a proper integral since $\lim_{x\to 0}\frac{\sin x}{x} = 1$.

### *Important Tips*

☞ *Let f(x) be bounded function defined on (a, b] such that a is the only point of infinite discontinuity of f(x) i.e., f(x)* $\to \infty$ *as x* $\to$ *a. Then the improper integral of f(x) on (a, b] is denoted by* $\int_a^b f(x)dx$ *and is defined as* $\int_a^b f(x)dx = \lim_{\varepsilon\to 0}\int_{a+\varepsilon}^b f(x)dx$. *Provided that the limit on right hand side exists. If l denotes the limit on right hand side, then the improper integral* $\int_a^b f(x)dx$ *is said to converge to l, when l is finite. If l = +* $\infty$ *or l =* $-\infty$, *then the integral is said to be a divergent integral.*

☞ *Let f(x) be bounded function defined on [a, b) such that b is the only point of infinite discontinuity of f(x) i.e. f(x)* $\to \infty$ *as x* $\to$ *b. Then the improper integral of f(x) on [a, b) is denoted by* $\int_a^b f(x)dx$ *and is defined as* $\int_a^b f(x)dx = \lim_{\varepsilon\to 0}\int_a^{b-\varepsilon} f(x)dx$

*Provided that the limit on right hand side exists finitely. If l denotes the limit on right hand side, then the improper integral* $\int_a^b f(x)dx$ *is said to converge to l, when l is finite.*

*If* $l=+\infty$ *or* $l=-\infty$, *then the integral is said to be a divergent integral.*

☞ *Let f(x) be a bounded function defined on (a, b) such that a and b are only two points of infinite discontinuity of f(x) i.e., f(a)* $\to\infty$, *f(b)* $\to\infty$.

*Then the improper integral of f(x) on (a, b) is denoted by* $\int_a^b f(x)dx$ *and is defined as*

$$\int_a^b f(x)dx = \lim_{\varepsilon\to 0}\int_{a+\varepsilon}^{c} f(x)dx + \lim_{\delta\to 0}\int_a^{b-\delta} f(x)dx,\ a<c<b$$

*Provided that both the limits on right hand side exist.*

☞ *Let f(x) be a bounded function defined [a, b]-{c},* $c\in$*[a, b] and c is the only point of infinite discontinuity of f(x) i.e. f(c)*$\to\infty$. *Then the improper integral of f(x) on [a, b] – {c} is denoted by* $\int_a^b f(x)dx$ *and is defined as* $\int_a^b f(x)dx = \lim_{x\to 0}\int_a^{c-x} f(x)dx + \lim_{\delta\to 0}\int_{c+\delta}^{b} f(x)dx$

*Provided that both the limits on right hand side exist finitely. The improper integral* $\int_a^b f(x)dx$ *is said to be convergent if both the limits on the right hand side exist finitely.*

☞ *If either of the two or both the limits on RHS are* $\pm\infty$, *then the integral is said to be divergent.*

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**Example: 32** The improper integral $\int_0^\infty e^{-x}dx$ is …… and the value is….

(a) Convergent, 1 (b) Divergent, 1 (c) Convergent, 0 (d) Divergent, 0

**Solution:** (a) $I=\int_0^\infty e^{-x}dx = \lim_{k\to\infty}\int_0^k e^{-x}dx \Rightarrow I=\lim_{k\to\infty}[-e^{-x}]_0^k = \lim_{k\to\infty}[-e^{-k}+e^0] \Rightarrow I=\lim_{k\to\infty}(1-e^{-k})=1-0=1$ $[\because \lim_{k\to\infty} e^{-k}=e^{-\infty}=0]$

Thus, $\lim_{k\to\infty}\int_0^k e^{-x}dx$ exists and is finite. Hence the given integral is convergent.

**Example: 33** The integral $\int_{-\infty}^0 \frac{1}{a^2+x^2}dx, a\neq 0$ is

(a) Convergent and equal to $\frac{\pi}{a}$ (b) Convergent and equal to $\frac{\pi}{2a}$

(c) Divergent and equal to $\frac{\pi}{a}$ (d) Divergent and equal to $\frac{\pi}{2a}$

**Solution:** (b) $I=\int_{-\infty}^0 \frac{dx}{a^2+x^2} = \lim_{k\to-\infty}\int_k^0 \frac{dx}{a^2+x^2}$

$\Rightarrow I=\lim_{k\to-\infty}\left[\frac{1}{a}\tan^{-1}\frac{x}{a}\right]_k^0 = \lim_{k\to-\infty}\left[\frac{1}{a}\tan^{-1}0-\frac{1}{a}\tan^{-1}\frac{k}{a}\right] \Rightarrow I=0-\frac{1}{a}\tan^{-1}(-\infty)=-\frac{1}{a}\left(\frac{-\pi}{2}\right)=\frac{\pi}{2a}$

Hence integral is convergent.

**Example: 34** The integral $\int_{-\infty}^\infty \frac{1}{e^{-x}+e^x}dx$ is

(a) Convergent and equal to $\pi/6$ (b) Convergent and equal to $\pi/4$

(c) Convergent and equal to $\pi/3$ (d) Convergent and equal to $\pi/2$

**Solution:** (d) $I=\int_{-\infty}^\infty \frac{1}{e^{-x}+e^x}dx = \int_{-\infty}^\infty \frac{e^x}{1+e^{2x}}dx$

Put $e^x=t \Rightarrow e^x dx = dt$

$\therefore\ I=\int_0^\infty \frac{1}{1+t^2}dt \Rightarrow I=[\tan^{-1}t]_0^\infty = [\tan^{-1}\infty-\tan^{-1}0] \Rightarrow I=\pi/2$, which is finite so convergent.

**Example: 35** $\int_1^2 \frac{x+1}{\sqrt{x-1}}dx$ is

(a) Convergent and equal to $\frac{14}{3}$ (b) Divergent and equal to $\frac{3}{14}$

(c) Convergent and equal to ∞ (d) Divergent and equal to ∞

**Solution:** (a) $I=\int_1^2 \sqrt{x-1}dx+\int_1^2 \frac{2}{\sqrt{x-1}}dx = \left[\frac{2}{3}(x-1)^{3/2}\right]_1^2+[4\sqrt{x-1}]_1^2 = 14/3$ which is finite so convergent.

**Example: 36** $\int_1^2 \frac{dx}{x^2-5x+4}dx$ is

(a) Convergent and equal to $\frac{1}{3}\log 2$ (b) Convergent and equal to 3/log2

(c) Divergent (d) None of these

**Solution:** (c) $I=\int_1^2 \frac{dx}{(x-1)(x-4)}=\frac{1}{3}\int_1^2\left(\frac{1}{x-4}-\frac{1}{x-1}\right)dx = \frac{1}{3}[\log 2-\infty]=-\infty$

So the given integral is not convergent.

## 6.11 Some Important results of Definite Integral

(1) If $I_n=\int_0^{\pi/4}\tan^n x dx$ then $I_n+I_{n-2}=\frac{1}{n-1}$

(2) If $I_n=\int_0^{\pi/4}\cot^n x dx$ then $I_n+I_{n-2}=\frac{1}{1-n}$

(3) If $I_n=\int_0^{\pi/4}\sec^n x dx$ then $I_n=\frac{(\sqrt{2})^{n-2}}{n-1}+\frac{n-2}{n-1}I_{n-2}$

(4) If $I_n=\int_0^{\pi/4}\text{cosec}^n x dx$ then $I_n=\frac{(\sqrt{2})^{n-2}}{n-1}+\frac{n-2}{n-1}I_{n-2}$

(5) If $I_n=\int_0^{\pi/2}x^n \sin x dx$ then $I_n+n(n-1)I_{n-2}=n(\pi/2)^{n-1}$

(6) If $I_n=\int_0^{\pi/2}x^n \cos x dx$ then $I_n+n(n-1)I_{n-2}=(\pi/2)^n$

(7) If $a>b>0$, then $\int_0^{\pi/2}\frac{dx}{a+b\cos x}=\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\sqrt{\frac{a+b}{a-b}}$

(8) If $0<a<b$ then $\int_0^{\pi/2}\frac{dx}{a+b\cos x}=\frac{1}{\sqrt{b^2-a^2}}\log\left|\frac{\sqrt{b+a}-\sqrt{b-a}}{\sqrt{b+a}+\sqrt{b-a}}\right|$

(9) If $a>b>0$ then $\int_0^{\pi/2}\frac{dx}{a+b\sin x}=\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\sqrt{\frac{a-b}{a+b}}$

(10) If $0<a<b$, then $\int_0^{\pi/2}\frac{dx}{a+b\sin x}=\frac{1}{\sqrt{b^2-a^2}}\log\left|\frac{\sqrt{b+a}+\sqrt{b-a}}{\sqrt{b+a}-\sqrt{b-a}}\right|$

(11) If $a>b, a^2>b^2+c^2$, then $\int_0^{\pi/2}\frac{dx}{a+b\cos x+c\sin x}=\frac{2}{\sqrt{a^2-b^2-c^2}}\tan^{-1}\frac{a-b+c}{\sqrt{a^2-b^2-c^2}}$

(12) If $a>b, a^2<b^2+c^2$, then $\int_0^{\pi/2}\frac{dx}{a+b\cos x+c\sin x}=\frac{1}{\sqrt{b^2+c^2-a^2}}\log\left|\frac{a-b+c-\sqrt{b^2+c^2-a^2}}{a-b+c+\sqrt{b^2+c^2-a^2}}\right|$

(13) If $a<b,\ a^2<b^2+c^2$ then $\int_0^{\pi/2}\frac{dx}{a+b\cos x+c\sin x}=\frac{-1}{\sqrt{b^2+c^2-a^2}}\log\left|\frac{b-a-c-\sqrt{b^2+c^2-a^2}}{b-a-c+\sqrt{b^2+c^2-a^2}}\right|$

***Important Tips***

☞ $\lim_{x\to 0}\left|\frac{\int_0^x f(x)dx}{x}\right|=f(0)$

☞ $\int_a^b f(x)dx=(b-a)\int_0^1 f[(b-a)t+a]dt$

## 6.12 Integration of Piecewise Continuous Functions

Any function $f(x)$ which is discontinuous at finite number of points in an interval $[a, b]$ can be made continuous in sub-intervals by breaking the intervals into these subintervals. If $f(x)$ is discontinuous at points $x_1,\ x_2,\ x_3..........x_n$ in $(a, b)$, then we can define subintervals $(a,x_1),(x_1,x_2)............(x_{n-1},\ x_n),(x_n,b)$ such that $f(x)$ is continuous in each of these subintervals. Such functions are called piecewise continuous functions. For integration of Piecewise continuous function. We integrate $f(x)$ in these sub-intervals and finally add all the values.

**Example: 37** $\int_{-10}^{20}[\cot^{-1}x]\,dx$ , where [.] denotes greatest integer function

(a) $30+\cot 1+\cot 3$ (b) $30+\cot 1+\cot 2+\cot 3$

(c) $8\ 30+\cot 1+\cot 2$ (d) None of these

**Solution:** (b) Let $I=\int_{-10}^{20}[\cot^{-1}x]\,dx$,

we know $\cot^{-1}x\in(0,\pi)\ \forall\ x\in R$

thus, $[\cot^{-1}x]=\begin{cases}3, & x\in(-\infty,\cot 3)\\ 2, & x\in(\cot 3,\cot 2)\\ 1 & x\in(\cot 2,\cot 1)\\ 0 & x\in(\cot 1,\infty)\end{cases}$

Hence, $I=\int_{-10}^{\cot 3}3\,dx+\int_{\cot 3}^{\cot 2}2\,dx+\int_{\cot 2}^{\cot 1}1\,dx+\int_{\cot 1}^{20}0\,dx=30+\cot 1+\cot 2+\cot 3$

**Example: 38** $\int_0^2[x^2-x+1]\,dx$ , where [.] denotes greatest integer function

(a) $\frac{7-\sqrt5}{2}$ (b) $\frac{7+\sqrt5}{2}$ (c) $\frac{\sqrt5-3}{2}$ (d) None of these

**Solution:** (a) Let $I=\int_0^2[x^2-x+1]\,dx=\int_0^{\frac{1+\sqrt5}{2}}[x^2-x+1]\,dx+\int_{\frac{1+\sqrt5}{2}}^2[x^2-x+1]\,dx=\int_0^{\frac{1+\sqrt5}{2}}1\,dx+\int_{\frac{1+\sqrt5}{2}}^2 2dx=\frac{7-\sqrt5}{2}$

# Assignment

## Fundamental Definite Integral

### Basic Level

**1.** $\int_0^{-1} e^{2\ln x} =$ **[MP PET 1990]**

(a) 0 (b) 1/2 (c) 1/3 (d) 1/4

**2.** $\int_1^2 e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx =$ **[MNR 1990; AMU 1999; UPSEAT 2000]**

(a) $\frac{e^2}{2} + e$ (b) $e - \frac{e^2}{2}$ (c) $\frac{e^2}{2} - e$ (d) None of these

**3.** $\int_2^3 \frac{dx}{x^2 - x} =$ **[EAMCET 2002]**

(a) $\log \frac{2}{3}$ (b) $\log \frac{1}{4}$ (c) $\log \frac{4}{3}$ (d) $\log \frac{8}{3}$

**4.** $\int_1^3 (x-1)(x-2)(x-3) dx =$ **[Karnataka CET 2002]**

(a) 3 (b) 2 (c) 1 (d) 0

**5.** $\int_1^{\sqrt{3}} \frac{1}{1+x^2} dx$ is equal to **[DCE 2002]**

(a) $\pi/12$ (b) $\pi/6$ (c) $\pi/4$ (d) $\pi/3$

**6.** $\int_1^e \frac{1}{x} dx$ is equal to **[SCRA 1996]**

(a) $\infty$ (b) 0 (c) 1 (d) log (1 + $e$)

**7.** The value of $\int_0^{2/3} \frac{dx}{4 + 9x^2}$ is **[Rajasthan PET 1992; MP PET 1997]**

(a) $\pi/12$ (b) $\pi/24$ (c) $\pi/4$ (d) 0

**8.** $\int_0^{2\pi} \sqrt{1 + \sin\frac{x}{2}}\, dx =$ **[MNR 1987; UPSEAT 2000]**

(a) 0 (b) 2 (c) 8 (d) 4

**9.** The value of $I = \int_0^{\pi/2} \frac{(\sin x + \cos x)^2}{\sqrt{1 + \sin 2x}} dx$ is

(a) 3 (b) – 1 (c) 2 (d) 0

**10.** $\int_0^{\pi/2} e^x \sin x\, dx$ is equal to **[Roorkee 1978; EAMCET 1991]**

(a) $\frac{1}{2}(e^{\pi/2} - 1)$ (b) $\frac{1}{2}(e^{\pi/2} + 1)$ (c) $\frac{1}{2}(1 - e^{\pi/2})$ (d) $2(e^{\pi/2} + 1)$

**11.** The value of $\int_1^2 \log x\, dx$ is **[Roorkee 1995]**

(a) $\log\frac{2}{e}$ (b) log 4 (c) $\log\frac{4}{e}$ (d) log 2

**12.** $\int_0^1 \frac{d}{dx}\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]dx$ is equal to **[Kerala (Engg.) 2002]**

(a) 0 (b) $\pi$ (c) $\pi/2$ (d) $\pi/4$

**13.** The value of $\int_{-2}^{2}(ax^3+bx+c)dx$ depends on **[MNR 1988; Rajasthan PET 1990]**

(a) The value of *a* (b) The value of *b* (c) The value of *c* (d) The values of *a* and *b*

**14.** $\int_0^{\pi}\frac{dx}{1+\sin x}=$ **[CEE 1993]**

(a) 0 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{3}{2}$

**15.** $\int_0^1 \cos^{-1}x\,dx=$ **[DSSE 1988]**

(a) 0 (b) 1 (c) 2 (d) None of these

**16.** If $I=\int_0^{\pi/4}\sin^2 x\,dx$ and $J=\int_0^{\pi/4}\cos^2 x\,dx$, then $I=$ **[SCRA 1989]**

(a) $\frac{\pi}{4}-J$ (b) $2J$ (c) $J$ (d) $\frac{J}{2}$

**17.** If $x(x^4+1)\phi(x)=1$, then $\int_1^2\phi(x)dx=$ **[SCRA 1986]**

(a) $\frac{1}{4}\log\frac{32}{17}$ (b) $\frac{1}{2}\log\frac{32}{17}$ (c) $\frac{1}{4}\log\frac{16}{17}$ (d) None of these

**18.** $\int_{1/4}^{1/2}\frac{dx}{\sqrt{x-x^2}}=$ **[SCRA 1986]**

(a) $\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

**19.** $\int_0^1\frac{dx}{\sqrt{1+x}-\sqrt{x}}=$ **[SCRA 1986]**

(a) $\frac{2\sqrt{2}}{3}$ (b) $\frac{4\sqrt{2}}{3}$ (c) $\frac{8\sqrt{2}}{3}$ (d) None of these

**20.** $\int_0^{2\pi}(\sin x+\cos x)dx=$ **[SCRA 1991]**

(a) 0 (b) 2 (c) – 2 (d) 1

**21.** $\int_0^3\frac{3x+1}{x^2+9}dx=$ **[EAMCET 2003]**

(a) $\log(2\sqrt{2})+\frac{\pi}{12}$ (b) $\log(2\sqrt{2})+\frac{\pi}{2}$ (c) $\log(2\sqrt{2})+\frac{\pi}{6}$ (d) $\log(2\sqrt{2})+\frac{\pi}{3}$

**22.** $\int_0^{\pi/4}\tan^2 x dx=$ **[Roorkee 1985]**

(a) $1-\frac{\pi}{4}$ (b) $1+\frac{\pi}{4}$ (c) $\frac{\pi}{4}-1$ (d) $\frac{\pi}{4}$

**23.** $\int_0^{\pi/2}\frac{x+\sin x}{1+\cos x}dx=$ **[MP PET 1989]**

(a) – log 2 (b) log 2 (c) $\pi/2$ (d) 0

**24.** If $\int_0^1 f(x)dx=M;\int_0^1 g(x)dx=N$. Which of the following is correct

(a) $\int_0^1 (f(x)+g(x))dx = M+N$ (b) $\int_0^1 (f(x)g(x))dx = MN$ (c) $\int_0^1 \frac{1}{f(x)}dx = \frac{1}{M}$ (d) $\int_0^1 \frac{f(x)}{g(x)}dx = \frac{M}{N}$

**25.** $\int_{-\pi/4}^{\pi/2} e^{-x}\sin x dx =$ **[CEE 1993]**

(a) $-\frac{1}{2}e^{-\pi/2}$ (b) $-\frac{\sqrt{2}}{2}e^{-\pi/4}$ (c) $-\sqrt{2}(e^{\pi/4}+e^{-\pi/4})$ (d) 0

**26.** $\int_1^e \frac{1+\log x}{x}dx =$ **[SCRA 1986]**

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{e}$ (d) None of these

**27.** If $\int_0^1 x\log\left(1+\frac{x}{2}\right)dx = a+b\log\frac{2}{3}$, then **[SCRA 1986]**

(a) $a=\frac{3}{2}, b=\frac{3}{2}$ (b) $a=\frac{3}{4}, b=-\frac{3}{4}$ (c) $a=\frac{3}{4}, b=\frac{3}{2}$ (d) $a=b$

**28.** The value of $\int_0^{\pi/4}\frac{1+\tan x}{1-\tan x}dx$ is **[SCRA 1986]**

(a) $-\frac{1}{2}\log 2$ (b) $\frac{1}{4}\log 2$ (c) $\frac{1}{3}\log 2$ (d) None of these

**29.** $\int_{-1}^0 \frac{dx}{x^2+2x+2}$ is equal to **[MP PET 2000]**

(a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) $-\pi/4$

**30.** $\int_0^{\pi/6}(2+3x^2)\cos 3x\, dx =$ **[DSSE 1985]**

(a) $\frac{1}{36}(\pi+16)$ (b) $\frac{1}{36}(\pi-16)$ (c) $\frac{1}{36}(\pi^2-16)$ (d) $\frac{1}{36}(\pi^2+16)$

**31.** The values of $\alpha$ which satisfy $\int_{\pi/2}^{\alpha}\sin x\, dx = \sin 2\alpha$ $(\alpha\in[0,2\pi])$ are equal to **[IIT Screening]**

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) $\frac{7\pi}{6}$ (d) All of the above

**32.** $\int_0^{\pi/4}(\cos x-\sin x)dx+\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx+\int_{2\pi}^{\pi/4}(\cos x-\sin x)dx$ is equal to **[Rajasthan PET 2000]**

(a) $\sqrt{2}-2$ (b) $2\sqrt{2}-2$ (c) $3\sqrt{2}-2$ (d) $4\sqrt{2}-2$

**33.** If $\int_0^a x dx \le a+4$, then **[Rajasthan PET 2000]**

(a) $0\le a\le 4$ (b) $-2\le a\le 4$ (c) $-2\le a\le 0$ (d) $a\le -2$ or $a\ge 4$

**34.** $\int_0^{\pi/2}\{x-[\sin x]\}dx$ is equal to **[AMU 1999]**

(a) $\pi^2/8$ (b) $\frac{\pi^2}{8}-1$ (c) $\frac{\pi^2}{8}-2$ (d) None of these

**35.** The value of $\int_0^1 \frac{x^4+1}{x^2+1}dx$ is **[MP PET 1998]**

(a) $\frac{1}{6}(3\pi-4)$ (b) $\frac{1}{6}(3-4\pi)$ (c) $\frac{1}{6}(3\pi+4)$ (d) $\frac{1}{6}(3+4\pi)$

**36.** If $I_m = \int_1^x (\log x)^m dx$ satisfies the relation $I_m = k - l\, I_{m-1}$, then

(a) $k=e$ (b) $l=m$ (c) $k=\frac{1}{e}$ (d) None of these

**37.** If $\int_{-1}^{4} f(x)dx = 4$ and $\int_{2}^{4}(3 - f(x))dx = 7$, then the value of $\int_{2}^{-1} f(x)dx$ is

(a) 2 (b) – 3 (c) – 5 (d) None of these

**38.** The value of $\int_3^5 \frac{x^2}{x^2 - 4} dx$ is **[Roorkee 1992]**

(a) $2 - \log_e\left(\frac{15}{7}\right)$ (b) $2 + \log_e\left(\frac{15}{7}\right)$

(c) $2 + 4\log_e 3 - 4\log_e 7 + 4\log_e 5$ (d) $2 - \tan^{-1}\left(\frac{15}{7}\right)$

**39.** Let $\int_0^1 f(x)dx = 1, \int_0^1 xf(x)dx = a$ and $\int_0^1 x^2 f(x)dx = a^2$, then the value of $\int_0^1 (x-a)^2 f(x)dx =$ **[IIT 1990]**

(a) 0 (b) $a^2$ (c) $a^2 - 1$ (d) $a^2 - 2a + 2$

**40.** $\int_0^1 \frac{e^x(x-1)}{(x+1)^3} dx =$ **[SCRA 1986]**

(a) $\frac{e}{4}$ (b) $\frac{e}{4} - 1$ (c) $\frac{e}{4} + 1$ (d) None of these

**41.** $\int_0^{\pi/2} \frac{\cos x}{1 + \cos x + \sin x} dx =$ **[Roorkee 1989]**

(a) $\frac{\pi}{4} + \frac{1}{2}\log 2$ (b) $\frac{\pi}{4} + \log 2$ (c) $\frac{\pi}{4} - \frac{1}{2}\log 2$ (d) $\frac{\pi}{4} - \log 2$

**42.** $\int_{\pi/4}^{\pi/2} e^x[\log \sin x + \cot x]dx$ is equal to **[AI CBSE 1991]**

(a) $e^{\pi/4}\log 2$ (b) $-e^{\pi/4}\log 2$ (c) $\frac{1}{2}e^{\pi/4}\log 2$ (d) $-\frac{1}{2}e^{\pi/4}\log 2$

**43.** $\int_0^{\infty}(a^{-x} - b^{-x})dx =$ **[EAMCET 1995]**

(a) $\frac{1}{\log a} - \frac{1}{\log b}$ (b) $\log a - \log b$ (c) $\log a + \log b$ (d) $\frac{1}{\log a} + \frac{1}{\log b}$

**44.** $\int_0^{1/2} |\sin \pi x|\, dx$ is equal to **[DCE 1998]**

(a) 0 (b) $\pi$ (c) $-\pi$ (d) $\frac{1}{\pi}$

**45.** $\int_0^1 \frac{x}{(1-x)^{3/4}} dx =$ **[EAMCET 1992]**

(a) $\frac{12}{5}$ (b) $\frac{16}{5}$ (c) $\frac{-16}{5}$ (d) None of these

**46.** $\int_1^e \log x dx =$ **[Karnataka CET 1994]**

(a) 1 (b) $e$ – 1 (c) $e$ + 1 (d) 0

**47.** Value of $\int_0^{1/2} \frac{dx}{\sqrt{x - x^2}}$ is **[Assam JEE 1998]**

(a) $\frac{1}{2}$ (b) $\pi$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

**48.** $\int_2^4 (3x-2)^2 dx$ equals **[MP PET 1989]**

(a) 102 (b) 104 (c) 100 (d) 98

**49.** $\int_1^4 \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)dx$ equals **[MP PET 1989]**

(a) 20/3 (b) 19/3 (c) 13/2 (d) 6

**50.** $\int_0^\infty \sec hx dx$ equals **[Karnataka CET 1996]**

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) $\frac{\pi}{2}+1$ (d) 1

**51.** If $\frac{d}{dx}[f(x)]=\phi(x)$, then the value of $\int_1^2 \phi(x)dx$ equals **[Rajasthan PET 1995]**

(a) $f(1)-f(2)$ (b) $\phi(2)-\phi(1)$ (c) $f(2)-f(1)$ (d) $\phi(1)-\phi(2)$

**52.** $\int_0^a \sqrt{a^2-x^2}dx$ equals **[EAMCET 1996]**

(a) $\frac{\pi a}{4}$ (b) $\frac{\pi a^2}{4}$ (c) $\frac{\pi a^2}{2}$ (d) $\frac{\pi a}{2}$

**53.** $\int_0^{\pi/2} \sqrt{1+\sin 2x}\, dx$ equals **[Rajasthan PET 1990]**

(a) 1 (b) 1/2 (c) 2 (d) None of these

**54.** $\int_0^{\pi/4} \frac{dx}{1+\cos 2x}$ equals **[Rajasthan PET 1987]**

(a) 1 (b) – 1 (c) 1/2 (d) – 1/2

**55.** Let $I_1=\int_1^2 \frac{1}{\sqrt{1+x^2}}dx$ and $I_2=\int_1^2 \frac{1}{x}dx$ . Then

(a) $I_1>I_2$ (b) $I_2>I_1$ (c) $I_1=I_2$ (d) $I_1>2I_2$

**56.** If for every integer $n$, $\int_n^{n+1} f(x)dx=n^2$ , then the value of $\int_{-2}^4 f(x)dx$ is

(a) 16 (b) 14 (c) 19 (d) None of these

**57.** If $I_n=\int_0^1 x^n e^{-x}dx$ for $n\in N$, then $I_n-nI_{n-1}=$

(a) $e$ (b) $1/e$ (c) $-1/e$ (d) None of these

**58.** $\int_0^3 x\sqrt{(1+x)}dx$ equals

(a) 9/2 (b) 27/4 (c) 116/15 (d) None of these

**59.** $\int_1^{\frac{4\sqrt{3}}{5}-1} \frac{x+2}{\sqrt{x^2+2x-3}}dx=$

(a) $\frac{2\sqrt{3}}{3}-\frac{1}{2}\log 3$ (b) $\frac{2\sqrt{3}}{3}+\frac{1}{2}\log 3$ (c) $\frac{2\sqrt{3}}{3}-\frac{1}{2}\log(\sqrt{3}+2)$ (d) $\frac{2\sqrt{3}}{3}+\frac{1}{2}\log(\sqrt{3}+2)$

**60.** If $I_1=\int_0^1 2^{x^2}dx, I_2=\int_0^1 2^{x^3}dx, I_3=\int_1^2 2^{x^2}dx$ and $I_4=\int_1^2 2^{x^3}dx$ then

(a) $I_1>I_2$ (b) $I_2>I_1$ (c) $I_3>I_4$ (d) $I_4>I_3$

**61.** If $f(x)=\begin{vmatrix} \sin x+\sin 2x+\sin 3x & \sin 2x & \sin 3x \\ 3+4\sin x & 3 & 4\sin x \\ 1+\sin x & \sin x & 1 \end{vmatrix}$, then the value of $\int_0^{\pi/2} f(x)dx$ is

(a) 3 (b) 2/3 (c) 1/3 (d) 0

**62.** If $\int_2^3 \frac{x^2+1}{(2x+1)(x^2-1)}dx = p\log\frac{7}{5}+q\log\frac{4}{3}+r\log 2$, then

(a) $p=-\frac{5}{6}, q=1, r=\frac{1}{3}$ (b) $p=\frac{5}{6}, q=1, r=\frac{1}{3}$ (c) $p=-\frac{5}{6}, q=-1, r=-\frac{1}{3}$ (d) $p=\frac{5}{6}, q=1, r=-\frac{1}{3}$

**63.** If $\frac{1}{\sqrt{a}}\int_1^a \left(\frac{3}{2}\sqrt{x}+1-\frac{1}{\sqrt{x}}\right)dx < 4$, then 'a' may take values

(a) 0 (b) 4 (c) 9 (d) $\frac{13+\sqrt{313}}{2}$

**64.** The value of $\int_0^{\frac{\pi}{2}} \frac{\cos 3x+1}{\cos 2x-1}dx$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) 0

**65.** $\int_0^2 (t-\log_2 a)dt$ equals

(a) $\log_2(2/a)$ (b) $2\log_2(2/a)$ (c) $2\log_4(2/a)$ (d) None of these

**66.** $\int_0^{4/\pi}\left(3x^2\sin\frac{1}{x}-x\cos\frac{1}{x}\right)dx =$

(a) $\frac{8\sqrt{2}}{\pi^3}$ (b) $\frac{32\sqrt{2}}{\pi^3}$ (c) $\frac{24\sqrt{2}}{\pi^3}$ (d) $\frac{\sqrt{2048}}{\pi^3}$

**67.** The value of $\int_a^b \frac{x}{|x|}dx, a<b<0$ is **[Orissa JEE 2003]**

(a) $-(|a|+|b|)$ (b) $|b|-|a|$ (c) $|a|-|b|$ (d) $|a|+|b|$

**68.** The value of $\alpha \in (-\pi, 0)$ satisfying $\sin\alpha + \int_\alpha^{2\alpha}\cos 2x\, dx = 0$ is

(a) $-\pi/2$ (b) $-\pi$ (c) $-\pi/3$ (d) 0

**69.** If $\int_0^{36}\frac{1}{2x+9}dx = \log k$, then $k$ is equal to

(a) 3 (b) 9/2 (c) 9 (d) 81

**70.** The value of $\lim_{x\to\pi/2}\frac{\int_{\pi/2}^{x} t\,dt}{\sin(2x-\pi)}$ is

(a) $\infty$ (b) $\pi/2$ (c) $\pi/4$ (d) $\pi/8$

**71.** Suppose that $f''(x)$ is continuous for all $x$ and $f(0)=f'(1)=1$. If $\int_0^1 t\, f''(t)dt = 0$, then the value of $f(1)$ is

(a) 2 (b) 3 (c) $4\frac{1}{2}$ (d) None of these

**72.** $I_{m,n} = \int_0^1 x^m(\ln x)^n dx =$ **[EAMCET 1994]**

(a) $\frac{n}{n+1}I_{m,n-1}$ (b) $\frac{-m}{n+1}I_{m,n-1}$ (c) $\frac{-n}{n+1}I_{m,n-1}$ (d) $\frac{m}{n+1}I_{m,n-1}$

## Advance Level

**73.** If $(n-m)$ is odd and $|m| \neq |n|$, then $\int_0^\pi \cos mx \sin nx\, dx$ is

(a) $\frac{2n}{n^2-m^2}$ (b) 0 (c) $\frac{2n}{m^2-n^2}$ (d) $\frac{2m}{n^2-m^2}$

**74.** The value of the definite integral $\int_0^1 \frac{dx}{x^2+2x\cos\alpha+1}$ for $0 < \alpha < \pi$ is equal to **[Kurukshetra CEE 2002]**

(a) $\sin\alpha$ (b) $\tan^{-1}(\sin\alpha)$ (c) $\alpha\sin\alpha$ (d) $\frac{\alpha}{2}(\sin\alpha)^{-1}$

**75.** If $f(y)=e^y, g(y)=y$ ; $y > 0$ and $F(t)=\int_0^t f(t-y)g(y)dy$ , then **[AIEEE 2003]**

(a) $F(t)=1-e^{-t}(1+t)$ (b) $F(t)=e^t-(1+t)$ (c) $F(t)=te^t$ (d) $F(t)=te^{-t}$

**76.** If $l(m,n)=\int_0^1 t^m(1+t)^n dt$, then the expression for $l(m,n)$ in terms of $l(m+1,n-1)$ is **[IIT Screening 2003]**

(a) $\frac{2^n}{m+1}-\frac{n}{m+1}l(m+1,n-1)$ (b) $\frac{n}{m+1}l(m+1,n-1)$

(c) $\frac{2^n}{m+1}+\frac{n}{m+1}l(m+1,n-1)$ (d) $\frac{m}{n+1}l(m+1,n-1)$

**77.** Let $f(x)$ be a function satisfying $f'(x)=f(x)$ with $f(0)=1$ and $g(x)$ be the function satisfying $f(x)+g(x)=x^2$ The value of intergral $\int_0^1 f(x)g(x)dx$ is equal to **[AIEEE 2003]**

(a) $\frac{1}{4}(e-7)$ (b) $\frac{1}{4}(e-2)$ (c) $\frac{1}{2}(e-3)$ (d) None of these

**78.** $\int_0^{\pi/2}\left(\frac{\theta}{\sin\theta}\right)^2 d\theta =$

(a) $\pi\log 2$ (b) $\frac{\pi}{\log 2}$ (c) $\pi$ (d) None of these

**79.** The value of $\int_0^{\pi}\left|\sin^3\theta\right| d\theta$ is **[UPSEAT 2003]**

(a) 0 (b) $\frac{3}{8}$ (c) $\frac{4}{3}$ (d) $\pi$

**80.** $\int_0^{\pi}\frac{\sin\left(n+\frac{1}{2}\right)x}{\sin x}dx, (n\in N)$ is equal to **[Kurukshetra CEE 1998]**

(a) $n\pi$ (b) $(2\pi+1)\frac{\pi}{2}$ (c) $\pi$ (d) 0

**81.** If $I$ is the greatest of the definite integrals $I_1=\int_0^1 e^{-x}\cos^2 x\,dx$ , $I_2=\int_0^1 e^{-x^2}\cos^2 x\,dx$ , $I_3=\int_0^1 e^{-x^2}dx$ , $I_4=\int_0^1 e^{-x^2/2}dx$ , then

(a) $I=I_1$ (b) $I=I_2$ (c) $I=I_3$ (d) $I=I_4$

**82.** $\int_0^{\pi/2} x\cot x\,dx$ equals **[Rajasthan PET 1997]**

(a) $-\frac{\pi}{2}\log 2$ (b) $\frac{\pi}{2}\log 2$ (c) $\pi\log 2$ (d) $-\pi\log 2$

**83.** The value of $\lim_{x\to\infty}\frac{\left(\int_0^x e^{x}dx\right)^2}{\int_0^x e^{2x^2}dx}$ is

(a) 1 (b) 2 (c) 3 (d) 0

**84.** If $a < \int_0^{2\pi} \frac{1}{10+3\cos x} dx < b$, then the ordered pair (*a*, *b*) is

(a) $\left(\frac{2\pi}{7}, \frac{2\pi}{3}\right)$ (b) $\left(\frac{2\pi}{13}, \frac{2\pi}{7}\right)$ (c) $\left(\frac{\pi}{10}, \frac{2\pi}{13}\right)$ (d) None of these

**85.** Let $a_n = \int_0^{\pi/2} \cos^n x \cos nx \, dx$ , then $a_{n+1} : a_n =$

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) None of these

**86.** If $I_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin^2 x} dx$ , then

(a) $I_n = \frac{n\pi}{2}$ (b) $I_1, I_2, I_3, I_4, ........ I_n........$ are in *A.P*

(c) $\sin(I_{16}) = 0$ (d) All of these

**87,** If $n \in N$ and $\int_0^1 e^x (x-1)^n \, dx = 2e - 5$ , then *n* =

(a) 1 (b) 2 (c) 3 (d) None of these

**88.** The value of $\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x) d(x - [x])$, (where [.] denotes the greatest integer function ) is

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $\frac{2}{n-1}$ (d) None of these

**89.** The points of intersection of $f_1(x) = \int_2^x (2t-5) dt$ and $f_2(x) = \int_0^x 2t \, dt$ , are **[IIT Screening]**

(a) $\left(\frac{6}{5}, \frac{36}{25}\right)$ (b) $\left(\frac{2}{3}, \frac{4}{9}\right)$ (c) $\left(\frac{1}{3}, \frac{1}{9}\right)$ (d) $\left(\frac{1}{5}, \frac{1}{25}\right)$

**90.** The value of integral $\int_0^1 e^{x^2} dx$ lies in interval **[CEE 1993]**

(a) (0, 1) (b) (– 1, 0) (c) (1, *e*) (d) None of these

**91.** The greatest value of the function $f(x) = \int_1^x |t| dt$ on the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$ is given by **[IIT Screening]**

(a) $\frac{3}{8}$ (b) $-\frac{1}{2}$ (c) $-\frac{3}{8}$ (d) $\frac{2}{5}$

**92.** The absolute value of $\int_{10}^{19} \frac{\cos x}{1+x^8} dx$ is

(a) Less than $10^{-7}$ (b) More than $10^{-7}$ (c) Less than $10^{-6}$ (d) Both (a) and (c)

**93.** $\int_0^{\pi/4} \sin x(x - [x]) dx$ is equal to

(a) $\frac{1}{2}$ (b) $1 - \frac{1}{\sqrt{2}}$ (c) 1 (d) None of these

**94.** $\int_{-1}^{10} \text{sgn}\,(x - [x]) dx$ equals

(a) 10 (b) 11 (c) 9 (d) $\frac{11}{2}$

**95.** If $I_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x} dx$ and $a_n = \int_0^{\pi/2} \left(\frac{\sin n\theta}{\sin \theta}\right)^2 d\theta$, then $a_{n+1} - a_n =$

(a) $I_n$ (b) $2I_n$ (c) $I_{n+1}$ (d) 0

**96.** If $f'(x) = f(x) + \int_0^1 f(x)\,dx$ and given $f(0) = 1$ then $f(x) =$

(a) $\frac{e^x}{2-e} + \left(\frac{1+e}{1-e}\right)$ (b) $\frac{2e^x}{3-e} + \left(\frac{1-e}{1+e}\right)$ (c) $\frac{e^x}{2-e}$ (d) $\frac{2e^x}{3-e}$

**97.** On the interval $\left[\frac{5\pi}{3}, \frac{7\pi}{4}\right]$, the greatest value of the function $f(x) = \int_{5\pi/3}^{x} (6\cos t - 2\sin t)\,dt =$

(a) $3\sqrt{3} + 2\sqrt{2} + 1$ (b) $3\sqrt{3} - 2\sqrt{2} - 1$ (c) Does not exist (d) None of these

**98.** If $f'''(x) = k$ in [0, $a$] then $\int_0^a f(x)dx - \left\{xf(x) - \frac{x^2}{2!}f'(x) + \frac{x^3}{3!}f''(x)\right\}_0^a =$

(a) $-\frac{ka^4}{12}$ (b) $\frac{ka^4}{24}$ (c) $-\frac{ka^4}{24}$ (d) None of these

**99.** The value of $\int_0^1 \frac{2^{2x+1} - 5^{2x-1}}{10^x}\,dx$ is

(a) $\frac{3}{5}\left[\frac{2}{\log_e\left(\frac{2}{5}\right)} + \frac{1}{2\log_e\left(\frac{5}{2}\right)}\right]$ (b) $-\frac{3}{5}\left[\frac{2}{\log_e\left(\frac{2}{5}\right)} + \frac{1}{2\log_e\left(\frac{5}{2}\right)}\right]$

(c) $\frac{3}{5}\left[\frac{2}{\log_e\left(\frac{2}{5}\right)} - \frac{1}{2\log_e\left(\frac{5}{2}\right)}\right]$ (d) None of these

**100.** If $\int_0^1 e^{x^2}(x-\alpha)\,dx = 0$, then **[MNR 1994]**

(a) $1 < \alpha < 2$ (b) $\alpha < 0$ (c) $0 < \alpha < 1$ (d) None of these

**101.** The value of $\lim_{x\to 0} \frac{\int_0^x \cos t^2}{x}\,dt$ is

(a) 0 (b) 1 (c) –1 (d) None of these

**102.** If $a$ be a positive integer, the number of value of $a$ satisfying $\int_0^{\pi/2} \left\{a^2\left(\frac{\cos 3x}{4} + \frac{3}{4}\cos x\right) + a\sin x - 20\cos x\right\} dx \le -\frac{a^2}{3}$ is

(a) Only one (b) Two (c) Three (d) Four

**103.** The expression $\frac{\int_0^n [x]\,dx}{\int_0^n \{x\}\,dx}$, where $[x]$ and $\{x\}$ are integral and fractional parts of $x$ and $n \in N$ is equal to

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n}$ (c) $n$ (d) $n-1$

**104.** If $f(x) = x^3$ and $\int_a^b f(x)dx = \frac{b-a}{6}\left[f(a) + f(b) + kf\left(\frac{a+b}{2}\right)\right]$, then $k$ =

(a) 0 (b) 2 (c) 4 (d) None of these

**105.** If $I_n = \int_0^{\pi/2} x^n \sin x\,dx$ and $n > 1$ then $I_n + n(n-1)I_{n-2}$ is equal to

(a) $n\left(\frac{\pi}{2}\right)^n$ (b) $(n-1)\left(\frac{\pi}{2}\right)^n$ (c) $n\left(\frac{\pi}{2}\right)^{n-1}$ (d) $(n-1)\left(\frac{\pi}{2}\right)^{n-1}$

**106.** The value of $\int_0^{\pi} e^{\sec x} \sec^3 x(\sin^2 x + \cos x + \sin x + \sin x \cos x)\,dx$ equals

(a) 0 (b) $e + \left(\frac{1}{e}\right)$ (c) $-e - \left(\frac{1}{e}\right)$ (d) $e$

**107.** If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$, then the constants $A$ and $B$ are respectively **[IIT 1995]**

(a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$ (c) $\frac{4}{\pi}$ and 0 (d) 0 and $-\frac{4}{\pi}$

**108.** If for non-zero $x$, $af(x) + bf\left(\frac{1}{x}\right) = \frac{1}{x} - 5$, where $a \neq b$, then $\int_1^2 f(x)\,dx =$ **[IIT 1996]**

(a) $\frac{1}{(a^2+b^2)}\left[a\log 2 - 5a + \frac{7}{2}b\right]$ (b) $\frac{1}{(a^2-b^2)}\left[a\log 2 - 5a + \frac{7}{2}b\right]$

(c) $\frac{1}{(a^2-b^2)}\left[a\log 2 - 5a - \frac{7}{2}b\right]$ (d) $\frac{1}{(a^2+b^2)}\left[a\log 2 - 5a - \frac{7}{2}b\right]$

**109.** If $u_n = \int_0^{\pi/2} \frac{\sin^2 nx}{\sin x}\,dx$, then $u_2 - u_1, u_3 - u_2, u_4 - u_3, .....$ are in

(a) A.P. (b) G.P. (c) H.P. (d) None

**110.** If $f(x) = \begin{cases} x, & \text{for } x < 1 \\ x - 1, & \text{for } x \geq 1 \end{cases}$, then $\int_0^2 x^2 f(x)\,dx$ is equal to

(a) 1 (b) $\frac{4}{3}$ (c) $\frac{5}{3}$ (d) $\frac{5}{2}$

**111.** If $I = \int_0^{1/2} \frac{1}{\sqrt{1 - x^{2n}}}\,dx$, then

(a) $I \leq \frac{\pi}{6}$ (b) $I \geq \frac{\pi}{2}$ (c) $I \geq 0$ (d) All of these

**112.** If $f(x)$ and $g(x)$ are two integrable functions defined on $[a, b]$, then $\left|\int_a^b f(x)g(x)\,dx\right|$ is

(a) Less than $\sqrt{\left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)}$ (b) Less than or equal to $\sqrt{\left(\int_a^b f^2(x)\,dx\right) + \left(\int_a^b g^2(x)\,dx\right)}$

(c) Less than or equal to $\sqrt{\int_a^b (f^2(x)\,dx)\left(\int_a^b g^2(x)\,dx\right)}$ (d) None of these

**113.** If $f(x) = a + bx + cx^2$ then $\int_0^4 f(x)\,dx$ has the value

(a) $\frac{1}{6}\left\{f(0) + 2f\left(\frac{1}{2}\right) + f(1)\right\}$ (b) $\frac{1}{6}\left\{3f(0) + 2f\left(\frac{1}{2}\right) + 3f(1)\right\}$ (c) $\frac{1}{6}\left\{f(0) + 4f\left(\frac{1}{2}\right) + f(1)\right\}$ (d) $\frac{1}{6}\left\{f(0) + f\left(\frac{1}{2}\right) + f(1)\right\}$

## *Definite integral by Substitution Method*

### *Basic Level*

**114.** The value of $\int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x}\,dx$ is **[Rajasthan PET 1995]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

**115.** $\int_0^a x^2 \sin x^3 \, dx$ equals **[Rajasthan PET 1995]**

(a) $(1-\cos a^3)$ (b) $3(1-\cos a^3)$ (c) $-\frac{1}{3}(1-\cos a^3)$ (d) $\frac{1}{3}(1-\cos a^3)$

**116.** $\int_0^\infty \frac{\log(1+x^2)}{1+x^2} dx =$

(a) $\pi \log \frac{1}{2}$ (b) $\pi \log 2$ (c) $2\pi \log \frac{1}{2}$ (d) $2\pi \log 2$

**117.** $\int_0^{\pi/4} \frac{dx}{\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x} =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) None of these

**118.** The value of the integral $\int_0^{\pi/4} \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ equals **[Kurukshetra CEE 1996]**

(a) 1 (b) 2 (c) 0 (d) 4

**119.** $\int_1^2 \frac{1}{x^2} e^{-1/x} dx =$ **[DCE 2001]**

(a) $\sqrt{e}+1$ (b) $\sqrt{e}-1$ (c) $\frac{\sqrt{e}+1}{e}$ (d) $\frac{\sqrt{e}-1}{e}$

**120.** $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx =$ **[SCRA 1987; MNR 1990; Rajasthan PET 2001]**

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi^2}{4}$ (d) $\frac{\pi^2}{32}$

**121.** The value of $\int_{1/e}^{\tan x} \frac{t\,dt}{1+t^2} + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)} =$ **[IIT Screening]**

(a) –1 (b) 1 (c) 0 (d) None of these

**122.** The value of the integral $\int_{1/\pi}^{2/\pi} \frac{\sin(1/x)}{x^2} dx =$ **[IIT 1990]**

(a) 2 (b) –1 (c) 0 (d) 1

**123.** $\int_0^{\pi/3} \frac{\cos x}{3+4\sin x} dx =$

(a) $\frac{1}{4}\log\left(\frac{3+2\sqrt{3}}{2}\right)$ (b) $\frac{1}{2}\log\left(\frac{3+2\sqrt{3}}{2}\right)$ (c) $\frac{1}{3}\log\left(\frac{3+2\sqrt{3}}{2}\right)$ (d) None of these

**124.** $\int_0^1 \frac{e^{-x}}{1+e^x} dx =$ **[Roorkee 1976]**

(a) $\log\left(\frac{1+e}{e}\right)-\frac{1}{e}+1$ (b) $\log\left(\frac{1+e}{2e}\right)-\frac{1}{e}+1$ (c) $\log\left(\frac{1+e}{2e}\right)+\frac{1}{e}-1$ (d) None of these

**125.** The value of the integral $\int_0^{\log 5} \frac{e^x\sqrt{e^x-1}}{e^x+3} dx =$

(a) $3+2\pi$ (b) $4-\pi$ (c) $2+\pi$ (d) None of these

**126.** If $I_1 = \int_e^{e^2} \frac{dx}{\log x}$ and $I_2 = \int_1^2 \frac{e^x}{x} dx$ , then **[Karnataka CET 2000]**

(a) $I_1 = I_2$ (b) $I_1 > I_2$ (c) $I_1 < I_2$ (d) None of these

**127.** $\int_0^{\pi/6} \frac{\sin x}{\cos^3 x} dx =$ **[SCRA 1979]**

(a) $\frac{2}{3}$ (b) $\frac{1}{6}$ (c) 2 (d) $\frac{1}{3}$

**128.** If $I_n = \int_0^{\pi/4} \tan^n \theta \, d\theta,$ then for any positive integer $n$, the value of $n(I_{n-1} + I_{n+1})$ is

**[AIEEE 2002; Rajasthan PET 1999; Karnataka CET 2000]**

(a) 1 (b) 2 (c) $\pi / 4$ (d) $\pi$

**129.** The value of $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}} dx$ is **[SCRA 1992]**

(a) $\frac{2}{\log 3}.(3^{\sqrt{2}} - 1)$ (b) 0 (c) $2.\frac{\sqrt{2}}{\log 3}$ (d) $\frac{3^{\sqrt{2}}}{\sqrt{2}}$

**130.** $\int_0^{\pi/2} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ is equal to **[SCRA 1986; Karnataka CET 1999]**

(a) $\pi ab$ (b) $\pi^2 ab$ (c) $\frac{\pi}{ab}$ (d) $\frac{\pi}{2ab}$

**131.** If $I_1 = \int_0^x e^{zx} e^{-z^2} dz$ and $I_2 = \int_0^x e^{-z^2/4} dz$ , then, **[MP PET 1990]**

(a) $I_1 = e^x I_2$ (b) $I_1 = e^{x^2} I_2$ (c) $I_1 = e^{x^2/2} I_2$ (d) None of these

**132.** $\int_1^x \frac{\log(x^2)}{x} dx =$ **[DCE 1999]**

(a) $(\log x)^2$ (b) $\frac{1}{2}(\log x)^2$ (c) $\frac{\log x^2}{2}$ (d) None of these

**133.** The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is **[SCRA 1980]**

(a) $\tan^{-1}\left(\frac{1-e}{1+e}\right)$ (b) $\tan^{-1}\left(\frac{e-1}{e+1}\right)$ (c) $\frac{\pi}{4}$ (d) $\tan^{-1} e - \frac{\pi}{4}$

**134.** $\int_0^{\pi/2} (\sin x - \cos x) \log(\sin x + \cos x) dx =$ **[SCRA 1986]**

(a) –1 (b) 1 (c) 0 (d) None of these

**135.** $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 3\cos x + 2} dx$ is equal to **[MNR 1981; Rajasthan PET 1990; MP PET 1990]**

(a) $\log\left(\frac{8}{9}\right)$ (b) $\log\left(\frac{9}{8}\right)$ (c) $\log(8, 9)$ (d) None of these

**136.** $\int_0^{\pi/2} \frac{\cos x}{(1+\sin x)(2+\sin x)} dx =$ **[UPSEAT 1999]**

(a) $\log \frac{4}{3}$ (b) $\log \frac{1}{3}$ (c) $\log \frac{3}{4}$ (d) None of these

**137.** $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx =$ **[Karnataka CET 1999]**

(a) $\frac{\pi}{2} - 2\log\sqrt{2}$ (b) $\frac{\pi}{2} + 2\log\sqrt{2}$ (c) $\frac{\pi}{4} - \log\sqrt{2}$ (d) $\frac{\pi}{4} + \log\sqrt{2}$

**138.** $\int_0^{\pi/4} \sec^7 \theta \sin^3 \theta \, d\theta =$

(a) $\frac{1}{12}$ (b) $\frac{3}{12}$ (c) $\frac{5}{12}$ (d) None of these

**139.** The correct evaluation of $\int_0^{\pi/2} \sin x \sin 2x$ is **[MP PET 1993, 2003]**

(a) $\frac{4}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

**140.** $\int_{\pi/4}^{\pi/2} \cos\theta \,\text{cosec}^2\theta \,d\theta =$ **[Roorkee 1978]**

(a) $\sqrt{2}-1$ (b) $1-\sqrt{2}$ (c) $\sqrt{2}+1$ (d) None of these

**141.** $\int_1^2 \frac{\cos(\log x)}{x} dx =$ **[MP PET 1990]**

(a) $\sin(\log 3)$ (b) $\sin(\log 2)$ (c) $\cos(\log 3)$ (d) None of these

**142.** $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx =$ **[Roorkee 1984]**

(a) $\frac{\pi}{4}+\frac{1}{2}\log 2$ (b) $\frac{\pi}{4}-\frac{1}{2}\log 2$ (c) $\frac{\pi}{2}+\log 2$ (d) $\frac{\pi}{2}-\log 2$

**143.** $\int_0^{\pi/4} \frac{4\sin 2\theta \, d\theta}{\sin^4\theta+\cos^4\theta} =$ **[SCRA 1986]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\pi$ (d) None of these

**144.** If $I_n = \int_0^{\pi/4} \tan^n\theta \, d\theta$, then $I_8 + I_6$ equals **[Kurukshetra CEE 1996]**

(a) 1/4 (b) 1/5 (c) 1/6 (d) 1/7

**145.** $\int_0^1 \sqrt{\frac{1-x}{1+x}} dx$ equals **[Rajasthan PET 1997; Karnataka CET 1993; Bihar CEE 1998; AIEEE 2004]**

(a) $\left(\frac{\pi}{2}-1\right)$ (b) $\left(\frac{\pi}{2}+1\right)$ (c) $\pi/2$ (d) $(\pi+1)$

**146.** The value of the integral $\int_{-\pi/4}^{\pi/4} \sin^{-4} x \, dx$ is **[IIT Screening; MP PET 2003]**

(a) $\frac{3}{2}$ (b) $-\frac{8}{3}$ (c) $\frac{3}{8}$ (d) $\frac{8}{3}$

**147.** $\int_0^{\pi/2} \frac{dx}{2+\cos x} =$ **[BIT Ranchi 1992; Rajasthan PET 1993]**

(a) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\sqrt{3}\tan^{-1}(\sqrt{3})$ (c) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $2\sqrt{3}\tan^{-1}(\sqrt{3})$

**148.** $\int_0^1 \tan^{-1} x dx =$ **[Karnataka CET 1993; Rajasthan PET 1997]**

(a) $\frac{\pi}{4}-\frac{1}{2}\log 2$ (b) $\pi-\frac{1}{2}\log 2$ (c) $\frac{\pi}{4}-\log 2$ (d) $\pi-\log 2$

**149.** $\int_0^{1/2} \frac{x\sin^{-1} x}{\sqrt{1-x^2}} dx$ **[IIT 1984]**

(a) $\frac{1}{2}+\frac{\sqrt{3}\pi}{12}$ (b) $\frac{1}{2}-\frac{\sqrt{3}\pi}{12}$ (c) $\frac{1}{2}-\frac{\sqrt{3}\pi}{12}$ (d) None of these

**150.** $\int_0^2 \sqrt{\frac{2+x}{2-x}}dx =$ **[MNR 1984; CEE 1993]**

(a) $\pi + 2$ (b) $\pi + \frac{3}{2}$ (c) $\pi + 1$ (d) None of these

**151.** $\int_0^1 \frac{\log x}{\sqrt{1-x^2}}\, dx =$ **[Ranchi BIT 1984]**

(a) $\frac{\pi}{2}\log 2$ (b) $\pi \log 2$ (c) $-\frac{\pi}{2}\log 2$ (d) $-\pi \log 2$

**152.** $\int_0^{\pi/2} \frac{\sin x\, \cos x}{1+\sin^4 x}\, dx =$ **[AISSE 1988]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

**153.** $\int_0^1 \frac{dx}{[ax+b(1-x)]^2} =$ **[SCRA 1986]**

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$ (c) $ab$ (d) $\frac{1}{ab}$

**154.** $\int_0^1 \log \sin\left(\frac{\pi}{2}x\right)dx$ is equal to **[Rajasthan PET 1997]**

(a) $-\log 2$ (b) $\log 2$ (c) $\frac{\pi}{2}\log 2$ (d) $\frac{-\pi}{2}\log 2$

**155.** If $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then $\lim_{n\to\infty} n[I_n + I_{n-2}]$ equals **[AIEEE 2002]**

(a) 1/2 (b) 1 (c) $\infty$ (d) 0

**156.** The value of $\int_0^1 (x^3+3e^x+4)(x^2+e^x)dx$ is **[Rajasthan PET 1987]**

(a) $(3e-2)/6$ (b) $(3e+2)/6$ (c) $(3e-2)^2/36$ (d) None of these

**157.** $\int_{-\pi/2}^{\pi/2} \cos^3 x(1+\sin x)^2 dx$ equals **[EAMCET 1996]**

(a) 8/5 (b) 5/8 (c) –8/5 (d) –5/8

**158.** $\int_0^{\pi/2} \frac{dx}{1+\sin x}$ equals **[MNR 1983; Rajasthan PET 1990; Kurukshetra CEE 1997]**

(a) 0 (b) 1 (c) –1 (d) 2

**159.** $\int_0^{\pi/2} \frac{\sin 2x}{a\sin^2 x + b\cos^2 x} dx$ equals **[Rajasthan PET 1991]**

(a) $\frac{1}{a-b}\log\frac{b}{a}$ (b) $\frac{1}{a-b}\log\frac{a}{b}$ (c) $\frac{1}{a-b}\log(ab)$ (d) $\frac{1}{a+b}\log\frac{a}{b}$

**160.** If $u_n = \int_0^{\pi/4} \tan^n x\, dx$, then $u_2+u_4, u_3+u_5, u_4+u_6.....$ are in **[MP PET 1990]**

(a) A.P. (b) G.P. (c) H.P. (d) None

**161.** $\int_1^{e^{37}} \frac{\pi \sin(\pi \ln x)}{x} dx$ is equal to

(a) 1 (b) 2 (c) $e$ (d) 37

**162.** $\int_0^1 \frac{dx}{x+\sqrt{x}}$ equals **[Rajasthan PET 1993]**

(a) 0 (b) log 2 (c) log 3 (d) log 4

**163.** $\int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}}dx$ **[Rajasthan PET 1994]**

(a) 2 (b) 1 (c) $\frac{\pi}{4}$ (d) $\frac{\pi^2}{8}$

**164.** $\int_2^4 \frac{\sqrt{(x^2-4)}}{x}dx =$ **[Rajasthan PET 1992]**

(a) $2(3\sqrt{3}-\pi)/3$ (b) $\pi$ (c) $2(3\sqrt{3}-\pi)$ (d) None of these

**165.** The value of $\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}dx$ is

(a) $\pi/4$ (b) $\pi/3$ (c) $\pi$ (d) $2\pi$

**166.** $\int_0^{\pi/2} \frac{dx}{1+2\sin x+\cos x}$ equals **[Rajasthan PET 1991]**

(a) $(1/2)\log 3$ (b) $\log 3$ (c) $(4/3)\log 3$ (d) None of these

**167.** $\int_0^1 \frac{dx}{(x^2+1)^{3/2}}$ is equal to **[Kurukshetra CEE 1991]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\sqrt{2}$

**168.** The value of $\int_0^1 \frac{x^3}{\sqrt{1-x^8}}dx$ is

(a) $\pi/2$ (b) $\pi/4$ (c) $\pi/6$ (d) $\pi/8$

**169.** The value of $\int_0^1 \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) 1 (d) 0

**170.** $\int_0^{\pi} \frac{dx}{a+b\cos x}$ equals **[Karnataka CET 1993]**

(a) $\frac{\pi}{\sqrt{a^2-b^2}}$ (b) $\frac{\pi}{ab}$ (c) $\frac{\pi}{\sqrt{a^2+b^2}}$ (d) $\pi(a+b)$

**171.** $\int_0^1 \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ equals **[Rajasthan PET 1987]**

(a) $\pi^2/4$ (b) $\pi/4$ (c) $\pi^2/8$ (d) $\pi\ /8$

**172.** If $\int_0^1 \frac{e^t\,dt}{t+1}=a$, then $\int_{b-1}^{b} \frac{e^{-1}\,dt}{t-b-1}$ is equal to **[MP PET 1990]**

(a) $ae^{-b}$ (b) $-ae^{-b}$ (c) $-be^{-a}$ (d) $ae^{b}$

**173.** The value of the integral $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}dx$ is **[MP PET 1990]**

(a) 6 (b) 0 (c) 3 (d) 4

**174.** $\int_0^{\pi/2} \frac{\sin 2x}{a^2+b^2\sin^2 x}dx =$

(a) 0 (b) $\pi/4$ (c) $\frac{1}{b^2}\log\left(\frac{a^2+b^2}{a^2}\right)$ (d) $\frac{1}{b^2}\log\left(a^2+b^2\right)$

175. $\int_0^{\pi/2} \frac{\sin x \cos x}{1+\sin x} dx$ is

(a) $\frac{3}{2} - \log 2$ (b) $1 - \log 2$ (c) $3 - \log 2$ (d) $3 + \log 2$

176. The value of the integral $\int_2^4 \frac{\sqrt{x^2-4}}{x^4} dx$ is **[MP PET 1990]**

(a) $\sqrt{\frac{3}{32}}$ (b) $\frac{\sqrt{3}}{32}$ (c) $\frac{32}{\sqrt{3}}$ (d) $-\frac{\sqrt{3}}{32}$

177. If $\int_{\log 2}^{x} \frac{1}{\sqrt{e^x - 1}} dx = \frac{\pi}{6}$, then $x$ is equal to **[MP PET 1990]**

(a) $e^2$ (b) 1/e (c) $\log 4$ (d) none of these

178. $\int_0^1 \frac{x^7}{\sqrt{1-x^4}} dx$ is equal to **[AMU 2000]**

(a) 1 (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{\pi}{3}$

179. $\int_0^{-1} \frac{(1-x^2)}{1+x^2+x^4} dx$ equals

(a) $(1/2) \log 2$ (b) $(1/2) \tan^{-1} 3$ (c) $(1/2) \log 3$ (d) none of these

180. If $\int_0^{\pi/2} \frac{d\theta}{13 - 4\sin^2\theta - 9\cos^2\theta} = \pi k$, then **[MP PET 1990]**

(a) $k = \frac{1}{3}$ (b) $k = \frac{1}{6}$ (c) $k = \frac{1}{12}$ (d) $k = \frac{1}{13}$

181. The value of $\int_1^4 \frac{dx}{x^2 - 2x + 10}$ is

(a) 0 (b) $\infty$ (c) $\pi/12$ (d) $\pi/6$

182. The value of $\int_0^{\pi/2} \cos x . e^{\sin x} dx$ is **[MP PET 1990]**

(a) 0 (b) 1 (c) $-1$ (d) $e - 1$

183. If $u_n = \int_0^{\pi/4} \tan^n x \, dx$, then $u_n + u_{n-2} =$ **[UPSEAT 2002]**

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $\frac{1}{2n-1}$ (d) $\frac{1}{2n+1}$

184. $\int_{a+c}^{b+c} f(x-c) dx$ is equal to

(a) $\int_c^b f(x-c) dx$ (b) $\int_a^b f(x) dx$ (c) $\int_a^b f(a+b+c+x) dx$ (d) none of these

185. $\int_0^{\pi/4} \frac{\cos x}{\sqrt{(1-2\sin^2 x}} dx$ is equal to

(a) $\pi/\sqrt{2}$ (b) $\pi/2\sqrt{2}$ (c) $\pi$ (d) $\pi/2$

186. The value of the integral $\int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$ is **[MP PET 1990]**

(a) $\log 2$ (b) $\log 3$ (c) $\frac{1}{4} \log 3$ (d) $\frac{1}{8} \log 3$

**187.** $I=\int_0^{\pi/2}\frac{\sin 2x}{1+4\cos^2 x}dx$ is equal to

(a) $\frac{1}{4}\log 2$ (b) $\frac{1}{4}\log 4$ (c) $\frac{1}{4}\log 3$ (d) $\frac{1}{4}\log 5$

**188.** $\int_0^{\pi}\frac{dx}{1+2\sin^2 x}=$

(a) $\pi/\sqrt{3}$ (b) $\pi/3\sqrt{3}$ (c) $\pi/3$ (d) None of these

**189.** If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right)+x^2 f(x)=0$ for all non-zero $x$, then $\int_{\sin\theta}^{\text{cosec}\,\theta} f(x)dx$ equals

(a) $\sin\theta+\text{cosec}\,\theta$ (b) $\sin^2\theta$ (c) $\text{cosec}^2\theta$ (d) 0

**190.** $\int_{\pi/6}^{\pi/4}\frac{1}{\sqrt{\cos x\sin^3 x}}dx$ is equal to

(a) 1 (b) $\frac{1}{3}$ (c) –2 (d) none of these

**191.** The tangent to the graph of the function $y=f(x)$ at the point with abscissa $x=a$ makes with $x$-axis an angle of $\pi/3$ and at the abscissa $x=b$ an angle of $\frac{\pi}{4}$. The value of the integral $\int_a^b f'(x)\,f''(x)dx$ is

(a) $\frac{1}{2}(1-\sqrt{3})$ (b) $\frac{1}{2}(1+\sqrt{3})$ (c) –1 (d) none

**192.** If $\frac{d[f(x)]}{dx}=g(x)$ for $a\le x\le b$, then $\int_a^b f(x)g(x)dx$ equals **[CEE 1993]**

(a) $f(b)-f(a)$ (b) $g(b)-g(a)$ (c) $\frac{[f(b)]^2-[f(a)]^2}{2}$ (d) $\frac{[g(b)]^2-[g(a)]^2}{2}$

**193.** The value of $\int_0^{\pi/2}\frac{1}{9\cos x+12\sin x}dx$ is

(a) $\frac{1}{15}\log_{10}6$ (b) $\frac{1}{15}\log_e 6$ (c) $\log\left(\frac{6}{15}\right)$ (d) none of these.

**194.** Let $I=\int_0^1\frac{e^x}{x+1}dx$, then the value of the integral $\int_0^1\frac{x\,e^{x^2}}{x^2+1}dx$ is

(a) $I^2$ (b) $\frac{1}{2}I$ (c) $2\,I$ (d) $\frac{1}{2}I^2$

## Advance Level

**195.** Let $\frac{d}{dx}f(x)=\left(\frac{e^{\sin x}}{x}\right)$, $x>0$. If $\int_1^4\frac{3}{x}e^{\sin x^3}dx=f(k)-f(1)$, then one of the possible values of $k$ is

(a) 15 (b) 16 (c) 63 (d) 64

**196.** $\int_8^{15}\frac{dx}{(x-3)\sqrt{x+1}}=$ **[Rajasthan PET 1991; UPSEAT 2003]**

(a) $\frac{1}{2}\log\frac{5}{3}$ (b) $\frac{1}{3}\log\frac{5}{3}$ (c) $\frac{1}{2}\log\frac{3}{5}$ (d) $\frac{1}{5}\log\frac{3}{5}$

**197.** $\int_0^{\pi/2} \frac{1+2\cos x}{(2+\cos x)^2} dx =$ **[CEE 1993]**

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) $\frac{1}{2}$ (d) None of these

**198.** $\int_0^{\pi/4} \frac{\sin x + \cos x}{9+16\sin 2x} dx =$ **[IIT 1983]**

(a) $\frac{1}{20}\log 3$ (b) $\log 3$ (c) $\frac{1}{20}\log 5$ (d) None of these

**199.** The value of $\int_{-4}^{-5} e^{(x+5)^2} dx + 3\int_{1/3}^{2/3} e^{9(x-2/3)^2} dx$ is **[MP PET 1990]**

(a) $e^5$ (b) $e^4$ (c) $3e^2$ (d) 0

**200.** $\int_0^{\pi/4} [\sqrt{\tan x} + \sqrt{\cot x}]dx$ equals

(a) $\sqrt{2}\pi$ (b) $\pi/2$ (c) $\pi/\sqrt{2}$ (d) $2\pi$

**201.** $\int_0^{2\pi} e^{x/2}.\sin\left(\frac{x}{2}+\frac{\pi}{4}\right) dx =$ **[Roorkee 1982]**

(a) 1 (b) $2\sqrt{2}$ (c) 0 (d) None of these

**202.** $\int_0^{\pi/4} \frac{\sec x}{1+2\sin^2 x}$ is equal to **[MNR 1994]**

(a) $\frac{1}{3}\left[\log(\sqrt{2}+1)+\frac{\pi}{2\sqrt{2}}\right]$ (b) $\frac{1}{3}\left[\log(\sqrt{2}+1)-\frac{\pi}{2\sqrt{2}}\right]$ (c) $3\left[\log(\sqrt{2}+1)-\frac{\pi}{2\sqrt{2}}\right]$ (d) $3\left[\log(\sqrt{2}+1)+\frac{\pi}{2\sqrt{2}}\right]$

**203.** If $I(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1} dx$, then

(a) $I(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$ (b) $I(m,n)=\int_0^\infty \frac{x^m}{(1+x)^{m+n}} dx$ (c) $I(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$ (d) Both (a) and (c)

**204.** If $I_n=\int_{\pi/4}^{\pi/2} (\tan x)^{-n} dx \ (n>1)$, then $I_n + I_{n+2} =$ **[MP PET 1990]**

(a) $\frac{1}{n-1}$ (b) $\frac{1}{n+1}$ (c) $-\frac{1}{n+1}$ (d) $\frac{1}{n}-1$

**205.** The value of $\int_0^\pi \left(\sum_{r=0}^{3} a_r \cos^{3-r} x \sin^r x\right) dx$ depends on **[MP PET 1990]**

(a) $a_0$ and $a_2$ (b) $a_1$ and $a_2$ (c) $a_0$ and $a_3$ (d) $a_1$ and $a_3$

**206.** The value of the integral $\int_0^3 \frac{dx}{\sqrt{x+1}+\sqrt{5x+1}}$ is **[MP PET 1990]**

(a) $\frac{11}{15}$ (b) $\frac{14}{15}$ (c) $\frac{2}{5}$ (d) None of these

**207.** $\int_0^{\pi/4} \cos^{3/2} 2\theta \cos\theta \, d\theta$ equal to

(a) $3/8\sqrt{2}$ (b) $3\pi/16\sqrt{2}$ (c) $3\pi/16$ (d) None of these

**208.** The value of the integral $\int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)}\, dx$ is **[MP PET 1990]**

(a) $\frac{\pi}{4}(\beta-\alpha)^2$ (b) $\frac{\pi}{2}(\beta-\alpha)^2$ (c) $\frac{\pi}{8}(\beta-\alpha)^2$ (d) None of these.

**209.** Let $I_1=\int_0^1 \frac{\tan^{-1} x}{x} dx$ and $I_2=\frac{1}{2}\int_0^{\pi/2} \frac{t}{\sin t} dt$, then **[MP PET 1990]**

(a) $I_1=I_2$ (b) $I_1<I_2$ (c) $I_1>I_2$ (d) None of these.

**210.** $\int_0^1 \log(\sqrt{1+x}+\sqrt{1-x})\ dx =$

(a) $\frac{1}{2}\left(\log\ 2-\frac{\pi}{2}+1\right)$ (b) $\frac{1}{2}\left(\log\ 2+\frac{\pi}{2}+1\right)$ (c) $\frac{1}{2}\left(\log\ 2+\frac{\pi}{2}-1\right)$ (d) None of these

**211.** Let $\frac{d}{dx}f(x)=\frac{e^{\sin x}}{x}$, $x>0$. If $\int_1^4 \frac{2e^{\sin x^2}}{x}dx=f(k)-f(I)$ then one of possible values of $k$ is

(a) 4 (b) 16 (c) 2 (d) None of these

**212.** $\int_0^1 \sin\left(2\tan^{-1}\sqrt{\frac{1+x}{1-x}}\right) dx =$ **[EAMCET 2003]**

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/2$ (d) $\pi$

**213.** $\int_0^{\pi} \frac{dx}{1-2a\cos x+a^2}$ equals **[Haryana CEE 1993; MNR 1997]**

(a) $\frac{\pi}{2(1-a^2)}$ (b) $\pi(1-a^2)$ (c) $\frac{\pi}{1-a^2}$ (d) None of these

## *Properties of Definite Integration*

### *Basic Level*

**214.** $\int_0^1 f(1-x)\ dx$ has the same value as the integral **[SCRA 1990]**

(a) $\int_0^1 f(x)\ dx$ (b) $\int_0^1 f(-x)\ dx$ (c) $\int_0^1 f(x-1)\ dx$ (d) $\int_{-1}^1 f(x)\ dx$

**215.** $\left[\sum_{n=1}^{10}\int_{-2n-1}^{2n} \sin^{27} x\ dx\right]+\left[\sum_{n=1}^{10}\int_{-2n}^{2n+1} \sin^{27} x\ dx\right]$ equals **[MP PET 2002]**

(a) $27^2$ (b) –54 (c) 36 (d) 0

**216.** Let $a, b, c$ bc non-zero real numbers such that $\int_0^3 (3ax^2+2bx+c)\ dx=\int_1^3 (3ax^2+2bx+c)\ dx$, then

(a) $a+b+c=3$ (b) $a+b+c=1$ (c) $a+b+c=0$ (d) $a+b+c=2$

**217.** $\int_0^1 |\sin 2\pi x|\ dx$ is equal to

(a) 0 (b) $-\frac{1}{\pi}$ (c) $\frac{1}{\pi}$ (d) $\frac{2}{\pi}$

**218.** $\int_0^2 |x-1|\ dx =$ **[UPSEAT 2003]**

(a) 0 (b) 2 (c) 1/2 (d) 1

**219.** $\int_{-1}^{2} |x|\, dx =$ **[DCE 1999]**

(a) 5/2 (b) 1/2 (c) 3/2 (d) 7/2

**220.** $\int_{0}^{3} |2-x|\, dx =$ **[Rajasthan PET 1999]**

(a) 2/7 (b) 5/2 (c) 3/2 (d) $-3/2$

**221.** The value of $\int_{1}^{5} (|x-3| + |1-x|)\, dx$ is **[IIT Screening]**

(a) 10 (b) $\frac{5}{6}$ (c) 21 (d) 12

**222.** $\int_{1/e}^{e} |\log x|\, dx =$ **[UPSEAT 2001]**

(a) $1-\frac{1}{e}$ (b) $2\left(1-\frac{1}{e}\right)$ (c) $e^{-1}-1$ (d) none of these

**223.** The correct evaluation of $\int_0^{\pi/2} \left|\sin\left(x-\frac{\pi}{4}\right)\right| dx$ is **[MP PET 1993]**

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$ (c) $-2+\sqrt{2}$ (d) 0

**224.** The value of $\int_0^1 |3x^2-1|\, dx$ is **[AMU 1999]**

(a) 0 (b) $4/3\sqrt{3}$ (c) 3/7 (d) 5/6

**225.** $\int_0^{\pi/2} |\sin x-\cos x|\, dx =$ **[Roorkee 1990; MP PET 2001; UPSEAT 2001]**

(a) 0 (b) $2(\sqrt{2}-1)$ (c) $\sqrt{2}-1$ (d) $2(\sqrt{2}+1)$

**226.** $\int_0^2 |(1-x)|\, dx =$ **[SCRA 1990; Rajasthan PET 2001]**

(a) 1 (b) 2 (c) 3 (d) 0

**227.** $\int_{-4}^{4} |x+2|\, dx =$

(a) 50 (b) 24 (c) 20 (d) None of these

**228.** $\int_0^{2\pi} |\sin x|\, dx =$

(a) 0 (b) 1 (c) 2 (d) 4

**229.** Let $f(x)=x-[x]$, for every real number $x$, where $[x]$ is the integer part of $x$, then $\int_{-1}^{1} f(x)dx$ is

(a) 1 (b) 2 (c) 0 (d) 1/2

**230.** Find the value of $\int_0^9 [\sqrt{x}+2]dx$, where [.] is the greatest integer function **[UPSEAT 2002]**

(a) 31 (b) 22 (c) 23 (d) None of these

**231.** If $[x]$denotes the greatest integer less than or equal to $x$, then the value of the integral $\int_0^2 x^2[x]dx$ is

(a) 5/3 (b) 7/3 (c) 8/3 (d) 4/3

**232.** The value of $I=\int_0^1 x\left|x-\frac{1}{2}\right| dx$ is

(a) 1/3 (b) 1/4 (c) 1/8 (d) None of these

**233.** The value of $\int_0^{\sqrt{2}} [x^2]\, dx$ where [.] is the greatest integer function

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$ (c) $\sqrt{2}-1$ (d) $\sqrt{2}-2$

**234.** $\int_0^{2\pi} (\sin x+|\sin x|)\, dx =$ **[Karnataka CET 2003]**

(a) 0 (b) 4 (c) 8 (d) 1

**235.** $\int_0^{\pi} |\sin x+\cos x|\, dx$ is equal to **[WB JEE 1994]**

(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) $1/\sqrt{2}$

**236.** The value of integral $\int_{-2}^{4} x[x]dx$ is

(a) $\frac{41}{2}$ (b) 20 (c) $\frac{21}{2}$ (d) None of these.

**237.** The value of $\int_{-2}^{3} |1-x^2|\,dx$ is

(a) $\frac{1}{3}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{28}{3}$

**238.** If $a < 0 < b$, then $\int_{a}^{b} x|x|\,dx =$

(a) $\frac{1}{2}(a^2+b^2)$ (b) $\frac{1}{3}(b^2-a^2)$ (c) $\frac{1}{3}(a^3+b^3)$ (d) None of these

**239.** The value of $\int_{-1}^{3}(|x-2|+[x])dx$ is ($[x]$ stands for greatest integer less than or equal to $x$)

(a) 7 (b) 5 (c) 4 (d) 3

**240.** $\int_{0}^{\pi/2} \frac{d\theta}{1+\tan\theta}$ is equal to **[Roorkee 1980; Karnataka CET 1993; MP PET 1996; DCE 1999]**

(a) $\pi$ (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$

**241.** $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}}$ is **[DCE 2001]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{12}$ (d) $\frac{\pi}{2}$

**242.** $\int_{0}^{\pi} \frac{x\tan x}{\sec x+\cos x}dx =$ **[MNR 1985; BIT Ranchi 1986; UPSEAT 2002]**

(a) $\frac{\pi^2}{4}$ (b) $\frac{\pi^2}{2}$ (c) $\frac{3\pi^2}{2}$ (d) $\frac{\pi^2}{3}$

**243.** The value of $\int_{0}^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}}dx$ is **[Karnataka CET 1999]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\pi$ (d) $2\pi$

**244.** $\int_{0}^{\pi} \frac{x\tan x}{\sec x+\tan x}$ is equal to **[MNR 1984]**

(a) $\frac{\pi}{2}-1$ (b) $\pi\left(\frac{\pi}{2}+1\right)$ (c) $\frac{\pi}{2}+1$ (d) $\pi\left(\frac{\pi}{2}-1\right)$

**245.** The value of $\int_{0}^{1} \frac{dx}{x+\sqrt{1-x^2}}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{1}{2}$ (d) $\frac{\pi}{4}$

**246.** The value of $\int_{0}^{\pi/2} \frac{dx}{1+\tan^3 x}$ is **[IIT 1993; DCE 2001]**

(a) 0 (b) 1 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

**247.** $\int_{0}^{\pi} x\sin x\,dx =$ **[SCRA 1980, 1991]**

(a) $\pi$ (b) 0 (c) 1 (d) $\pi^2$

**248.** $\int_0^{\pi/2} \frac{1}{1+\sqrt{\tan x}} dx =$ **[Rajasthan PET 1995; Kurukshetra CEE 1998 ]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) 1

**249.** $\int_0^{\pi/2} (a\cos^2 x + b\sin^2 x)dx$ equals **[Ranchi BIT 1994]**

(a) $(a+b)\pi/4$ (b) $(a+b)\pi/2$ (c) $(a+b)\pi/3$ (d) None of these

**250.** $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ equals **[Rajasthan PET 1996; Kerala (Engg.) 2002]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

**251.** $\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx =$ **[Haryana CEE 1997; Assam JEE 1999; IIT 1999; Karnataka CET 2000]**

(a) $a$ (b) $\frac{a}{2}$ (c) $2a$ (d) 0

**252.** $\int_0^{\pi/2} \frac{\cos x - \sin x}{1+\sin x \cos x} dx =$

(a) 2 (b) –2 (c) 0 (d) None of these

**253.** $\int_0^{\pi} |\cos x| dx =$ **[MP PET 1998]**

(a) $\pi$ (b) 0 (c) 2 (d) 1

**254.** $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx =$ **[MNR 1989; UPSEAT 2002]**

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) None of these

**255.** $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}} dx =$ **[MP PET 1990, 1995; IIT 1983; MNR 1990]**

(a) $\pi$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

**256.** $\int_0^{\pi/2} \log \sin x \, dx =$ **[MP PET 1994; Rajasthan PET 1995, 96, 97]**

(a) $-\left(\frac{\pi}{2}\right)\log 2$ (b) $\pi \log \frac{1}{2}$ (c) $-\pi \log \frac{1}{2}$ (d) $\frac{\pi}{2}\log 2$

**257.** The value of $\int_0^{\pi/2} \frac{e^{x^2}}{e^{x^2}+e^{\left(\frac{\pi}{2}-x\right)^2}} dx$ is **[AMU 1999]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $e^{\pi^2/16}$ (d) $e^{\pi^2/4}$

**258.** The maximum and minimum value of the integral $\int_0^{\pi/2} \frac{dx}{(1+\sin^2 x)}$ are

(a) $\frac{\pi}{4}$ (b) $\pi$ (c) $\frac{\pi}{2}$ (d) Both (a) and (c)

**259.** $\int_0^{\pi/2} \log \tan x \, dx$ **[MP PET 1999; Rajasthan PET 2001, 02; Karnataka CET 1999, 2000, 01, 02]**

(a) $\frac{\pi}{2}\log_e^2$ (b) $-\frac{\pi}{2}\log_e^2$ (c) $\pi \log_e^2$ (d) 0

**260.** $\int_0^{\pi/2} \sin 2x \log \tan x \, dx =$ **[Kerala (Engg.) 2002; AI CBSE 1990; Karnataka CET 1996, 98]**

(a) 1 (b) –1 (c) 0 (d) None of these

**261.** $\int_0^{\pi/4} \log(1+\tan x)\, dx =$ **[SCRA 1986; Karnataka CET 2000; IIT 1997]**

(a) $\frac{\pi}{4}\log 2$ (b) $\frac{\pi}{4}\log\frac{1}{2}$ (c) $\frac{\pi}{8}\log 2$ (d) $\frac{\pi}{8}\log\frac{1}{2}$

**262.** $\int_0^{2\pi} \frac{\sin 2\theta}{a-b\cos\theta} d\theta =$ **[Roorkee 1988]**

(a) 1 (b) 2 (c) $\frac{\pi}{4}$ (d) 0

**263.** $\int_0^{\pi} x \sin^3 x \, dx =$ **[CEE 1993]**

(a) $\frac{4\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) 0 (d) None of these

**264.** If $f$ and $g$ are continuous function on $[0, a]$ satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$ then $\int_0^a f(x) g(x)\, dx =$ **[IIT 1989]**

(a) $\int_0^a f(x)\, dx$ (b) $\int_a^0 f(x)\, dx$ (c) $2\int_0^a f(x)\, dx$ (d) None of these

**265.** If $\int_0^{\pi} x f(\cos^2 x + \tan^2 x) dx = k \int_0^{\pi/2} f(\cos^2 x + \tan^4 x) dx$ , then the value of $k$ is

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) $-\frac{\pi}{2}$ (d) None of these

**266.** The value of $\int_{\pi/4}^{3\pi/4} \frac{\phi}{1+\sin\phi} d\phi$ , is **[AI CBSE 1990; IIT 1993]**

(a) $\pi \tan\frac{\pi}{8}$ (b) $\log\tan\frac{\pi}{8}$ (c) $\tan\frac{\pi}{8}$ (d) None of these

**267.** The value of $\int_0^{\pi} e^{\cos^2 x} \cos^5 3x \, dx$ is **[Bihar CEE 1994]**

(a) 1 (b) –1 (c) 0 (d) None of these

**268.** If $\int_{-1}^{1} f(x)\, dx = 0$ , then **[SCRA 1990]**

(a) $f(x) = f(-x)$ (b) $f(-x) = -f(x)$ (c) $f(x) = 2f(x)$ (d) None of these

**269.** $\int_{-\alpha}^{\alpha} f(x) dx =$ **[MP PET 1994]**

(a) $2\int_0^{\alpha} f(x) dx$ (b) $\int_{-\alpha}^{\alpha} f(-x) dx$ (c) 0 (d) None of these

**270.** The value $\int_{-2}^{2} \left[ p \ln\left(\frac{1+x}{1-x}\right) + q \ln\left(\frac{1-x}{1+x}\right)^{-2} + r \right] dx$ depends on **[Orissa JEE 2003]**

(a) The value of $p$ (b) The value of $q$ (c) the value of $r$ (d) The value of $p$ and $q$

**271.** $\int_{-\pi/2}^{\pi/2} \log\left(\frac{2-\sin x}{2+\sin x}\right) dx =$

(a) 0 (b) 1 (c) 2 (d) None of these

**272.** $\int_{-1}^{1} \log\left(\frac{1+x}{1-x}\right) dx =$ **[MP PET 1995]**

(a) 2 (b) 1 (c) 0 (d) $\pi$

**273.** To find the numerical value of $\int_{-2}^{2}(px^2+qx+s)\,dx$ it is necessary to know the values of constants **[IIT 1992]**

(a) $p$ (b) $q$ (c) $s$ (d) $p$ and $s$

**274.** $\int_{-a}^{a}\sin x\, f(\cos x)dx =$ **[Rajasthan PET 1997]**

(a) $2\int_{0}^{a}\sin x\, f(\cos x)dx$ (b) 0 (c) 1 (d) None of these

**275.** $\int_{-\pi/2}^{\pi/2}\frac{\sin x}{1+\cos^2 x}e^{-\cos^2 x}dx$ is equal to **[AMU 1999]**

(a) $2e^{-1}$ (b) 1 (c) 0 (d) None of these

**276.** $\int_{-3}^{3}\frac{x^2\sin 2x}{x^2+1}dx =$

(a) 0 (b) 1 (c) $2\log_e 3$ (d) None of these

**277.** $\int_{-\frac{1}{2}}^{\frac{1}{2}}\cos x\ln\frac{1+x}{1-x}dx$ is equal to **[AMU 2000; MNR 1998]**

(a) 0 (b) 1 (c) 2 (d) ln 3

**278.** $\int_{-1/2}^{1/2}(\cos x)\left[\log\left(\frac{1-x}{1+x}\right)\right]dx =$ **[Karnataka CET 2002]**

(a) 0 (b) 1 (c) $e^{1/2}$ (d) $2e^{1/2}$

**279.** The value of $\int_{-\pi/2}^{\pi/2}(3\sin x+\sin^3 x)dx$ is **[MP PET 2003]**

(a) 3 (b) 2 (c) 0 (d) $\frac{10}{3}$

**280.** $\int_{-1}^{1}\log\frac{2-x}{2+x}dx =$ **[Roorkee 1986; Kurukshetra CEE 1998]**

(a) 2 (b) 1 (c) –1 (d) 0

**281.** $\int_{-1}^{1}x|\,x|\,dx =$ **[MP PET 1990]**

(a) 1 (b) 0 (c) 2 (d) –2

**282.** $\int_{-2}^{2}|x|\,dx =$ **[MP PET 2000]**

(a) 0 (b) 1 (c) 2 (d) 4

**283.** The value of $\int_{-1}^{1}(\sqrt{1+x+x^2}-\sqrt{1-x+x^2})dx$ is

(a) 0 (b) 1 (c) –1 (d) None of these

**284.** $\int_{-1}^{1}\sin^3 x\cos^2 x\,dx =$ **[MNR 1991; UPSEAT 2000]**

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) 2

**285.** $\int_{-1}^{1}\sin^{11}x\,dx$ is equal to **[MNR 1995]**

(a) $\frac{10}{11}\cdot\frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}$ (b) $\frac{10}{11}\cdot\frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{3}\cdot\frac{\pi}{2}$ (c) 1 (d) 0

**286.** If. $f:R\rightarrow R$ and $g:R\rightarrow R$ are one to one, real valued functions, then the value of the integral $\int_{-\pi}^{\pi}[f(x)+f(-x)][g(x)-g(-x)]dx$ is

[DCE 2001]

(a) 0 (b) $\pi$ (c) 1 (d) None of these

**287.** $\int_{-1}^{1} x \tan^{-1} x\, dx$ equals [Rajasthan PET 1997]

(a) $\left(\frac{\pi}{2}-1\right)$ (b) $\left(\frac{\pi}{2}+1\right)$ (c) $(\pi-1)$ (d) 0

**288.** If $f(x)$ is an odd function of $x$, then $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(\cos x)\, dx$ is equal to [MP PET 1998]

(a) 0 (b) $\int_0^{\frac{\pi}{2}} f(\cos x) dx$ (c) $2\int_0^{\frac{\pi}{2}} f(\sin x) dx$ (d) $\int_0^{\pi} f(\cos x) dx$

**289.** $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2 dx$ is equal to (where $p$ and $q$ are integers ) [IIT 1992]

(a) $-\pi$ (b) 0 (c) $\pi$ (d) $2\pi$

**290.** The value of $\int_{-1}^{1} \frac{\sin x - x^2}{3-|x|} dx$ [Roorkee 1995]

(a) 0 (b) $2\int_0^1 \frac{\sin x}{3-|x|} dx$ (c) $2\int_0^1 \frac{-x^2}{3-|x|} dx$ (d) $2\int_0^1 \frac{\sin x - x^2}{3-|x|} dx$

**291.** $\int_{-\pi/2}^{\pi/2} \sqrt{\frac{1}{2}(1-\cos 2x)} dx =$

(a) 0 (b) 2 (c) 1/2 (d) None of these

**292.** $\int_{-1}^{1} x^{17} \cos^4 x\, dx =$ [MP PET 1990]

(a) – 2 (b) – 1 (c) 0 (d) 2

**293.** If $f(x) = \frac{e^x}{1+e^x}$, $I_1 = \int_{f(-a)}^{f(a)} xg\{x(1-x)\}dx$ and $I_2 = \int_{f(-a)}^{f(a)} g\{x(1-x)\}dx$, then the value of $\frac{I_2}{I_1}$ is [AIEEE 2004]

(a) 1 (b) – 3 (c) – 1 (d) 2

**294.** $\int_{-\pi/2}^{\pi/2} \sqrt{(\cos x - \cos^3 x)} dx =$ [Rajasthan PET 1991]

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 0

**295.** Let $m$ be any integer. Then the integral $\int_0^{\pi} \frac{\sin 2m x}{\sin x} dx$ equals

(a) 0 (b) $\pi$ (c) 1 (d) None of these

**296.** $\int_0^{2a} f(x) dx =$ [Rajasthan PET 2002]

(a) $2\int_0^a f(x) dx$ (b) 0 (c) $\int_0^a f(x) dx + \int_0^a f(2a-x) dx$ (d) $\int_0^a f(x) dx + \int_0^{2a} f(2a-x) dx$

**297.** If $\int_0^{2a} f(x) dx = 2\int_0^a f(x) dx$, then [SCRA 1986]

(a) $f(2a-x) = -f(x)$ (b) $f(2a-x) = f(x)$ (c) $f(a-x) = -f(x)$ (d) $f(a-x) = f(x)$

**298.** $\int_0^{\pi} \frac{\cos x\, dx}{[\cos(x/2)+\sin(x/2)]^3}$ equals

(a) 1 (b) – 1 (c) 0 (d) 2

**299.** The value of $\int_0^{2\pi} \cos^{99} x\, dx$ is

(a) 1 (b) – 1 (c) 99 (d) 0

**300.** The value of $\int_0^{2\pi} |\sin^3 x|\, dx$ is

(a) 0 (b) 3/8 (c) 8/3 (d) $\pi$

**301.** $\int_0^{\pi} \log \sin^2 x dx =$ **[MP PET 1997]**

(a) $2\pi \log_e\left(\frac{1}{2}\right)$ (b) $\pi \log_e 2 + c$ (c) $\frac{\pi}{2} \log_e\left(\frac{1}{2}\right) + c$ (d) None of these

**302.** $\int_0^{2\pi} \sin^5 x\, dx$ equals

(a) 0 (b) 16/15 (c) 32/15 (d) None of these

**303.** The value of $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx$ is **[Rajasthan PET 1988]**

(a) 1 (b) 0 (c) $\pi/4$ (d) None of these

**304.** $\int_0^{\pi/2} \frac{\sin 8x \log(\cot x)}{\cos 2x} dx$ equals

(a) 0 (b) $\pi/4$ (c) $\pi/2$ (d) None of these

**305.** $\int_0^{\pi} \sin 2x \sin^3 x\, dx$ is equal to **[RPET 1993]**

(a) 0 (b) 1 (c) 2 (d) 4

**306.** If $f(x) = f(2 - x)$, then $\int_{0.5}^{1.5} x\, f(x) dx$ equals **[AMU 1999]**

(a) $\int_0^1 f(x) dx$ (b) $\int_{0.5}^{1.5} f(x) dx$ (c) $2\int_{0.5}^{1.5} f(x) dx$ (d) 0

**307.** The value of the integral $\int_{\frac{1}{n}}^{\frac{an-1}{n}} \frac{\sqrt{x}}{\sqrt{a-x} + \sqrt{x}} dx$ is **[AMU 2002]**

(a) $\frac{a}{2}$ (b) $\frac{na+2}{2n}$ (c) $\frac{na-2}{2n}$ (d) None of these

**308.** Let $I_1 = \int_a^{\pi-a} xf(\sin x) dx, I_2 = \int_a^{\pi-a} f(\sin x) dx$, then $I_2$ is equal to **[AMU 2000]**

(a) $\frac{\pi}{2} I_1$ (b) $\pi I_1$ (c) $\frac{2}{\pi} I_1$ (d) $2I_1$

**309.** Let $f$ be a positive function. Let $I_1 = \int_{1-k}^{k} x\, f\{x(1-x)\} dx$, $I_2 = \int_{1-k}^{k} f\{x(1-x)\} dx$ where $2k - 1 > 0$. Then $I_1 / I_2$ is **[IIT 1997]**

(a) 2 (b) $k$ (c) 1/2 (d) 1

**310.** The value of $\int_2^3 \frac{dx}{\sqrt{(x-2)(3-x)}}$ is

(a) 1 (b) $\pi/2$ (c) $\pi$ (d) $2\pi$

**311.** If $I = \int_0^{100\pi} \sqrt{(1 - \cos 2x)} dx$, then the value of $I$ is

(a) $100\sqrt{2}$ (b) $200\sqrt{2}$ (c) $50\sqrt{2}$ (d) None of these

**312.** $\int_{\pi}^{10\pi} |\sin x|\, dx$ is **[AIEEE 2002]**

(a) 20 (b) 8 (c) 10 (d) 18

**313.** $\int_0^{1000} e^{x-[x]} dx$ is **[AMU 2002]**

(a) $e^{1000}-1$ (b) $\frac{e^{1000}-1}{e-1}$ (c) $1000(e-1)$ (d) $\frac{e-1}{1000}$

**314.** If $n$ is a positive integer and $[x]$ is the greatest integer not exceeding $x$, then $\int_0^x \{x-[x]\}dx$ equals

(a) $n^2/2$ (b) $n(n-1)/2$ (c) $n/2$ (d) $\frac{n^2}{2}-n$

**315.** The value of $\int_{-\frac{\pi}{2}}^{199\frac{\pi}{2}} \sqrt{(1+\cos 2x)}dx$ equals **[AMU 1999]**

(a) $50\sqrt{2}$ (b) $100\sqrt{2}$ (c) $150\sqrt{2}$ (d) $200\sqrt{2}$

**316.** If $\int_0^{50\pi}(\sin^4 x+\cos^4 x)dx = k\int_0^{\pi/2}\left(\frac{3}{4}+\frac{1}{4}\cos 4x\right)dx$, then $k =$

(a) 200 (b) 100 (c) 50 (d) 25

**317.** $\int_0^{\pi} xf(\sin x)\,dx =$ **[IIT 1982; Kurukshetra CEE 1993]**

(a) $\pi\int_0^{\pi} f(\sin x)dx$ (b) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$ (c) $\frac{\pi}{2}\int_0^{\pi/2} f(\sin x)dx$ (d) None of these

**318.** If $\int_0^{\pi} xf(\sin x)dx = A\int_0^{\pi/2} f(\sin x)dx$, then $A$ is **[AIEEE 2004]**

(a) $2\pi$ (b) $\pi$ (c) $\frac{\pi}{4}$ (d) 0

**319.** The value of the definite integral $\int_0^1 \frac{x\,dx}{x^3+16}$ lies in the interval $[a, b]$. The smallest such interval is

(a) $\left[0, \frac{1}{17}\right]$ (b) $[0, 1]$ (c) $\left[0, \frac{1}{27}\right]$ (d) None of these

**320.** If $f(x)$ is a periodic function with period $T$, then

(a) $\int_a^b f(x)dx = \int_a^{b+T} f(x)dx$ (b) $\int_a^b f(x)dx = \int_{a+T}^{b} f(x)dx$ (c) $\int_a^b f(x)dx = \int_{a+T}^{b+T} f(x)dx$ (d) $\int_a^b f(x)dx = \int_{a+T}^{b+2T} f(x)dx$

**321.** If $f(x)$ is an odd function defined on $[-T/2, T/2]$ and has period $T$, then $\phi(x) = \int_a^x f(t)\,dt$ is

(a) A periodic function with period $T/2$
(b) A periodic function with period $T$
(c) Not a periodic function
(d) A periodic function with period $T/4$

## Advance Level

**322.** If $[x]$ denotes the greatest integer less than or equal to $x$, then the value of $\int_1^5 [|x-3|]\,dx$ is

(a) 1 (b) 2 (c) 4 (d) 8

**323.** $\int_{-2}^{2} |[x]|\,dx =$ **[EAMCET 2003]**

(a) 1 (b) 2 (c) 3 (d) 4

**324.** If for a real number $y$, $[y]$ is the greatest integer less than or equal to $y$, then the value of the integral $\int_{\pi/2}^{3\pi/2} [2\sin x]\,dx$ is

**[IIT 1999]**

(a) $-\pi$ (b) 0 (c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

**325.** The value of $\int_{\pi}^{2\pi}[2\sin x]\,dx$, where [.] represents the greatest integer function is **[IIT 1995]**

(a) $-\pi$ (b) $-2\pi$ (c) $-\frac{5\pi}{3}$ (d) $\frac{5\pi}{3}$

**326.** If $f(x)=\min\{|x-1|,|x|,|x+1|\}$, then $\int_{-1}^{1}f(x)\,dx$ equals

(a) 1 (b) 0 (c) 2 (d) None of these

**327.** The value of $\int_0^2[x^2-x+1]\,dx$, (where $[x]$ denotes the greatest integer function) is given by

(a) $\frac{5-\sqrt{5}}{2}$ (b) $\frac{6-\sqrt{5}}{2}$ (c) $\frac{7-\sqrt{5}}{2}$ (d) $\frac{8-\sqrt{5}}{2}$

**328.** $\int_{-2}^{2}\min(x-[x],-x-[-x])\,dx$ equals ([$x$] represent greatest integer less than or equal to $x$)

(a) 2 (b) 1 (c) 4 (d) 0

**329.** Let $a, b, c$ be non-zero real numbers such that $\int_0^{-1}(1+\cos^8 x)(ax^2+bx+c)dx=\int_0^{2}(1+\cos^8 x)(ax^2+bx+c)dx$. Then the quadratic equation $ax^2+bx+c=0$ has

(a) No root in (0, 2) (b) At least one root in (0, 2) (c) A double root in (0, 2) (d) None of these

**330.** $\int_0^{\pi/2}f(\sin 2x)\sin x\,dx=\int_0^{\pi/2}f(\sin 2x)\cos x\,dx=\sqrt{2}\int_0^{\pi/4}f(\cos 2x)\cos x\,dx$ **[IIT 1996]**

(a) Question is true (b) Question is false (c) Some data is missing (d) None of these

**331.** If $g(x)=\int_0^x\cos^4 t\,dt$ then $g(x+\pi)$ equals **[IIT 1997 Re-Exam; DCE 2001; UPSEAT 2001]**

(a) $g(x)+g(\pi)$ (b) $g(x)-g(\pi)$ (c) $g(x).g(\pi)$ (d) $g(x)/g(\pi)$

**332.** $\int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x+b^2\sin^2 x}=$ **[Karnataka CET 2003]**

(a) $\frac{\pi}{ab}$ (b) $\frac{\pi}{2ab}$ (c) $\frac{\pi^2}{ab}$ (d) $\frac{\pi^2}{2ab}$

**333.** The value of the integral $I=\int_0^1 x(1-x)^n dx$ is **[AIEEE 2003]**

(a) $\frac{1}{n+1}$ (b) $\frac{1}{n+2}$ (c) $\frac{1}{n+1}-\frac{1}{n+2}$ (d) $\frac{1}{n+1}+\frac{1}{n+2}$

**334.** $\int_0^1\tan^{-1}\left(\frac{1}{x^2-x+1}\right)dx$ is **[Orissa JEE 2003]**

(a) $\ln 2$ (b) $-\ln 2$ (c) $\frac{\pi}{2}+\ln 2$ (d) $\frac{\pi}{2}-\ln 2$

**335.** $\int_0^1\tan^{-1}(1-x+x^2)\,dx=$ **[IIT 1998]**

(a) $\log 2$ (b) $\log\frac{1}{2}$ (c) $\pi\log 2$ (d) $\frac{\pi}{2}\log\frac{1}{2}$

**336.** $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is **[AIEEE 2002]**

(a) $\pi^2/4$ (b) $\pi^2$ (c) 0 (d) $\pi/2$

**337.** The value of the integral $\int_{-\pi}^{\pi} \sin mx \sin nx \, dx$ for $m \neq n (m, n \in I)$, is

(a) 0 (b) $\pi$ (c) $\frac{\pi}{2}$ (d) $2\pi$

**338.** The value of $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx, a > 0$, is

(a) $\pi$ (b) $a\pi$ (c) $\frac{\pi}{2}$ (d) $2\pi$

## *Summation of series by integration*

### *Basic Level*

**339.** $\lim_{n\to\infty} \frac{1^4+2^4+3^4+.....+n^4}{n^5} - \lim_{n\to\infty} \frac{1^3+2^3+3^3+.....+n^3}{n^5}$ **[AIEEE 2003]**

(a) $\frac{1}{30}$ (b) Zero (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

**340.** $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{\sqrt{n}}{\sqrt{r}(3\sqrt{r}+4\sqrt{n})^2} =$

(a) $\frac{1}{7}$ (b) $\frac{1}{10}$ (c) $\frac{1}{14}$ (d) None of these

**341.** $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{(r^2 n - m)^{1/3}}{r\,n} =$

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) 0

**342.** The value of $\lim_{n\to\infty} \left[\frac{(2n)!}{n! n^n}\right]^{1/n}$ is equal to

(a) $4e$ (b) $e/4$ (c) $4/e$ (d) None of these

**343.** $\lim_{n\to\infty} \left[\frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+........+\frac{1}{n+n}\right]$ is equal to **[Karnataka CET 1993]**

(a) $\log_e 3$ (b) 0 (c) $\log_e 2$ (d) 1

**344.** $\lim_{n\to\infty} \left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)........\left(1+\frac{n^2}{n^2}\right)\right]^{1/n} =$ ........ **[WB JEE 1992, 93]**

(a) $2e^{(\pi+4)/2}$ (b) $2e^{\pi/4-1}$ (c) $2e^{(\pi-4)/2}$ (d) None of these

**345.** $\lim_{n\to\infty} \frac{1}{n^2} \sum_{r=1}^{n} r e^{r/n} =$ **[EAMCET 1992]**

(a) 0 (b) 1 (c) $e$ (d) $2e$

**346.** $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$ equals **[IIT 1997]**

(a) $1+\sqrt{5}$ (b) $-1+\sqrt{5}$ (c) $-1+\sqrt{2}$ (d) $1+\sqrt{2}$

**347.** $\lim_{n\to\infty} \frac{1}{n} \sum_{r=n+1}^{2n} \log_e \left(1+\frac{r}{n}\right)$ equals

(a) $\log\left(\frac{27}{4e}\right)$ (b) $\log\left(\frac{27}{e^2}\right)$ (c) $\log\left(\frac{4}{e}\right)$ (d) None of these

**348.** If $f$ is continuous then $\lim_{n\to\infty} \frac{1}{n}\left[f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+f\left(\frac{3}{n}\right)+....+f\left(\frac{n}{n}\right)\right]$ is nothing but

(a) $\int_0^1 f\left(\frac{1}{x}\right)dx$ (b) $\int_0^1 x\, f(x)dx$ (c) $\int_0^1 \frac{1}{x} f\left(\frac{1}{x}\right)dx$ (d) $\int_0^1 f(x)dx$

**349.** $\lim_{n\to\infty}\left[\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}+....+\frac{1}{nb}\right]$ is equal to

(a) $\log\left(\frac{b}{a}\right)$ (b) $\log\left(\frac{a}{b}\right)$ (c) $\log a$ (d) $\log b$

**350.** $\lim_{n\to\infty} \frac{1}{n}\left[\tan\frac{\pi}{4n}+\tan\frac{2\pi}{4n}+....\tan\frac{n\pi}{4n}\right]=$

(a) $\frac{1}{\pi}\log 2$ (b) $\frac{2}{\pi}\log 2$ (c) $\frac{4}{\pi}\log 2$ (d) None of these

**351.** $\lim_{n\to\infty}\left[\frac{1}{1+n^3}+\frac{4}{8+n^3}+.....\frac{r^2}{r^3+n^3}+......+\frac{1}{2n}\right]=$

(a) $\frac{1}{2}\log 2$ (b) $\frac{1}{3}\log 2$ (c) $\log\frac{1}{2}$ (d) None of these

**352.** $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\sqrt{\left(\frac{n+r}{n-r}\right)}=$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{2}+1$ (c) $\pi$ (d) None of these

**353.** $\lim_{n\to\infty}\left[\frac{1}{\sqrt{(n^2-1)}}+\frac{1}{\sqrt{(n^2-2^2)}}+......\frac{1}{\sqrt{[n^2-(n-1)^2]}}\right]=.......$

(a) 0 (b) $\pi/2$ (c) $\pi$ (d) None of these

## Advance Level

**354.** If $na=1$ always and $n\to\infty$ then the value of $\Pi\{1+(ar)^2\}^{1/r}$ is

(a) 1 (b) $e^{\pi^2/8}$ (c) $e^{\pi^2/24}$ (d) $e^{-\pi^2/12}$

**355.** The estimated value of $\frac{1}{1001}+\frac{1}{1002}+\frac{1}{1003}+....+\frac{1}{2000}$ is

(a) 1 (b) $\log_e 3$ (c) $\log_e 2$ (d) None of these

## Gamma function

## Basic Level

**356.** $\int_0^{\pi/2}\sin^2 x\cos^3 x\, dx=$ **[Rajasthan PET 1984, 2003]**

(a) 0 (b) $\frac{2}{15}$ (c) $\frac{4}{15}$ (d) None of these

**357.** The value of $\int_0^{\pi/2}(\sqrt{\sin\theta}\cos\theta)^3 d\theta$ is **[AMU 1999]**

(a) 2/9 (b) 2/15 (c) 8/45 (d) 5/2

**358.** $\int_{-\pi/2}^{\pi/2}\sin^4 x\cos^6 x\,dx =$ **[EAMCET 2002]**

(a) $\frac{3\pi}{64}$ (b) $\frac{3\pi}{572}$ (c) $\frac{3\pi}{256}$ (d) $\frac{3\pi}{128}$

**359.** $\int_0^a x^4\sqrt{a^2-x^2}\,dx =$

(a) $\frac{\pi}{32}$ (b) $\frac{\pi}{32}a^6$ (c) $\frac{\pi}{16}a^6$ (d) $\frac{\pi}{8}a^6$

**360.** The value of $\int_0^{2\pi/3}\cos^4(3x/4)\,dx$ is

(a) $\pi/8$ (b) $9\pi/64$ (c) $9\pi/128$ (d) $\pi/4$

## *Walli's formula*

### Basic Level

**361.** $\int_0^{\pi/2}\sin^5 x\,dx =$

(a) $\frac{8}{15}$ (b) $\frac{4}{15}$ (c) $\frac{8\sqrt{\pi}}{15}$ (d) $\frac{8\pi}{15}$

**362.** Let $f(x)=\begin{vmatrix}\sec x & \cos x & \sec^2 x+\cot x\operatorname{cosec} x\\ \cos^2 x & \cos^2 x & \operatorname{cosec}^2 x\\ 1 & \cos^2 x & \cos^2 x\end{vmatrix}$, then $\int_0^{\pi/2} f(x)dx =$ **[IIT 1987]**

(a) $\frac{\pi}{4}+\frac{8}{15}$ (b) $\frac{\pi}{4}-\frac{8}{15}$ (c) $-\frac{\pi}{4}-\frac{8}{15}$ (d) $-\frac{\pi}{4}+\frac{8}{15}$

**363.** $\int_0^{\pi/2}\sin^{2m} x\,dx =$

(a) $\frac{2m\,!}{(2^m.m\,!)}\cdot\frac{\pi}{2}$ (b) $\frac{(2m)!}{(2^m.m\,!)^2}\cdot\frac{\pi}{2}$ (c) $\frac{2m\,!}{2^m(m\,!)^2}\cdot\frac{\pi}{2}$ (d) None of these

**364.** $\int_0^{\pi}\cos^3 x\,dx =$ **[Rajasthan 1995; MP PET 1996]**

(a) – 1 (b) 0 (c) 1 (d) $\pi$

**365.** $\int_0^{\pi}\sin^5\left(\frac{x}{2}\right)dx$ equals **[Kurukshetra CEE 1996]**

(a) 16/15 (b) 32/15 (c) 8/15 (d) 5/6

**366.** The value of $\int_a^{a+(\pi/2)}(\sin^4 x+\cos^4 x)dx$ is

(a) Independent of $a$ (b) $a\left(\frac{\pi}{2}\right)^2$ (c) $\frac{3\pi}{8}$ (d) $\frac{3\pi a^2}{8}$

**367.** $\int_0^a x(2ax-x^2)^{3/2}dx =$

(a) $a^5\left[\frac{3\pi}{16}-1\right]$ (b) $a^5\left[\frac{3\pi}{16}+1\right]$ (c) $a^5\left[\frac{3\pi}{16}-\frac{1}{5}\right]$ (d) None of these

## *Leibnitz's rule*

### Basic Level

**368.** The least value of the function $F(x)=\int_{5\pi/4}^{x}(3\sin u+4\cos u)du$ on the interval $[5\pi/4, 4\pi/3]$, is

(a) $\sqrt{3}+\frac{3}{2}$ (b) $-2\sqrt{3}+\frac{3}{2}+\frac{1}{\sqrt{2}}$ (c) $\frac{3}{2}+\frac{1}{\sqrt{2}}$ (d) None of these

**369.** The function $L(x)=\int_1^x \frac{dt}{t}$ satisfies the equation **[IIT 1996; DCE 2001]**

(a) $L(x+y)=L(x)+L(y)$ (b) $L\left(\frac{x}{y}\right)=L(x)+L(y)$ (c) $L(xy)=L(x)+L(y)$ (d) None of these

**370.** If $\int_{\pi/2}^{x}\sqrt{3-2\sin^2 u}\,du+\int_0^y \cos t\,dt=0$, then $\frac{dy}{dx}=$

(a) $\frac{\sqrt{4-3\sin^2 x}}{\cos y}$ (b) $-\frac{\sqrt{3-2\sin^2 x}}{\cos y}$ (c) $\sqrt{3-2\sin^2 x}+\cos y$ (d) None of these

**371.** If $\int_0^x f(t)dt = x+\int_x^1 t\,f(t)dt$, then the value of $f(1)$ is **[IIT 1998]**

(a) 1/2 (b) 0 (c) 1 (d) –1/2

**372.** If $f(t)=\int_{-t}^{t}\frac{dx}{1+x^2}$, then $f'(1)$ is **[Roorkee 2000]**

(a) Zero (b) $\frac{2}{3}$ (c) –1 (d) 1

**373.** If $f(t)=\int_x^1 \frac{dt}{1+t^2}$ and $I_2=\int_1^{1/x}\frac{dt}{1+t^2}$ for $x > 0$, then

(a) $I_1=I_2$ (b) $I_1>I_2$ (c) $I_2=\cot^{-1}x-\pi/4$ (d) Both (a) and (c)

**374.** If $\int_0^t \frac{bx\cos 4x-a\sin 4x}{x^2}dx=\frac{a\sin 4t}{t}-1$ where $0<t<\frac{\pi}{4}$, then the value of $a$, $b$ are equal to

(a) $\frac{1}{4},1$ (b) – 1, 4 (c) 2, 2 (d) 2, 4

**375.** The equation of tangent to be curve $y=\int_{x^2}^{x^3}\frac{dt}{\sqrt{1+t^2}}$ at $x = 1$ is equal to

(a) $x\sqrt{3}-y+(\sqrt{3}+1)=0$ (b) $x\sqrt{3}-y+1=0$ (c) $x-y\sqrt{2}-1=0$ (d) None of these

**376.** If $f(x)=\int_2^{x^2}\frac{(\sin^{-1}\sqrt{t})^2}{\sqrt{t}}dt$ then the value of $(1-x^2)\{f''(x)\}^2-2f'(x)$ at $x=\frac{1}{\sqrt{2}}$ is

(a) $2-\pi$ (b) $3+\pi$ (c) $4-\pi$ (d) None of these

**377.** If $f(x)=\int_0^x t\sin t\,dt$, then $f'(x)=$ **[MNR 1982; Karnataka CET 1999]**

(a) cos $x$ + $x$ sin $x$ (b) $x$ sin $x$ (c) $x$ cos $x$ (d) None of these

**378.** Let $f(x)=\int_1^x \frac{\log_e t}{1+t}dt$, and $f(x)+f\left(\frac{1}{x}\right)=k(\log_e x)^2$, then $k$ =

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

**379.** The equation $\int_0^x (t^2-8t+13)\,dt = x\sin\left(\frac{a}{x}\right)$ has a solution if $\sin\left(\frac{a}{6}\right)$ is

(a) Zero (b) – 1 (c) 1 (d) None of these

## Advance Level

**380.** The difference between the greatest and least values of the function $\phi(x)=\int_0^x (t+1)\,dt$ on [2, 3] is

(a) 3 (b) 2 (c) 7/2 (d) 11/2

**381.** If $f(x)=\int_{x^2}^{x^2+1} e^{-t^2}dt$, then $f(x)$ increases in **[IIT Screening 2003]**

(a) (2, 2) (b) No value of $x$ (c) (0, ∞) (d) (– ∞, 0)

**382.** The value of $\int_0^{n\pi+v} |\sin x|\,dx$ is

(a) $2n+1+\cos v$ (b) $2n+1-\cos v$ (c) $2n+1$ (d) $2n+\cos v$

**383.** The value of $\int_0^{\sin^4 x} \sin^{-1}\sqrt{t}\,dt+\int_0^{\cos^2 x}\cos^{-1}\sqrt{t}\,dt$ is **[MP PET 2001]**

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{\pi}{4}$ (d) None of these

**384.** The point of extreme of $\int_0^{x^2} \frac{t^2-5t+4}{2+e^t}dt$ are **[IIT Screening]**

(a) $x=-2$ (b) $x=1$ (c) $x=0$ (d) All of these

**385.** Let $g(x)=\int_0^x f(t)\,dt$ where $\frac{1}{2}\le f(t)\le 1, t\in[0,1]$ and $0\le f(t)\le\frac{1}{2}$ for $t\in(1,2]$, then **[IIT Screening 2000]**

(a) $-\frac{3}{2}\le g(2)<\frac{1}{2}$ (b) $0\le g(2)<2$ (c) $\frac{3}{2}<g(2)\le\frac{5}{2}$ (d) 2 < $g$(2) < 4

**386.** If $f(x)=\int_0^{\sin x}\cos^{-1}t\,dt+\int_0^{\cos x}\sin^{-1}t\,dt, 0<x<\frac{\pi}{2}$, then $f\left(\frac{\pi}{4}\right)=$

(a) 0 (b) $\pi\sqrt{2}$ (c) 1 (d) $1+\frac{\pi}{2\sqrt{2}}$

**387.** The function $f(x)=\int_0^x t(t-1)(t-2)dt$ is minimum, when

(a) $x$ = 0, 1 (b) $x$ = 1, 2 (c) $x$ = 0, 2 (d) None of these

## *Integration with Infinite Function*

## Basic Level

**388.** $\int_0^\infty \log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2} =$ **[Rajasthan PET 2000, 2002]**

(a) $\pi \log 2$ (b) $-\pi \log 2$ (c) $\frac{\pi}{2}\log 2$ (d) $\frac{-\pi}{2}\log 2$

**389.** $\int_0^\infty \frac{x\,dx}{(1+x)(1+x^2)} =$ **[Karnataka CET 2003]**

(a) 0 (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) 1

**390.** Given that $\int_0^\infty \frac{x^2dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} = \frac{\pi}{2(a+b)(b+c)(c+a)}$ then the value of $\int_0^\infty \frac{x^2dx}{(x^2+4)(x^2+9)}$ is **[Karnataka CET 1993]**

(a) $\frac{\pi}{60}$ (b) $\frac{\pi}{20}$ (c) $\frac{\pi}{40}$ (d) $\frac{\pi}{80}$

**391.** $\int_0^\infty \frac{dx}{(x+\sqrt{x^2+1})^3} =$ **[EAMCET 1992]**

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $-\frac{3}{8}$ (d) None of these

**392.** The value of the integral $\int_0^\infty \frac{x\log x}{(1+x^2)^2}dx$ is **[Karnataka CET 1997; AMU 2000]**

(a) 0 (b) log 7 (c) 5 log 13 (d) None of these

**393.** $\int_0^\infty \frac{1}{1+e^x}dx =$

(a) log 2 – 1 (b) log 2 (c) log 4 – 1 (d) – log 2

**394.** $\int_0^\infty \frac{\sin(\tan^{-1}x)}{1+x^2}dx$ equals **[Rajasthan PET 1988]**

(a) 0 (b) $\pi$ (c) 1 (d) $\pi/2$

**395.** $\int_0^\infty \frac{x}{1+x^4}dx$ equals **[Rajasthan PET 1994]**

(a) $\pi/8$ (b) $\pi/4$ (c) $\pi/2$ (d) $\pi$

## Advance Level

**396.** The value of the integral $I = \int_1^\infty \frac{x^2-2}{x^3\sqrt{x^2-1}}dx$ is

(a) 0 (b) 2/3 (c) 4/3 (d) None of these

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# Answer Sheet

***Assignment (Basic and Advance Level)***

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| c | c | c | d | a | c | b | c | c | b | c | c | c | c | b | a | a | d | b | a |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| a | a | c | a | a | a | c | a | b | d | d | d | b | a | a | b | c | b | a | b |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| c | c | a | d | b | a | c | b | a | a | c | b | c | c | b | c | c | c | b | a |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 | 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| c | a | a | b | b | b | b | c | a | c | a | c | a | d | b | a | d | a | c | c |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 | 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |
| d | b | d | b | a | d | b | a | a | c | c | d | b | b | c | b | b | c | b | c |
| 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 109 | 110 | 111 | 112 | 113 | 114 | 115 | 116 | 117 | 118 | 119 | 120 |
| b | d | d | c | c | c | c | b | c | c | d | c | c | b | d | b | a | b | d | d |
| 121 | 122 | 123 | 124 | 125 | 126 | 127 | 128 | 129 | 130 | 131 | 132 | 133 | 134 | 135 | 136 | 137 | 138 | 139 | 140 |
| b | d | d | b | b | a | b | a | a | d | d | a | b | c | b | a | a | c | d | a |
| 141 | 142 | 143 | 144 | 145 | 146 | 147 | 148 | 149 | 150 | 151 | 152 | 153 | 154 | 155 | 156 | 157 | 158 | 159 | 160 |
| b | b | c | d | a | b | c | a | b | a | c | d | d | a | b | d | a | b | b | c |
| 161 | 162 | 163 | 164 | 165 | 166 | 167 | 168 | 169 | 170 | 171 | 172 | 173 | 174 | 175 | 176 | 177 | 178 | 179 | 180 |
| b | d | a | a | a | a | b | d | c | a | c | b | a | c | b | b | c | b | c | c |
| 181 | 182 | 183 | 184 | 185 | 186 | 187 | 188 | 189 | 190 | 191 | 192 | 193 | 194 | 195 | 196 | 197 | 198 | 199 | 200 |
| c | d | a | b | b | c | d | a | d | d | c | c | b | b | d | a | c | a | d | c |
| 201 | 202 | 203 | 204 | 205 | 206 | 207 | 208 | 209 | 210 | 211 | 212 | 213 | 214 | 215 | 216 | 217 | 218 | 219 | 220 |
| c | a | d | b | d | d | b | c | a | c | b | b | c | a | d | a | d | d | a | b |
| 221 | 222 | 223 | 224 | 225 | 226 | 227 | 228 | 229 | 230 | 231 | 232 | 233 | 234 | 235 | 236 | 237 | 238 | 239 | 240 |
| d | b | b | b | b | a | c | d | a | a | b | c | c | b | c | a | d | c | a | d |
| 241 | 242 | 243 | 244 | 245 | 246 | 247 | 248 | 249 | 250 | 251 | 252 | 253 | 254 | 255 | 256 | 257 | 258 | 259 | 260 |
| c | a | a | d | d | d | a | b | a | c | a | c | c | c | c | a | a | d | d | c |
| 261 | 262 | 263 | 264 | 265 | 266 | 267 | 268 | 269 | 270 | 271 | 272 | 273 | 274 | 275 | 276 | 277 | 278 | 279 | 280 |
| c | d | b | a | b | a | c | b | b | c | a | c | d | b | c | a | a | a | c | d |
| 281 | 282 | 283 | 284 | 285 | 286 | 287 | 288 | 289 | 290 | 291 | 292 | 293 | 294 | 295 | 296 | 297 | 298 | 299 | 300 |
| b | d | a | a | d | a | a | c | d | c | b | c | d | c | a | c | b | c | d | c |
| 301 | 302 | 303 | 304 | 305 | 306 | 307 | 308 | 309 | 310 | 311 | 312 | 313 | 314 | 315 | 316 | 317 | 318 | 319 | 320 |
| a | a | b | a | a | b | c | c | c | c | b | d | c | c | d | b | b | b | a | c |
| 321 | 322 | 323 | 324 | 325 | 326 | 327 | 328 | 329 | 330 | 331 | 332 | 333 | 334 | 335 | 336 | 337 | 338 | 339 | 340 |
| b | b | d | c | c | d | c | b | b | a | a | d | c | d | a | b | a | c | d | c |
| 341 | 342 | 343 | 344 | 345 | 346 | 347 | 348 | 349 | 350 | 351 | 352 | 353 | 354 | 355 | 356 | 357 | 358 | 359 | 360 |
| b | c | c | c | b | b | a | d | a | b | b | b | b | c | c | b | c | c | b | d |
| 361 | 362 | 363 | 364 | 365 | 366 | 367 | 368 | 369 | 370 | 371 | 372 | 373 | 374 | 375 | 376 | 377 | 378 | 379 | 380 |
| a | c | b | b | a | c | c | b | c | b | a | d | d | a | c | d | b | d | c | c |
| 381 | 382 | 383 | 384 | 385 | 386 | 387 | 388 | 389 | 390 | 391 | 392 | 393 | 394 | 395 | 396 | | | | |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **d** | **b** | **c** | **d** | **b** | **d** | **c** | **a** | **c** | **a** | **a** | **a** | **b** | **c** | **b** | **a** |