Short Answer Type Questions – II

[3 marks]

Que 1. If sin A = $\frac{3}{4}$, calculate cos A and tan A.

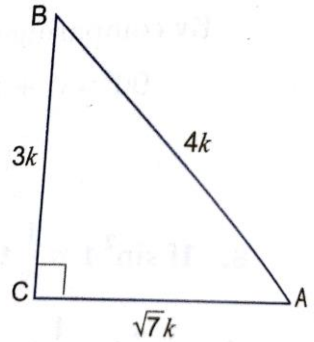

Fig. 10.3

Sol. Let us first draw a right ΔABC in which $\angle C = 90°$.

Now, we know that

$$\sin A = \frac{perpendicular}{Hypotenuse} = \frac{BC}{AB} = \frac{3}{4}$$

Let BC = 3k and AB = 4k, where k is a positive number.

Then, by Pythagoras Theorem, we have

$$AB^2 = BC^2 + AC^2 \quad \Rightarrow (4k)^2 = (3k)^2 + AC^2$$

$$\Rightarrow \quad 16k^2 - 9k^2 = AC^2 \quad \Rightarrow 7k^2 = AC^2$$

$$\therefore \quad AC = \sqrt{7}k$$

$$\therefore \quad \cos A = \frac{AC}{AB} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4} \quad \text{and} \quad \tan A = \frac{BC}{AC} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}.$$

Que 2. Given 15 cot A = 8, find sin A and sec A.

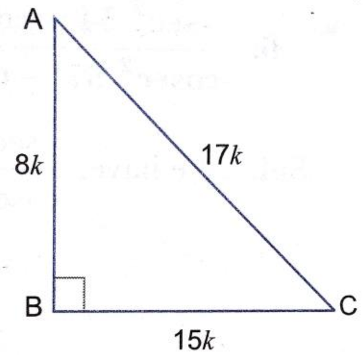

Fig. 10.4

Sol. Let us first draw a rightΔABC, in which $\angle B = 90°$.

Now, we have, 15 cot A = 8

$$\therefore \quad cot\ A = \frac{8}{15} = \frac{AB}{BC} = \frac{Base}{Perpendicular}$$

Let AB = 8k and BC = 15k

Then, AC = $\sqrt{(AB)^2 + (BC)^2}$ (By Pythagoras Theorem)

$= \sqrt{(8k)^2 + (15k)^2} = \sqrt{64k^2 + 225k^2} = \sqrt{289k^2} = 17k$

$$\therefore \quad \sin A = \frac{Perpendicular}{Hypotenuse} = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

And, sec A = $\frac{Hypotenuse}{Base} = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8}$.

Que 3. In Fig. 10.5, find tan P – cot R.

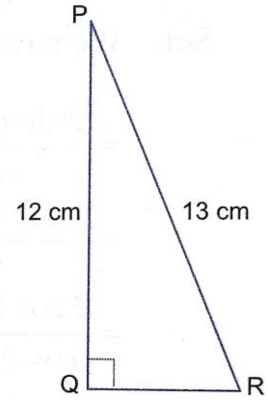

Fig. 10.5

Sol. Using Pythagoras Theorem, we have

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (13)^2 = (12)^2 + QR^2$

$\Rightarrow 169 = 144 + QR^2$

$\Rightarrow QR^2 = 169 – 144 = 25 \quad \Rightarrow QR = 5$ cm

Now, tan P = $\frac{QR}{PO} = \frac{5}{12}$ and cot R = $\frac{QR}{PQ} = \frac{5}{12}$

$$\therefore \quad \tan P - cot\ R = \frac{5}{12} - \frac{5}{12} = 0.$$

Que 4. If sin $\boldsymbol{\theta}$ + cos $\boldsymbol{\theta} = \sqrt{3}$, then prove that tan $\boldsymbol{\theta}$ + cot $\boldsymbol{\theta}$ = 1.

Sol. sin θ + cos $\theta = \sqrt{3}$

$\Rightarrow (\sin \theta + \cos \theta)^2 = 3$

$\Rightarrow \sin^2 \theta + \cos^2 \theta + 2 \sin\theta \cos \theta = 3$

$\Rightarrow 2 \sin \theta \cos \theta = 2 \quad (\because \sin^2 \theta + \cos^2 \theta = 1)$

$\Rightarrow \sin \theta. \text{Cos}\ \theta = 1 = \sin^2 \theta + \cos^2 \theta$

$\Rightarrow \qquad 1 = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\Rightarrow \qquad 1 = \tan\theta + \cot\theta$

Therefore tan θ + cot θ = 1

Que 5. Prove that $\frac{1-\sin\theta}{1+\sin\theta} = (\boldsymbol{sec\ \theta} - \mathbf{tan}\ \boldsymbol{\theta})^2$

Sol. LHS $= \frac{1-\sin\theta}{1+\sin\theta}$

$= \frac{1-\sin\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$ [Rationalising the denominator]

$= \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\vartheta} - \frac{\sin\theta}{\cos\theta}\right)^2$

$= (sec\ \theta - \tan\theta)^2 = RHS$

Without using tables, evaluate the following (6 to 10).

Que 6. $\frac{sec^2\ 54° - cot^2\ 36°}{cosec^2\ 57° - \tan^2 33°} + 2\sin^2 38°.sec^2\ 52° - \sin^2 45°.$

Sol. We have, $\frac{sec^2\ 54° - cot^2\ 36°}{cosec^2\ 57° - \tan^2 33°} + 2\sin^2 38°.sec^2\ 52° - \sin^2 45°.$

$= \frac{sec^2\ (90°-36°) - cot^2\ 36°}{cosec^2\ (90°-33°) - \tan^2 33°} + 2\sin^2 38°.sec^2\ (90° - 38°) - \sin^2 45°$

$= \frac{cosec^2\ 36° - cot^2\ 36°}{sec^2\ 33° - \tan^2 33°} + 2\sin^2 38°.cosec^2\ 38 - \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{1} + 2.1 - \frac{1}{2} = 3 - \frac{1}{2} = \frac{5}{2}.$

Que 7. $\frac{2\sin 68°}{\cos 22°} - \frac{2\ cot\ 15°}{5\tan 75°} - \frac{3\tan 45°.\tan 20°.\tan 40°.\tan 50°.\tan 70°}{5}.$

Sol. We have $\frac{2\sin 68°}{\cos 22°} - \frac{2\ cot\ 15°}{5\tan 75°} - \frac{3\tan 45°.\tan 20°.\tan 40°.\tan 50°.\tan 70°}{5}$

$= \frac{2\sin(90°-22°)}{\cos 22°} - \frac{2\ cot\ 15°}{5\tan(90°-15°)}$

$- \frac{3\tan 45°.\tan 20°.\tan 40°.\tan(90°-40°).\tan(90°-20°)}{5}$

$= \frac{2\cos 22°}{\cos 22°} - \frac{2\ cot\ 15°}{5\ cot\ 15°} - \frac{3\tan 45°,\tan 20°.\tan 40°.cot\ 40°.cot\ 20°}{5}$

$= 2 - \frac{2}{5} - \frac{3\tan 45°.(\tan 20°.cot\ 20°).(\tan 40°.cot\ 40°)}{5}$

$$2 - \frac{2}{5} - \frac{3}{5}.1.1.1 = 2 - \frac{2}{5} - \frac{3}{5} = 2 - 1 = 1.$$

Que 8. $\frac{\sin^2 20° + \sin^2 70°}{\cos^2 20° + \cos^2 70°} + \left[\frac{\sin(90°-\theta\sin\theta}{\tan\theta} + \frac{\cos(90°-\theta).\cos\theta}{cot\ \theta}\right].$

Sol. We have $\frac{\sin^2 20° + \sin^2 70°}{\cos^2 20° + \cos^2 70°} + \left[\frac{\sin(90°-\theta\sin\theta}{\tan\theta} + \frac{\cos(90°-\theta).\cos\theta}{cot\ \theta}\right]$

$$= \frac{\sin^2 20°+\sin^2 (90°-20°)}{\cos^2 20°+\cos^2 (90°-20°)} + \left[\frac{\cos\theta\sin\theta}{\tan\theta} + \frac{\cos\theta\sin\theta}{cot\ \theta}\right]$$

$$= \frac{\sin^2 20°+\cos^2\ 20°}{\cos^2 20°+\sin^2 20°} + \left[\frac{\cos\theta\sin\theta}{\frac{\sin\theta}{\cos\theta}} + \frac{\cos\theta\sin\theta}{\frac{\cos\theta}{\sin\theta}}\right]$$

$$= \frac{1}{1} + \left[\cos^{2\ \theta\ +\ \sin^2\theta}\right] = 1 + 1 = 2.$$

Que 9. Evaluate sin 25° cos 65° + cos 25° sin 65°.

Sol. sin 25°. Cos 65° + cos 25°. Sin 65°

$$= \sin(90° - 65°).\cos 65° + \cos(90° - 65°).\sin 65°$$
$$= \cos 65°.\cos 65° + \sin 65°.\sin 65°$$
$$= \cos^2 65° + \sin^2 65° = 1.$$

Que 10. Without using tables, evaluate the following:

$$\mathbf{3\cos 68°.cosec\ 22° - \frac{1}{2}\tan 43°.\tan 47°.\tan 12°.\tan 60°.\tan 78°.}$$

Sol. We have,

$$3\cos 68°.cosec\ 22° - \frac{1}{2}\tan 43°.\tan 47°.\tan 12°.\tan 60°.\tan 78°.$$

$$= 3\cos(90° - 22°).cosec\ 22° - \frac{1}{2}.\{\tan 43°.\tan(90° - 43°)\}$$

$$.\{\tan 12°.\tan(90° - 12°).\tan 60°$$

$$= 3\sin 22°.cosec\ 22° - \frac{1}{2}(\tan 43°.cot\ 43°).(\tan 12°.cot\ 12°).\tan 60°$$

$$= 3 \times 1 - \frac{1}{2} \times 1 \times 1 \times \sqrt{3} = 3 - \frac{\sqrt{3}}{2} = \frac{6-\sqrt{3}}{2}.$$

Que 11. If sin 3 $\boldsymbol{\theta}$ = cos ($\boldsymbol{\theta}$ – 6°) where 3 $\boldsymbol{\theta}$ and $\boldsymbol{\theta}$ – 6° are both acute angles, find the value of $\boldsymbol{\theta}$.

Sol. According to question:

sin 3θ = cos (θ – 6°)

$\Rightarrow \quad \cos(90° - 3\theta) = \cos(\theta - 6°) \qquad [\because \cos(90° - \theta) = \sin\theta]$

$\Rightarrow \quad 90° - 3\theta = \theta - 6° \qquad\qquad [comparing\ the\ angles]$

$\Rightarrow \quad 4\theta = 90° + 6 = 96° \qquad \Rightarrow \qquad \theta = \frac{96}{4} = 24°$

Hence, θ = 24°

Que 12. If sec $\boldsymbol{\theta = x + \frac{1}{4x}}$, prove that sec $\boldsymbol{\theta}$ + tan $\boldsymbol{\theta}$ = 2x or $\boldsymbol{\frac{1}{2x}}$.

Sol. Let sec θ + tan θ = λ …(i)

We know that, $\sec^2 \theta - \tan^2 \theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1 \qquad \Rightarrow \lambda\,(sec\ \theta - \tan\theta = 1$

sec θ – tan θ = $\frac{1}{\lambda}$ …(ii)

Adding equations (i) and (i), we get

$2\ sec\ \theta = \gamma + \frac{1}{\lambda} \qquad \Rightarrow \qquad 2\left(x + \frac{1}{4x}\right) = \lambda + \frac{1}{\lambda}$

$\Rightarrow 2x + \frac{1}{2x} = \lambda + \frac{1}{\lambda}$

On comparing, we get λ = 2x or $\lambda = \frac{1}{2x}$

$\Rightarrow \ sec\ \theta + \tan\theta = 2x\ \ or\ \ \frac{1}{2x}$

Que 13. Find an acute angle$\boldsymbol{\theta}$, when $\boldsymbol{\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{5}}}$.

Sol. We have,

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}} \qquad \Rightarrow \qquad \frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

[Dividing numerator & denominator of the LHS by cosθ]

$$\Rightarrow \quad \frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

On comparing we get

$\Rightarrow \quad \tan\theta = \sqrt{3} \qquad \Rightarrow \quad \tan\theta = \tan 60° \qquad \Rightarrow \ \theta = 60°$

Que 14. The altitude AD of a $\boldsymbol{\Delta ABC}$, in which $\boldsymbol{\angle A}$ is an obtuse angle has length 10 cm. If BD = 10 cm and CD = 10$\boldsymbol{\sqrt{3}}$ cm, determine $\boldsymbol{\angle A}$.

Sol. ΔABD is a right triangle right angled at D, such that AD = 10 cm and BD = 10 cm.

Let $\angle BAD = \theta$

$\therefore \quad \tan\theta = \frac{BD}{AD} \qquad \Rightarrow \tan\theta = \frac{10}{10} = 1$

$\Rightarrow \tan\theta = \tan 45° \quad \Rightarrow \theta = \angle BAD = 45°$ …(i)

ΔACD is a right triangle right angled at D such that AD = 10 cm and DC = $10\sqrt{3}\ cm$.
Let $\angle CAD = \phi$

$\therefore \quad \tan\phi = \frac{CD}{AD} \qquad \Rightarrow \tan\phi = \frac{10\sqrt{3}}{10} = \sqrt{3}$

$\Rightarrow \tan\phi = \tan 60° \quad \Rightarrow \phi = \angle CAD = 60°$ …(ii)

From (i) & (ii), we have

$\angle BAC = \angle BAD + \angle CAD = 45° + 60° = 105°$

Que 15. If cosec $\boldsymbol{\theta} = \frac{13}{12}$, evaluate $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

Sol. Given cosec $\theta = \frac{13}{12}$, then sin $\theta = \frac{12}{13}$

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{12}{13}\right)^2 = \frac{169-144}{169} = \frac{25}{169}$$

$$\cos\theta = \frac{5}{13}$$

Now, $$\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}} = \frac{24-15}{48-45} = \frac{9}{3} = 3$$