Limits and Derivatives

Question 1.
The expansion of $\log(1 - x)$ is
(a) $x - x^2/2 + x^3/3 - \ldots$
(b) $x + x^2/2 + x^3/3 + \ldots$
(c) $-x + x^2/2 - x^3/3 + \ldots$
(d) $-x - x^2/2 - x^3/3 - \ldots$

Answer: (d) $-x - x^2/2 - x^3/3 - \ldots$
$\log(1 - x) = -x - x^2/2 - x^3/3 - \ldots$

Question 2.
The value of $\lim_{x\to a} (a \times \sin x - x \times \sin a)/(ax^2 - xa^2)$ is
(a) $= (a \times \cos a + \sin a)/a^2$
(b) $= (a \times \cos a - \sin a)/a^2$
(c) $= (a \times \cos a + \sin a)/a$
(d) $= (a \times \cos a - \sin a)/a$

Answer: (b) $= (a \times \cos a - \sin a)/a^2$
Given,
$\lim_{x\to a} (a \times \sin x - x \times \sin a)/(ax^2 - xa^2)$
When we put x = a in the expression, we get 0/0 form.
Now apply L. Hospital rule, we get
$\lim_{x\to a} (a \times \cos x - \sin a)/(2ax - a^2)$
$= (a \times \cos a - \sin a)/(2a \times a - a^2)$
$= (a \times \cos a - \sin a)/(2a^2 - a^2)$
$= (a \times \cos a - \sin a)/a^2$
So, $\lim_{x\to a} (a \times \sin x - x \times \sin a)/(ax^2 - xa^2) = (a \times \cos a - \sin a)/a^2$

Question 3.
$\lim_{x\to -1} [1 + x + x^2 + \ldots + x^{10}]$ is

(a) 0
(b) 1
(c) -1
(d) 2

Answer: (b) 1
Given, $\text{Lim}_{x\to-1} [1 + x + x^2 + \ldots\ldots\ldots + x^{10}]$
$= 1 + (-1) + (-1)^2 + \ldots\ldots\ldots + (-1)^{10}$
$= 1 - 1 + 1 - \ldots\ldots + 1$
$= 1$

Question 4.
The value of the limit $\text{Lim}_{x\to0} \{\log(1 + ax)\}/x$ is
(a) 0
(b) 1
(c) a
(d) 1/a

Answer: (c) a
Given, $\text{Lim}_{x\to0} \{\log(1 + ax)\}/x$
$= \text{Lim}_{x\to0} \{ax - (ax)^2 /2 + (ax)^3 /3 - (ax)^4 /4 + \ldots\ldots\}/x$
$= \text{Lim}_{x\to0} \{ax - a^2 x^2 /2 + a^3 x^3 /3 - a^4 x^4 /4 + \ldots\ldots\}/x$
$= \text{Lim}_{x\to0} \{a - a^2 x /2 + a^3 x^2 /3 - a^4 x^3 /4 + \ldots\ldots\}$
$= a - 0$
$= a$

Question 5.
The value of the limit $\text{Lim}_{x\to0} (\cos x)^{\cot^2 x}$ is
(a) 1
(b) e
(c) $e^{1/2}$
(d) $e^{-1/2}$

Answer: (d) $e^{-1/2}$
Given, $\text{Lim}_{x\to0} (\cos x)^{\cot^2 x}$
$= \text{Lim}_{x\to0} (1 + \cos x - 1)^{\cot^2 x}$
$= e^{\text{Lim}_{x\to0}} (\cos x - 1) \times \cot^2 x$

$= e^{\text{Lim}_{x\to 0} (\cos x - 1)/\tan^2 x}$
$= e^{-1/2}$

Question 6.
Then value of $\text{Lim}_{x\to 1} (1 + \log x - x)\}/(1 - 2x + x^2)$ is
(a) 0
(b) 1
(c) 1/2
(d) -1/2

Answer: (d) -1/2
Given, $\text{Lim}_{x\to 1} (1 + \log x - x)\}/(1 - 2x + x^2)$
$= \text{Lim}_{x\to 1} (1/x - 1)\}/(-2 + 2x)$ {Using L. Hospital Rule}
$= \text{Lim}_{x\to 1} (1 - x)/\{2x(x - 1)\}$
$= \text{Lim}_{x\to 1} (-1/2x)$
$= -1/2$

Question 7.
The value of $\lim_{y\to 0} \{(x + y) \times \sec (x + y) - x \times \sec x\}/y$ is
(a) $x \times \tan x \times \sec x$
(b) $x \times \tan x \times \sec x + x \times \sec x$
(c) $\tan x \times \sec x + \sec x$
(d) $x \times \tan x \times \sec x + \sec x$

Answer: (d) $x \times \tan x \times \sec x + \sec x$
Given, $\lim_{y\to 0} \{(x + y) \times \sec (x + y) - x \times \sec x\}/y$
$= \lim_{y\to 0} \{x \sec (x + y) + y \sec (x + y) - x \times \sec x\}/y$
$= \lim_{y\to 0} [x\{ \sec (x + y) - \sec x\} + y \sec (x + y)]/y$
$= \lim_{y\to 0} x\{ \sec (x + y) - \sec x\}/y + \lim_{y\to 0} \{y \sec (x + y)\}/y$
$= \lim_{y\to 0} x\{1/\cos (x + y) - 1/\cos x\}/y + \lim_{y\to 0} \{y \sec (x + y)\}/y$
$= \lim_{y\to 0} [\{\cos x - \cos (x + y)\} \times x/\{y \times \cos (x + y) \times \cos x\}] + \lim_{y\to 0} \{y \sec (x + y)\}/y$
$= \lim_{y\to 0} [\{2\sin (x + y/2) \times \sin(y/2)\} \times 2x/\{2y \times \cos (x + y) \times \cos x\}] + \lim_{y\to 0} \{y \sec (x + y)\}/y$
$= \lim_{y\to 0} \{\sin (x + y/2) \times \lim_{y\to 0} \{\sin(y/2)/(2y/2)\} \times \lim_{y\to 0} \{ x/\{y \times \cos (x + y) \times \cos x\}] + \sec x$
$= \sin x \times 1 \times x/\cos^2 x + \sec x$
$= x \times \tan x \times \sec x + \sec x$
So, $\lim_{y\to 0} \{(x + y) \times \sec (x + y) - x \times \sec x\}/y = x \times \tan x \times \sec x + \sec x$

Question 8.
$\text{Lim}_{x \to 0} (e^{x^2} - \cos x)/x^2$ is equals to
(a) 0
(b) 1
(c) 2/3
(d) 3/2

Answer: (d) 3/2
Given, $\text{Lim}_{x \to 0} (e^{x^2} - \cos x)/x^2$
$= \text{Lim}_{x \to 0} (e^{x^2} - \cos x - 1 + 1)/x^2$
$= \text{Lim}_{x \to 0} \{(e^{x^2} - 1)/x^2 + (1 - \cos x)\}/x^2$
$= \text{Lim}_{x \to 0} \{(e^{x^2} - 1)/x^2 + \text{Lim}_{x \to 0} (1 - \cos x)\}/x^2$
$= 1 + 1/2$
$= (2 + 1)/2$
$= 3/2$

Question 9.
The expansion of a^x is
(a) $a^x = 1 + x/1! \times (\log a) + x^2 /2! \times (\log a)^2 + x^3 /3! \times (\log a)^3 + \ldots\ldots\ldots\ldots$
(b) $a^x = 1 - x/1! \times (\log a) + x^2 /2! \times (\log a)^2 - x^3 /3! \times (\log a)^3 + \ldots\ldots\ldots\ldots$
(c) $a^x = 1 + x/1 \times (\log a) + x^2 /2 \times (\log a)^2 + x^3 /3 \times (\log a)^3 + \ldots\ldots\ldots\ldots$
(d) $a^x = 1 - x/1 \times (\log a) + x^2 /2 \times (\log a)^2 - x^3 /3 \times (\log a)^3 + \ldots\ldots\ldots\ldots$

Answer: (a) $a^x = 1 + x/1! \times (\log a) + x^2 /2! \times (\log a)^2 + x^3 /3! \times (\log a)^3 + \ldots\ldots\ldots\ldots$
$a^x = 1 + x/1! \times (\log a) + x^2 /2! \times (\log a)^2 + x^3 /3! \times (\log a)^3 + \ldots\ldots\ldots\ldots$

Question 10.
The value of the limit $\text{Lim}_{n \to 0} (1 + an)^{b/n}$ is
(a) e^a
(b) e^b
(c) e^{ab}
(d) $e^{a/b}$

Answer: (c) e^{ab}
Given, $\text{Lim}_{n \to 0} (1 + an)^{b/n}$
$= e^{\text{Lim}_{n \to 0}(an \times b/n)}$

$= e^{Lim_{n\to 0}(ab)}$
$= e^{ab}$

Question 11.
The value of $Lim_{x\to 0}$ cos x/(1 + sin x) is
(a) 0
(b) -1
(c) 1
(d) None of these

Answer: (c) 1
Given, $Lim_{x\to 0}$ cos x/(1 + sin x)
= cos 0/(1 + sin 0)
= 1/(1 + 0)
= 1/1
= 1

Question 12.
Lim $tan_{x\to \pi/4}$ tan 2x × tan(π/4 – x) is
(a) 0
(b) 1
(c) 1/2
(d) 3/2

Answer: (c) 1/2
Given, Lim $tan_{x\to \pi/4}$ tan 2x × tan(π/4 – x)
= Lim $tan_{h\to 0}$ tan 2(π/4 – x) × tan(-h)
= Lim $tan_{h\to 0}$ -cot 2h/(-cot h)
= Lim $tan_{h\to 0}$ tan h/tan 2h
= (1/2) × Lim $tan_{h\to 0}$ (tan h/h)/(2h/tan 2h)
= (1/2) × {Lim $tan_{h\to 0}$ (tan h/h)}/{Lim $tan_{h\to 0}$ (2h/tan 2h)}
= (1/2) × 1
= 1/2

Question 13.
$Lim_{x\to 2}$ $(x^3 – 6x^2 + 11x – 6)/(x^2 – 6x + 8)$ =
(a) 0
(b) 1

(c) 1/2
(d) Limit does not exist

Answer: (c) 1/2
When x = 2, the expression
$(x^3 – 6x^2 + 11x – 6)/(x^2 – 6x + 8)$ assumes the form 0/0
Now,
$\text{Lim}_{x\to2} (x^3 – 6x^2 + 11x – 6)/(x^2 – 6x + 8) = \text{Lim}_{x\to2} \{(x – 1) \times (x – 2) \times (x – 3)\}/\{(x – 2) \times (x – 4)\}$
$= \text{Lim}_{x\to2} \{(x – 1) \times (x – 3)\}/(x – 4)$
$= \{(2 – 1) \times (2 – 3)\}/(2 – 4)$
$= 1/2$

Question 14.
The value of the limit $\text{Lim}_{x\to2} (x – 2)/\sqrt{(2 – x)}$ is
(a) 0
(b) 1
(c) -1
(d) 2

Answer: (a) 0
Given, $\text{Lim}_{x\to2} (x – 2)/\sqrt{(2 – x)}$
$= \text{Lim}_{x\to2} -(2 – x)/\sqrt{(2 – x)}$
$= \text{Lim}_{x\to2} -\{\sqrt{(2 – x)} \times \sqrt{(2 – x)}\}/\sqrt{(2 – x)}$
$= \text{Lim}_{x\to2} -\sqrt{(2 – x)}$
$= -\sqrt{(2 – 2)}$
$= 0$

Question 15.
The derivative of the function $f(x) = 3x^3 – 2x^3 + 5x – 1$ at x = -1 is
(a) 0
(b) 1
(c) -18
(d) 18

Answer: (d) 18
Given, function $f(x) = 3x^3 – 2x^2 + 5x – 1$
Differentiate w.r.t. x, we get
$df(x)/dx = 3 \times 3 \times x^2 – 2 \times 2 \times x + 5$
$\Rightarrow df(x)/dx = 9x^2 – 4x + 5$

$\Rightarrow \{df(x)/dx\}_{x=-1} = 9 \times (-1)^2 - 4 \times (-1) + 5$
$\Rightarrow \{df(x)/dx\}_{x=-1} = 9 + 4 + 5$
$\Rightarrow \{df(x)/dx\}_{x=-1} = 18$

Question 16.
$\text{Lim}_{x\to 0} \sin^2(x/3)/x^2$ is equals to
(a) 1/2
(b) 1/3
(c) 1/4
(d) 1/9

Answer: (d) 1/9
Given, $\text{Lim}_{x\to 0} \sin^2 (x/3)/ x^2$
$= \text{Lim}_{x\to 0} [\sin^2 (x/3)/ (x/3)^2 \times \{(x/3)^2 /x^2\}]$
$= \text{Lim}_{x\to 0} [\{\sin (x/3)/ (x/3)\}^2 \times \{(x^2 /9)/x^2\}]$
$= 1 \times 1/9$
$= 1/9$

Question 17.
The expansion of ax is
(a) $a^x = 1 + x/1! \times (\log a) + x^2 /2! \times (\log a)^2 + x^3 /3! \times (\log a)^3 + \ldots\ldots\ldots\ldots$
(b) $a^x = 1 - x/1! \times (\log a) + x^2 /2! \times (\log a)^2 - x^3 /3! \times (\log a)^3 + \ldots\ldots\ldots\ldots$
(c) $a^x = 1 + x/1 \times (\log a) + x^2 /2 \times (\log a)^2 + x^3 /3 \times (\log a)^3 + \ldots\ldots\ldots\ldots$
(d) $a^x = 1 - x/1 \times (\log a) + x^2 /2 \times (\log a)^2 - x^3 /3 \times (\log a)^3 + \ldots\ldots\ldots\ldots$

Answer: (a) $a^x = 1 + x/1! \times (\log a) + x^2 /2! \times (\log a)^2 + x^3 /3! \times (\log a)^3 + \ldots\ldots\ldots\ldots$
$ax = 1 + x/1! \times (\log a) + x^2 /2! \times (\log a)^2 + x^3 /3! \times (\log a)^3 + \ldots\ldots\ldots\ldots$

Question 18.
Differentiation of $\cos \sqrt{x}$ with respect to x is
(a) $\sin x /2\sqrt{x}$
(b) $-\sin x /2\sqrt{x}$
(c) $\sin \sqrt{x} /2\sqrt{x}$
(d) $-\sin \sqrt{x} /2\sqrt{x}$

Answer: (d) $-\sin \sqrt{x} /2\sqrt{x}$
Let $y = \cos \sqrt{x}$
Put $u = \sqrt{x}$

$du/dx = 1/2\sqrt{x}$
Now, $y = \cos u$
$dy/du = -\sin u$
$dy/dx = (dy/du) \times (du/dx)$
$= -\sin u \times (1/2\sqrt{x})$
$= -\sin \sqrt{x} /2\sqrt{x}$

Question 19.
Differentiation of log(sin x) is
(a) cosec x
(b) cot x
(c) sin x
(d) cos x

Answer: (b) cot x
Let $y = \log(\sin x)$
Again let $u = \sin x$
$du/dx = \cos x$
Now, $y = \log u$
$dy/du = 1/u = 1/\sin x$
Now, $dy/dx = (dy/du) \times (du/dx)$
$\Rightarrow dy/dx = (1/\sin x) \times \cos x$
$\Rightarrow dy/dx = \cos x/\sin x$
$\Rightarrow dy/dx = \cot x$

Question 20.
$\text{Lim}_{x\to\infty} \{(x + 5)/(x + 1)\}^x$ equals
(a) e^2
(b) e^4
(c) e^6
(d) e^8

Answer: (c) e^6
Given, $\text{Lim}_{x\to\infty} \{(x + 5)/(x + 1)\}x$
$= \text{Lim}_{x\to\infty} \{1 + 6/(x + 1)\}x$
$= e^{\text{Lim}_{x\to\infty} 6x/(x + 1)}$
$= e^{\text{Lim}_{x\to\infty} 6/(1 + 1/x)}$
$= e^{6/(1 + 1/\infty)}$

$= e^{6/(1 + 0)}$
$= e^{6}$
