Linear Inequations (in one variable)

Exercise 4A

Question 1.

State, true or false:

(i) $x < -y \Rightarrow -x > y$

(ii) $-5x \geq 15 \Rightarrow x \geq -3$

(iii) $2x \leq -7 \Rightarrow \frac{2x}{-4} \geq \frac{-7}{-4}$

(iv) $7 > 5 \Rightarrow \frac{1}{7} < \frac{1}{5}$

Solution:

(i) $x < -y \Rightarrow -x > y$

The given statement is true.

(ii) $-5x \geq 15 \Rightarrow \frac{-5x}{5} \geq \frac{15}{5} \Rightarrow x \leq -3$

The given statement is false.

(iii) $2x \leq -7 \Rightarrow \frac{2x}{-4} \geq \frac{-7}{-4}$

The given statement is true.

(iv) $7 > 5 \Rightarrow \frac{1}{7} < \frac{1}{5}$

The given statement is true.

Question 2.

State, whether the following statements are true or false:

(i) $a < b$, then $a - c < b - c$ (ii) If $a > b$, then $a + c > b + c$

(iii) If $a < b$, then $ac > bc$

(iv) If $a > b$, then $\frac{a}{c} < \frac{b}{c}$

(v) If $a - c > b - d$, then $a + d > b + c$

(vi) If $a < b$, and $c > 0$, then $a - c > b - c$

Where a, b, c and d are real numbers and $c \neq 0$.

Solution:
(i) $a < b \Rightarrow a - c < b - c$ The given statement is true.
(ii) If $a > b \Rightarrow a + c > b + c$
The given statement is true.
(iii) If $a < b \Rightarrow ac < bc$ The given statement is false.
(iv) If $a > b \Rightarrow \frac{a}{c} > \frac{b}{c}$
The given statement is false.
(v) If $a - c > b - d \Rightarrow a + d > b + c$
The given statement is true.
(vi) If $a < b \Rightarrow a - c < b - c$ (Since, $c > 0$)
The given statement is false.

Question 3.
If $x \in N$, find the solution set of inequations.
(i) $5x + 3 \leq 2x + 18$
(ii) $3x - 2 < 19 - 4x$

Solution:
(i) $5x + 3 \leq 2x + 18$
$5x - 2x \leq 18 - 3$
$3x \leq 15$
$x \leq 5$

Since, $x \in N$, therefore solution set is {1, 2, 3, 4, 5}.
(ii) $3x - 2 < 19 - 4x$
$3x + 4x < 19 + 2$
$7x < 21$
$x < 3$
Since, $x \in N$, therefore solution set is {1, 2}.

Question 4.

If the replacement set is the set of whole numbers, solve:
(i) $x + 7 \leq 11$
(ii) $3x - 1 > 8$
(iii) $8 - x > 5$
(iv) $7 - 3x \geq -\frac{1}{2}$
(v) $x - \frac{3}{2} < \frac{3}{2} - x$
(vi) $18 \leq 3x - 2$

Solution:

(i) $x + 7 \leq 11$
$x \leq 11 - 7$
$x \leq 4$

Since, the replacement set = W (set of whole numbers)
⇒ Solution set = {0, 1, 2, 3, 4}

(ii) $3x - 1 > 8$
$3x > 8 + 1$
$x > 3$
Since, the replacement set = W (set of whole numbers)
⇒ Solution set = {4, 5, 6, …}

(iii) $8 - x > 5$
$-x > 5 - 8$
$-x > -3$
$x < 3$

Since, the replacement set = W (set of whole numbers)
⇒ Solution set = {0, 1, 2}

(iv) $7 - 3x \geq -\frac{1}{2}$

$-3x \geq -\frac{1}{2} - 7$

$-3x \geq -\frac{15}{2}$

$x \leq \frac{5}{2}$

Since, the replacement set = W (set of whole numbers)
∴ Solution set = {0, 1, 2}

(v) $x - \frac{3}{2} < \frac{3}{2} - x$

$x + x < \frac{3}{2} + \frac{3}{2}$

$2x < 3$

$x < \frac{3}{2}$

Since, the replacement set = W (set of whole numbers)
∴ Solution set = {0, 1}

(vi) $18 \leq 3x - 2$
$18 + 2 \leq 3x$
$20 \leq 3x$
$x \geq \frac{20}{3}$

Since, the replacement set = W (set of whole numbers)
∴ Solution set = {7, 8, 9, …}

Question 5.
Solve the inequation:
$3 - 2x \geq x - 12$ given that $x \in N$.

Solution:
$3 - 2x \geq x - 12$
$-2x - x \geq -12 - 3$
$-3x \geq -15$
$x \leq 5$
Since, $x \in N$, therefore,
Solution set = {1, 2, 3, 4, 5}

Question 6.
If $25 - 4x \leq 16$, find:
(i) the smallest value of x, when x is a real number,
(ii) the smallest value of x, when x is an integer.

Solution:
$25 - 4x \leq 16$
$-4x \leq 16 - 25$
$-4x \leq -9$

$x \geq \frac{9}{4}$
$x \geq 2.25$

(i) The smallest value of x, when x is a real number, is 2.25.
(ii) The smallest value of x, when x is an integer, is 3.

Question 7.

If the replacement set is the set of real numbers, solve:

(i) $-4x \geq -16$

(ii) $8 - 3x \leq 20$

(iii) $5 + \frac{x}{4} > \frac{x}{5} + 9$

(iv) $\frac{x+3}{8} < \frac{x-3}{5}$

Solution:

(i) $-4x \geq -16$

$x \leq 4$

Since, the replacement set of real numbers.

$\therefore$ Solution set = $\{x : x \in R \text{ and } x \leq 4\}$

(ii) $8 - 3x \leq 20$

$-3x \leq 20 - 8$

$-3x \leq 12$

$x \geq -4$

Since, the replacement set of real numbers.

$\therefore$ Solution set = $\{x : x \in R \text{ and } x \geq -4\}$

(iii) $5 + \frac{x}{4} > \frac{x}{5} + 9$

$\frac{x}{4} - \frac{x}{5} > 9 - 5$

$\frac{x}{20} > 4$

$x > 80$

Since, the replacement set of real numbers.

$\therefore$ Solution set = $\{x : x \in R \text{ and } x > 80\}$

(iv) $\frac{x+3}{8} < \frac{x-3}{5}$

$5x + 15 < 8x - 24$

$5x - 8x < -24 - 15$

$-3x < -39$

$x > 13$

Since, the replacement set of real numbers.

$\therefore$ Solution set = $\{x : x \in R \text{ and } x > 13\}$

Question 8.

Find the smallest value of x for which $5 - 2x < 5\frac{1}{2} - \frac{5}{3}x$, where x is an integer.

Solution:

$5 - 2x < 5\frac{1}{2} - \frac{5}{3}x$

$-2x + \frac{5}{3}x < \frac{11}{2} - 5$

$\frac{-x}{3} < \frac{1}{2}$

$-x < \frac{3}{2}$

$x > \frac{-3}{2}$

$x > -1.5$

Thus, the required smallest value of x is -1.

Question 9.
Find the largest value of x for which
$2(x - 1) \leq 9 - x$ and $x \in W$.

Solution:

$2(x - 1) \leq 9 - x$
$2x - 2 \leq 9 - x$
$2x + x \leq 9 + 2$
$3x \leq 11$
$x \leq \frac{11}{3}$
$x \leq 3.67$

Since, $x \in W$, thus the required largest value of x is 3.

Question 10.

Solve the inequation: $12 + 1\frac{5}{6}x \leq 5 + 3x$ and $x \in R$.

Solution:

$$12 + 1\frac{5}{6}x \leq 5 + 3x$$
$$\frac{11}{6}x - 3x \leq 5 - 12$$
$$\frac{-7}{6}x \leq -7$$
$$x \geq 6$$
$\therefore$ Solution set = $\{x : x \in R \text{ and } x \geq 6\}$

Question 11.
Given x ∈ {integers}, find the solution set of:
$-5 \leq 2x - 3 < x + 2$

Solution:

$-5 \leq 2x - 3 < x + 2$

$\Rightarrow -5 \leq 2x - 3$	and	$2x - 3 < x + 2$
$\Rightarrow -5 + 3 \leq 2x$	and	$2x - x < 2 + 3$
$\Rightarrow -2 \leq 2x$	and	$x < 5$
$\Rightarrow x \geq -1$	and	$x < 5$

Since, $x \in$ {integers}
$\therefore$ Solution set = {-1, 0, 1, 2, 3, 4}

Question 12.
Given x ∈ {whole numbers}, find the solution set of:
$-1 \leq 3 + 4x < 23$

Solution:

$-1 \leq 3 + 4x < 23$

$\Rightarrow -1 \leq 3 + 4x$	and	$3 + 4x < 23$
$\Rightarrow -4 \leq 4x$	and	$4x < 20$
$\Rightarrow x \geq -1$	and	$x < 5$

Since, $x \in$ {whole numbers}
$\therefore$ Solution set = {0, 1, 2, 3, 4}

Exercise 4B

Question 1.

Represent the following inequalities on real number lines:

(i) $2x - 1 < 5$

(ii) $3x + 1 \geq -5$

(iii) $2(2x - 3) \leq 6$

(iv) $-4 < x < 4$

(v) $-2 \leq x < 5$

(vi) $8 \geq x > -3$

(vii) $-5 < x \leq -1$

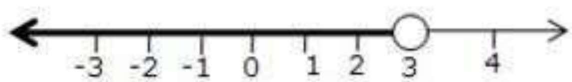
Solution:

(i) $2x - 1 < 5$

$2x < 6$

$x < 3$

Solution on number line is:

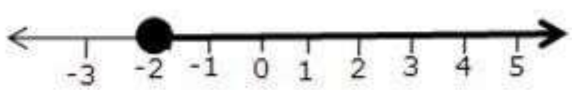


(ii) $3x + 1 \geq -5$

$3x \geq -6$

$x \geq -2$

Solution on number line is:



(iii) $2(2x - 3) \leq 6$

$2x - 3 \leq 3$

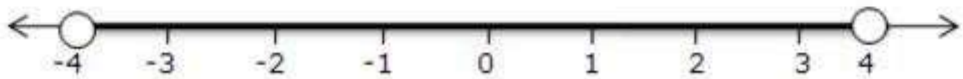
$2x \leq 6$

$x \leq 3$

Solution on number line is:

(iv) $-4 < x < 4$

Solution on number line is:



(v) $-2 \le x < 5$

Solution on number line is:

(vi) $8 \ge x > -3$

Solution on number line is:

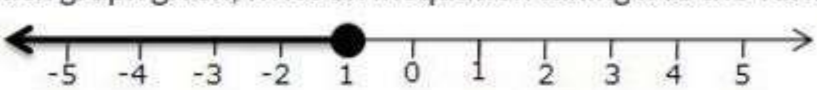
(vii) $-5 < x \le -1$

Solution on number line is:

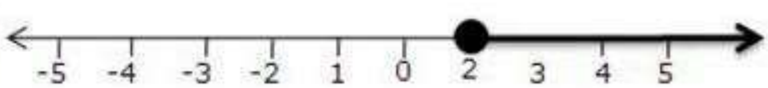
Question 2.

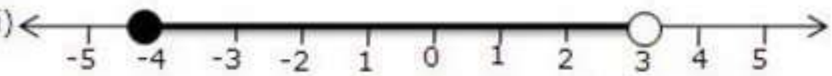
For each graph given, write an inequation taking x as the variable:

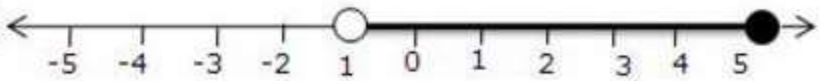
(i)



(ii)



(iii) 

(iv) 

Solution:

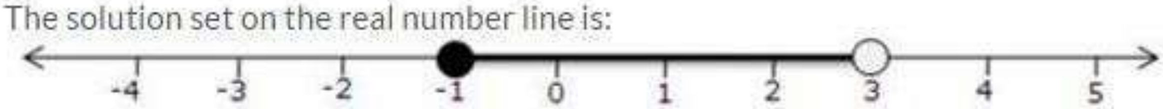
(i) $x \le -1, x \in R$
(ii) $x \ge 2, x \in R$
(iii) $-4 \le x < 3, x \in R$
(iv) $-1 < x \le 5, x \in R$

Question 3.

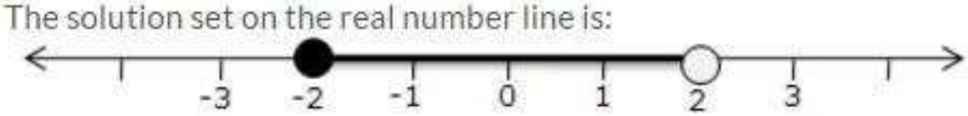
For the following inequations, graph the solution set on the real number line:
(i) $-4 \le 3x - 1 < 8$
(ii) $x - 1 < 3 - x \le 5$

Solution:

(i) $-4 \le 3x - 1 < 8$
$-4 \le 3x - 1$ and $3x - 1 < 8$
$-1 \le x$ and $x < 3$
The solution set on the real number line is:



(ii) $x - 1 < 3 - x \le 5$
$x - 1 < 3 - x$ and $3 - x \le 5$
$2x < 4$ and $-x \le 2$
$x < 2$ and $x \ge -2$
The solution set on the real number line is:



Question 4.

Represent the solution of each of the following inequalities on the real number line:

(i) $4x - 1 > x + 11$

(ii) $7 - x \leq 2 - 6x$

(iii) $x + 3 \leq 2x + 9$

(iv) $2 - 3x > 7 - 5x$

(v) $1 + x \geq 5x - 11$

(vi) $\frac{2x + 5}{3} > 3x - 3$

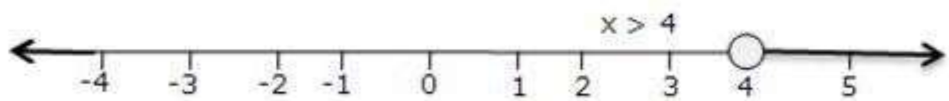
Solution:

(i) $4x - 1 > x + 11$

$3x > 12$

$x > 4$

The solution on number line is:

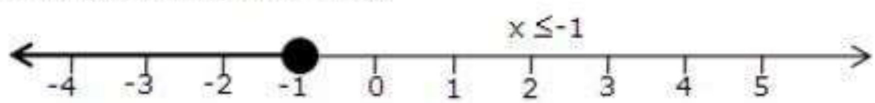


(ii) $7 - x \leq 2 - 6x$

$5x \leq -5$

$x \leq -1$

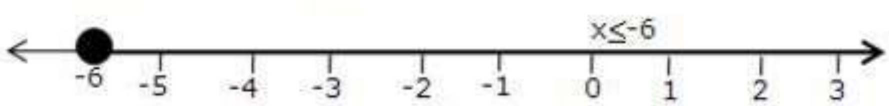
The solution on number line is:



(iii) $x + 3 \leq 2x + 9$

$-6 \leq x$

The solution on number line is:



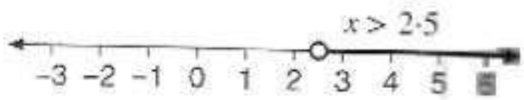
$(iv) 2 - 3x > 7 - 5x$

$2x > 5$

$x > \frac{5}{2}$

$x > 2.5$

The solution on number line is:

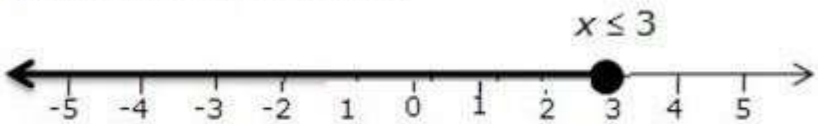


$(v) 1 + x \geq 5x - 11$

$12 \geq 4x$

$3 \geq x$

The solution on number line is:



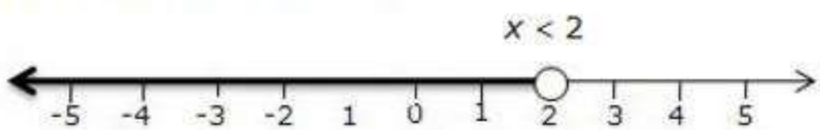
$(vi) \frac{2x+5}{3} > 3x - 3$

$2x + 5 > 9x - 9$

$-7x > -14$

$x < 2$

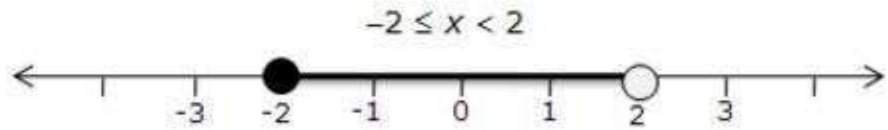
The solution on number line is:



Question 5.
$x \in$ {real numbers} and $-1 < 3 - 2x \leq 7$, evaluate x and represent it on a number line.

Solution:
$-1 < 3 - 2x \leq 7$
$-1 < 3 - 2x$ and $3 - 2x \leq 7$
$2x < 4$ and $-2x \leq 4$

x < 2 and x ≥ -2
Solution set = {-2 ≤ x < 2, x ∈ R}
Thus, the solution can be represented on a number line as:



Question 6.
List the elements of the solution set of the inequation
-3 < x − 2 ≤ 9 − 2x; x ∈ N.

Solution:
-3 < x − 2 ≤ 9 − 2x
-3 < x − 2 and x − 2 ≤ 9 − 2x
-1 < x and 3x ≤ 11

-1 < x ≤ $\frac{11}{3}$
Since, x ∈ N
∴ Solution set = {1, 2, 3}

Question 7.

Find the range of values of x which satisfies
$-2\frac{2}{3} \le x + \frac{1}{3} < 3\frac{1}{3}$; x ∈ R.
Graph these values of x on the number line.

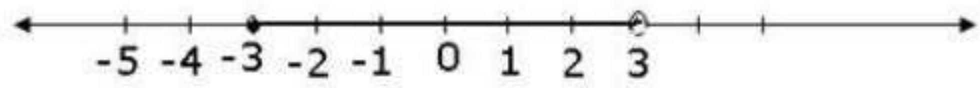
Solution:

$-2\frac{2}{3} \le x + \frac{1}{3}$ and $x + \frac{1}{3} < 3\frac{1}{3}$
$\Rightarrow -\frac{8}{3} \le x + \frac{1}{3}$ and $x + \frac{1}{3} < \frac{10}{3}$
$\Rightarrow -\frac{8}{3} - \frac{1}{3} \le x$ and $x < \frac{10}{3} - \frac{1}{3}$
$\Rightarrow -\frac{9}{3} \le x$ and $x < \frac{9}{3}$

$\Rightarrow -3 \leq x$ and $x < 3$

$\therefore -3 \leq x < 3$

The required graph of the solution set is:



Question 8.

Find the values of x, which satisfy the inequation:

$-2 \leq \frac{1}{2} - \frac{2x}{3} < 1\frac{5}{6}$; $x \in N$.

Graph the solution on the number line.

Solution:

$-2 \leq \frac{1}{2} - \frac{2x}{3} < 1\frac{5}{6}$

$-2 \leq \frac{1}{2} - \frac{2x}{3}$ and $\frac{1}{2} - \frac{2x}{3} < 1\frac{5}{6}$

$\frac{-5}{2} \leq -\frac{2x}{3}$ and $-\frac{2x}{3} < \frac{8}{6}$

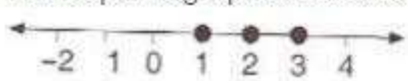
$\frac{15}{4} \geq x$ and $x > -2$

$3.75 \geq x$ and $x > -2$

Since, $x \in N$

$\therefore$ Solution set = {1, 2, 3}

The required graph of the solution set is:



Question 9.

Given x ∈ {real numbers}, find the range of values of x for which $-5 \leq 2x - 3 < x + 2$ and represent it on a number line.

Solution:

$-5 \leq 2x - 3 < x + 2$

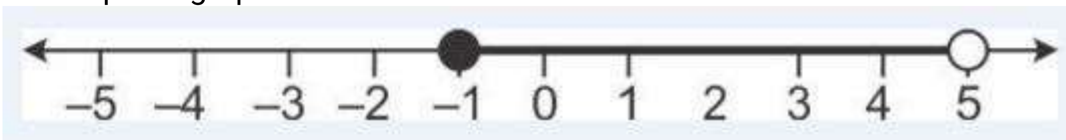
$-5 \leq 2x - 3$ and $2x - 3 < x + 2$

$-2 \leq 2x$ and $x < 5$

$-1 \leq x$ and $x < 5$

Required range is $-1 \leq x < 5$.

The required graph is:



Question 10.
If 5x – 3 ≤ 5 + 3x ≤ 4x + 2, express it as a ≤ x ≤ b and then state the values of a and b.

Solution:
5x – 3 ≤ 5 + 3x ≤ 4x + 2
5x – 3 ≤ 5 + 3x and 5 + 3x ≤ 4x + 2

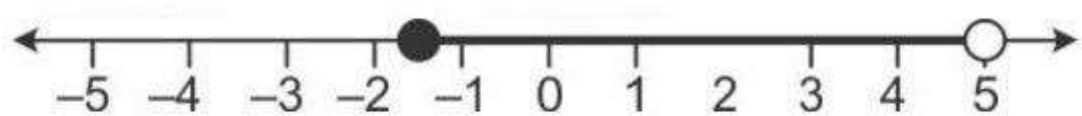
2x ≤ 8 and -x ≤ -3
x ≤ 4 and x ≥ 3

Thus, 3 ≤ x ≤ 4.
Hence, a = 3 and b = 4.

Question 11.
Solve the following inequation and graph the solution set on the number line:
2x – 3 < x + 2 ≤ 3x + 5, x ∈ R.

Solution:
2x – 3 < x + 2 ≤ 3x + 5
2x – 3 < x + 2 and x + 2 ≤ 3x + 5
x < 5 and -3 ≤ 2x
x < 5 and -1.5 ≤ x

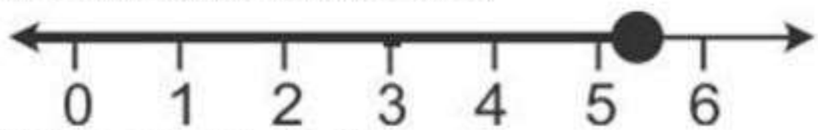
Solution set = {-1.5 ≤ x < 5}
The solution set can be graphed on the number line as:



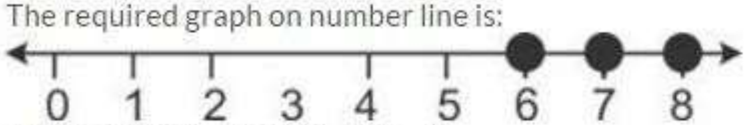
Question 12.
Solve and graph the solution set of:
(i) 2x – 9 < 7 and 3x + 9 ≤ 25, x ∈ R (ii) 2x – 9 ≤ 7 and 3x + 9 > 25, x ∈ I
(iii) x + 5 ≥ 4(x – 1) and 3 – 2x < -7, x ∈ R

Solution:

(i) $2x - 9 < 7$ and $3x + 9 \leq 25$
$2x < 16$ and $3x \leq 16$
$x < 8$ and $x \leq 5\frac{1}{3}$
$\therefore$ Solution set = $\{x \leq 5\frac{1}{3}, x \in R\}$
The required graph on number line is:



(ii) $2x - 9 \leq 7$ and $3x + 9 > 25$
$2x \leq 16$ and $3x > 16$
$x \leq 8$ and $x > 5\frac{1}{3}$
$\therefore$ Solution set = $\{5\frac{1}{3} < x \leq 8, x \in I\} = \{6, 7, 8\}$
The required graph on number line is:



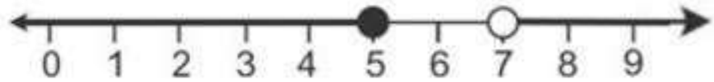
(iii) $x + 5 \geq 4(x - 1)$ and $3 - 2x < -7$
$9 \geq 3x$ and $-2x < -10$
$3 \geq x$ and $x > 5$
$\therefore$ Solution set = Empty set

Question 13.
Solve and graph the solution set of:
(i) $3x - 2 > 19$ or $3 - 2x \geq -7, x \in R$
(ii) $5 > p - 1 > 2$ or $7 \leq 2p - 1 \leq 17, p \in R$

Solution:
(i) $3x - 2 > 19$ or $3 - 2x \geq -7$
$3x > 21$ or $-2x \geq -10$
$x > 7$ or $x \leq 5$

Graph of solution set of $x > 7$ or $x \leq 5$ = Graph of points which belong to $x > 7$ or $x \leq 5$ or both.

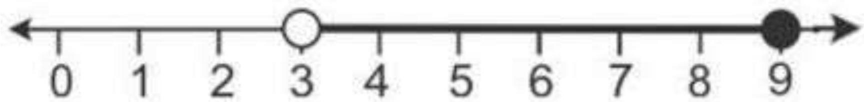
Thus, the graph of the solution set is:



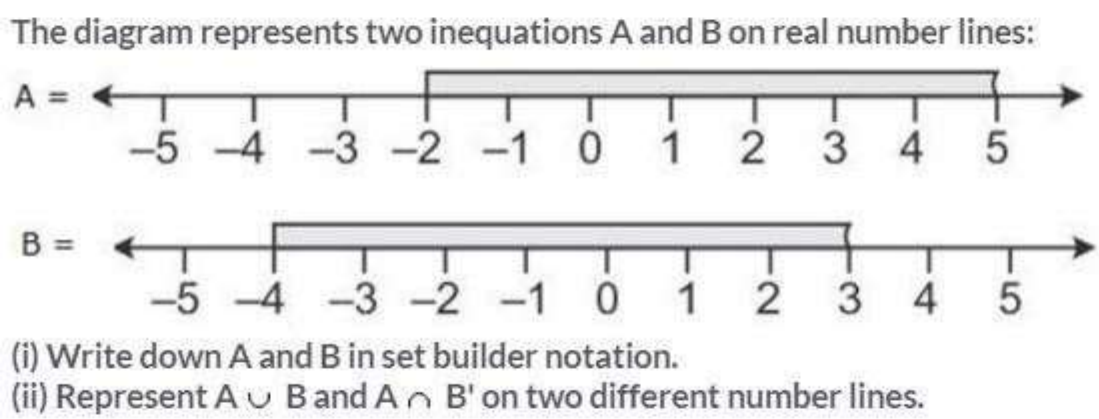
(ii) $5 > p - 1 > 2$ or $7 \leq 2p - 1 \leq 17$
$6 > p > 3$ or $8 \leq 2p \leq 18$
$6 > p > 3$ or $4 \leq p \leq 9$

Graph of solution set of $6 > p > 3$ or $4 \leq p \leq 9$
= Graph of points which belong to $6 > p > 3$ or $4 \leq p \leq 9$ or both
= Graph of points which belong to $3 < p \leq 9$

Thus, the graph of the solution set is:



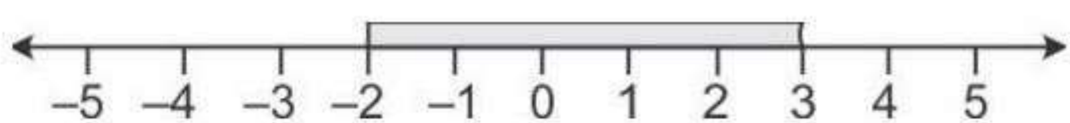
Question 14.


The diagram represents two inequations A and B on real number lines:

(i) Write down A and B in set builder notation.
(ii) Represent $A \cup B$ and $A \cap B'$ on two different number lines.

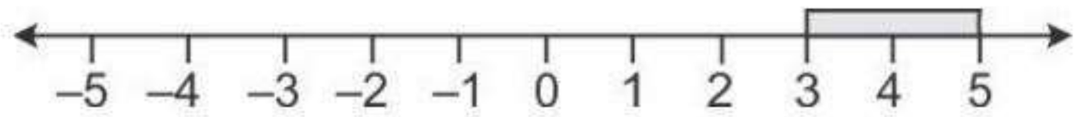
Solution:

(i) $A = \{x \in R: -2 \leq x < 5\}$
$B = \{x \in R: -4 \leq x < 3\}$
(ii) $A \cap B = \{x \in R: -2 \leq x < 5\}$
It can be represented on number line as:



$B' = \{x \in R: 3 < x \leq -4\}$
$A \cap B' = \{x \in R: 3 \leq x < 5\}$

It can be represented on number line as:

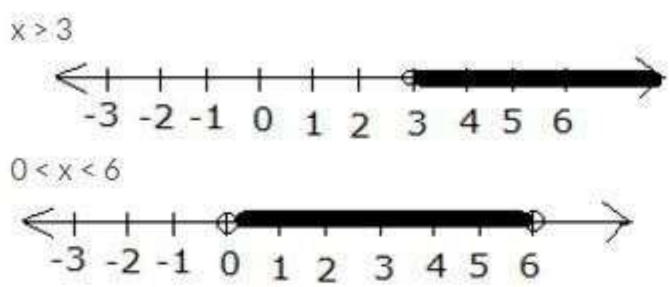


Question 15.
Use real number line to find the range of values of x for which:
(i) $x > 3$ and $0 < x < 6$
(ii) $x < 0$ and $-3 \leq x < 1$
(iii) $-1 < x \leq 6$ and $-2 \leq x \leq 3$

Solution:
(i) $x > 3$ and $0 < x < 6$

Both the given inequations are true in the range where their graphs on the real number lines overlap.

The graphs of the given inequations can be drawn as:

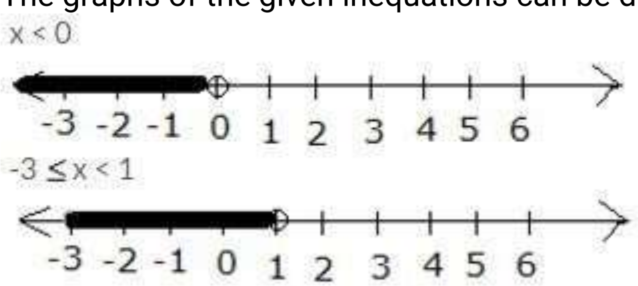


From both graphs, it is clear that their common range is
$3 < x < 6$

(ii) $x < 0$ and $-3 \leq x < 1$

Both the given inequations are true in the range where their graphs on the real number lines overlap.

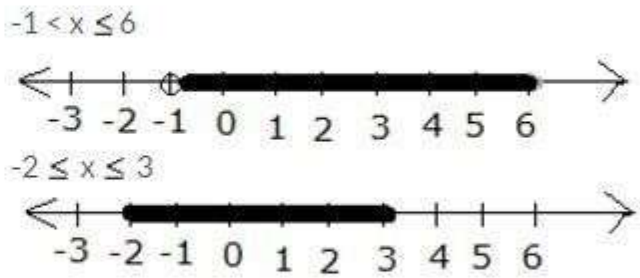
The graphs of the given inequations can be drawn as:



From both graphs, it is clear that their common range is
-3 ≤ x < 0

(iii) -1 < x ≤ 6 and -2 ≤ x ≤ 3
Both the given inequations are true in the range where their graphs on the real number lines overlap.

The graphs of the given inequations can be drawn as:

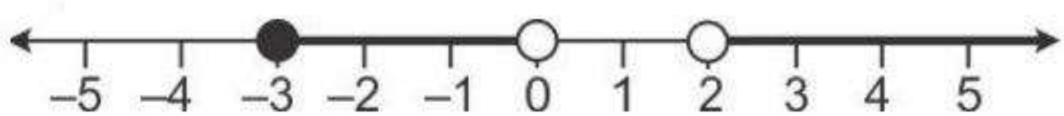


From both graphs, it is clear that their common range is
-1 < x ≤ 3

Question 16.
Illustrate the set {x: -3 ≤ x < 0 or x > 2, x ∈ R} on the real number line.

Solution:
Graph of solution set of -3 ≤ x < 0 or x > 2
= Graph of points which belong to -3 ≤ x < 0 or x > 2 or both

Thus, the required graph is:



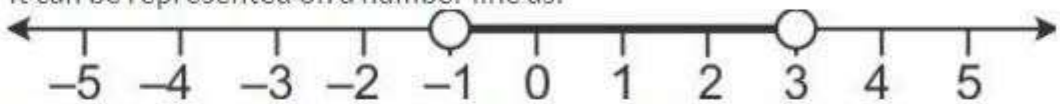
Question 17.
Given A = {x: -1 < x ≤ 5, x ∈ R} and B = {x: -4 ≤ x < 3, x ∈ R}

Represent on different number lines:
(i) A ∩ B
(ii) A' ∩ B
(iii) A – B

Solution:

(i) $A \cap B = \{x: -1 < x < 3, x \in R\}$

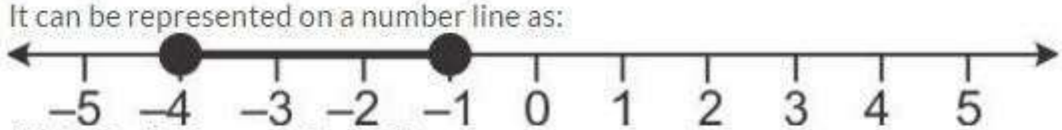
It can be represented on a number line as:



(ii) Numbers which belong to B but do not belong to A = B - A

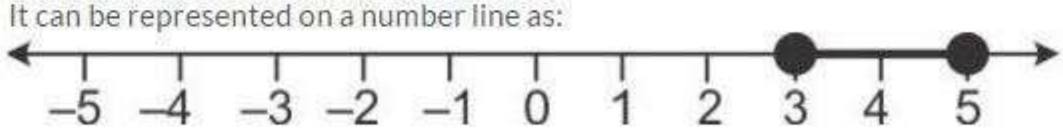
$A' \cap B = \{x: -4 \leq x \leq -1, x \in R\}$

It can be represented on a number line as:



(iii) $A - B = \{x: 3 \leq x \leq 5, x \in R\}$

It can be represented on a number line as:



Question 18.

P is the solution set of $7x - 2 > 4x + 1$ and Q is the solution set of $9x - 45 \geq 5(x - 5)$; where $x \in R$. Represent:

(i) $P \cap Q$

(ii) $P - Q$

(iii) $P \cap Q'$

on different number lines.

Solution:

$P = \{x : 7x - 2 > 4x + 1, x \in R\}$

$7x - 2 > 4x + 1$

$7x - 4x > 1 + 2$

$3x > 3$

$x > 1$

and

$Q = \{x : 9x - 45 \geq 5(x - 5), x \in R\}$

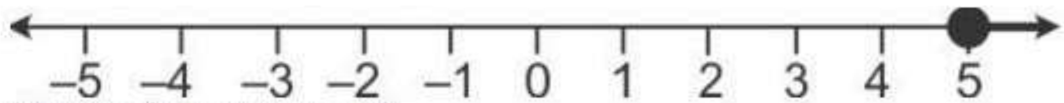
$9x - 45 \geq 5x - 25$

$9x - 5x \geq -25 + 45$

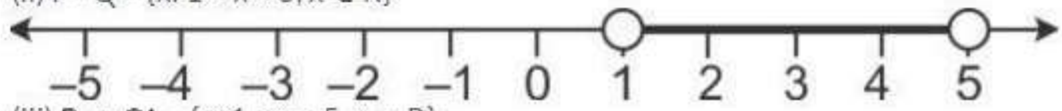
$4x \geq 20$

$x \geq 5$

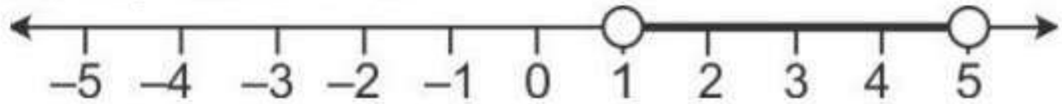
(i) $P \cap Q = \{x : x \geq 5, x \in R\}$



(ii) P - Q = {x: 1 < x < 5, x ∈ R}



(iii) P ∩ Q′ = {x: 1 < x < 5, x ∈ R}



Question 19.

Find the range of values of x, which satisfy:

$$-\frac{1}{3} \leq \frac{x}{2} + 1\frac{2}{3} < 5\frac{1}{6}$$

Graph, in each of the following cases, the values of x on the different real number lines:
(i) x ∈ W (ii) x ∈ Z (iii) x ∈ R

Solution:

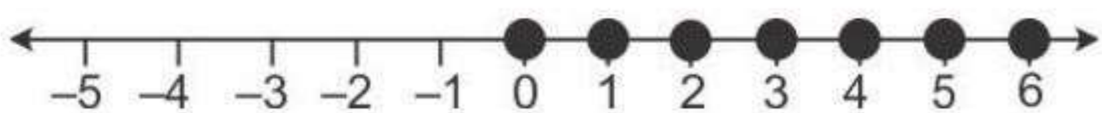
$$-\frac{1}{3} \leq \frac{x}{2} + 1\frac{2}{3} < 5\frac{1}{6}$$

$$-\frac{1}{3} - \frac{5}{3} \leq \frac{x}{2} < \frac{31}{6} - \frac{5}{3}$$

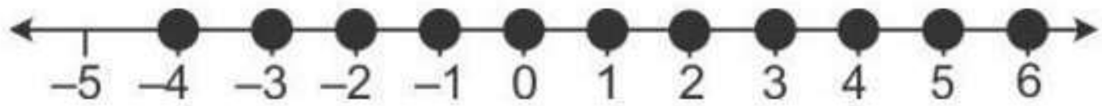
$$-\frac{6}{3} \leq \frac{x}{2} < \frac{21}{6}$$

$$-4 \leq x < 7$$

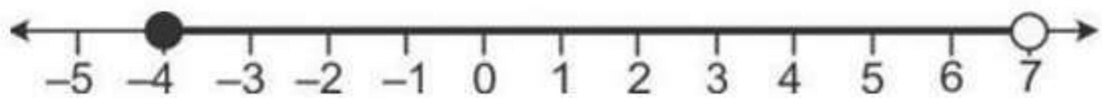
(i)If x ∈ W, range of values of x is {0, 1, 2, 3, 4, 5, 6}.



(ii) If x ∈ Z, range of values of x is {-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6}.



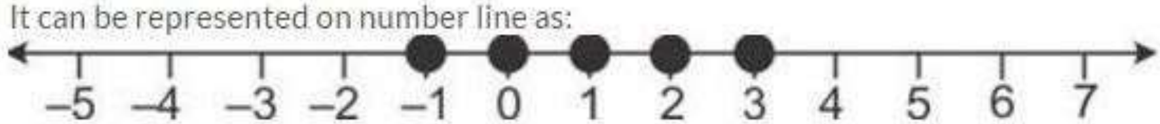
(iii)If $x \in R$, range of values of x is $-4 \leq x < 7$.



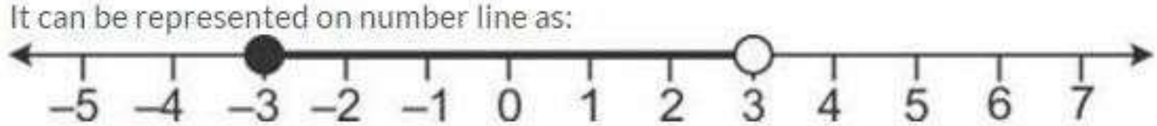
Question 20.
Given: A = {x: -8 < 5x + 2 ≤ 17, x ∈ I}, B = {x: -2 ≤ 7 + 3x < 17, x ∈ R}
Where R = {real numbers} and I = {integers}. Represent A and B on two different number lines. Write down the elements of A ∩ B.

Solution:

A = {x: -8 < 5x + 2 ≤ 17, x ∈ I}
= {x: -10 < 5x ≤ 15, x ∈ I}
= {x: -2 < x ≤ 3, x ∈ I}
It can be represented on number line as:



B = {x: -2 ≤ 7 + 3x < 17, x ∈ R}
= {x: -9 ≤ 3x < 10, x ∈ R}
= {x: -3 ≤ x < 3.33, x ∈ R}
It can be represented on number line as:



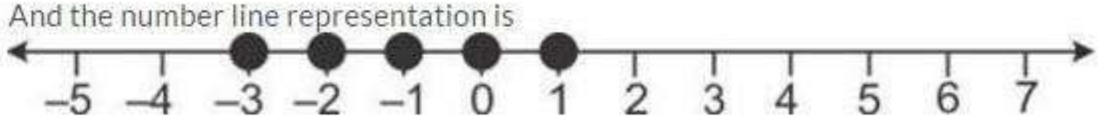
A ∩ B = {-1, 0, 1, 2, 3}

Question 21.

Solve the following inequation and represent the solution set on the number line 2x – 5 ≤ 5x +4 < 11, where x ∈ I

Solution:

$2x - 5 \leq 5x + 4$ and $5x + 4 < 11$
$2x - 5x \leq 4 - 5$ and $5x < 11 - 4$
$3x \leq -1$ and $5x < 7$
$x \geq -1$ and $x < \frac{7}{5}$
$x \geq -1$ and $x < 1.4$
Since $x \in$ I, the solution set is $\{-3, -2 - 1, 0, 1\}$
And the number line representation is

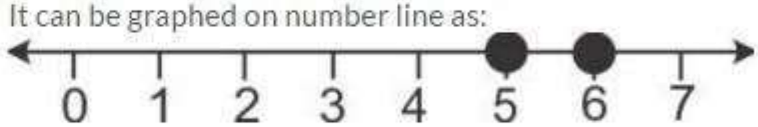


Question 22.

Given that $x \in$ I, solve the inequation and graph the solution on the number line:
$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$$

Solution:

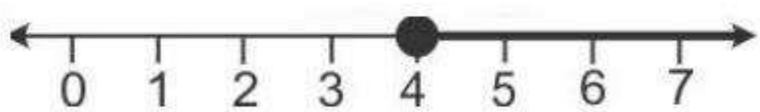
$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$$
$$3 \geq \frac{3x - 12 + 2x}{6} \geq 2$$
$$18 \geq 5x - 12 \geq 12$$
$$30 \geq 5x \geq 24$$
$$6 \geq x \geq 4.8$$
Solution set = {5, 6}
It can be graphed on number line as:



Question 23.
Given:
A = {x: $11x - 5 > 7x + 3$, $x \in$ R} and
B = {x: $18x - 9 \geq 15 + 12x$, $x \in$ R}.
Find the range of set $A \cap B$ and represent it on number line.

Solution:
A = {x: $11x - 5 > 7x + 3$, $x \in$ R}
= {x: $4x > 8$, $x \in$ R}

$= \{x: x > 2, x \in R\}$
$B = \{x: 18x - 9 \geq 15 + 12x, x \in R\}$
$= \{x: 6x \geq 24, x \in R\}$
$= \{x: x \geq 4, x \in R\}$
$A \cap B = \{x: x \geq 4, x \in R\}$
It can be represented on number line as:



Question 24.

Find the set of values of x, satisfying:
$7x + 3 \geq 3x - 5$ and $\frac{x}{4} - 5 \leq \frac{5}{4} - x$, where $x \in N$.

Solution:

$7x + 3 \geq 3x - 5$
$4x \geq -8$
$x \geq -2$
$\frac{x}{4} - 5 \leq \frac{5}{4} - x$
$\frac{x}{4} + x \leq \frac{5}{4} + 5$
$\frac{5x}{4} \leq \frac{25}{4}$
$x \leq 5$

Since, $x \in N$
$\therefore$ Solution set = {1, 2, 3, 4, 5}

Question 25.

Solve:
(i) $\frac{x}{2} + 5 \leq \frac{x}{3} + 6$, where x is a positive odd integer.
(ii) $\frac{2x + 3}{3} \geq \frac{3x - 1}{4}$, where x is a positive even integer.

Solution:

(i) $\frac{x}{2} + 5 \leq \frac{x}{3} + 6$

$\frac{x}{2} - \frac{x}{3} \leq 6 - 5$

$\frac{x}{6} \leq 1$

$x \leq 6$

Since, x is a positive odd integer
∴ Solution set = {1, 3, 5}

(ii) $\frac{2x+3}{3} \geq \frac{3x-1}{4}$

$8x + 12 \geq 9x - 3$

$-x \geq -15$

$x \leq 15$

Since, x is a positive even integer
∴ Solution set = {2, 4, 6, 8, 10, 12, 14}

Question 26.

Solve the inequation:
$-2\frac{1}{2} + 2x \leq \frac{4x}{5} \leq \frac{4}{3} + 2x$, $x \in$ W. Graph the solution set on the number line.

Solution:

$-2\frac{1}{2} + 2x \leq \frac{4x}{5} \leq \frac{4}{3} + 2x$

$-2\frac{1}{2} \leq \frac{4x}{5} - 2x \leq \frac{4}{3}$

$-\frac{5}{2} \leq -\frac{6x}{5} \leq \frac{4}{3}$

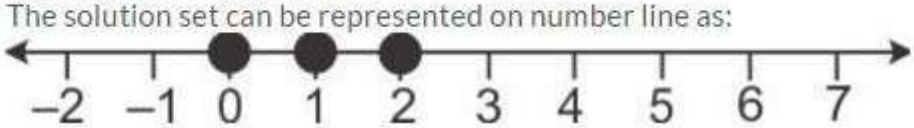
$\frac{25}{12} \geq x \geq -\frac{10}{9}$

$2.083 \geq x \geq -1.111$

Since, $x \in W$

$\therefore$ Solution set = {0, 1, 2}

The solution set can be represented on number line as:



Question 27.
Find three consecutive largest positive integers such that the sum of one-third of first, one-fourth of second and one-fifth of third is atmost 20.

Solution:
Let the required integers be x, x + 1 and x + 2.
According to the given statement,

$$\frac{1}{3}x + \frac{1}{4}(x+1) + \frac{1}{5}(x+2) \leq 20$$

$$\frac{20x + 15x + 15 + 12x + 24}{60} \leq 20$$

$$47x + 39 \leq 1200$$

$$47x \leq 1161$$

$$x \leq 24.702$$

Thus, the largest value of the positive integer x is 24.
Hence, the required integers are 24, 25 and 26.

Question 28.
Solve the given inequation and graph the solution on the number line.
$2y - 3 < y + 1 \leq 4y + 7, y \in R$

Solution:

$2y - 3 < y + 1 \leq 4y + 7, y \in R$

$\Rightarrow 2y - 3 - y < y + 1 - y \leq 4y + 7 - y$

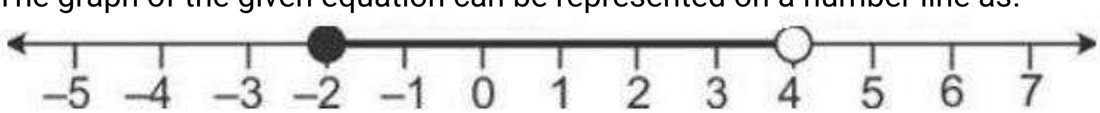
$\Rightarrow y - 3 < 1 \leq 3y + 7$

$\Rightarrow y - 3 < 1$ and $1 \leq 3y + 7$

$\Rightarrow y < 4$ and $3y \geq 6 \Rightarrow y \geq -2$

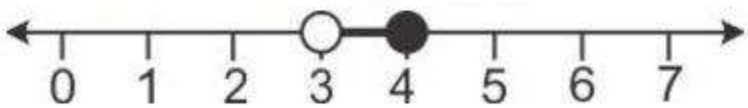
$\Rightarrow -2 \leq y < 4$

The graph of the given equation can be represented on a number line as:



Question 29.
Solve the inequation:
$3z - 5 \leq z + 3 < 5z - 9, z \in R$.
Graph the solution set on the number line.

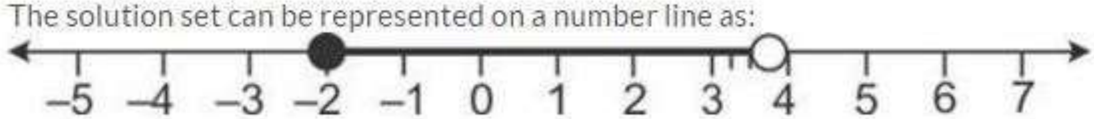
Solution:
$3z - 5 \leq z + 3 < 5z - 9$
$3z - 5 \leq z + 3$ and $z + 3 < 5z - 9$
$2z \leq 8$ and $12 < 4z$
$z \leq 4$ and $3 < z$
Since, z R
$\therefore$ Solution set = $\{3 < z \leq 4, x \in R\}$
It can be represented on a number line as:



Question 30.
Solve the following inequation and represent the solution set on the number line.
$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in R$

Solution:

$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}$
Multiply by 6, we get
$\Rightarrow -18 < -3 - 4x \leq 5$
$\Rightarrow -15 < -4x \leq 8$
Dividing by -4, we get
$\Rightarrow \frac{-15}{-4} > x \geq \frac{8}{-4}$
$\Rightarrow -2 \leq x < \frac{15}{4}$
$\Rightarrow x \in \left[-2, \frac{15}{4}\right)$
The solution set can be represented on a number line as:



Question 31.

Solve the following inequation and represent the solution set on the number line:

$4x - 19 < \frac{3x}{5} - 2 \le \frac{-2}{5} + x, x \in R$

Solution:

Consider the given inequation:

$$4x - 19 < \frac{3x}{5} - 2 \le \frac{-2}{5} + x, x \in R$$

$$\Rightarrow 4x - 19 + 2 < \frac{3x}{5} - 2 + 2 \le \frac{-2}{5} + x + 2, x \in R$$

$$\Rightarrow 4x - 17 < \frac{3x}{5} \le x + \frac{8}{5}, x \in R$$

$$\Rightarrow 4x - \frac{3x}{5} < 17 \text{ and } \frac{-8}{5} \le x - \frac{3x}{5}, x \in R$$

$$\Rightarrow \frac{20x - 3x}{5} < 17 \text{ and } \frac{-8}{5} \le \frac{5x - 3x}{5}, x \in R$$

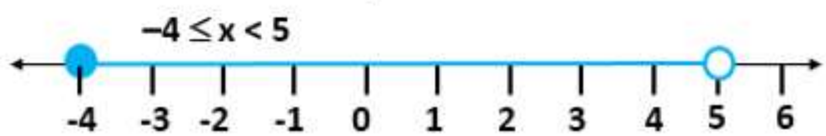
$$\Rightarrow \frac{17x}{5} < 17 \text{ and } \frac{-8}{5} \le \frac{2x}{5}, x \in R$$

$$\Rightarrow \frac{x}{5} < 1 \text{ and } -4 \le x, x \in R$$

$$\Rightarrow x < 5 \text{ and } -4 \le x, x \in R$$

$\Rightarrow -4 \le x < 5$; where $x \in R$

The solution set can be represented on a number line as follows:



Question 32.

Solve the following in equation, write the solution set and represent it on the number line:

$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$

Solution:

The given inequation is

$$-\frac{x}{3} \le \frac{x}{2} - 1\frac{1}{3} < \frac{1}{6}, x \in R$$

$$\Rightarrow -\frac{x}{3} \le \frac{x}{2} - \frac{4}{3} < \frac{1}{6}$$

Now,

$$-\frac{x}{3} \le \frac{x}{2} - \frac{4}{3}$$

$$\Rightarrow -\frac{x}{3} - \frac{x}{2} \le -\frac{4}{3}$$

$$\Rightarrow \frac{2x+3x}{6} \ge \frac{4}{3}$$

$$\Rightarrow \frac{5x}{6} \ge \frac{4}{3}$$

$$\Rightarrow 5x \ge 8$$

$$\Rightarrow x \ge \frac{8}{5}$$

$$\Rightarrow x \ge 1.6$$

$$\frac{x}{2} - \frac{4}{3} < \frac{1}{6}$$

$$\Rightarrow \frac{x}{2} < \frac{1}{6} + \frac{4}{3}$$

$$\Rightarrow \frac{x}{2} < \frac{1+4\times 2}{6}$$

$$\Rightarrow \frac{x}{2} < \frac{1+8}{6}$$

$$\Rightarrow \frac{x}{2} < \frac{9}{6}$$

$$\Rightarrow \frac{x}{2} < \frac{3}{2}$$

$$\Rightarrow x < 3$$

$\therefore$ Solution set = $\{x : 1.6 \le x < 3\}$

It can be represented on a number line as follows :

Question 33.

Find the values of x, which satify the inequation

$$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2, x \in W, .$$

Graph the solution set on the number line.

Solution:

We need to find the values of x, such that

x satisfies the inequation $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2, x \in W$

Consider the given inequation:

$$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \le 2$$

$$\Rightarrow \frac{-17}{6} < \frac{3-4x}{6} \le \frac{12}{6}$$

$$\Rightarrow \frac{17}{6} > \frac{4x-3}{6} \ge \frac{-12}{6}$$

$$\Rightarrow 17 > 4x - 3 \ge -12$$

$$\Rightarrow -12 \le 4x - 3 < 17$$

$$\Rightarrow -12 + 3 \le 4x - 3 + 3 < 17 + 3$$

$$\Rightarrow -9 \le 4x < 20$$

$$\Rightarrow -\frac{9}{4} \le \frac{4x}{4} < \frac{20}{4}$$

$$\Rightarrow -\frac{9}{4} \le x < 5$$

Since $x \in W$, the values of x are 0, 1, 2, 3, 4.

And the required line is

Question 34.

Solve the following in equation and write the solution set:
$13x - 5 < 15x + 4 < 7x + 12, x \in R$

Solution:

$13x - 5 < 15x + 4 < 7x + 12,\ x \in R$

We have

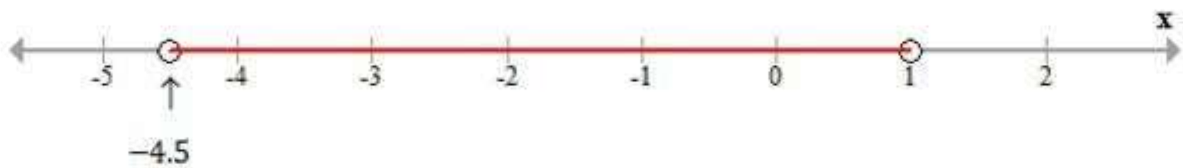
$13x - 5 < 15x + 4$	and	$15x + 4 < 7x + 12$
$\Rightarrow 13x < 15x + 9$		$\Rightarrow 15x < 7x + 8$
$\Rightarrow 0 < 2x + 9$		$\Rightarrow 8x < 8$
$\Rightarrow -9 < 2x$		$\Rightarrow x < 1$
$\Rightarrow -\frac{9}{2} < x$		$\Rightarrow x < 1$

$\therefore -\frac{9}{2} < x < 1$

i.e. $-4.5 < x < 1$

$\therefore$ Solution set $= \{x : x \in R \text{ and } -4.5 < x < 1\}$

The required line is



Question 35.

Solve the following inequation, write the solution set and represent it on the number line.

-3(x – 7) ≥ 15 – 7x > x+1/3, x R.

Solution:

$$-3(x-7) \geq 15 - 7x > \frac{x+1}{3}, x \in R$$

$$\Rightarrow -3(x-7) \geq 15 - 7x \text{ and } 15 - 7x > \frac{x+1}{3}$$

$$\Rightarrow -3x + 21 \geq 15 - 7x \text{ and } 45 - 21x > x + 1$$

$$\Rightarrow -3x + 7x \geq 15 - 21 \text{ and } 45 - 1 > x + 21x$$

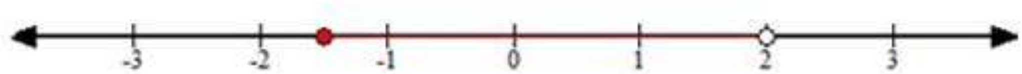
$$\Rightarrow 4x \geq -6 \text{ and } 44 > 22x$$

$$\Rightarrow x \geq \frac{-3}{2} \text{ and } 2 > x$$

$$\Rightarrow x \geq -1.5 \text{ and } 2 > x$$

$\therefore$ The solution set is $\{x : x \in R, -1.5 \leq x < 2\}$.

The solution set is represented on number line as follows:



Question 36.

Solve the following inequation and represent the solution set on a number line.

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \le 7\frac{1}{2},\ x \in I$$

Solution:

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \le 7\frac{1}{2},\ x \in I$$

$$-8\frac{1}{2} < -\frac{1}{2} - 4x$$
$$\Rightarrow \frac{-15}{2} < -\frac{1}{2} - 4x$$
$$\Rightarrow \frac{-15}{2} + \frac{1}{2} < -4x$$
$$\Rightarrow \frac{-14}{2} < -4x$$
$$\Rightarrow -7 < -4x$$
$$\Rightarrow 7 > 4x$$
$$\Rightarrow x < \frac{7}{4}$$

$$-\frac{1}{2} - 4x \le 7\frac{1}{2}$$
$$\Rightarrow -\frac{1}{2} - 4x \le \frac{15}{2}$$
$$\Rightarrow -4x \le \frac{15}{2} + \frac{1}{2}$$
$$\Rightarrow -4x \le 8$$
$$\Rightarrow x \ge -2$$

So,

$$\frac{7}{4} > x \ge -2$$

As, $x \in I$

$$x = \{-2, -1, 0, 1\}$$