Chapter 1

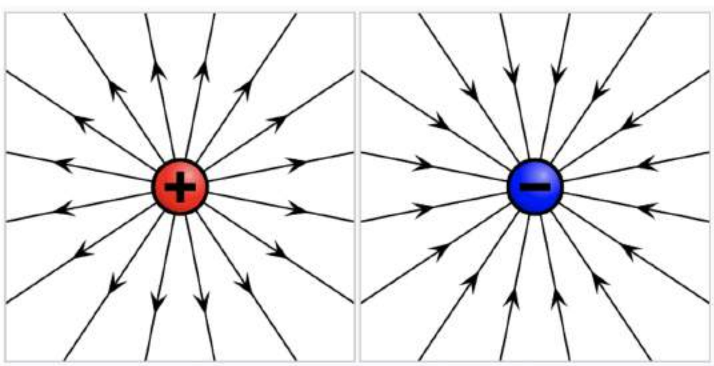
Electric Charges and Fields

Introduction to Electric Charges

What is Electric Charge?

Electric charge is the basic physical property of matter that causes it to experience a force when kept in an electric or magnetic field.

- An electric charge is associated with an electric field and the moving electric charge generates a magnetic field. A combination of electric and magnetic fields is known as the **electromagnetic field.**
- Interaction of the charges generates an electromagnetic force which is the foundation of Physics. Electric Charge comes from the name of electricity, which is coined from the Greek word 'elektron' meaning amber.



Electric field induced by a positive electric charge (left) and a field induced by a negative electric charge (right).

Types of Electric Charges

Two kinds of electric charges are there:

1. Positive (+) charge
2. Negative (-) charge

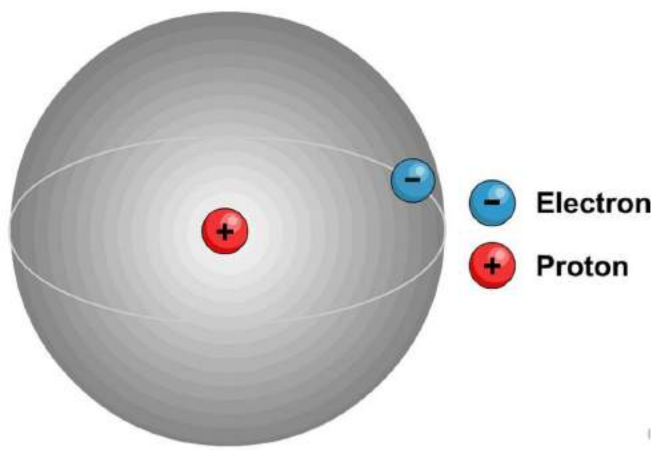
Positive Charge: When an object has a positive charge it means that it has more protons than electrons.

Negative Charge: When an object has a negative charge it means that it has more electrons than protons.

When there is an identical number of positive and negative charges, the negative and positive charges would cancel out each other and the object would become **neutral**.

Important Facts About Electric Charges

- Atoms are the building blocks of the universe. Whatever you see around you can be divided into smaller and smaller parts until you finally reach a part you cannot divide further. This building block is what we call an Atom.
  Inside an atom are protons, electrons and neutrons. Out of the three, electrons and protons fit the definition of an electric charge.
- The protons are positively charged, the electrons are negatively charged, and the neutrons are neutral. A majority of the mass of the atom is concentrated into a very tiny space in the centre called the nucleus and the electrons revolve around this heavy nucleus.
- This means that electrons are held very loosely compared to protons. Therefore from henceforth the movement of charges here will be restricted to the movement of electrons. Since the atoms are made up of protons and electrons, we can safely conclude that all things are made up of electric charges.



Note:
Quantity of negative charge on an electron = quantity of positive charge on a proton.

- The charge of one proton is equal in strength to the charge of one electron. When the number of protons in an atom equals the number of electrons, the atom itself has no overall charge, it is neutral.
- Charge of a material body or particle is the property (acquired or natural) due to which it produces and experiences electrical and magnetic effects. Some of the naturally charged particles are **electrons, proton, α-particle** etc.
- Benjamin Franklin introduced the concept of positive and negative charges.

Is Electric Charge a Vector Quantity?

- No, electric charge is a **scalar quantity**.
- Apart from having a 'magnitude' and 'direction', for a quantity to be termed a vector (which we will study in detail later) it should also obey the laws of vector addition such as triangle law of vector addition and parallelogram law of vector addition, only then the quantity is said to be a vector quantity.
- In the case of an electric current, when two currents meet at a junction, the resultant current of these *will be an algebraic sum and not the vector sum*. Therefore, an electric current is a scalar quantity although it possesses magnitude and direction.

How to measure an Electric charge?

The electric charge is measured using a coulomb.
"One coulomb is the quantity of charge transferred in one second."

Mathematically, the definition of a coulomb is represented as:
$\mathbf{Q = I.t}$

In the equation, Q is the electric charge, I is the electric current and t is the time.

Unit of Electric Charge

A charge is a derived physical quantity. The charge is measured in **coulomb** in the S.I. unit.
In practice we use:

- millicoulomb mC (10^{-3} C)
- microcoulomb μC (10^{-6} C)
- nanocoulombs nC (10^{-9} C)
- C.G.S unit of charge = electrostatic unit = esu
- 1 coulomb = 3×10^9 esu of charge

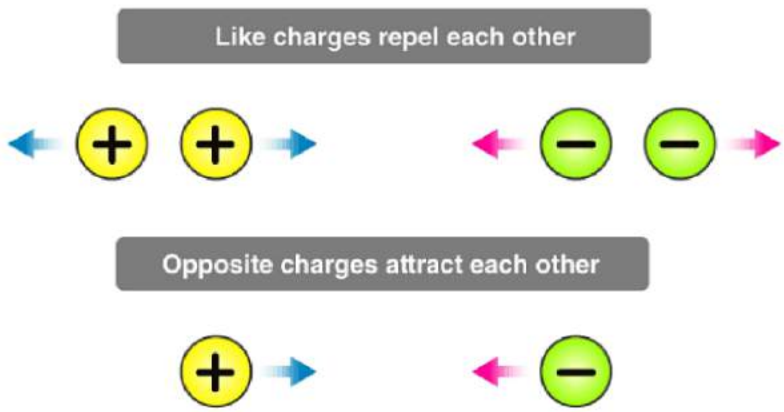
• Dimensional formula of charge = $[M^0L^0T^1A^1]$

Note:

• Charge of a single electron = -1.602 × 10-19 C
• Charge of a single proton = + 1.602 × 10-19 C
• Charge of a single neutron = 0 C

Properties of Electric Charge

1. Like charges repel each other and unlike charges attract each other. (As Electric Charge comes in two varieties, which are called "plus" and "minus".)

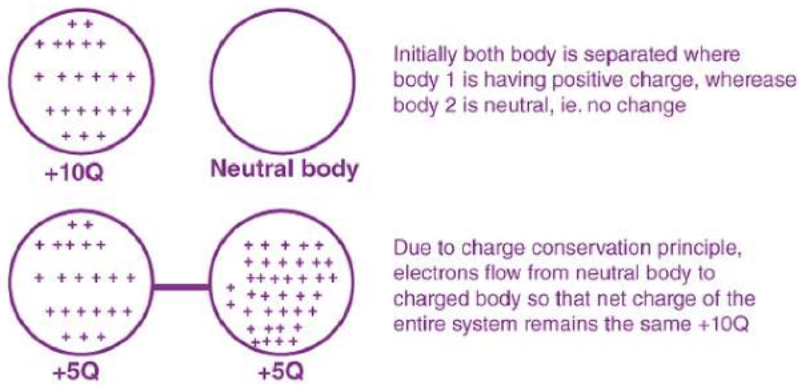


2. Electric Charge is a scalar quantity: It follows scalar laws of operations, i.e. it adds algebraically and represents the excess of electrons in a negatively charged atom or a deficiency of electrons in a positively charged atom.

3. A charge is transferable: Electric charge can be transferred from one body to another, but there is a restriction to the charge transfer. Only electrons are transferred from one body to another because protons are tightly bound to the nucleus of every atom. Hence, the body which loses electrons in the transfer becomes positively charged, and the body which receives electrons becomes negatively charged.

• A neutral body has a number of electrons = number of protons
• A positively charged body has a number of electrons < number of protons
• A negatively charged body has a number of electrons > number of protons.

4. Charge is always conserved: In an isolated system, the total charge (sum of positive and negative) remains constant whatever charge transfer takes place in the system internally. It is called the principle of charge conservation.



Conservation of charge

5. Charge is quantized: Charge on anybody always exists in integral multiples of a fundamental unit of electric charge. This unit is equal to the magnitude of the charge on one electron ($1e = 1.6 \times 10^{-19}$ C). So charge on anybody $Q = \pm\, ne$, where n is an integer and e is the charge on a single electron. This was proved by **Millikan's oil drop experiment.**

- Recently, the existence of particles of charge $+(2/3)$ e and $-(1/3)$ e has been postulated. These particles are called **quarks,** but still, this is not considered as the quantum of charge because these are unstable (They have a very short span of life.)

6. Charge is always associated with mass: Yes! Electrons, Protons and Neutrons also have masses.

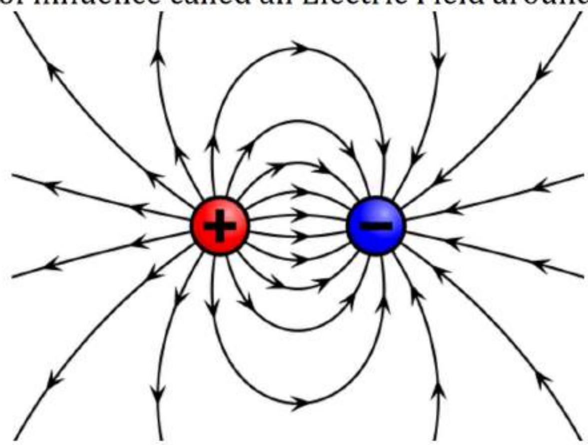
Their value is determined, experimentally, to be following:

- Mass of an electron = $\mathbf{9.109 \times 10^{-31}}$ Kg = 5.49×10^{-4} amu
- Mass of a proton = $\mathbf{1.6726 \times 10^{-27}}$ Kg = 1.007 amu
- Mass of a neutron = $\mathbf{1.6749 \times 10^{-27}}$ Kg = 1.008 amu

- It is recommended to remember these values in Kg (SI units). Also, please note that the mass of a neutron is slightly greater than the mass of a proton.
- This also shows that the mass of a negatively charged body is greater than the mass of a positively charged identical body as it would have an excess number of electrons than the positively charged bodies.

7. Charge is relativistically invariant: This means that **charge is independent of the frame of reference,** i.e., the charge on a body does not change whatever be its speed. This property is worth mentioning as in contrast to charge, the mass of a body depends on its speed and increases with an increase in speed. You will be exposed to this property later when you will learn The Special Theory of Relativity.

8. A charge at rest produces an only an electric field around itself: A charge at rest creates a region of influence called an Electric Field around itself in space.

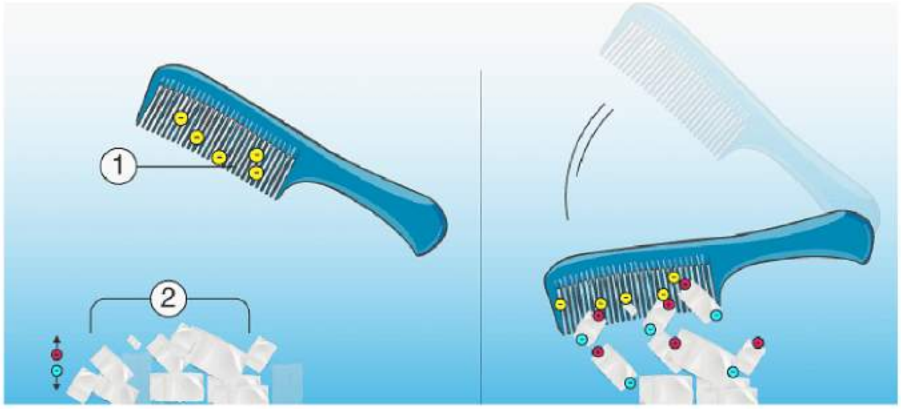


Electric field lines

- While a charge having uniform motion (constant velocity) produces electric as well as the magnetic field around itself.
- Accelerated charges produce a special combination of electric and magnetic fields called electromagnetic waves. We will study the electric field in detail in the coming section.

Activity to Understand Charges

- Have you felt the sudden painful jolt you get when you brush someone's arm? If you have a woollen blanket then you should definitely do this, switch off the light and brush your clothes with the woollen blanket.
- The number of sparks that go off will amaze you. Believe it or not, this very phenomenon is also responsible for all the lightning strikes on our planet. So what is it? It is known as Static Electricity.
- Static electricity can be a nuisance or even a danger. The energy that makes your hair to stand on end can also damage electronic equipment's and cause explosions.



The comb attracting small pieces of paper with static electricity

What is Static Electricity?

Static electricity refers to an imbalance between the electric charges in a body, specifically the imbalance between the negative and the positive charges on a body.

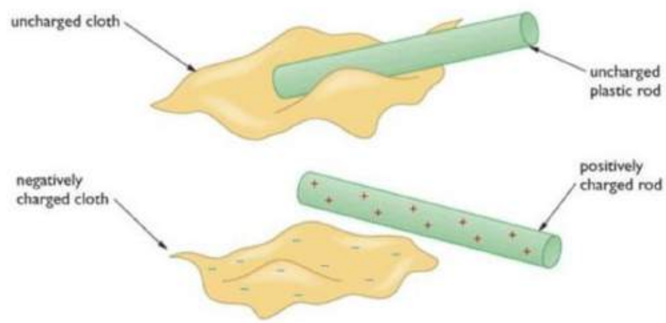
- The imbalance in the charge is introduced by physical means. One of the most common causes of static electricity is contact between solid objects. It was mentioned earlier that the movement of protons is not possible and the only movement of electric charge seen in static electricity is electrons.
- Electrons in materials are held extremely loosely meaning that they can be exchanged through simple contact like rubbing.
- The image below is an example of rubbing a glass rod with silk which causes static electricity. When two objects are rubbed together to create static electricity, one object gives up electrons and becomes more positively charged while the other material collects electrons and becomes more negatively charged.
- We should keep in mind that the rules such as like charges repel and unlike charges attract is applicable here.

Charging by Induction

Most objects are electrically neutral, which means that they have an equal number of positive and negative charges. In order to charge an object, one has to alter the charge balance of positive and negative charges. There are three ways to do it: friction, conduction and induction.

Charging by Friction

The charging by friction process involves rubbing of one particle on another resulting in electrons moving from one surface to another. This method is useful for charging insulators.



Rubbing a neutral rod with a neutral piece of cloth can result in them becoming charged

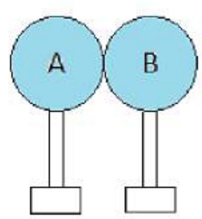
Charging by Conduction

The charging by conduction process involves touching of a charged particle to a conductive material. This way, the charges are transferred from the charged material to the conductor. This method is useful for charging conductors.

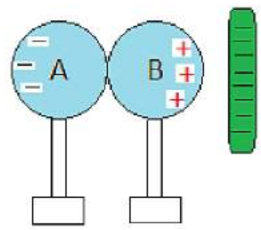
Charging By Induction

- Charging by induction occurs when we bring a charged object near a conductor. It is not for no reasons that we say near.
- The charged object does not actually touch the conductor. The charged object is just allowed to get close to the conductor.
- As a result, the conductor will be charged. We say that a charge has been induced in the conductor. We will make this clear with an experiment.

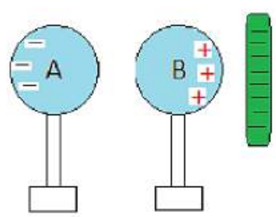
Experiment



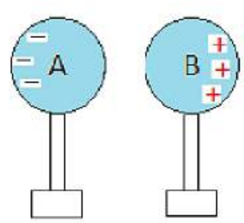
- Consider the two spheres above made with a metal.
- Since metal is a good conductor, it is a good choice for this experiment.
  (i) They touch each other, so they become a single conductor.
  (ii) We put them on insulated stands so charges or electricity does not travel to the ground.
  (iii) The two spheres right now form a neutral system. This means that there is the same number of electrons and protons in each sphere.
- We get a rod that is negatively charged and we put it next to the two spheres. The rod is shown on the right in green. The lines inside the rod represent negative charges.



- Electrons in sphere B are repelled by the rod and move to sphere A to create an excessive charge called also net charge. This situation creates also a net charge in sphere B.
- We say that a charge has been induced on the spheres.
- We can separate the spheres while the rod is still there.



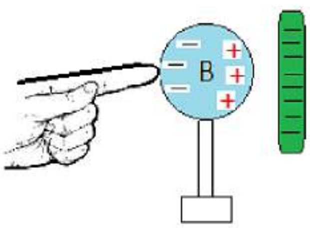
- Finally, we can remove the rod completely.



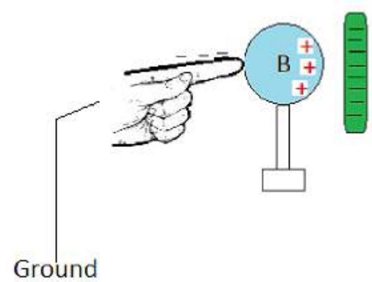
- The spheres will keep their charges and this is what we mean by charging by induction.

- The charges on the spheres are equal and opposite.
- The charges are equal because for each single electron that goes to A and therefore creates a single negative net charge, it leaves B with a single positive charge.

In our example above, 3 electrons went to A creating a situation where B has a positive charge with 3 protons.
Keep in mind that spheres A and B could have billions and billions of electrons. The reason that sphere A is charged now is because it has an excess of 3 electrons although this is a small charge. By the same token, the reason sphere B is charged is because it has 3 more protons now than electrons.
Charging by induction and grounding Use only 1 sphere this time and induce a charge again with a charged rod. Then, put your finger where the electrons are.



When you touch the metal with your finger, electrons leave the sphere by means of your finger and enter the ground.



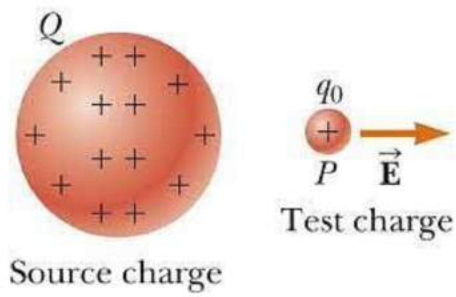
When we allow charges or electricity to leave a conductor by touching it, we are grounding the conductor.

Electric Field & Electric Field Lines

Electric Field

The concept of a field was developed by Michael Faraday. An electric field intensity or simply, electric field is said to exist in the region of space around a charged object. When another charged object, (the test charge) enters this space, we say the test charge experiences an electric force, F_e due to this field.

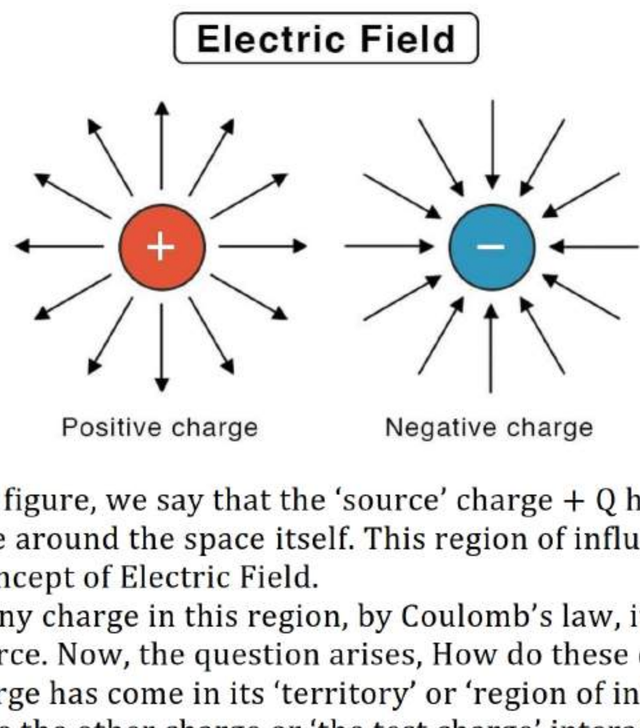
Definition: We define the electric field due to the source charge at the location of the test charge to be the electric force on the test charge per unit charge.



In simple words, Electrostatic force per unit positive test charge is defined as Electric Field due to the source charge.

$\vec{E}$ is a vector quantity and its direction is same as that of force on the test charge. The unit and dimensions for Electric Field would be **Newton/Coulomb** and $[M^1 L^1 T^{-3} I^{-1}]$ respectively.

Note: The electric field is the property of its source. Presence of test charge is not necessary for the Electric field (due to source charge) to exist. It exists with or without the test charge. The test charge is used to detect and measure the Electric field due to source charge.

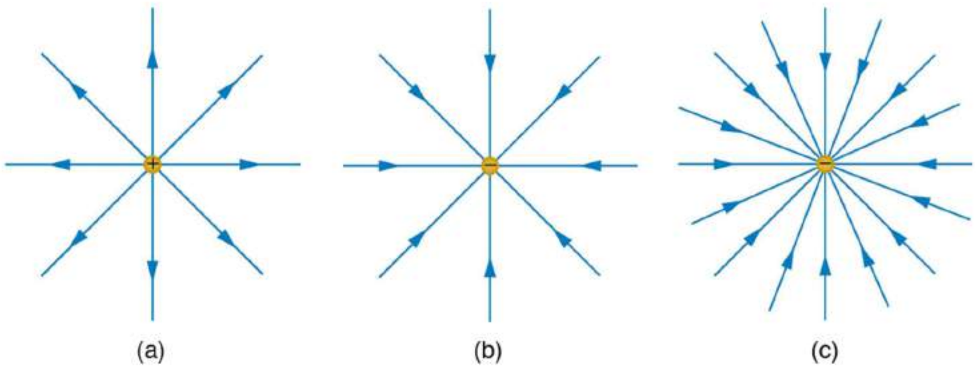


Note: Observe the figure, we say that the 'source' charge + Q has created a region of influence around the space itself. This region of influence is visualized by defining the concept of Electric Field.

Now, if we bring any charge in this region, by Coulomb's law, it will experience an electrostatic force. Now, the question arises, How do these charges realise that the other charge has come in its 'territory' or 'region of influence'?

It happens because the other charge or 'the test charge' interacts with the Electric Field of the source charge and thus, electrostatic force is exerted on each

other.
Conventionally, we only take the test charge to be positive. Therefore, a positive source charge would repel a 'positive test charge' and a negative source charge would attract a 'positive test charge'. Thus, the electric field can be visualized in space as following:

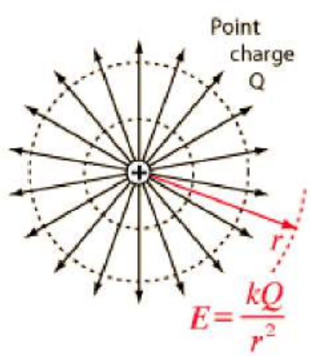


The direction of electric field is radially outwards for a positive charge and is radially inwards for a negative charge as shown in the figure above based on the direction of electrostatic force on 'positive test charge'.

There are some points to always to be kept in mind. These are

1. It is important to note that with every charged particle, there is an electric field associated which extends up to infinity.
2. No charged particle experiences force due to its own electric field.

Electric Field Strength due to Point Charge



As discussed earlier, if we find the electric field due to a point charge at a distance r from it. Its magnitude can be given as

$$\vec{E} = \frac{\vec{F_e}}{q}$$

By definition,
Electrostatic force on test charge +q,

$$F_e = K\frac{(+Q)(+q)}{r^2}\hat{r}$$

Now,

$$\vec{E} = K\frac{(+Q)(+q)}{qr^2}\hat{r} = \frac{KQ}{r^2}\hat{r}$$

Note: That if the source charge is negative i.e. –Q then we can visualise the electric field by

$$\vec{E} = K\frac{(-Q)(+q)}{qr^2}\hat{r} = \frac{K(-Q)}{r^2}\hat{r} = \frac{KQ}{r^2}(-\hat{r})$$

This means, we can simply reverse the Electric field vector's direction.
Thus, Electric Field due to a point charge of magnitude + Q at a distance r from it is given by the expression KQ/r^2 and its direction is along the line joining the source charge and the point of consideration.

Electric Field Lines

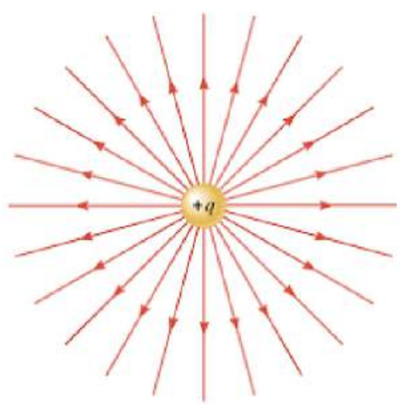
- The concept of Electric field intensity or Electric field is visualized using Electric Field Lines.
- Electric field due to point charges we sketched the 'region of influence' or 'electric field' using lines. If the source charge is positive, then the lines are radially outwards and if the source charge is negative then the lines are radially inwards. These are not physical lines in the space, these are imaginary lines called Electric Fields lines.
- Any charge creates a region of influence called 'Electric Field' which is visualized by 'Electric Field Lines' and whenever other charge particle enters into this region, it experiences an 'electrostatic force' expressed by Coulomb's Law.

Q. Why have we defined the concept of electric field? Is it really necessary? If eventually, we are measuring the electrostatic force, why can't we do it directly using Coulomb's Force?
Ans.

- When charges are stationary, the concept of electric field is convenient, but not really necessary. Electric field in electrostatics is an elegant way of characterising the electrical environment of a system of charges.

- The true physical significance of the concept of electric field, however, emerges only when we go beyond electrostatics and deal with time dependent electromagnetic phenomena.
- Suppose we consider the force between two distant charges q_1, q_2 in accelerated motion. The greatest speed with which a signal or information can go from one point to another is c, the speed of light. Thus, the effect of any motion of q_1 on q_2 cannot arise instantaneously. There will be some time delay between the effect (force on q_2) and the cause (motion of q_1). It is precisely here that the notion of electric field (strictly, electromagnetic field) is natural and very useful.
- The field picture is this: the accelerated motion of charge q_1 produces electromagnetic waves, which then propagate with the speed c, reach q_2 and cause a force on q_2. The notion of field elegantly accounts for the time delay. Thus, even though electric and magnetic fields can be detected only by their effects (forces) on charges, they are regarded as physical entities, not merely mathematical constructs. They have an independent dynamics of their own, i.e., they evolve according to laws of their own. They can also transport energy. Thus concept of field is now among the central concepts in physics.

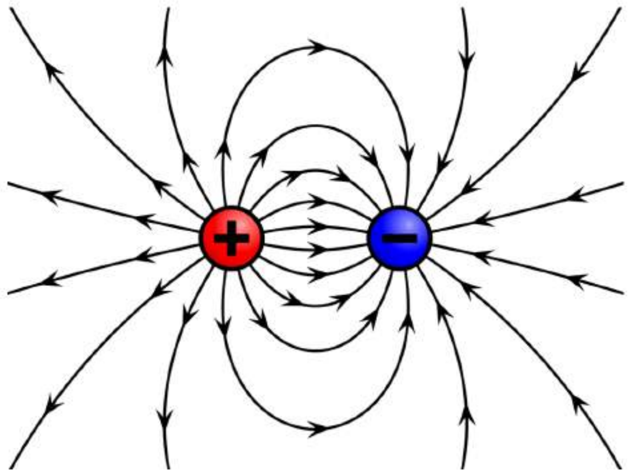
Concept of Electric Field Lines



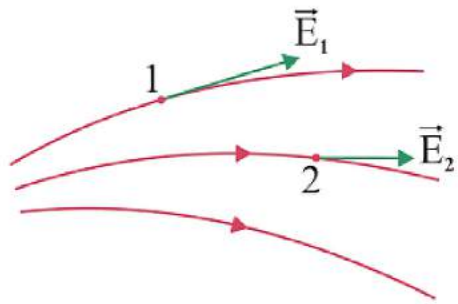
Remember, this diagram, it represent electric field or the 'region of influence' of charge +q around the space. Now, the arrows in this diagram are called the electric field lines. Without these lines, we would not be able to visualize the concept of electric field. In this figure, each arrow indicates the electric field, i.e., the force acting on a unit positive charge, placed at the tail of that arrow. Connect the arrows pointing in one direction and the resulting figure represents a field line. We thus get many field lines, all pointing outwards from the positive point charge.

Properties of Electric Field Lines

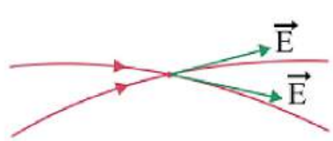
- Field lines start from positive charges and end at negative charges. If there is a single charge, they may start or end at infinity.



- In a charge-free region, electric field lines can be taken to be continuous curves without any breaks.
- Tangent drawn to an electric field line represents the direction of electric field at that point.



- Two field lines can never cross each other. (If they did, the field at the point of intersection will not have a unique direction, which is absurd.)



- Electrostatic field lines do not form any closed loops. This follows from the conservative nature of electric field.
- The number of field lines per unit area passing through a small cross-sectional area perpendicular to the electric field is called as density of field lines.
- Relative Density of Electric Field Lines represent the magnitude of Electric Field intensity 'E', which is strong near the charge, as the density of field lines is more near the charge and the lines are closer.
- Away from the charge, the field gets weaker and the density of field lines is less, resulting in well - separated lines. Some may draw more lines but the number of lines is not important. It is the relative density of lines in different regions which is important.

Note: That electric field lines of +2Q charge are twice in number than that of +Q. So, irrespective of the number of lines in each representation, the ratio must be maintained to 2.

- Relative density of field lines is inversely proportional to the square of distance
- Mathematically, Number of lines per unit area at distance r > number of lines per unit area at distance 2r > number of lines per unit area at a distance 3r measured from S.

Relative density of field lines at distance r/Relative density of field lines at distance 2r = 4/1

Similarly,

Relative density of field lines at distance 2r/Relative density of field lines at distance 3r = 9/4

Drawing Field Lines

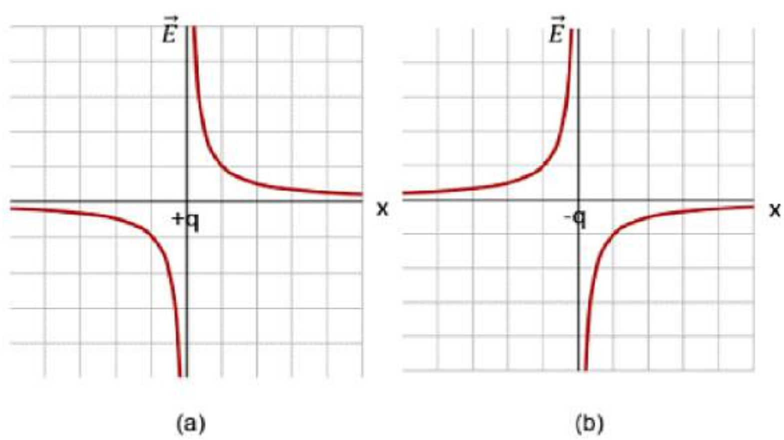
- Electric field lines are a way of pictorially mapping the electric field around a configuration of charges.
- An electric field line is, in general, a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point.
- An arrow on the curve is obviously necessary to specify the direction of electric field from the two possible directions indicated by a tangent to the curve. A field line is a space curve, i.e., a curve in three dimensions.

Note: That the red dots represent positive charge and the blue dots represent the negative charge and try to verify all the properties of electric field lines listed above.

Superposition of Electric Fields

- If we are dealing with many charges (let's assume n) then electric field at a point p is the vector sum
  $\overrightarrow{E_{net}} = K\ \sum_i^n \frac{Q_i}{r_i^2}\ \widehat{r_i}$
- Where r_i is the is the distance from the i^{th} source charge Q_i, to the point P and $\widehat{r_i}$ is a unit vector directed from Q_i toward P. If some more charge are added, more terms are added to the summation.
- However, there is no change to the terms that were already there, provided that the original charges do not move. If we know the electric fields generated by two different sets of charges separately, the electric field generated by both together is simply the **vector sum** of the two separate fields.
- The two fields, which each occupy three dimensional space, are superimposed on one another. Because it has this property, the electric field is said to satisfy the **principle of superposition**.

Graph of Electric Field Due to Binary Charge



(a) Electric field versus x for a positive point charge kept at the origin. Note that the electric field at positive x is positive, because it is in positive direction. At negative x it is negative, because it is in negative direction.
(b) Electric field versus x for a negative point charge kept at the origin. Note that the electric field at positive x is negative, because it is in negative direction. At negative x it is positive, because it is in positive direction.

Gold Leaf Electroscope

What is an Electroscope?

An electroscope is a scientific device that is used to detect the presence of an electric charge on a body.

- In the year 1600, British physician William Gilbert invented the first electroscope with a pivoted needle called **versorium.**
- The electroscope detects the charge based on the Coulomb electrostatic force which causes the motion of the test charge.
- An electroscope can be regarded as a crude voltmeter as the electric charge of an object is equal to its capacitance.
- An instrument that is used to measure the charge quantitatively is known as an **electrometer**.

Working of Electroscope

The working principle of an electroscope is based on the atomic structure of elements, charge induction, the internal structure of metal elements and the idea that like charges repel each other while unlike charges attract each other.

An electroscope is made up of a metal detector knob on top which is connected to a pair of metal leaves hanging from the bottom of the connecting rod. When no charge is present the metals leaves hang loosely downward. But, when an object with a charge is brought near an electroscope, one of the two things can happen.

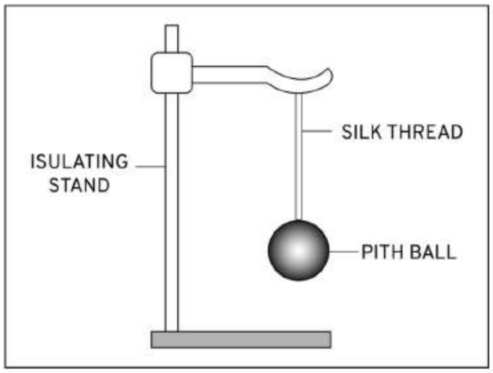
- When the charge is positive, electrons in the metal of the electroscope are attracted to the charge and move upward out of the leaves. This results in the leaves having a temporary positive charge and because like charges repel, the leaves separate. When the charge is removed, the electrons return to their original positions and the leaves relax.
- When the charge is negative, the electrons in the metal of the electroscope repel and move toward the leaves on the bottom. This causes the leaves to gain a temporary negative charge and because like charges repel, the leaves again separate. Then when the charge is removed, the electrons return to their original position and the leaves relax.

An electroscope responds to the presence of a charge through the movement of electrons either into or away from, the leaves. In both cases, the leaves separate. It is important to note that the electroscope cannot determine if the charged object is positive or negative – it is only responding to the presence of an electrical charge.

Types of electro scope

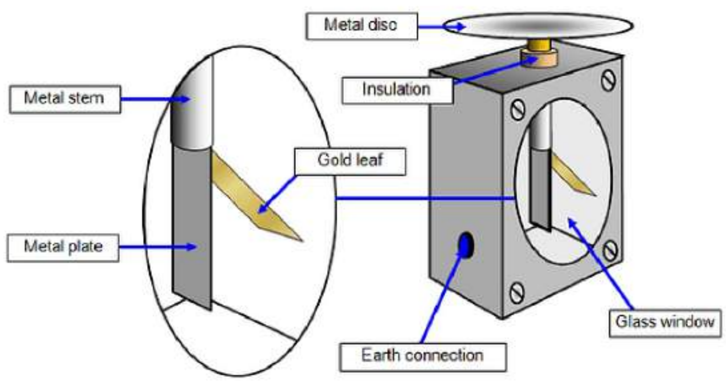
There are two classical types of electroscopes and they are as follows:

1. Pith-ball electroscope: Pith-ball electroscope was invented by John Canton in the year 1754. It consists of one or two small light balls that are a lightweight non-conductive substance called pith. In order to find if the object is charged or not, it is brought near an uncharged pith ball. If the ball gets attracted towards the object it means the object is charged.



2. Gold-leaf electroscope:

It is an instrument for detecting and measuring **static electricity or voltage**. A metal disc is connected to a narrow metal plate and a thin piece of gold leaf is fixed to the plate. The whole of this part of the electroscope is insulated from the body of the instrument. A glass front prevents air draughts but allows you to watch the behaviour of the leaf.



Gold Leaf Electroscope

When a charge is put on the disc at the top it spreads down to the plate and leaf. This means that both the leaf and plate will have the same charge. Similar charges repel each other and so the leaf rises away from the plate - the bigger the charge the more the leaf rises.

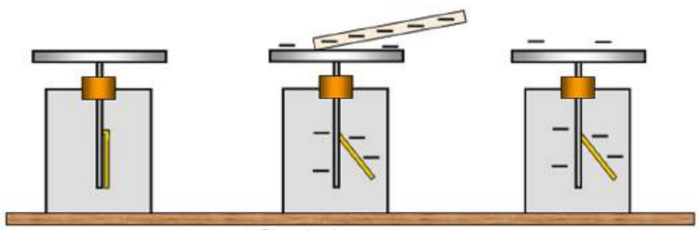
The leaf can be made to fall again by touching the disc - you have earthed the electroscope. An earth terminal prevents the case from becoming live.

Methods of Charging

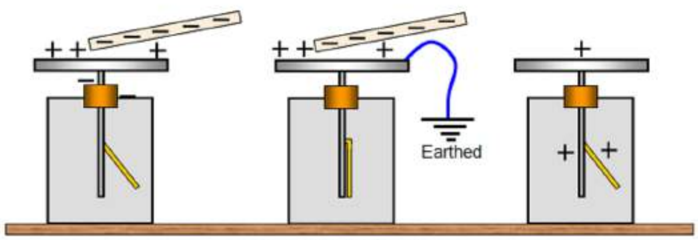
The electroscope can be charged in two ways:

- **By contacting:** A charged rod is touched on the surface of the disc and some of the charges is transferred to the electroscope. This is not a very effective method of charging the electroscope.
- **By induction:** A charged rod is brought up to the disc and then the electroscope is earthed, the rod is then removed.

The two methods give the gold leaf opposite charges.



Charging by contact



Charging by induction

The above diagrams show you how the charges spread over the plate and gold leaf in different conditions.

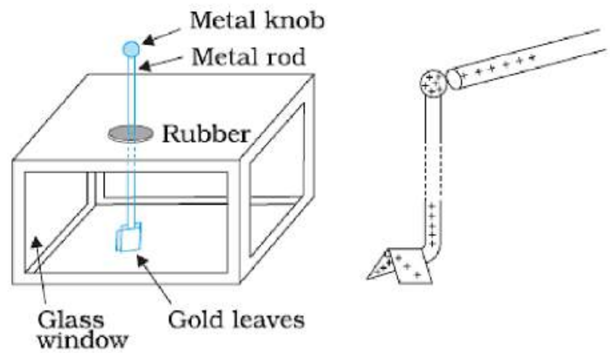
Construction of Gold Leaf Electroscope

It consists of a metal rod that is fitted in an insulating box. The metal rod has a metal knob at its top. Two gold leaves are also attached at the bottom end of the rod.

Working of Gold Leaf Electroscope

- Since electroscope is used to detect the presence of charge. So through it, we can find whether a body is charged or uncharged.
- There fore the body to be detected is brought close enough to the metal knob. When a charged object touches the knob at the top of the rod, charge flows through the rod onto the leaves.

- Both the gold leaves will have the same charge and hence as a result they will repel and diverge.
- The degree of divergence is an indicator of the amount of charge i.e., the more the charge, the more will be the divergence.



Uses of Electroscope

The various benefits of an electroscope are:

- They are useful to analyze the electrostatic charges and any ionizing radiation present in a body.
- The nature of the electrical charge is measurable using an electroscope.
- Also, with the help of an electroscope, we can easily compare the magnitudes of two different charges.

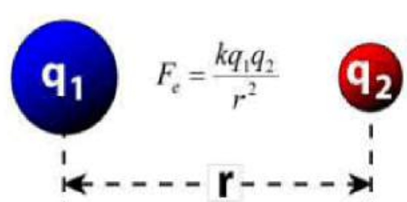
Coulomb's Law & Its Applications

Coulomb's Law

Charles Augustin de Coulomb, in 1785 through his experiments, found out that the twopoint charges 'q_1' and 'q_2' kept at a distance 'r' in a medium exert an electrostatic force 'F' on each other. The value of force F is given by

$$F = \frac{k|q_1||q_2|}{r^2}$$

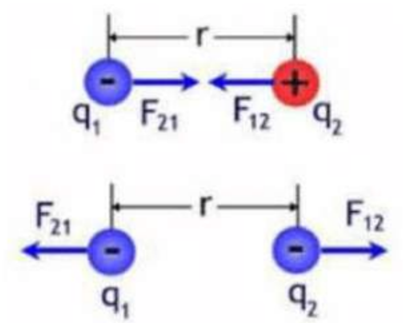
This law gives the net electrostatic force experienced by q_1 due to q_2 and vice versa.



Where,
F gives the magnitude of electrostatic force,
'q1 and 'q2 are the magnitudes of the two interacting point charges,
K is electrostatic constant which depends upon the medium surrounding the two charges. For vacuum, it is equal to 1/4πε0, the symbol ε0 is called 'epsilon naught' and it represents permittivity of vacuum.
$K = 1/4\pi\varepsilon_0 = 9 \times 10^9\ Nm^2/c^2$
$\varepsilon_0 = 8.854 \times 10^{-12}\ C^2/Nm^2$
If the point charges are kept in a medium with permittivity ε, then the electrostatic force between the point charges will be:

$$F = \frac{k|q_1||q_2|}{r^2} = \frac{1}{4\pi\varepsilon}\frac{|q_1||q_2|}{r^2}$$

This force F acts along the line joining the two charges and is **repulsive** if q_1 and q_2 are of the **same sign** and it is **attractive** if they are of **opposite sign** because like charges repel each other and unlike charges attract each other.
Refer following cases:



Coulomb's Law Formula

Coulomb's Formula

$$F = \frac{1}{4\pi\varepsilon_o\varepsilon_r} \cdot \frac{q_1 q_2}{d^2} = \frac{1}{4\pi\varepsilon_o K} \cdot \frac{q_1 q_2}{d^2}$$

$$F = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{d^2}$$

In Short: $F \propto q_1q_2/d^2$
where,

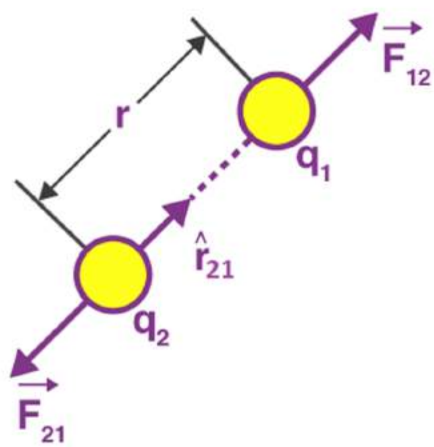
- **ε is absolute permittivity**,

- K or ε_r is the **relative permittivity** or **specific inductive capacity**
- ε_0 is the **permittivity of free space**.
- K or ε_r is also called a dielectric constant of the medium in which the two charges are placed.

History of Coulomb's Law

A French physicist Charles Augustin de Coulomb in 1785 coined a tangible relationship in mathematical form between two bodies that have been electrically charged. He published an equation for the force causing the bodies to attract or repel each other which is known as Coulomb's law or **Coulomb's inverse-square law**.

Coulomb's Law in Vector Form



$$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}; \quad \vec{F}_{12} = -\vec{F}_{21}$$

Here F_{12} is the force exerted by q_1 on q_2 and F_{21} is the force exerted by q_2 on q_1. Coulomb's law holds for stationary charges only which are point sized. This law obeys Newton's third law

$$\left(ie\ \vec{F}_{12} = -\vec{F}_{21}\right)$$

Force on a charged particle due to a number of point charges is the resultant of forces due to individual point charges i.e.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots$$

What is 1 Coulomb of Charge?

A coulomb is that charge which repels an equal charge of the same sign with a force of 9×10^9 N, when the charges are one meter apart in a vacuum. Coulomb force is the conservative mutual and internal force.

The value of ε_0 is 8.86×10^{-12} C^2/Nm^2 (or) 8.86×10^{-12} Fm^{-1}

Note: Coulomb force is true only for **static charges**.

Coulomb's Law – Conditions for Stability

If q is slightly displaced towards A, F_A increases in magnitude while F_B decreases in magnitude. Now the net force on q is toward A so it will not return to its original position. So for axial displacement, the equilibrium is unstable.

If q is displaced perpendicular to AB, the force F_A and F_B bring the charge to its original position. So for perpendicular displacement, the equilibrium is stable.

Key Points on Coulomb's Law

1. If the force between two charges in two different media is the same for different separations, $F = \frac{1}{K}\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$ = constant.

2. Kr^2 = constant or $K_1 r_1^2 = K_2 r_2^2$

3. If the force between two charges separated by a distance 'r_0' in a vacuum is the same as the force between the same charges separated by a distance 'r' in a medium, then from Coulomb's Law; $Kr^2 = r_0^2$

4. Two identical conductors having charges q_1 and q_2 are put to contact and then separated after which each will have a charge equal to $q_1 + q_2/2$. If the charges are q_1 and $-q_2$, then each will have a charge equal to $q_1 - q_2/2$.

5. Two spherical conductors having charges q_1 and q_2 and radii r_1 and r_2 are put to contact and then separated the charges of the conductors after contact is;

$q_1 = [r_1/(r_1 + r_2)]\,(q_1 + q_2)$ and $q_2 = [r_2/(r_1 + r_2)]\,(q_1 + q_2)$

6. If the force of attraction or repulsion between two identical conductors having charges q_1 and q_2 when separated by a distance d is F. Also if they are put to contact and then separated by the same distance the new force between them is

$$F = \frac{F(q_1+q_2)^2}{4q_1 q_2}$$

7. If charges are q_1 and $-q_2$ then, $F = F(q_1 + q_2)^2 / 4q_1q_2$
8. Between two-electrons separated by a certain distance: Electrical force/Gravitational force = 10^{42}
9. Between two protons separated by a certain distance: Electrical force/Gravitational force = 10^{36}
10. Between a proton and an electron separated by a certain distance: Electrical force/Gravitational force = 10^{39}
11. The relationship between the velocity of light, the permeability of free space and permittivity of free space is given by the expression $c = 1 / \sqrt{(\mu_o \varepsilon_o)}$
12. If Coulomb's law is applied to two identical balls of mass m are hung by silk thread of length 'l' from the same hook and carry similar charges q then;

- The distance between balls = $\left[\frac{q^2 2l}{4\pi\epsilon_o mg}\right]^{\frac{1}{3}}$
- The tension in the thread = $\sqrt{f^2 + (mg)^2}$
- If the total system is kept in space then the angle between threads is 180°

  and tension in a thread is given by $T = \frac{1}{4\pi\in_0}\frac{q^2}{4\ell^2}$
- A charge Q is divided into q and (Q – q). Then electrostatic force between them is maximum when

$$\frac{q}{Q} = \frac{1}{2} \ (or) \ \frac{q}{(Q-q)} = 1$$

Application of The Coulombs Law

- To calculate the distance and force between the two charges.
- The electric field can be calculated using the coulombs law

$$E = \frac{F}{Q_T}\left(\frac{N}{C}\right)$$

Where E = Strength of the electric field
F = Electrostatic force
Q_T = Test charge in coulombs

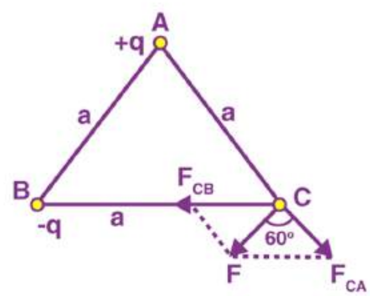
- To calculate the force on one point due to the presence of several points (Theorem of superposition).

Problems on Coulombs Law

Problem 1: Charges of magnitude 100 microcoulomb each are located in vacuum at the corners A, B and C of an equilateral triangle measuring 4 meters on each side. If the charge at A and C are positive and the charge B negative, what is the magnitude and direction of the total force on the charge at C?

Sol. The situation is shown in fig. Let us consider the forces acting on C due to A and B.

Now, from Coulomb's law, the force of repulsion on C due to A i.e., FCA in direction AC is given by



$$F_{CA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{a^2} \text{ along AC}$$

The force of attraction on C due to B i.e., F_{CB} in direction CB is given by

$$F_{CB} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{a^2} \text{ along CB}$$

Thus the two forces are equal in magnitude. The angle between them is 120º. The resultant force F is given by

$$F = \sqrt{F_{CA}^2 + F_{CB}^2 + 2F_{CA} \times F_{CB} \cos 120^\circ}$$

$$= \frac{q^2}{4\pi\varepsilon a^2} = \frac{9 \times 10^9 \times (100 \times 10^{-6})^2}{4^2} = 5.625 \text{ Newton}$$

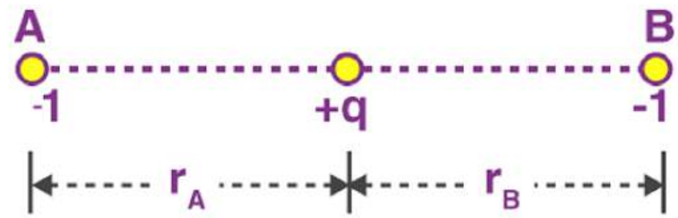
This force is parallel to AB.

Problem 2: The negative point charges of unit magnitude and a positive point charge q are placed along the straight line. At what position and for what value of q will the system be in equilibrium? Check whether it is stable, unstable or

neutral equilibrium.

Sol. The two negative charges A and B of unit magnitude are shown in fig. Let the positive charge q be at a distance r_A from A and at a distance r_B from B.

Now, from coulombs law, Force on q due to A



FqA = $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{{r_A}^2}$ towards A

Force on q due to B

FqB= $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{{r_B}^2}$ towards B.

These two forces acting on q are opposite and collinear. For the equilibrium of q, the two forces must also be equal i.e.

$|F_qA| = |F_qB|$

or

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{{r_A}^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{{r_B}^2}$$

Hence rA = rB

So for the equilibrium of q, it must be equidistant from A & B i.e. at the middle of AB

Now for the equilibrium of the system, A and B must be in equilibrium. For the equilibrium of A

Force on A by q = $\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{{r_A}^2}$ towards q

Force on A by B = $\frac{1}{4\pi\varepsilon_0}\cdot\frac{(1)(1)}{(r_A+r_B)^2}$

= $\frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{(2r_A)^2}$ away from q

The two forces are opposite and collinear. For equilibrium the forces must be equal, opposite and collinear. Hence

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{{r_A}^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{1}{(2r_A)^2}$$

or q = 1/4 in magnitude of either charge.
It can also be shown that for the equilibrium of B, the magnitude of q must be 1/4 of the magnitude of either charge.

Problem 3: A positive charge of 6×10^{-6} C is 0.040m from the second positive charge of 4×10^{-6} C. Calculate the force between the charges.
Given
$q_1 = 6 \times 10^{-6}$ C
$q_2 = 4 \times 10^{-6}$ C
r = 0.040 m
Sol.

$$F_e = k\frac{q_1 q_2}{r^2}$$

$$F_e = \frac{8.99\times10^9\left(6\times10^{-6}\right)\left(4\times10^{-6}\right)}{(0.04^2)}$$

$$F_e = \frac{8.99\times10^9\left(2.4\times10^{-11}\right)}{1.6\times10^{-3}}$$

$$F_e = \frac{0.21576}{1.6\times10^{-3}}$$

$F_e = 134.85$ N

Problem 4: Two-point charges, $q_1 = +9$ μC and $q_2 = 4$ μC, are separated by a distance r = 12 cm. What is the magnitude of the electric force?
given
$k = 8.988 \times 10^9$ Nm^{2}C^{-2}
q1 = 9 ×10-6 C
q2 = 4 ×10-6 C
Sol:

$$F_e = k\frac{q_1 q_2}{r^2}$$

$$F_e = \frac{8.99\times10^9\left(9\times10^{-6}\right)\left(4\times10^{-6}\right)}{(0.12^2)}$$

$$F_e = \frac{8.99\times10^9\left(3.6\times10^{-11}\right)}{0.0144}$$

$$F_e = \frac{0.32364}{0.0144}$$

$F_e = 22.475$ N

Limitations of Coulomb's Law

Coulomb's Law is derived under certain assumptions and can't be used freely like other general formulas. The law is limited to following points:

- We can use the formula if the charges are static (in rest position)
- The formula is easy to use while dealing with charges of regular and smooth shape, and it becomes too complex to deal with charges having irregular shapes
- The formula is only valid when the solvent molecules between the particle are sufficiently larger than both the charges

Forces Between Multiple Charges

A charge is an inherent property of every atom, an atom is said to be charged if it has an irregular number of electrons and protons, an atom is said to be positively charged if it has less number of electrons than protons, and negatively charged if it has more number of electrons than protons.
The bodies get charged differently, the most common way of charging a body is to rub. If you rub a plastic comb with your hair, the comb attains electrons from hair, now if we get tiny pieces of paper close to the comb attracts the pieces like a magnet attracting iron fillings, this is because the electrons attract the positive charge on the paper. This is the force of charges in action.

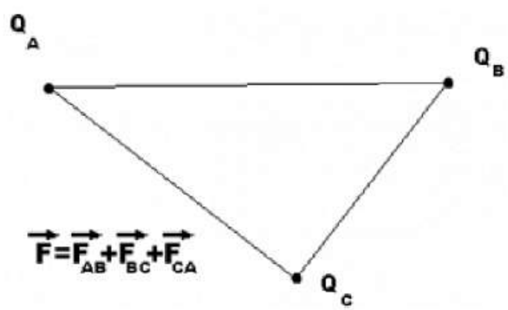
How to calculate the magnitude of the force between two charges

We can find the force between any two charges by Coulomb's law. Coulomb's law states that two charged bodies will attract or repel each other with a force that proportional to the product of their masses and inversely proportional to the square of the distance between them,
Let's get an equation out of this,

$$F = k * \frac{Q1 * Q2}{d^2}$$

Where F is the force of attraction or repulsion depending upon the charges,
K is the coulombs constant, for air it is 9×10^9 $kg \cdot m^3 \cdot s^{-2} \cdot C^{-2}$.
Q1 and Q2 are the magnitudes of two charges

d is the distance between the two charges,
This is only applicable for two charged particles, how will we find a force on one charge due to multiple charges?
Let's consider 3 charges Q_A, Q_B, and Q_C.



We could get the net force acting on a charge by calculating the vector sum of all the forces acting on the charge, this is called the superposition theorem.
Considering the above example of 3 point charges Qa, Qb and Qc with a position vector of r1, r2 and r3. Then the force experienced by one charge due to the other charges is given by,

$$\vec{F} = \vec{F}_{AB} + \vec{F}_{BC} + \vec{F}_{CA}$$

This can be written as,

$$\overrightarrow{F_1} = \sum_{j=1}^{n} F_{ij} \text{ (where } j \neq i \text{)}$$

By applying this to our current situation of 3 point charges we will get,

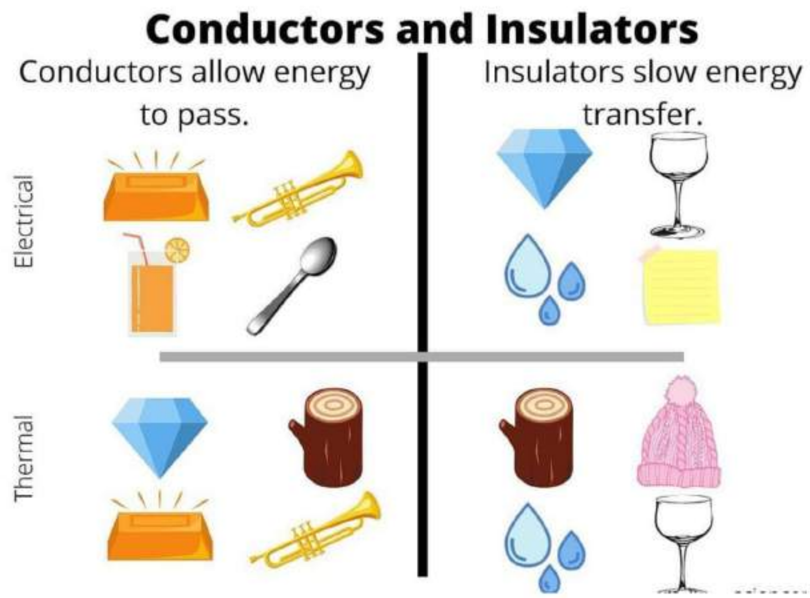
$$\overrightarrow{F_1} = \frac{1}{4\pi\in}\left[\frac{Q_A Q_B}{r_{AB}^2}\hat{r}_{AB} + \frac{Q_A Q_C}{r_{AC}^2}\hat{r}_{AC}\right]$$

This is a combination of the coulombs law and the superposition theorem, and any electro static force can be derived using coulombs law and the superposition theorem this way.

- The force acting on a charge is directly proportional to the magnitude of the charge and inversely proportional to the square of the distance between them.
- The force acting on a point charge due to multiple charges is given by the vector sum of all individual forces acting on the charges.

Conductors & Insulators

Any object can be broadly classified in either of the following two categories on the basis of their electrical properties:
(i) Conductors
(ii) Insulators



(i) Conductors: The materials or substances which allow electricity to flow through them are called **Conductors**. Conductors are able to conduct electricity because they allow electrons to flow inside them very easily.
The general property of conductor is to allow the transition of heat or light from one source to another. Metals, humans, earth and animal bodies fall in the category of conductors. This category generally comprises of metals but may sometimes contain non-metals too.

Example: Carbon in the form of graphite. Conductors have free electrons on its surface which allows current to pass through, that's why conductors are able to conduct electricity.

APPLICATIONS OF CONDUCTORS

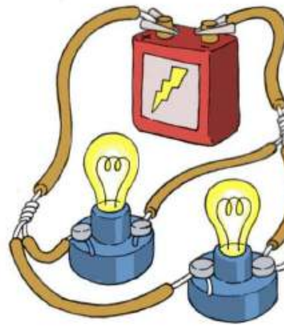


Fig: Use of conductors in lightning a bulb

Conductors are quite useful in many ways and used in many real life applications like:

- Mercury is used in thermometer to check temperature of body.
- Aluminium is used in making foils to store food and also in production of fry pans to store heat quickly.
- Iron is used in vehicle engine to conduct heat.
- The plate of an iron is made up of steel to absorb heat briskly.
- Conductors are used in car radiators to eradicate heat away from the engine.

Wood

Plastic

Rubber

Fig: Insulators

(ii) Insulators: The materials or substances which resist or don't allow the current to flow through them are called **Insulators.** Insulators are mostly solid in nature and

are used in a variety of systems. Insulators don't allow the flow of heat as well. The property which makes insulators different from conductors is its resistivity. Wood, cloth, glass, mica, and quartz are some good examples of insulators. Insulators are also called **Protectors** as they give protection against heat, sound and of course passage of electricity.
Insulators don't have any electrons in its and that's why insulators don't conduct electricity.

Examples

- Glass is the best insulator as it has the highest resistivity.
- Plastic is a good insulator and is used in making number of things.
- Rubber which is used to make tyres, fire-resistant clothes and slipper is a very good insulator.

APPLICATIONS OF INSULATORS

Fig: An insulator is used to protect wire opening

Being resistive to flow of electron, insulators are used worldwide in a number of ways.

Some are as follows

- Thermal Insulators, disallow heat to move from one place to another and is used in making thermoplastic bottles, in fireproofing ceilings and walls.
- Sound Insulators help in controlling noise level, as they are good in absorbance of sound and are used in buildings, conference halls, and buildings to make them noise free.
- Electrical Insulators, which hinders flow of electron or passage of current through them are extensively used in circuit boards, high-voltage systems and also in coating electric wire and cables.

Gauss Law & Its Applications

Gauss Law states that the total electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity. The electric flux in an area is defined as the electric field multiplied by the area of the surface projected in a plane and perpendicular to the field.

What is Gauss Law?

According to the Gauss law, the total flux linked with a closed surface is $1/\varepsilon 0$ times the charge enclosed by the closed surface.

$$[latex] \oint \vec{E}.\vec{d}s = \frac{1}{\epsilon_0} q [/latex]$$

For example, a point charge q is placed inside a cube of edge 'a'. Now as per Gauss law, the flux through each face of the cube is $q/6\varepsilon_0$.
The electric field is the basic concept to know about electricity. Generally, the electric field of the surface is calculated by applying Coulomb's law, but to calculate the electric field distribution in a closed surface, we need to understand the concept of Gauss law. It explains the electric charge enclosed in a closed or the electric charge present in the enclosed closed surface.

Gauss Law Formula

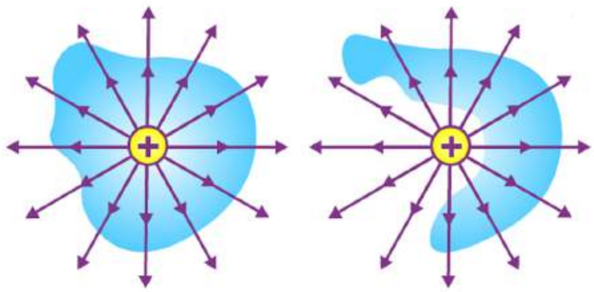
As per the Gauss theorem, the total charge enclosed in a closed surface is proportional to the total flux enclosed by the surface. Therefore, if ϕ is total flux and ϵ_0 is electric constant, the total electric charge Q enclosed by the surface is;
$Q = \phi \epsilon_0$
The Gauss law formula is expressed by;
$\phi = Q/\epsilon_0$
Where,
Q = total charge within the given surface,
ε_0 = the electric constant.

The Gauss Theorem

The net flux through a closed surface is directly proportional to the net charge in the volume enclosed by the closed surface.
$\Phi = \rightarrow E.d \rightarrow A = q_{net}/\varepsilon_0$
In simple words, the Gauss theorem relates the 'flow' of electric field lines (flux) to the charges within the enclosed surface. If there are no charges enclosed by a surface, then the net electric flux remains zero.
This means that the number of electric field lines entering the surface is equal to the field lines leaving the surface.

The Gauss theorem statement also gives an important corollary:

The electric flux from any closed surface is only due to the sources (positive charges) and sinks (negative charges) of electric fields enclosed by the surface. Any charges outside the surface do not contribute to the electric flux. Also, only electric charges can act as sources or sinks of electric fields. Changing magnetic fields, for example, cannot act as sources or sinks of electric fields.



Gauss Law in Magnetism

The net flux for the surface on the left is non-zero as it encloses a net charge. The net flux for the surface on the right is zero since it does not enclose any charge.
Note: The Gauss law is only a restatement of the Coulombs law. If you apply the Gauss theorem to a point charge enclosed by a sphere, you will get back the Coulomb's law easily.

Applications of Gauss Law

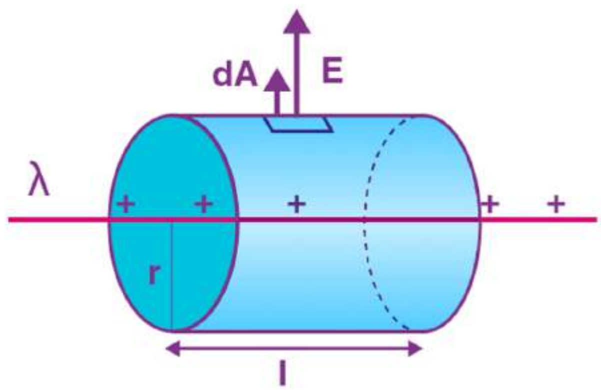
1. In the case of a charged ring of radius R on its axis at a distance x from the centre of the ring.

$$E = [latex] \frac{1}{4\pi\in_0} \frac{qx}{\left(R^2+x^2\right)^{3/2}} [/latex]. Atthecentre, x = 0 and E = 0$$

2. In case of an infinite line of charge, at a distance 'r'. $E = (1/4 \times \pi r\varepsilon_0)(2\pi/r) = \lambda/2\pi r\varepsilon_0$. Where λ is the linear charge density.
3. The intensity of the electric field near a plane sheet of charge is $E = \sigma/2\varepsilon_0 K$ where σ = surface charge density.
4. The intensity of the electric field near a plane charged conductor $E = \sigma/K\varepsilon_0$ in a medium of dielectric constant K. If the dielectric medium is air, then $E_{air} = \sigma/\varepsilon_0$.
5. The field between two parallel plates of a condenser is $E = \sigma/\varepsilon_0$, where σ is the surface charge density.

Electric Field due to Infinite Wire – Gauss Law Application

Consider an infinitely long line of charge with the charge per unit length being λ. We can take advantage of the cylindrical symmetry of this situation. By symmetry, The electric fields all point radially away from the line of charge, there is no component parallel to the line of charge.
We can use a cylinder (with an arbitrary radius (r) and length (l)) centred on the line of charge as our Gaussian surface.



Applications of Gauss Law – Electric Field due to Infinite Wire

As you can see in the above diagram, the electric field is perpendicular to the curved surface of the cylinder. Thus, the angle between the electric field and area vector is zero and $\cos\theta = 1$
The top and bottom surfaces of the cylinder lie parallel to the electric field. Thus the angle between area vector and the electric field is 90 degrees and $\cos\theta = 0$.
Thus, the electric flux is only due to the curved surface
According to Gauss Law,
$\Phi = \rightarrow E.d \rightarrow A$
$\Phi = \Phi_{\text{curved}} + \Phi_{\text{top}} + \Phi_{\text{bottom}}$
$\Phi = \rightarrow E\,.\,d \rightarrow A = \int E\,.\,dA \cos 0 + \int E\,.\,dA \cos 90° + \int E\,.\,dA \cos 90°$
$\Phi = \int E\,.\,dA \times 1$
Due to radial symmetry, the curved surface is equidistant from the line of charge and the electric field in the surface has a constant magnitude throughout.
$\Phi = \int E\,.\,dA = E\int dA = E\,.2\pi rl$
The net charge enclosed by the surface is:
$q_{\text{net}} = \lambda.l$
Using Gauss theorem,
$\Phi = E \times 2\pi rl = q_{\text{net}}/\varepsilon_0 = \lambda l/\varepsilon_0$

$E \times 2\pi rl = \lambda l/\varepsilon_0$
$E = \lambda/2\pi r\varepsilon_0$

Problems on Gauss Law

Problem 1: A uniform electric field of magnitude E = 100 N/C exists in the space in X-direction. Using the Gauss theorem calculate the flux of this field through a plane square area of edge 10 cm placed in the Y-Z plane. Take the normal along the positive X-axis to be positive.

Solution: The flux $\Phi = \int E.\cos\theta\, ds$.
As the normal to the area points along the electric field, $\theta = 0$.
Also, E is uniform so, $\Phi = E.\Delta S = (100\ N/C)\ (0.10m)^2 = 1\ N\text{-}m^2$.

Problem 2: A large plane charge sheet having surface charge density $\sigma = 2.0 \times 10^{-6}$ C-m-2 lies in the X-Y plane. Find the flux of the electric field through a circular area of radius 1 cm lying completely in the region where x, y, z are all positive and with its normal making an angle of 60^0 with the Z-axis.

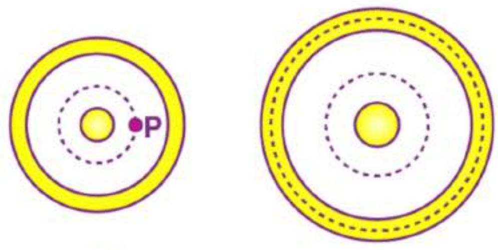
Solution: The electric field near the plane charge sheet is $E = \sigma/2\varepsilon_0$ in the direction away from the sheet. At the given area, the field is along the Z-axis.
The area $= \pi r^2 = 3.14 \times 1\ cm^2 = 3.14 \times 10^{-4}\ m^2$.
The angle between the normal to the area and the field is 60°.

Hence, according to Gauss theorem, the flux = [latex]$\vec{E}.\Delta\vec{S}$[/latex] =
$E.\Delta S \cos\theta = \sigma/2\varepsilon_0 \times pr^2 \cos 60^\circ$
[latex]$\frac{2.0\times10^{-6}C/m^2}{2\times8.85\times10^{-12}C^2/N-m^2} \times (3.14 \times 10^{-4}m^2)\frac{1}{2}$[/latex] = 17.5 $N\text{-}m^2C^{-1}$.

Problem 3: A charge of 4×10^{-8} C is distributed uniformly on the surface of a sphere of radius 1 cm. It is covered by a concentric, hollow conducting sphere of radius 5 cm.

- Find the electric field at a point 2 cm away from the centre.
- A charge of 6×10^{-8}C is placed on the hollow sphere. Find the surface charge density on the outer surface of the hollow sphere.

Solution:

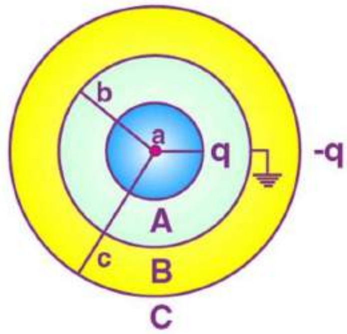


(i) (ii)

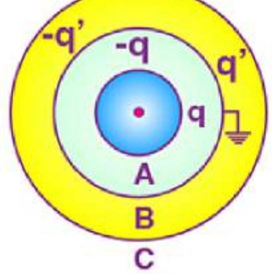
(a) Let us consider the figure (i).
Suppose, we have to find the field at point P. Draw a concentric spherical surface through P. All the points on this surface are equivalent and by symmetry, the field at all these points will be equal in magnitude and radial in direction.

The flux through this surface = $[latex]\oint \vec{E}\,.\,\overrightarrow{dS}\,[/latex]$
$= [latex]\oint EdS = E\oint dS[/latex] = 4\pi x2E.$
where x = 2 cm = 2×10^{-2} m.
From Gauss law, this flux is equal to the charge q contained inside the surface divided by ε_0. Thus,
$\Rightarrow 4\pi x^2 E = q/\varepsilon_0$ or, $E = q/4\pi\varepsilon_0 x^2$
$= (9 \times 10^9) \times [(4 \times 10^{-8})/ (4 \times 10^{-4})] = 9 \times 10^5$ N C^{-1}.
(b) Let us consider the figure (ii).
Take the Gaussian surface through the material of the hollow sphere. As the electric field in a conducting material is zero, the flux $[latex]\oint \vec{E}\,.\,d\vec{S}\,[/latex]$ through this Gaussian surface is zero. Using Gauss law, the total charge enclosed must be zero. Hence, the charge on the inner surface of the hollow sphere is 4×10^{-8}C.
But the total charge given to this hollow sphere is 6×10^{-8} C. Hence, the charge on the outer surface will be 10×10^{-8}C.

Problem 4: The figure shows three concentric thin spherical shells A, B and C of radii a, b, and c respectively. The shells A and C are given charges q and -q respectively and the shell B is earthed. Find the charges appearing on the surfaces of B and C.



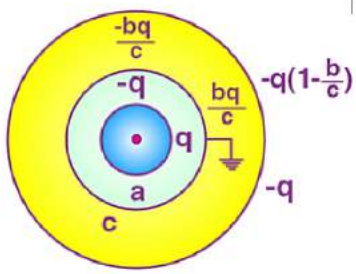
Solution: As shown in the previous worked out example, the inner surface of B must have a charge -q from the Gauss law. Suppose, the outer surface of B has a charge q'. The inner surface of C must have a charge -q' from Gauss law. As the net charge on C must be -q, its outer surface should have a charge q' – q. The charge distribution is shown in the figure.



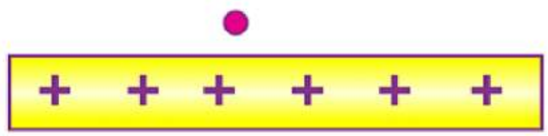
The potential at B,

- Due to the charge q on A = $q/4\pi\varepsilon_0 b$,
- Due to the charge -q on the inner surface of B = $-q/4\pi\varepsilon_0 b$,
- Due to the charge q' on the outer surface of B = $q'/4\pi\varepsilon_0 b$,
- Due to the charge -q', on the inner surface of C = $-q'/4\pi\varepsilon_0 c$,
- Due to the charge q' – q on the outer surface of C = $(q' - q)/4\pi\varepsilon_0 c$.

The net potential is, $V_B = q'/4\pi\varepsilon_0 b - q/4\pi\varepsilon_0 c$
This should be zero as the shell B is earthed. Thus, $q' = q \times b/c$
The charges on various surfaces are as shown in the figure:

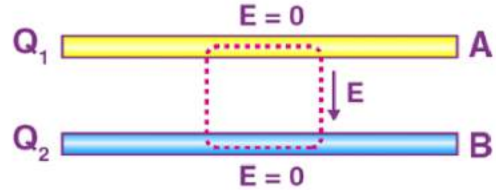


Problem 5: A particle of mass 5×10^{-6}g is kept over a large horizontal sheet of charge of density 4.0×10^{-6} C/m^2 (figure). What charge should be given to this particle so that if released, it does not fall down? How many electrons are to be removed to give this charge? How much mass is decreased due to the removal of these electrons?

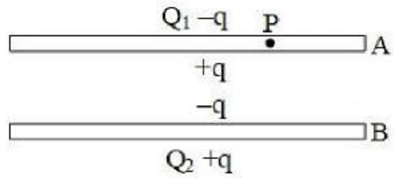


Solution: The electric field in front of the sheet is,
$E = \sigma/2\varepsilon_0 = (4.0 \times 10^{-6})/(2 \times 8.85 \times 10^{-12}) = 2.26 \times 10^5$ N/C
If a charge q is given to the particle, the electric force qE acts in the upward direction. It will balance the weight of the particle if
$q \times 2.26 \times 10^5$ N/C $= 5 \times 10^{-9}$ kg $\times 9.8$ m/s^2
or, $q = [4.9 \times 10^{-8}]/[2.26 \times 10^5]$C $= 2.21 \times 10^{-13}$ C
The charge on one electron is 1.6×10^{-19}C. The number of electrons to be removed;
$= [2.21 \times 10^{-13}]/[1.6 \times 10^{-19}] = 1.4 \times 10^6$
Mass decreased due to the removal of these electrons $= 1.4 \times 10^6 \times 9.1 \times 10^{-31}$ kg $=$ 1.3×10^{-24} kg.

Problem 6: Two conducting plates A and B are placed parallel to each other. A is given a charge Q1 and B a charge Q2. Find the distribution of charges on the four surfaces.
Solution:



Consider a Gaussian surface as shown in figure (a). Two faces of this closed surface lie completely inside the conductor where the electric field is zero.
The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero.
The total flux of the electric field through the closed surface is, therefore, zero. From Gauss law, the total charge inside the closed surface should be zero. The charge on the inner surface of A should be equal and opposite to that on the inner surface of B.

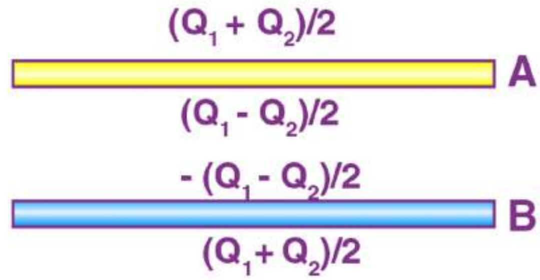


(b)

The distribution should be like the one shown in figure (b). To find the value of q, consider the field at a point P inside the plate A. Suppose, the surface area of the plate (one side) is A.
Using the equation $E = \sigma/2\varepsilon_0$, the electric field at P;

- Due to the charge $Q1 - q = (Q1 - q)/2A\varepsilon_0$ (downward),
- Due to the charge $+q = q/2A\varepsilon_0$ (upward),
- Due to the charge $-q = q/2A\varepsilon_0$ (downward),
- Due to the charge $Q2 + q = (Q2 + q)/2A\varepsilon_0$ (upward).

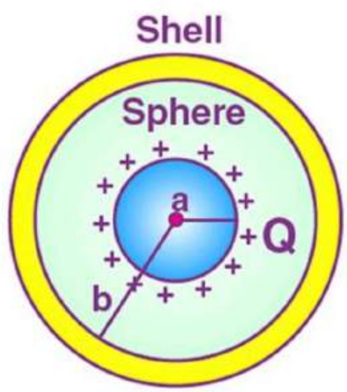
The net electric field at P due to all the four charged surfaces is (in the downward direction)
$(Q1 - q)/2A\varepsilon_0 - q/2A\varepsilon_0 + q/2A\varepsilon_0 - (Q2 + q)/2A\varepsilon_0$
As the point P is inside the conductor, this field is should be zero.
Hence, $Q1 - q - Q2 - q = 0$
or $q = (Q1 - Q2)/2$ ..... (i)
Thus, $Q1 - q = (Q1 + Q2)/2$ ...... (ii)
and $Q2 + q = [Q1 + Q]2/2$
Using these equations, the distribution shown in the figure (a, b) can be redrawn as in the figure.



This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost surfaces get equal charges and the facing surfaces get equal and opposite charges.

Problem 7: A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of hollow shell be V. What will be the new potential difference between the same two surfaces if the shell is given a charge -3Q?

Solution: In case of a charged conducting sphere



$V_{in} = V_c = V_s = 1/4\pi\varepsilon_0$

and $V_{out} = 1/4\pi\varepsilon_0$

So if a and b are the radii of a sphere and spherical shell respectively, the potential at their surfaces will be;

$V_{sphere} = 1/4\pi\varepsilon_0\, [Q/a]$ and $V_{shell} = 1/4\pi\varepsilon_0\, [Q/b]$ and so according to the given problem;

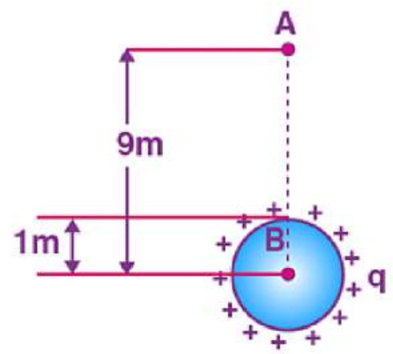
$V = V'_{sphere} - V'_{shell} = Q/4\pi\varepsilon 0\, [1/a - 1/b] = V$ .... (1)

Now when the shell is given a charge (-3Q) the potential at its surface and also inside will change by;

$V_0 = 1/4\pi\varepsilon_0\, [-3Q/b]$

So that now,
V'sphere = $1/4\pi\varepsilon_0$ [Q/a + V_0] and V'shell = $1/4\pi\varepsilon_0$ [Q/b + V_0]
Hence, V'sphere – V'shell = Q/4πε0 [1/a – 1/b] = V [from Eqn. (1)]
i.e., if any charge is given to external shell the potential difference between sphere and shell will not change.
This is because by the presence of charge on the outer shell, potential everywhere inside and on the surface of the shell will change by the same amount and hence the potential difference between sphere and shell will remain unchanged.

Problem 8: A very small sphere of mass 80 g having a charge q is held at height 9 m vertically above the centre of a fixed non conducting sphere of radius 1 m, carrying an equal charge q. When released it falls until it is repelled just before it comes in contact with the sphere. Calculate the charge q. [g = 9.8 m/s²]
Solution: Keeping in mind that here both electric and gravitational potential energy is changing and for an external point, a charged sphere behaves as the whole of its charge were concentrated at its centre.
Applying the law of conservation of energy between initial and final position, we have

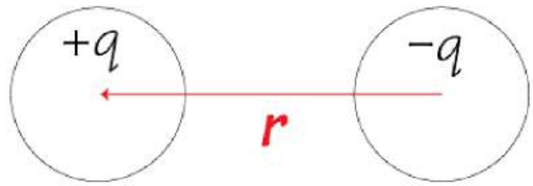


$1/4\pi\varepsilon_0 \times (q.q/9) + mg \times 9 = 1/4\pi\varepsilon_0 \times (q^2/1) + mg \times 1$
or, $q2 = (80 \times 10^{-3} \times 9.8)/10^9 = 28\mu C$.

Electric Dipole

Introduction

- **Definition:** When two charges of **equal magnitude** and **opposite sign** are separated by a **very small distance**, then the arrangement is called electric dipole.
- Total charge of the dipole is zero but electric field of the dipole is not zero as charges q and -q are separated by some distance and electric field due TO them when added is not zero.



$$\mu = q\mathbf{r}$$

We define a quantity called Dipole Moment $\vec{p}$ for such a system such that:

$\vec{p} = q\vec{d}$

where $\vec{d}$, conventionally, represents the direction from –q to +q.
Axis of a dipole is the line joining –q to +q

- Midpoint of the axis of the dipole is called the **centre of the dipole**.
- All the distances in the space are measured from the centre of the dipole.
- Perpendicular bisector of the axis of the dipole is called the **equatorial line of the dipole**.

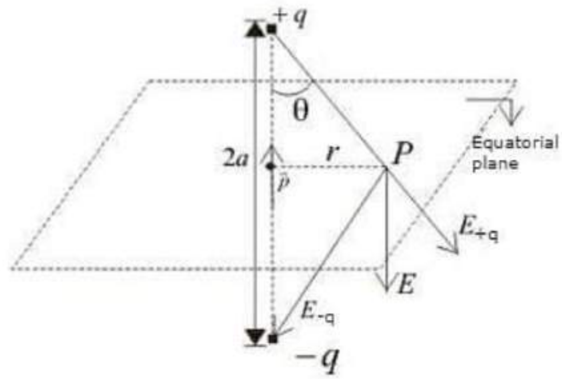
Q. Why are we defining a dipole? Why can't we treat it as simply two charge system? Why are we giving it special treatment?

Ans. Dipole is commonly occurring system in nature. We need to generalize our results in context with the dipole system to avoid repeated single point charge calculations using Coulomb's Law. After these results we would be able to directly apply simplified results derived here to dipole systems.

Calculation of Electric Field due to an Electric Dipole

1. Field of an electric dipole at points in equatorial plane

We now find the magnitude and direction of electric field due to dipole.



• P point in the equatorial plane of the dipole at a distance r from the centre of the dipole. Then electric field due to -q and +q are

$$E_{-q} = \frac{-q\hat{P}}{4\pi\epsilon_0(r^2 + a^2)} \dots\dots(1a)$$

$$E_{+q} = \frac{q\hat{P}}{4\pi\epsilon_0(r^2 + a^2)} \dots\dots(1b)$$

and they are equal in magnitude. Note that $\hat{P}$ is the unit vector along the dipole axis (from -q to + q)

• From figure we can see the direction of $\vec{E}_{+q}$ and $\vec{E}_{-q}$.

Their components normal (perpendicular) to dipole cancel away and components along the dipole add up.

• Dipole moment vector points from negative charge to positive charge so in vector form.

• $\vec{E} = -(\vec{E}_{+q} + \vec{E}_{-q}) \cos\theta$

• Substituting the values of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ calculated above also, by geometry, $\cos\theta = \frac{a}{\sqrt{r^2+a^2}}$,

$$E = -\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r^2 + a^2} + \frac{1}{r^2 + a^2}\right]\frac{a}{\sqrt{r^2 + a^2}}$$

$$\vec{E} = -\frac{2qa}{4\pi\varepsilon_0(r^2 + a^2)}\hat{p}$$

Now, very frequently we measure electric field at large distances from the dipole ie. r>>a

Therefore, by approximation,

$$\vec{E}_{equator} = -\frac{2qa\hat{P}}{4\pi\varepsilon_0 r^3}$$

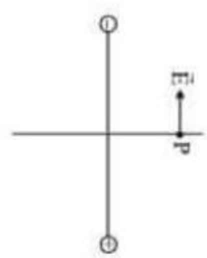
We know that, by definition,

$$\vec{P} = q(2a)\hat{P}$$

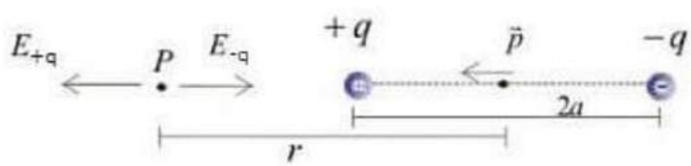
Hence,

$$\vec{E}_{equator} = -\frac{K\vec{P}}{r^3}$$

Observe, the - sign, it represents that electric field at the equator is in the opposite direction to the dipole moment of the electric dipole i.e. +q to −q.



2. Field of an electric dipole for points on the axis



- Let P be the point at a distance r from the centre of the dipole on side of charge +q as shown in the figure

$$E_{-q} = \frac{-q\hat{P}}{4\pi\epsilon_0 (r+a)^2}$$

$$E_{+q} = \frac{q\hat{P}}{4\pi\epsilon_0 (r-a)^2}$$

Where $\hat{P}$ is the unit vector along the dipole axis (from -q to + q)

$$\vec{E} = \vec{E}_{+q} + \vec{E}_{-q}$$

Thus,

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{P}$$

or

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ar}{(r^2-a^2)}\right]\hat{P}$$

for r >> a

$$\vec{E}_{axis} = \frac{4qa\hat{P}}{4\pi\epsilon_0 r^3} = \frac{2K\vec{P}}{r^3}$$

As we know that, by definition of dipole moment,

$$\vec{P} = q(2a)\hat{P}$$

- Unit of dipole moment is **Coulomb meter (Cm)**.
- Thus, in a nutshell, in terms of electric dipole moment, electric field due to a dipole at large distances(r >> a)

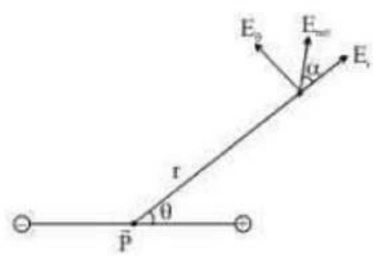
(i) At point on equatorial plane (r >> a)

$$\vec{E}_{equator} = -\frac{K\vec{P}}{r^3}$$

(ii) At point on dipole axis (r>>a)

$$\vec{E}_{axis} = \frac{4qa\hat{P}}{4\pi\epsilon_0 r^3} = \frac{2K\vec{P}}{r^3}$$

Note: Dipole field at large distances falls off as $1/r^3$.
Now, we can generalize the calculation of electric field at any general point in space due to the dipole using the above results.

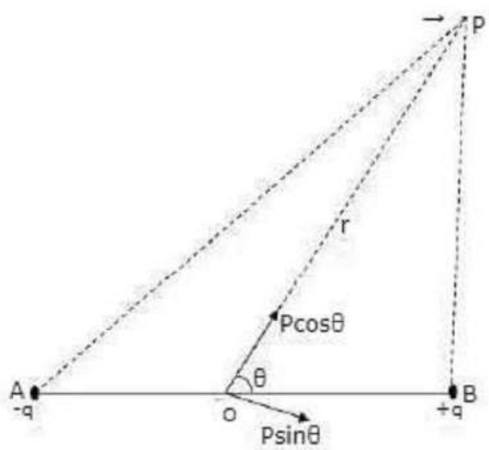


Any general point in space, can be located using the polar coordinates r and θ, where the origin can be placed at the center of the dipole, as shown in the above figure.

Now, for any general point P in space located at distance r from centre and inclined at an angle θ with the axis of the dipole, we can imagine components of the original dipole with dipole moment $\vec{P}$ such that the P lies on the equator of one component and on the axis of the other component.

Now, Lets express our dipole moment $\vec{P}$ as,

$\vec{P} = \vec{P}_{axial} + \vec{P}_{equatorial}$



Where $\vec{P}_{axial}$ is the component of the original dipole moment, such that point P is located on the axis of this dipole, i.e.

$\vec{P}_{axial} = \vec{P} \cos\theta \ \widehat{P_a}$

Now, at P,

$\vec{E}_P = \vec{E}_{axial} + \vec{E}_{equatorial}$

We know that,

$\vec{E}_{axial} = \frac{2K\vec{P}_{axial}}{r^3} \widehat{P_a}$ and $\vec{E}_{equator} = -\frac{K\vec{P}_{equatorial}}{r^3} \widehat{P_e}$

Thus,

$\vec{E}_P = \frac{2K\vec{P}_{axial}}{r^3} \widehat{P_a} + -\frac{K\vec{P}_{equatorial}}{r^3} \widehat{P_e}$

Therefore,

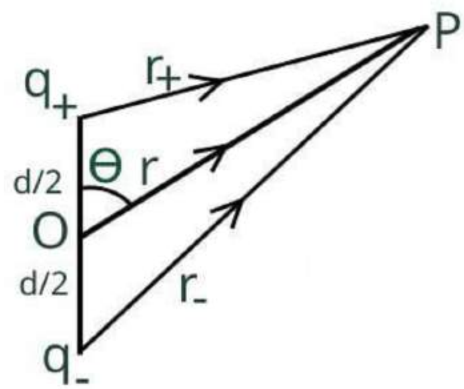
$$\vec{E}_P = \frac{2KP\cos\theta}{r^3} \widehat{P_a} + -\frac{K P\sin\theta}{r^3} \widehat{P_e}$$

One of the component will be along the axial component of electric dipole i.e.

$\widehat{P_a}$ and the other component will be along the equatorial component of electric dipole i.e. $\widehat{P_\theta}$.

ELECTRIC FIELD DUE TO A DIPOLE

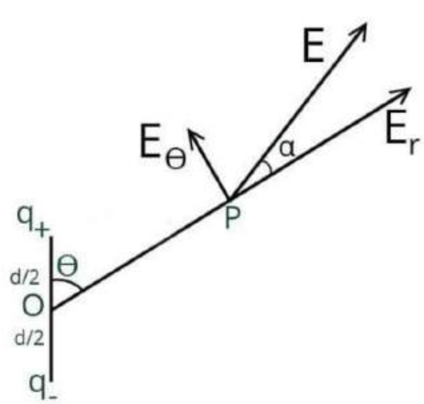
The electric field due to a pair of equal and opposite charges at any test point can be calculated using the Coulomb's law and the superposition principle. Let the test point P be at a distance r from the center of the dipole. The distance between +q and -q is d. We have shown the situation in the diagram below.



If $\vec{E_+}$ and $\vec{E_-}$ be the electric field at point P due to the positive and the negative charges separately then the total electric field $\vec{E}$ at Point P can be calculated by using the superposition principle.

$\vec{E} = \vec{E_+} + \vec{E_-}$

Please note that the directions of $\vec{E_+}$ and $\vec{E_-}$ are along $\vec{r_+}$ and $\vec{r_-}$ respectively. This is the most general form of the electric field due to a dipole. However, we will express this vector in terms of radial and inclination vectors as shown in the diagram below.



In order to calculate the electric field in the polar coordinate, we will use the expression of the electric potential due to an electric dipole which we have calculated earlier.

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{pcos\theta}{r^2}\right]$$

Here p is the magnitude of the dipole moment and is given by qd
We can easily derive the electric field due to this dipole by calculating the negative gradient of this electric potential. In polar coordinate electric field will be independent of azimuthal (ϕ) coordinate.

$$E_r = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\epsilon_0}\left[\frac{2pcos\theta}{r^3}\right]$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{1}{4\pi\epsilon_0}\left[\frac{psin\theta}{r^3}\right]$$

$$\boxed{\vec{E} = \frac{p}{4\pi\epsilon_0}\left[\frac{2cos\theta}{r^3}\hat{r} + \frac{sin\theta}{r^3}\hat{\theta}\right]}$$

The resultant electric field at point P is

$$E = \sqrt{E_r^2 + E_\theta^2}$$

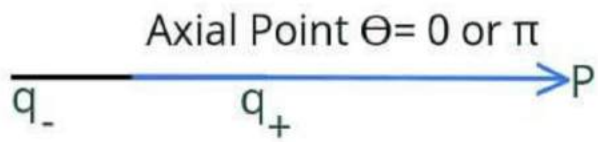
$$= \frac{1}{4\pi\epsilon_0}\sqrt{\left(\frac{2pcos\theta}{r^3}\right)^2 + \left(\frac{psin\theta}{r^3}\right)^2}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3cos^2\theta + 1}$$

As shown in the diagram, the resultant electric field makes an angle \alpha with the radial vector. Then

$$tan\alpha = \frac{E_\theta}{E_r} = \frac{tan\theta}{2}$$

ELECTRIC FIELD AT AN AXIAL POINT



In this case, the test point P is on the axis of the dipole. Consequently θ= 0 or π. The electric field at point P is

$$\vec{E} = \pm\frac{2p}{4\pi\epsilon_0 r^3}\hat{r}$$

SIMPLIFIED DERIVATION

The electric field at point P due to the positive charge is

$$\vec{E_+} = \frac{q}{4\pi\epsilon_0\left(r-\frac{d}{2}\right)^2}\hat{r}$$

Electric field at point P due to negative charge is

$$\vec{E_-} = -\frac{q}{4\pi\epsilon_0\left(r+\frac{d}{2}\right)^2}\hat{r}$$

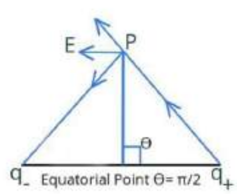
Total electric field due to the dipole at axial point P is

$$\vec{E} = \vec{E_+} + \vec{E_-} = \frac{2qrd}{4\pi\epsilon_0\left(r^2-\frac{d^2}{4}\right)^2}\hat{r}$$

At a relatively large distance r>>d/2 and we can approximate the electric field as

$$\vec{E} = \frac{2qd}{4\pi\epsilon_0 r^3}\hat{r} = \frac{2p}{4\pi\epsilon_0 r^3}\hat{r}$$

ELECTRIC FIELD AT AN EQUATORIAL POINT



In this case, the test point P is on the perpendicular bisector of the dipole. Consequently θ= π/2. The electric field at point P is

$$\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}\hat{\theta}$$

SIMPLIFIED DERIVATION

The electric field at point P due to positive charge is

$$\vec{E_+} = \frac{q}{4\pi\epsilon_0\left(r^2 - \frac{d^2}{4}\right)}\hat{r}_+$$

Electric field at point P due to negative charge is

$$\vec{E_-} = -\frac{q}{4\pi\epsilon_0\left(r + \frac{d^2}{4}\right)}\hat{r}_-$$

Total electric field due to the dipole at equatorial point P is

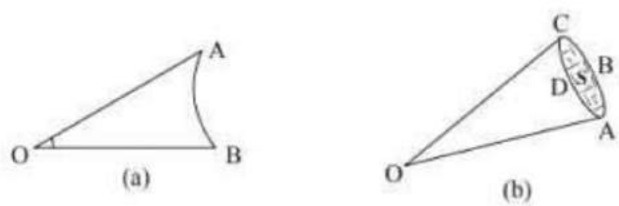
$$\vec{E} = \vec{E_+} + \vec{E_-} = \frac{qd}{4\pi\epsilon_0\left(r^2 + \frac{d^2}{4}\right)}\hat{\theta}$$

At a relatively large distance r>>d/2 and we can approximate the electric field as

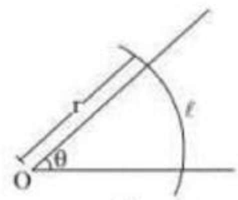
$$\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}\hat{\theta}$$

CONCEPT OF SOLID ANGLE

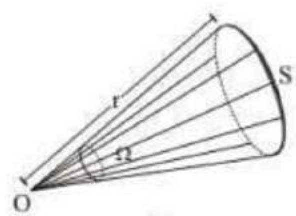
Solid angle is a generalisation of the plane angle: In figure we show a plane curve AB. The end points A and B are joined to the point O. We say that the curve AB subtends an angle or a plane angle at O. An angle is formed at O by the two lines OA and OB passing through O. We say that the curve AB subtends an angle or a plane angle at O. An angle is formed at O by the two lines OA and OB passing through O.



To construct a solid angle, we start with a surface S (fig.) and join all the points on the periphery such as A, B, C, D etc., with the given point O. We then say that a solid angle is formed at O and that the surface S has subtended the solid angle. The solid angle is formed by the lines joining the points on the periphery with O. The whole figure looks like a cone. As a typical example, think of the paper containers used by Mungfali Wala.

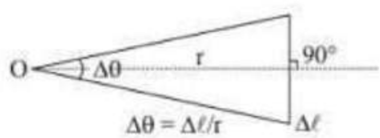


(a)

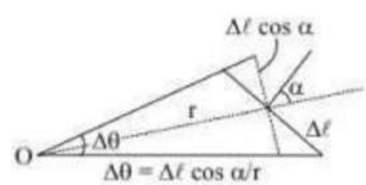


(b)

How do we measure a solid angle? Let us consider how do we measure a plane angle. See fig. We draw a circle of any radius r with the centre at O and measure the length l of the arc intercepted by the angle.
The angle θ is then defined as $\theta = 1/\mathrm{r}$. In order to measure a solid angle at the point O (fig.), we draw a sphere of any radius r with O as the centre and measure the area S of the part of the sphere intercepted by the cone. The solid angle Ω is then defined as $\Omega = \mathrm{S}/\mathrm{r}^2$
Note: That this definition makes the solid angle a dimensionless quantity. It is independent of the radius of the sphere drawn.

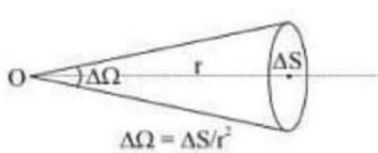


(a)

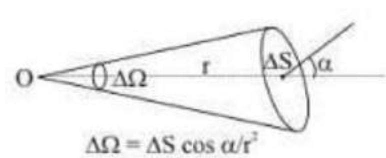


(b)

Next, consider a plane angle subtended at a point O by a small line segment $\Delta\ell$ (fig.). Suppose, the line joining O to the middle point of $\Delta\ell$ is perpendicular to $\Delta\ell$.As the segmentis small, we can approximately write.
$\Delta\theta = \Delta\ell/\mathrm{r}$
As $\Delta\ell$ gets smaller, the approximation becomes better. Now suppose, the line joining O to $\Delta\ell$ is not perpendicular to $\Delta\ell$ (fig.). Suppose, this line makes an angle α with the perpendicular to $\Delta\ell$. The angle subtended by $\Delta\ell$ at O is
$\Delta\theta = \Delta\ell\cos\alpha/\mathrm{r}$



(a)



(b)

Similarly, if a small plane area ΔS (fig.) subtends a solid angle $\Delta\ell$ at O in such a way that the line joining O to ΔS is normal to ΔS, we can write$\Delta\Omega = \Delta \mathrm{S} / \mathrm{r}^2$.
But if the line joining O to ΔS makes an angle α with the normal to ΔS (fig.), we

should write

$\Delta\Omega = \Delta S \cos\alpha / r^2$

A complete circle subtends an angle

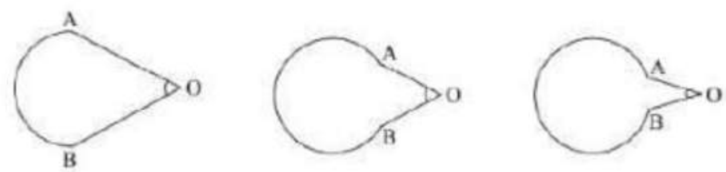
$\theta = \ell / r = 2\pi r / r = 2\pi$

at the centre. In fact, any closed curve subtends an angle 2π at an y of the internal points. Similarly, a complete sphere subtends a solid angle,

$\Omega = S/r^2 = 4\pi r^2/r^2 = 4\pi$

at the centre. Also, any closed surface subtends a solid angle 4π at any internal point.

How much is the angle subtended by a closed plane curve at an external point?



Dipole in Uniform & Non-Uniform Electric Field

INTRODUCTION TO DIPOLE IN UNIFORM EXTERNAL FIELD

If a dipole is kept in an external electric field, it experiences a rotating effect. By external electric field, we mean electric field that is not induced by dipole itself. The rotating effect is also called torque on the dipole. How we can calculate the torque on a dipole and what are its applications? This can be done by calculating the net torque on opposite charges of the dipole.

Dipole in Uniform External Field

To find torque on a dipole from an external field, consider there is electric dipole placed in an uniform external field. The uniform external electric field is produced externally and is not induced by dipole.

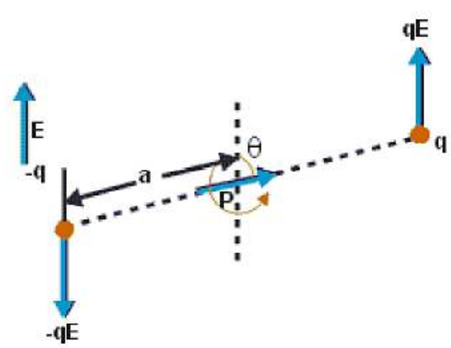


Fig: An electric dipole placed in non-uniform external electric field

The external electric field $\vec{E}$ will produce electric force $\vec{F} = q\vec{E}$ on positive charge in upward direction (same direction as $\vec{E}$) and on negative charge in downward direction (opposite direction to $\vec{E}$).

We can see that the dipole is in transitional equilibrium as net force on the dipole is zero.

What about the rotational equilibrium? Is it also zero? If that was the case, then the dipole would have been stationary in position, but experimentally it is found that the dipole rotates with some angular velocity.

This is because, both the electrostatic force that is, $\vec{F} = q\vec{E}$ acts a torque in a clockwise direction, thereby making the dipole to rotate in a uniform external electric field.

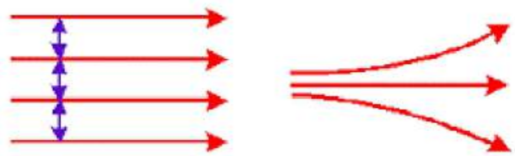


Fig: Uniform and Non-uniform electric field

Torque always acts in a couple, and its magnitude equals to the product of force and its arm. Arm is the distance between the point where the force acts and the point which rotates the dipole. In the dipole placed in the uniform external electric field, we take origin as the point. Torque is denoted by the symbol $\vec{\tau}$ and as it has a direction, it is a vector quantity.

Mathematically,

Magnitude of torque = $q\,E \times 2a \sin\theta$

$\tau = 2\,q\,a\,E \sin\theta$

Since, the dipole moment ($p = 2qa$)

$\tau = p\,E \sin\theta$

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{P}} \times \vec{\boldsymbol{E}}$$

The vector form of torque is the cross product of dipole moment and electric field. To understand what cross product is, let's take an example.

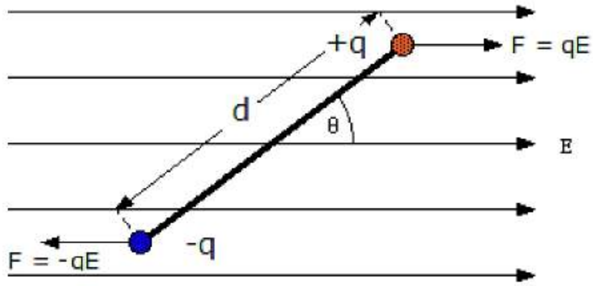


Fig: Torque rotates the dipole in uniform electric field

The net external force acting on the dipole will be zero, hence it will not translate in space. But the same cannot be said about the net torque on the dipole.
In short,

$$\vec{F}_{dipole} = +qE - qE = 0$$

$$\tau_{dipole} \neq 0$$

$$\vec{\tau} = \vec{P} \times \vec{E}$$

Where $\vec{P}$ represents the dipole moment of the electric dipole and $\vec{E}$ represents the external electric field in which the dipole is kept.

Observations in net force and torque

Taking the nature of electric field and position of the dipole, following remarks will come out:

- If the dipole $\vec{P}$ and external electric field $\vec{E}$ are parallel, that is, angle between them is zero, then the dipole will feel zero torque. That is, no rotational effect.
- If the external electric field $\vec{E}$ is non-uniform, then net force on the dipole $\neq 0$, and torque will create rotation. Hence, it would be a combined rotational and translational motion.
- If the dipole external electric field $\vec{E}$ are antiparallel, that is, angle between them is non-zero, then the dipole will feel zero torque.

When the electric dipole $\vec{P}$ and electric field $\vec{E}$ are parallel, the direction of net force will be in direction of increasing electric field.

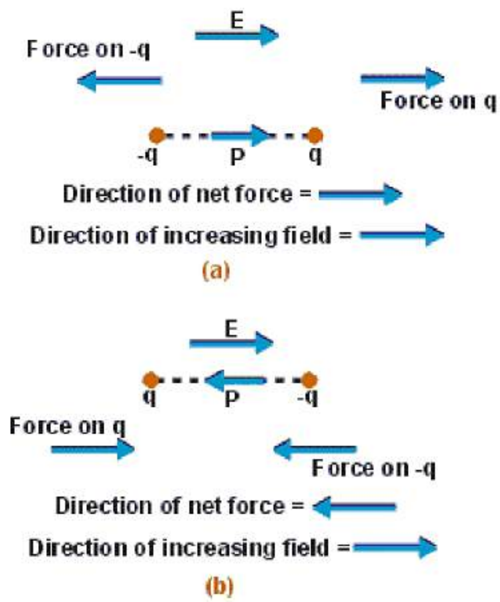


Fig: Direction of net force depends on orientation of electric dipole

- When the electric dipole $\vec{P}$ and electric field $\vec{E}$ are anti-parallel, then the direction of net force will be in direction of decreasing electric field.
- Force and Torque on a dipole placed in a uniform external field $\vec{E}$ varies with the orientation of dipole in free space

PHYSICAL SIGNIFICANCE

Fig: Comb attract dry paper piece

When we comb our dry hair and bring it near to some paper pieces, we find that the comb attracts the paper pieces. The comb gains charge, from our hair by the process of rubbing and induce a charge in the uncharged paper. In another way, the comb polarizes the pieces of paper that is, generate a net dipole moment in the direction of electric field. Also, since the electric field is non-uniform, the paper pieces move in the direction of the comb.

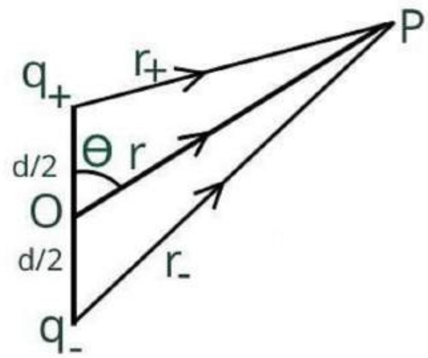
Electric Field due to an Electric Dipole

WHAT IS AN ELECTRIC DIPOLE

An electric dipole is defined as a pair of equal and opposite charges separated by a distance. However, a continuous charge distribution can also be approximated as an electric dipole from a large distance. These dipoles are characterized by their dipole moment, a vector quantity defined as the charge multiplied by their separation and the direction of this vector quantity is from the -Ve charge to the +ve charge. The total charge corresponding to a dipole is always zero. As the positive and negative charge centers are separated by a finite distance, the electric field at a test point does not cancel out completely leading to a finite electric field. Similarly, we also get finite electric potential due to a dipole.

ELECTRIC FIELD DUE TO A DIPOLE

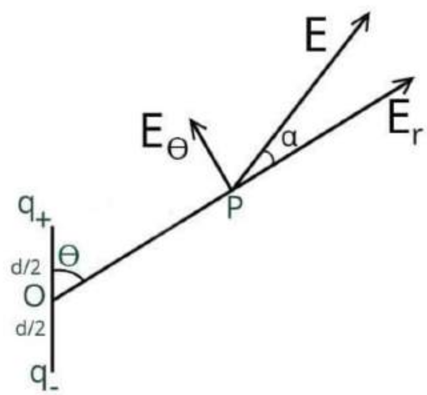
The electric field due to a pair of equal and opposite charges at any test point can be calculated using the Coulomb's law and the superposition principle. Let the test point P be at a distance r from the center of the dipole. The distance between +q and -q is d. We have shown the situation in the diagram below.



If $\vec{E_+}$ and $\vec{E_-}$ be the electric field at point P due to the positive and the negative charges separately then the total electric field $\vec{E}$ at Point P can be calculated by using the superposition principle.

$\vec{E} = \vec{E_+} + \vec{E_-}$

Please note that the directions of $\vec{E_+}$ and $\vec{E_-}$ are along $\vec{r_+}$ and $\vec{r_-}$ respectively. This is the most general form of the electric field due to a dipole. However, we will express this vector in terms of radial and inclination vectors as shown in the diagram below.



In order to calculate the electric field in the polar coordinate, we will use the expression of the electric potential due to an electric dipole which we have calculated earlier.

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left[\frac{p cos\theta}{r^2}\right]$$

Here p is the magnitude of the dipole moment and is given by qd

We can easily derive the electric field due to this dipole by calculating the negative gradient of this electric potential. In polar coordinate electric field will be independent of azimuthal (ϕ) coordinate.

$$E_r = -\frac{\partial V}{\partial r} = \frac{1}{4\pi\epsilon_0}\left[\frac{2p cos\theta}{r^3}\right]$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{1}{4\pi\epsilon_0}\left[\frac{p sin\theta}{r^3}\right]$$

$$\boxed{\vec{E} = \frac{p}{4\pi\epsilon_0}\left[\frac{2cos\theta}{r^3}\hat{r} + \frac{sin\theta}{r^3}\hat{\theta}\right]}$$

The resultant electric field at point P is

$E = \sqrt{E_r^2 + E_\theta^2}$

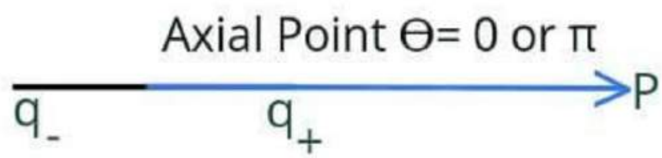
$= \frac{1}{4\pi\epsilon_0}\sqrt{\left(\frac{2pcos\theta}{r^3}\right)^2 + \left(\frac{psin\theta}{r^3}\right)^2}$

$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3cos^2\theta + 1}$

As shown in the diagram, the resultant electric field makes an angle \alpha with the radial vector. Then

$tan\alpha = \frac{E_\theta}{E_r} = \frac{tan\theta}{2}$

ELECTRIC FIELD AT AN AXIAL POINT



In this case, the test point P is on the axis of the dipole. Consequently θ= 0 or π. The electric field at point P is

$\vec{E} = \pm\frac{2p}{4\pi\epsilon_0 r^3}\hat{r}$

SIMPLIFIED DERIVATION

The electric field at point P due to the positive charge is

$\vec{E_+} = \frac{q}{4\pi\epsilon_0\left(r - \frac{d}{2}\right)^2}\hat{r}$

Electric field at point P due to negative charge is

$\vec{E_-} = -\frac{q}{4\pi\epsilon_0\left(r + \frac{d}{2}\right)^2}\hat{r}$

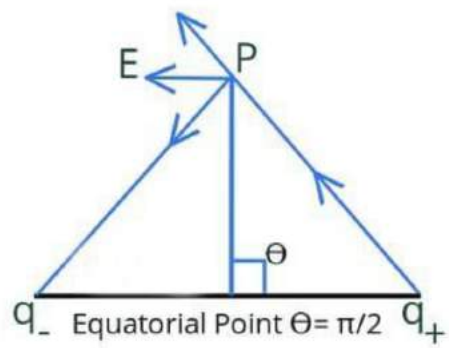
Total electric field due to the dipole at axial point P is

$\vec{E} = \vec{E_+} + \vec{E_-} = \frac{2qrd}{4\pi\epsilon_0\left(r^2 - \frac{d^2}{4}\right)^2}\hat{r}$

At a relatively large distance r>>d/2 and we can approximate the electric field as

$\vec{E} = \frac{2qd}{4\pi\epsilon_0 r^3}\hat{r} = \frac{2p}{4\pi\epsilon_0 r^3}\hat{r}$

ELECTRIC FIELD AT AN EQUATORIAL POINT



In this case, the test point P is on the perpendicular bisector of the dipole. Consequently θ= π/2. The electric field at point P is

$$\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}\hat{\theta}$$

SIMPLIFIED DERIVATION

The electric field at point P due to positive charge is

$$\vec{E_+} = \frac{q}{4\pi\epsilon_0\left(r^2 - \frac{d^2}{4}\right)}\hat{r}_+$$

Electric field at point P due to negative charge is

$$\vec{E_-} = -\frac{q}{4\pi\epsilon_0\left(r + \frac{d^2}{4}\right)}\hat{r}_-$$

Total electric field due to the dipole at equatorial point P is

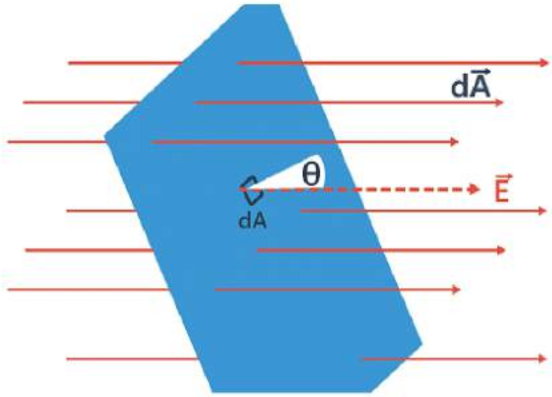
$$\vec{E} = \vec{E_+} + \vec{E_-} = \frac{qd}{4\pi\epsilon_0\left(r^2 + \frac{d^2}{4}\right)}\hat{\theta}$$

At a relatively large distance r>>d/2 and we can approximate the electric field as

$$\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}\hat{\theta}$$

Electric Flux

Analogous with flow of water and concept of flux



Consider flow of a liquid with velocity v, through small flat surface dA, in a direction normal to the surface. The rate of flow of liquid is given by the volume crossing the area per unit time vdA and represents the flux of liquid flowing across the plane. If the normal to the surface is not parallel to the direction of flow of liquid, i.e., to v, but makes an angle θ with it, the projected area in a plane perpendicular to v is vdA cosθ. There fore the flux going out of the surface d A isV. ñ dA.

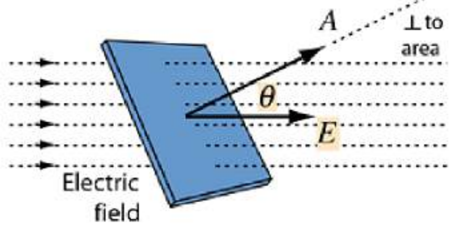
For the case of the electric field, we define an analogous quantity and call it electric flux.

We should however note that there is no flow of a physically observable quantity unlike the case of liquid flow. In the picture of electric field lines described above, we saw that the number of field lines crossing a unit area, placed normal to the field at a point is a measure of the strength of electric field at that point.

This means that if we place a small planar element of area ΔA normal to E at a point, the number of field lines crossing it is proportional to E ΔA. Now suppose we tilt the area element by angle θ. Clearly, the number of field lines crossing the area element will be smaller. The projection of the area element normal to E is ΔA cos θ.

Thus, the number of field lines crossing ΔA is proportional to E.ΔA.cosθ. When θ = 90° , field lines will be parallel to ΔA and will not cross it at all (Figure).

$$\text{flux} = \Phi = EA\cos\theta$$



where E is the magnitude of the electric field (having units of V/m), A is the area of the surface, and θ is the angle between the electric field lines and the normal (perpendicular) to A.

Note: That that an area element should be treated as a vector. It has a magnitude and also a direction. How to specify the direction of a planar area? Clearly, the normal to the plane specifies the orientation of the plane. **Thus, the direction of a planar area vector is along its normal.**
But a normal can point in two directions. Which direction do we choose as the direction of the vector associated with the area element?
Conventionally, the vector associated with every area element of a closed surface is taken to be in the direction of the outward normal.

(a) (b)

In above diagram, note that:

- Represents an open surface, thus, it is irrelevant to define 'outward' normal. You can choose any of the two normal at the surface.
- The surface in b is a closed one, it is important to define ' outward normal' as the direction of area vector (as shown in the figure), conventionally.
- For infinitely small area element dA

  $$d\Phi_E = \vec{E}.\overrightarrow{dA} = EdA\cos\theta$$

  And if the Electric field holds different value on different points on the surface, then, we must add up the electric flux from all of them individually,
- For finite number of surfaces (let' s assume

  $$\emptyset_{net} = \sum_{i=1}^{n} \emptyset_i = \sum_{i=1}^{n} \vec{E_i}.\vec{A_i} = \vec{E_1}.\vec{A_1} + \vec{E_2}.\vec{A_2} \ldots\ldots\ldots\ldots + \vec{E_n}.\vec{A_n}$$

  n),
- For infinite addition of infinitesimal surfaces i.e. integration,

  $$\emptyset_{net} = \int d\emptyset = \int \vec{E}.\overrightarrow{dA}$$

- For closed surfaces,

  $$\emptyset_{net} = \oint d\emptyset = \oint \vec{E}.\overrightarrow{dA}$$

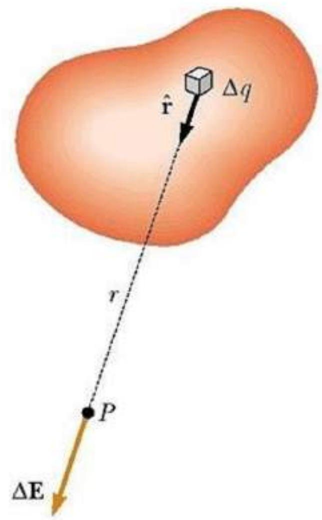
Electric Field due to Continuous Charge Distribution

Introduction

With the help of **Coulomb's Law** and **Superposition Principle**, we can easily find out the electric field due to the system of charges or discrete system of charges. The word discrete means every charge is different and has the existence of its own. Suppose, a system of charges having charges as q_1, q_2, q_3....... up to qn. We can easily find out the net charge by adding charges algebraically and net electric field by using the principle of superposition.

This is because:

- Discrete system of charges is easier to solve
- Discrete system of charges do not involve calculus in calculations



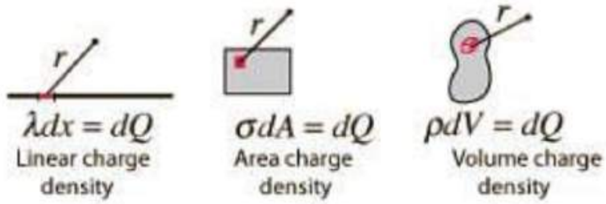
$$\vec{E}_{net} \approx k_e \sum_i \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

Considering the charge distribution as continuous, the total field at P in the limit $\Delta q_i \to 0$ is

$$\vec{E} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_1^2} \hat{r}_i \quad = k_e \int \frac{dq}{r^2} \hat{r}$$

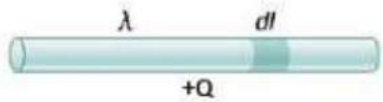
This means a combination of infinite point charges kept together forming a **linear, surface** or a **volumetric** shape constitutes a continuous charge system with **linear, surface or volumetric charge** density respectively.

Refer to the following figure:



Thus, there are three types of continuous charge distribution system.

1. Linear Charge Distribution: A body having a finite charge distributed along its length i.e. along one dimension will have a linear charge distribution. In this case, we define the Linear Charge Distribution denoted by lower case Greek letter lambda (λ).



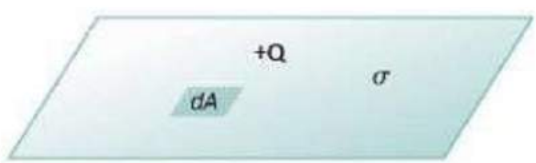
Observe the rod given above of length L, a charge of +Q is distributed along the length of the rod. A small element dl will have a charge dq on itself. In this case, we define linear charge density of the rod.

$\lambda = \lim_{\Delta l \to \infty} \frac{\Delta q}{\Delta l} = \frac{dq}{dl} = \frac{Q}{L}$ [Linear Charge density for the rod]

$dq = \lambda dl$ [Charge on infinitely small element dl]

$Q = \int dq = \int dl$ [Total charge on the rod]

2. Surface Charge Distribution: a body having a finite charge distributed along its area or surface will have a Surface Charge Distribution. In this case, we define the Surface Charge Distribution denoted by lower case Greek letter Sigma (σ).

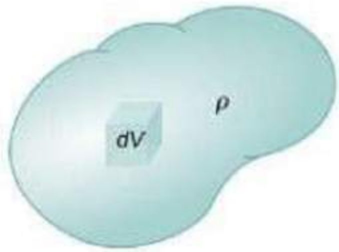


$\sigma = \lim_{\Delta A \to \infty} \frac{\Delta q}{\Delta A} = \frac{dq}{dA} = \frac{Q}{A}$ [Surface Charge density for the sheet]

$dq = \sigma dA$ [Charge on infinitely small element d A]

$Q = \int dq = \int \sigma dA$ [Total charge on the sheet]

3. Volume Charge Distribution: a body having a finite charge distributed along its volume will have a Volumetric Charge Distribution. In this case, we define the Volumetric Charge Distribution denoted by lower case Greek letter rho (ρ).



$\rho = \lim_{\Delta V \to \infty} \frac{\Delta q}{\Delta V} = \frac{dq}{dV} = \frac{Q}{V}$ [Volumetric Charge density]
$dq = \rho dV$ [Charge on infinitely small volume element dV]
$Q = \int dq = \int \rho dV$ [Total charge on the body]

Linear Charge Density

When the charge is non-uniformly distributed over the length of a conductor, it is called linear charge distribution. It is also called linear charge density and is denoted by the symbol λ (Lambda).
Mathematically linear charge density is $\lambda = dq/dl$
The unit of linear charge density is C/m. If we consider a conductor of length 'L' with surface charge density λ and take an element dl on it, then small charge on it will be $dq = \lambda l$
So, the electric field on small charge element dq will be

$$dE = \frac{kdq}{r^2}$$

$$dE = \frac{k\lambda\, dl}{r^2}$$

To calculate the net electric field we will integrate both sides with proper limit, that is

$$\int dE = \int_0^L \frac{k\lambda\, dl}{r^2}$$

$$\int dE = \frac{k}{r^2} \int_0^L \lambda\; dl$$

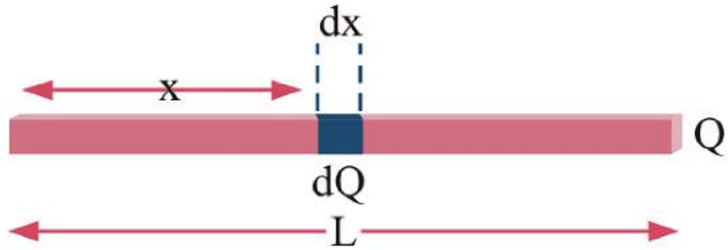


Fig: We take small element x and integrate it in case of linear charge density

Surface Charge Density

When the charge is uniformly distributed over the surface of the conductor, it is called Surface Charge Density or Surface Charge Distribution. It is denoted by the symbol σ (sigma) symbol and is the unit is C/m^2.
It is also defined as charge/ per unit area. Mathematically surface charge density is $\sigma = dq/ds$
where dq is the small charge element over the small surface ds. So, the small charge on the conductor will be $dq = \sigma ds$
The electric field due to small charge at some distance 'r' can be evaluated as

$$dE = \frac{kdq}{r^2}$$

$$dE = \frac{k\sigma ds}{r^2}$$

Integrating both sides with proper limits we get

$$\int dE = \int_0^S \frac{k\sigma\, ds}{r^2}$$

$$\int dE = \frac{k}{r^2}\int_0^S \sigma\ ds$$

Volume Charge Density

When the charge is distributed over a volume of the conductor, it is called Volume Charge Distribution. It is denoted by symbol ρ (rho). In other words charge per unit volume is called Volume Charge Density and its unit is C/m3. Mathematically, volume charge density is $\rho = dq/dv$
where dq is small charge element located in small volume dv. To find total charge we will integrate dq with proper limits. The electric field due to dq will be
$dq = \rho\ dv$

$$dE = \frac{kdq}{r^2}$$

$$dE = \frac{k\,\rho\,dv}{r^2}$$

Integrating both sides with proper limits we get

$$\int dE = \int_0^V \frac{v\,\rho\,dv}{r^2}$$

$$\int dE = \frac{k}{r^2}\int_0^V \rho\,dv$$

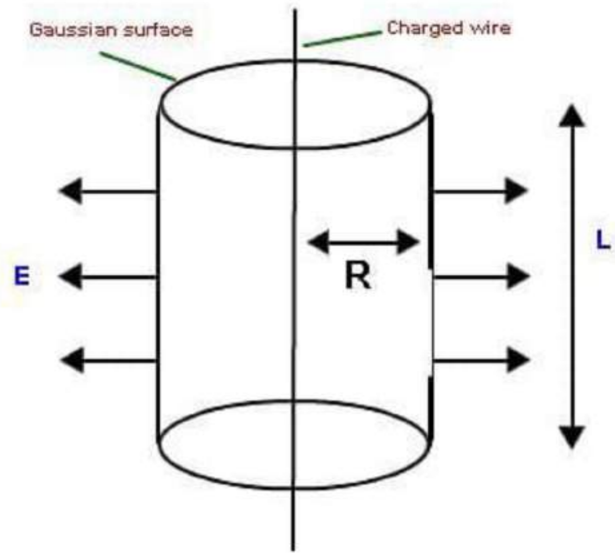


Fig: We can easily find electric field in different geometries using charge distribution system

Steps to calculate Electric Field Intensity due to continuous charge body:

(i) Identify the type of charge distribution and compute the charge density λ, σ or ρ.
(ii) Divide the charge distribution into infinitesimal charges dq, each of which will act as a tiny point charge.
(iii) The amount of charge dq, i.e., within a small element dl, dA or dV is
$dq = \lambda\, dl$ (charge distributed in length)
$dq = \sigma\, dA$ (charge distributed over a surface)
$dq = \rho\, dV$ (charge distributed throughout a volume)

(iv) Draw at point P the dE vector produced by the charge dq. The magnitude of dE is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

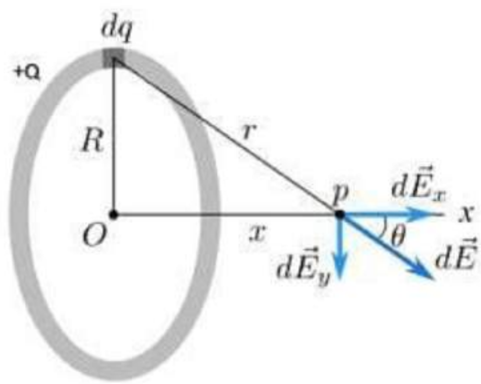
(v) Resolve the dE vector into its components. Identify any special symmetry features to show whether any component(s) of the field that are not canceled by other components.
(vi) Write the distance r and any trigonometric factors in terms of given coordinates and parameters.
(vii) The electric field is obtained by summing over all the infinitesimal contributions.

$$\vec{E} = \int d\vec{E} = \int \frac{dq}{4\pi\varepsilon_0 r^2}$$

(viii) Perform the indicated integration over limit of integration that includes all the source charges.

Electric Field calculation due to Uniformly Distributed Continuous Charge

(a) Electric Field on axis of a uniformly charged circular ring:



Consider a uniformly charged circular ring with a total charge +Q distributed uniformly along its length. We need to evaluate the net electric field due to this charged ring at a point P which is located x distance from its centre on its axis. Conclude that the charge is distributed linearly throughout the length of the ring, hence we will define linear charge density λ for this ring,

λ = Total charge on the ring/Total Length = $Q/2\pi r$

Now, we consider an infinitely small length element dl on the ring,
Infinitesimal charge on element dl,

$dq = \lambda dl$

Now, we write the expression of Electric Field at point P due to dq

$$d\vec{E} = \int \frac{K\text{dq}}{r^2} = \int \frac{K\lambda\text{dl}}{r^2}$$

This, infinitely small electric field vector will be inclined at an angle θ with the axis of the ring (x axis), as shown in diagram.

We need to imagine components of $\overrightarrow{dE}$ along the x and y axis i.e. $\overrightarrow{dE_x}$ and $\overrightarrow{dE_y}$.

By resolving $\overrightarrow{dE}$ we get,

$$\overrightarrow{dE_x} = \overrightarrow{dE}\cos\theta$$

$$\overrightarrow{dE_y} = \overrightarrow{dE}\sin\theta$$

Observe and imagine, that $\overrightarrow{dE_y}$ will cancel out if we take each and every element of the ring into consideration.

Therefore net electric field at P,

$$\overrightarrow{E_p} = \int \overrightarrow{dE_x} = \int \overrightarrow{dE}\cos\theta = \int \frac{K\text{dq}}{r^2}\cos\theta = \int_0^{2\pi R} \frac{K\lambda\text{dl}}{r^2}\cos\theta$$

Now, by geometry,

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{(x^2+R^2)}}$$

Thus,

$$\overrightarrow{E_p} = \int \overrightarrow{dE_x} = \int_0^{2\pi R} \frac{K\lambda\text{dl}}{r^2}\cos\theta = \int_0^{2\pi R} \frac{K\lambda\text{dl}}{(x^2+R^2)} \frac{x}{\sqrt{(x^2+R^2}}$$

$$\overrightarrow{E_p} = \int_0^{2\pi R} \frac{Kx\lambda\text{dl}}{(x^2+R^2)^{3/2}}$$

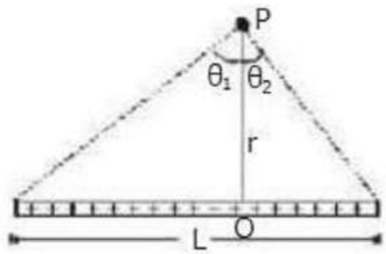
$$\overrightarrow{E_p} = \frac{Kx\lambda}{(x^2+R^2)^{3/2}} \int_0^{2\pi R} dl$$

Replacing with the value of λ defined above,

$$\overrightarrow{E_p} = \frac{KQx}{(x^2+R^2)^{3/2}}$$

(b) Electric field strength at a general point due to a uniformly charged rod:

As shown in figure, if P is any general point in the surrounding of rod, to find the electric field strength at P, again we consider an element on rod of length dx at a distance x from point O as shown in figure.



Now if dE be the electric field at P due to the element, then it can be given as

$$dE = \frac{Kdq}{(x^2 + r^2)}$$

Here

$$dq = \frac{Q}{L}dx = \lambda dx$$

Now we resolve electric field in components. Electric field strength in x-direction due to dq at P is,

$dE_x = dE\sin\theta$

$$dE_x = \frac{Kdq}{(x^2 + r^2)}\sin\theta = \frac{K\lambda dx}{(x^2 + r^2)}\sin\theta$$

$$= \frac{KQ\sin\theta}{L(x^2 + r^2)}dx$$

Here we have $x = r\tan\theta$

and $dx = r\sec^2\theta d\theta$

$$dE_x = \frac{KQ}{L}\frac{r\sec^2\theta d\theta}{r^2\sec^2\theta}\sin\theta$$

Net electric field strength due to dq at point P in x-direction is

$$E_x = \int dE_x = \frac{KQ}{Lr}\int_{-\theta_2}^{\theta_1}\sin\theta\, d\theta$$

or $E_x = \frac{KQ}{Lr}[-\cos\theta]_{-\theta_2}^{\theta_1}$

$$\boldsymbol{E_x = \frac{KQ}{Lr}[\cos\theta_2 - \cos\theta_1] = \frac{K\lambda}{r}[\cos\theta_2 - \cos\theta_1]}$$

Similarly, the electric field strength at point P due to dq in y-direction is

$dE_y = dE\cos\theta$

$$dE_y = \frac{KQdx}{L(r^2 + x^2)} \times \cos\theta$$

Again we have $x = r\tan\theta$

And $dx = r\sec^2\theta d\theta$

Thus we have,

$$dE_y = \frac{KQ}{L}\cos\theta \times \frac{r\sec^2\theta}{r^2\sec^2\theta} = \frac{KQ}{Lr}\cos\theta d\theta$$

Net electric field strength at P due to dq in y-direction is

$$E_y = \int dE_y = \frac{KQ}{Lr}\int_{-\theta_2}^{\theta_1}\cos\theta\, d\theta$$

$$E_y = \frac{KQ}{Lr}[+\sin\theta]_{-\theta_2}^{\theta_1}$$

$$\boldsymbol{E_y = \frac{KQ}{Lr}[\sin\theta_1 + \sin\theta_2] = \frac{K\lambda}{r}[\sin\theta_1 + \sin\theta_2]}$$

Thus electric field at a general point in the surrounding of a uniformly charged rod which subtends angles $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$ at the two corners of the rod from the point of consideration can be given as In parallel direction,

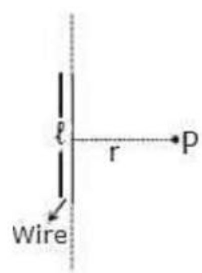
$$\boldsymbol{E_x = \frac{K\lambda}{r}[\cos\theta_2 - \cos\theta_1]}$$

In perpendicular direction ,

$$\boldsymbol{E_y = \frac{K\lambda}{r}[\sin\theta_1 + \sin\theta_2]}$$

We can use this generalized finite relation to calculate the Electric Field due to following systems too:

(i) Infinitely long uniformly charged rod with charge density λ:



For infinite rod, $\theta_1 \to 90°$ and $\theta_2 \to 90°$

Therefore, for infinitely long uniformly charged rod,

$$E_x = \frac{K\lambda}{r}[\cos 90° - \cos 90°] = 0$$

While,

$$\boldsymbol{E_y = \frac{K\lambda}{r}[\sin 90° + \sin 90°] = \frac{2K\lambda}{r}}$$

(ii) Electric field due to semi-infinite wire:

For this case,

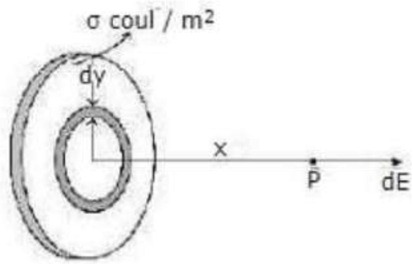
$$\theta_2 = \frac{\pi}{2}, \qquad \theta_2 = 0°$$

$$\therefore E_x = \frac{K\lambda}{r};\ E_y = \frac{K\lambda}{r}$$

$$E_{net}\ at\ P = \frac{\sqrt{2}K\lambda}{r}$$

(c) Electric field strength due to a uniformly surface charged disc:

If there is a disc of radius R, charged on its surface with surface charge density s C/m^2, we wish to find electric field strength due to this disc at a distance x from the centre of disc on its axis at point P shown in figure.



Note: Identify that the electric charge is distributed over the surface of the non-conducting disc, hence we would define a surface charge density σ for this disc.
σ = Total Charge/Total Area = $Q/\pi R^2$
To find electric field at point P due to this disc, we consider an elemental ring of radius y and width dy in the disc as shown in figure. Now the charge on this elemental ring dq can be given as
$dq = \sigma\ (dA)$
where dA is the area of the ring element on the disc,
also we can imagine ring element to be a small rectangle with width dy. Thus,
$dA = 2\pi y dy$
$dq = \sigma(2\pi y dy)$
Now we know that electric field strength due to a ring of radius R. Charge Q at a distance x from its centre on its axis can be given as

$$\overrightarrow{E_p} = \frac{KQx}{(x^2+R^2)^{3/2}}$$

Here due to the elemental ring electric field strength dE at point P can be given as

$$\overrightarrow{dE_p} = \frac{K(dQ)x}{(x^2+R^2)^{3/2}} = \frac{Kx\sigma(2\pi y dy)}{(x^2+R^2)^{3/2}}$$

Net electric field at point P due to this disc is given by integrating above expression from O to R as

$$E = \int dE = \int_0^R \frac{K\sigma 2\pi xydy}{(x^2+y^2)^{3/2}}$$

$$E = K\sigma\pi x \int_0^R \frac{2ydy}{(x^2+y^2)^{3/2}}$$

Now, using integration by substitution we can solve the above integral as,

$$E = K\sigma\pi x[-\frac{1}{\sqrt{x^2+y^2}}]_0^R$$

$$\boldsymbol{E = \frac{\sigma}{2\epsilon_0}[1 - \frac{x}{\sqrt{x^2+R^2}}]}$$

By geometry,

$$\frac{x}{\sqrt{x^2+R^2}} = \cos\,\theta$$

Hence,

$$\boldsymbol{E = \frac{\sigma}{2\epsilon_0}[1 - \cos\,\theta]}$$

Please note that $\boldsymbol{\theta}$ is the angle subtended by the disc at point P which is x distance far from the center.

Case: (i) If $x << R \Rightarrow \cos\boldsymbol{\theta} \to 1$

Physically, this would mean that the disc has its radius $R \to \infty$, that is the disc can be effectively imagined as infinitely long sheet of charge,

Thus, Electric field due to infinitely long plane sheet of charge at a distance x would be,

$$E_{sheet} = \frac{\sigma}{2\epsilon_0}[1 - \cos(\frac{\pi}{2})]$$

$$\boldsymbol{E_{sheet} = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\pi R^2\epsilon_0}}$$

i.e. behaviour of the disc is like **infinite sheet**.

Case: (ii) If $x >> R$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{x\sqrt{\frac{R^2}{x^2}+1}}\right] = \frac{\sigma}{2\varepsilon_0}[1-(1+\frac{R^2}{x^2})^{-1/2}]$$

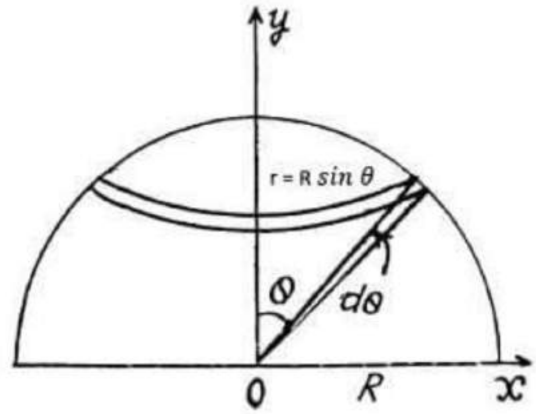
Now, using binomial approximation,

$$E = \frac{\sigma}{2\varepsilon_0}[1 - 1 + \frac{1}{2}\frac{R^2}{x^2} + \text{higher order terms}]$$

$$= \frac{\sigma}{4\varepsilon_0}\frac{R^2}{x^2} = \frac{\sigma\pi R^2}{4\pi\varepsilon_0 x^2} = \frac{Q}{4\pi\varepsilon_0 x^2} = \frac{KQ}{x^2}$$

i.e. behaviour of the disc is like a point charge.

Electric Field Strength due to a uniformly charged Hollow Hemispherical Cup:
Figure shows a hollow hemisphere, uniformly charged with surface charge density σ C/m^2 . To find electric field strength at its centre C, we consider an elemental ring on its surface of angular width dq at an angle q from its axis as shown. The surface area of this ring will be



$dA = 2\pi r \times Rd\theta$
By geometry, $dA = 2\pi R \sin\theta \times Rd\theta$
Charge on this elemental ring is
$dq = \sigma dA = \sigma .\, 2\pi R \sin\theta \times Rd\theta$
Now due to this ring electric field strength at centre C can be given as,

$$dE = \frac{Kdq(R\cos\ \theta)}{(R^2\sin^2\ \theta + R^2\cos^2\ \theta)^{3/2}}$$

$$dE = \frac{K\sigma 2\pi R^2 \sin\theta\ \ d\theta\ (R\cos\ \theta)}{(R)^3}$$

Net electric field at centre can be obtained by integrating this expression between limits 0 to $\pi/2$.

$$E_C = \int dE = \pi K\sigma \int_0^{\pi/2} \sin\ 2\theta d\theta$$

$$= \frac{\sigma}{4\varepsilon_0}[-\frac{\cos\ 2\theta}{2}]_0^{\pi/2} = \frac{\sigma}{4\epsilon_0}\left[\frac{1}{2}+\frac{1}{2}\right] = \frac{\sigma}{4\epsilon_0}$$

Hence, Electric Field intensity at centre C, due to uniformly charged nonconducting hemispherical shell is,

$$\boldsymbol{E_C = \frac{\sigma}{4\epsilon_0}}$$

Above given continuous charged systems are most frequently used ones. It is recommended to remember the procedure and results by heart.

Continuous Charge Distribution

INTRODUCTION TO CONTINUOUS CHARGE DISTRIBUTION

With the help of Coulomb's Law and Superposition Principle, we can easily find out the electric field due to the system of charges or discrete system of charges. The word discrete means every charge is different and has the existence of its own. Suppose, a system of charges having charges as q_1, q_2, q_3....... up to q_n. We can easily find out the net charge by adding charges algebraically and net electric field by using the principle of superposition.
This is because:

- Discrete system of charges is easier to solve
- Discrete system of charges do not involve calculus in calculations

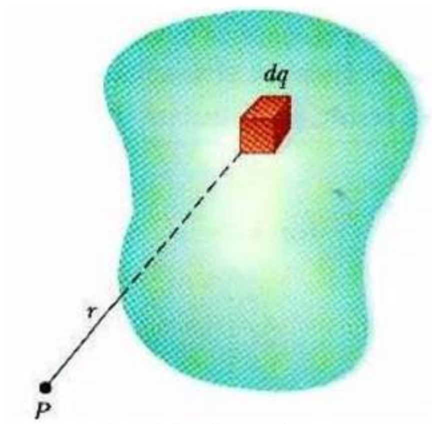


Fig: A system in which charge is distributed over a conductor, is called continuous charge distribution system

But how to calculate electrostatics terms in continuous charge system? For an Example if there is a rod with charge q, uniformly distributed over it and we wish to find the electric field at some distance 'r' due it. It would be illogical and irrelevant to simply add electric field using principle of superposition as the charge is uniformly distributed over the rod. So we take a small element of the rod and integrate it with proper limits.
We consider element, based on how density of charge is centered on the material or object. If the charge is uniformly distributed over the surface of the conductor, then it is called Surface Density. If the charge varies linearly along the length of the conductor, then it is called Linear Charge Density. And if the charge changes with volume of the conductor, then it is called Volume Charge Density.

WHAT IS CONTINUOUS CHARGE DISTRIBUTION?

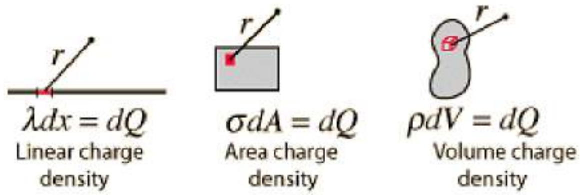


Fig: Types of Charge Distribution

The continuous charge distribution system is a system in which the charge is uniformly distributed over the conductor. In continuous charge system, infinite numbers of charges are closely packed and have minor space between them. Unlikely from the discrete charge system, the continuous charge distribution is uninterrupted and continuous in the conductor. There are three types of the continuous charge distribution system.

- Linear Charge Distribution
- Surface Charge Distribution
- Volume Charge Distribution

Volume charge density:

$$\rho_v = \lim_{\Delta V \to 0} \frac{\Delta q}{\Delta V} = \frac{dq}{dV} \qquad (\text{C/m}^3)$$

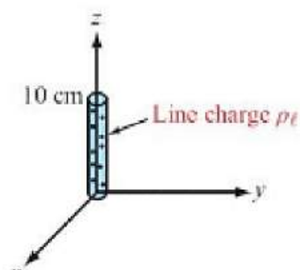
Total Charge in a Volume

$$Q = \int_V \rho_v \, dV \qquad (\text{C})$$

Surface and Line Charge Densities

$$\rho_s = \lim_{\Delta s \to 0} \frac{\Delta q}{\Delta s} = \frac{dq}{ds} \qquad (\text{C/m}^2)$$

$$\rho_\ell = \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l} = \frac{dq}{dl} \qquad (\text{C/m})$$



(a) Line charge distribution

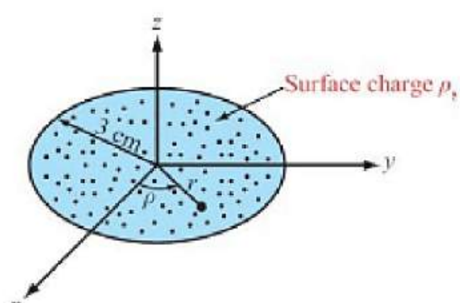


Fig: Types of charge distribution system

LINEAR CHARGE DENSITY

When the charge is non-uniformly distributed over the length of a conductor, it is called linear charge distribution. It is also called linear charge density and is denoted by the symbol λ (Lambda).

Mathematically linear charge density is $\lambda = dq/dl$

The unit of linear charge density is C/m. If we consider a conductor of length 'L' with surface charge density λ and take an element dl on it, then small charge on it will be $dq = \lambda l$

So, the electric field on small charge element dq will be

$$dE = \frac{kdq}{r^2}$$

$$dE = \frac{k\lambda\, dl}{r^2}$$

To calculate the net electric field we will integrate both sides with proper limit, that is

$$\int dE = \int_0^L \frac{k\lambda\, dl}{r^2}$$

$$\int dE = \frac{k}{r^2}\int_0^L \lambda\ dl$$

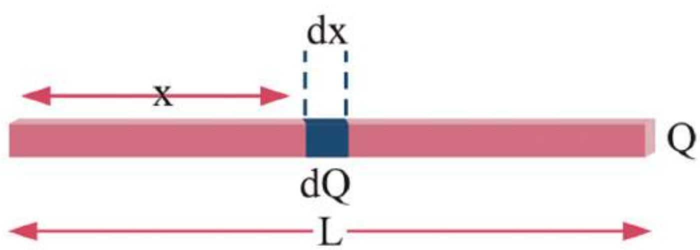


Fig: We take small element x and integrate it in case of linear charge density

SURFACE CHARGE DENSITY

When the charge is uniformly distributed over the surface of the conductor, it is called Surface Charge Density or Surface Charge Distribution. It is denoted by the symbol σ (sigma) symbol and is the unit is C/m^2.

It is also defined as charge/ per unit area. Mathematically surface charge density is σ

$= dq/ds$
where dq is the small charge element over the small surface ds. So, the small charge on the conductor will be $dq = \sigma ds$
The electric field due to small charge at some distance 'r' can be evaluated as

$$dE = \frac{kdq}{r^2}$$

$$dE = \frac{k\sigma ds}{r^2}$$

Integrating both sides with proper limits we get

$$\int dE = \int_0^S \frac{k\sigma\, ds}{r^2}$$

$$\int dE = \frac{k}{r^2}\int_0^S \sigma\ ds$$

VOLUME CHARGE DENSITY

When the charge is distributed over a volume of the conductor, it is called Volume Charge Distribution. It is denoted by symbol ρ (rho). In other words charge per unit volume is called Volume Charge Density and its unit is C/m3. Mathematically, volume charge density is $\rho = dq/dv$
where dq is small charge element located in small volume dv. To find total charge we will integrate dq with proper limits. The electric field due to dq will be
$dq = \rho\ dv$

$$dE = \frac{kdq}{r^2}$$

$$dE = \frac{k\,\rho\, dv}{r^2}$$

Integrating both sides with proper limits we get

$$\int dE = \int_0^V \frac{v\,\rho\, dv}{r^2}$$

$$\int dE = \frac{k}{r^2}\int_0^V \rho\, dv$$

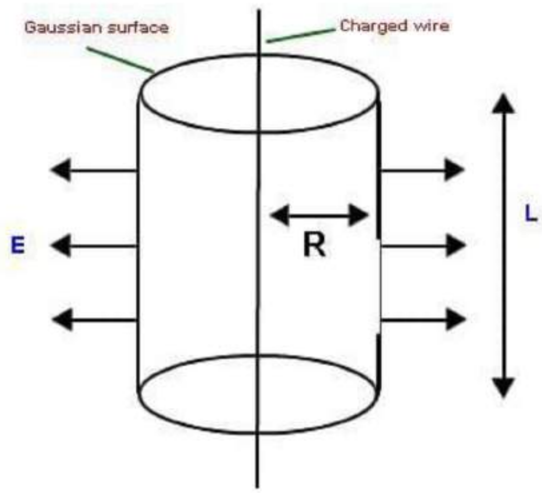
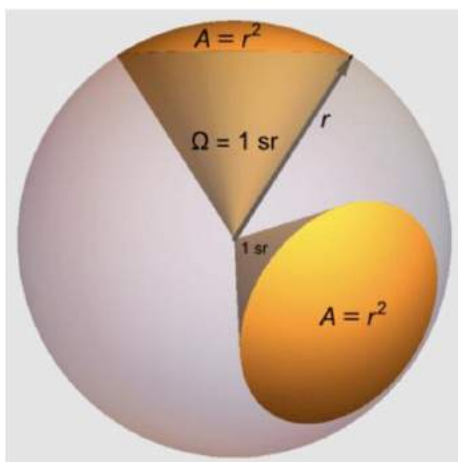


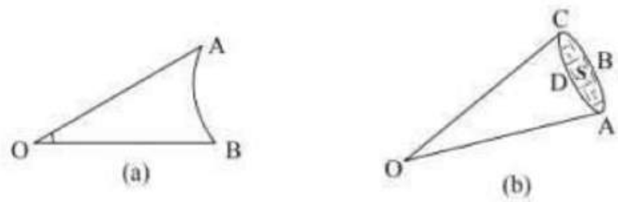
Fig: We can easily find electric field in different geometries using charge distribution system

Concept of Solid Angle

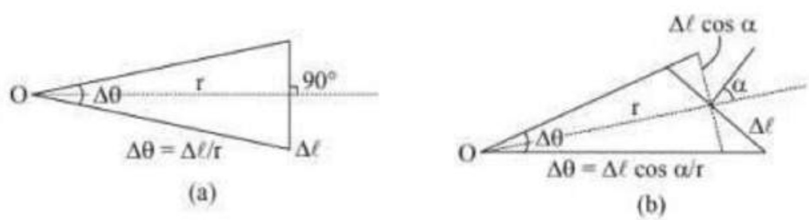
CONCEPT OF SOLID ANGLE



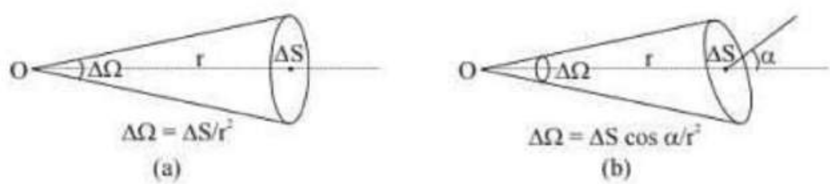
Solid angle is a generalisation of the plane angle: In figure we show a plane curve AB. The end points A and B are joined to the point O. We say that the curve AB subtends an angle or a plane angle at O. An angle is formed at O by the two lines OA and OB passing through O. We say that the curve AB subtends an angle or a plane angle at O. An angle is formed at O by the two lines OA and OB passing through O.



How do we measure a solid angle? Let us consider how do we measure a plane angle. See fig. We draw a circle of any radius r with the centre at O and measure the length l of the arc intercepted by the angle.
The angle θ is then defined as $\theta = 1/\mathrm{r}$. In order to measure a solid angle at the point O (fig.), we draw a sphere of any radius r with O as the centre and measure the area S of the part of the sphere intercepted by the cone. The solid angle Ω is then defined as $\Omega = \mathrm{S}/\mathrm{r}^2$
Note: That this definition makes the solid angle a dimensionless quantity. It is independent of the radius of the sphere drawn.



Next, consider a plane angle subtended at a point O by a small line segment $\Delta\ell$ (fig.). Suppose, the line joining O to the middle point of $\Delta\ell$ is perpendicular to $\Delta\ell$.As the segmentis small, we can approximately write.
$\Delta\theta = \Delta\ell/\mathrm{r}$
As $\Delta\ell$ gets smaller, the approximation becomes better. Now suppose, the line joining O to $\Delta\ell$ is not perpendicular to $\Delta\ell$ (fig.). Suppose, this line makes an angle α with the perpendicular to $\Delta\ell$. The angle subtended by $\Delta\ell$ at O is
$\Delta\theta = \Delta\ell\cos\alpha/\mathrm{r}$



Similarly, if a small plane area ΔS (fig.) subtends a solid angle $\Delta\ell$ at O in such a way that the line joining O to ΔS is normal to ΔS, we can write$\Delta\,\Omega = \Delta\mathrm{S}\,/\,\mathrm{r}^2$.
But if the line joining O to ΔS makes an angle α with the normal to ΔS (fig.), we should write

$\Delta\Omega = \Delta S\cos\alpha/r^2$

A complete circle subtends an angle

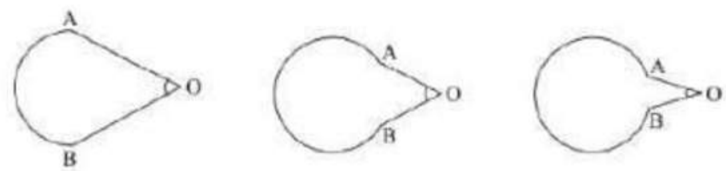
$\theta = \ell/r = 2\pi r/r = 2\pi$

at the centre. In fact, any closed curve subtends an angle 2π at an y of the internal points. Similarly, a complete sphere subtends a solid angle,

$\Omega = S/r^2 = 4\pi r^2/r^2 = 4\pi$

at the centre. Also, any closed surface subtends a solid angle 4π at any internal point.

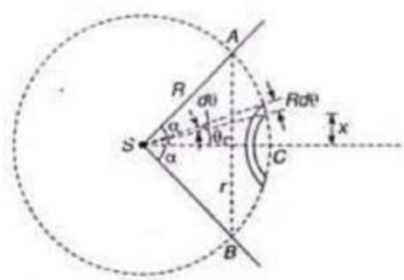
How much is the angle subtended by a closed plane curve at an external point?



APPLICATION OF SOLID ANGLE

Q. Fraction of light emerging from an isotropic point source through a conical region having semi vertex angle α and with its apex at the source.

Ans. Let us consider a sphere of radius R with its centre at the source S.



Let AB be the section (circular) where the cone ASB intercepts the sphere. If ΔS be the area of the spherical portion ACB (lying within the conical region) then, the solid angle

$\Omega = \Delta S/R^2$

Let SC be the symmetry axis of the portion of the sphere ACB.

If x be the distance of a thin circular strip then its area

$dS = 2\pi\, x\, Rd\theta$

$= 2\pi\, (R\sin\theta)\, Rd\theta$

Total Area,

$\Delta S = \int dS$

$= \int_0^\alpha 2\pi R^2 \sin\theta d\theta$

$= \mathbf{2\pi R^2\,[1 - \cos\alpha]}$

Therefore, area of the curved surface ΔS which subtends an angle α at the center,

$\Delta S = 2\pi R^2 [1 - \boldsymbol{\cos \alpha}]$

Also, Solid Angle

$\boldsymbol{\Omega} = S/R^2 = \boldsymbol{2\pi [1 - \cos \alpha]}$

This relation between plane angle α and solid angle $\boldsymbol{\Omega}$ is advised to be remembered.

If Ω steradian be the solid angle for the cone then, the fraction of light passing through the cone will be

$$f = \frac{\Omega}{4\pi} = \frac{2\pi[1 - \cos\ \alpha]}{4\pi} = \frac{[1 - \cos\ \alpha]}{2}$$