CBSE Class 10th Mathematics
Basic Sample Paper - 07

Maximum Marks:
Time Allowed: 3 hours

General Instructions:

a. All questions are compulsory
b. The question paper consists of 40 questions divided into four sections A, B, C & D.
c. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises 6 questions of 4 marks each.
d. There is no overall choice. However internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

Section A

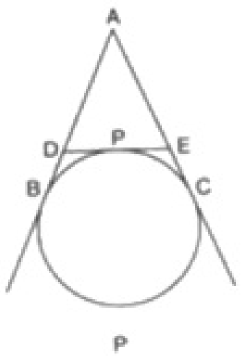
1. If the HCF of 210 and 55 is expressible in the form $210 \times 5 + 55y$, then y =
   a. –19
   b. –29
   c. 19
   d. 29
2. For any positive integer 'a' and 3, there exist unique integers 'q' and 'r' such that a = 3q + r where 'r' must satisfy
   a. 1 < r < 3
   b. $0 < r \leqslant 3$
   c. $0 \leqslant r < 3$
   d. 0 < r < 3
3. The decimal expansion of $\frac{21}{24}$ will terminate after:
   a. 1 decimal place

b. 3 decimal places

c. None of these

d. 2 decimal places

4. In the given figure, AB = 8 cm. If PE = 3 cm, then the measure of AE is



a. 3 cm

b. 11 cm

c. 5 cm

d. 7 cm

5. In the given data if n = 230, l = 40, cf = 76, h = 10, f = 65, then its median is

a. 48

b. 40

c. 47

d. 46

6. A letter of English alphabets is chosen at random. The probability that the letter chosen is a consonant is

a. $\frac{2}{26}$

b. $\frac{1}{26}$

c. $\frac{21}{26}$

d. $\frac{5}{26}$

7. The degree of the polynomial $5x^3 - 3x^2 - x + \sqrt{2}$ is

a. 2

b. 3

c. 1

d. 0

8. The zeroes of the quadratic polynomial $x^2 + 9x + 20$ are

a. – 4 and 5

b. – 4 and – 5

c. 4 and 5

d. 4 and – 5

9. The triangle whose vertices are (– 3, 0), (1, – 3) and (4, 1) is __________ triangle.

a. Obtuse triangle

b. equilateral

c. right angled isosceles

d. scalene

10. Points (1, 0) and (– 1, 0) lies on

a. line x + y = 0

b. y – axis

c. x – axis

d. line x – y =0

11. Fill in the blanks:

The distance of the point P(2, 3) from the X-axis is ________.

12. Fill in the blanks:

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ are a system of two simultaneous linear equations. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then the system has _______ solution.

OR

Fill in the blanks:

$a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ are a system of two simultaneous linear equations. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then the system has _______ solutions.

13. Fill in the blanks:

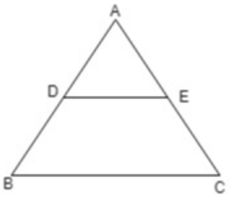
The value of trigonometric function $\cot^2\theta - \frac{1}{\sin^2\theta} =$ _______.

14. Fill in the blanks:

The value of $\sin^2 30^o + \cos^2 45^o + \cos^2 30^o$ is _______.

15. Fill in the blanks:

In the given figure, if $\frac{AD}{DB} = \frac{3}{5}$ and AC = 4.8cm, then the value of AE is ________.



16. Find the value of $\cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$?

OR

Evaluate $\frac{\sin 41^\circ}{\cos 49^\circ}$.

17. Radius of a circle is 1m. If diameter is increased by 100% then find the percentage increase in its area.

18. Two unbiased coins are tossed simultaneously. Find the probability of getting at least one head?

19. P and Q are the points on sides AB and AC, respectively of $\triangle ABC$, If AP = 3 cm , PB = 6 cm, AQ = 5 cm and QC = 10cm, Show that BC = 3PQ.

20. Find the 25$^{\text{th}}$ term of the AP : $-5, -\frac{5}{2}, 0, \frac{5}{2}, \ldots$.

Section B

21. It is known that a box of 600 electric bulbs contains 12 defective bulbs. One bulb is taken out at random from this box. What is the probability that it is a non-defective bulb?

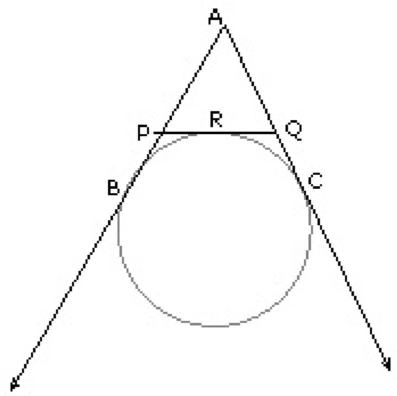
22. Cards marked with numbers 13, 14, 15, ..., 60 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that number on the card drawn is

    i. divisible by 5
    ii. a number is a perfect square

23. Prove that the perpendicular at the point of contact of the tangent to a circle passes through the centre.

OR

If AB, AC and PQ are tangents in the given figure and AB = 25 cm, Find the perimeter of $\triangle$ APQ.



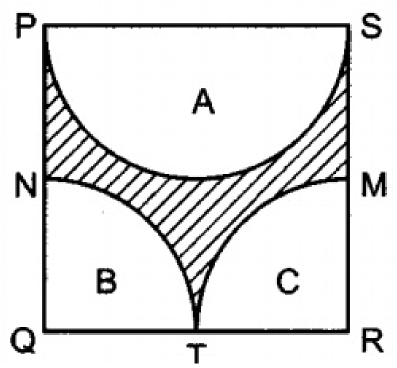
24. If $\sin\theta = \frac{a}{b}$, find $\sec\theta + \tan\theta$ in terms of a and b.

OR

Prove the identity:

$\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

25. In given figure, PQRS is a square of side 14 cm. Region A is a semicircle on PS as diameter. Region B and C are quadrants of a circles with centres Q and R respectively each having radius 7 cm. Find area of the shaded part.



26. Read the following passage and answer the question that follows:
Kavya went to a fair in her village. She wanted to enjoy rides on the Giant wheel and play Hoopla (a game in which you throw a ring on the item kept in a stall, and if the ring covers any object completely, you get it). She asked the rates of both rides to stall owner. He said each ride costs Rs 3, and a game of Hoopla cost Rs. 4. Her father gave her only Rs. 20 and also told her that the number of times she played Hoopla should

be half the number of rides she had on the Giant wheel.

(a) Represent the situation by two equations.

(b) Find the solution to this pair of equations.

Section C

27. Find all the zeros of the polynomial f (x) = $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if being given that two of its zeros are $\sqrt{2}$. and $-\sqrt{2}$

28. Draw a circle of radius 4.2 cm. Draw a pair of tangents to this circle inclined to each other at an angle of 45°.

OR

Draw a pair of tangents to a circle of radius 3 cm which are inclined to each other at angle of 60°.

29. A cone of radius 4 cm is divided into two parts by drawing a plane through the mid point of its axis and parallel to its base. Compare the volumes of two parts.

30. Find the value of the following without using trigonometric tables:

$\frac{\cos 50^\circ}{2\sin 40^\circ} + \frac{4(\operatorname{cosec}^2 59^\circ - \tan^2 31^\circ)}{3\tan^2 45^\circ} - \frac{2}{3}\tan 12^\circ \tan 78^\circ . \sin 90^\circ$

OR

Without using trigonometric tables, evaluate the following:

$\frac{\sec 39^\circ}{\operatorname{cosec} 51^\circ} + \frac{2}{\sqrt{3}} \tan 17^\circ \tan 38^\circ \tan 60^\circ \tan 52^\circ \tan 73^\circ - 3(\sin^2 31^\circ + \sin^2 59^\circ)$

31. For a morning walk, three persons steps off together. The measure of their steps is 80,85 and 90 cm respectively.What is the minimum distance each should walk so that all can cover the same distance in complete steps?

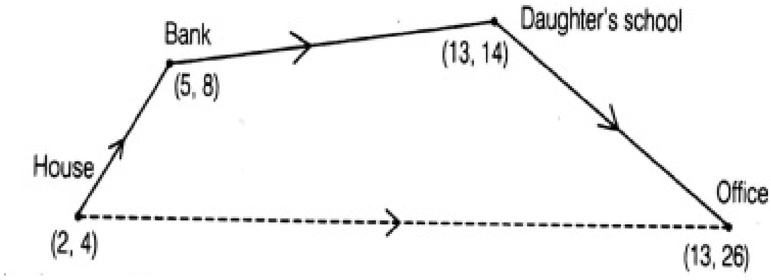
Which value is preferred in this situation?

OR

A mason has to fit a bathroom with square marble tiles of the largest possible size. The size of the bathroom is 10 ft. by 8 ft. What would be the size (in inches) of the tile required that has to be cut and how many such tiles are required?

32. The two tangents from an external point P to a circle with centre O are PA and PB. If $\angle APB = 70°$, what is the value of $\angle AOB$?

33. Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km., answer the following questions:



i. Calculate the distance between Ayush house and bank.
ii. Calculate the distance between bank and Ayush daughter's school.
iii. What is the extra distance travelled by Ayush in reaching his office? (Assume that all distance covered are in straight lines).

34. Find the value of k for which the following system of equations have infinitely many solutions:

$2x - 3y = 7$

$(k + 2)x - (2k + 1)y = 3(2k - 1)$

Section D

35. A two-digit positive number is six times the sum of its digits and is also equal to 6 less than thrice the product of its digits. Find the number.

36. Find the middle term of the sequence formed by all three-digit numbers which leave a remainder 3 when divided by 4. Also, find the sum of all numbers on both sides of the middle term.

OR

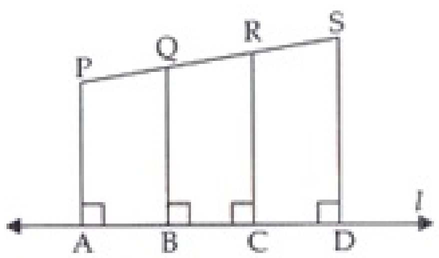
Find the sum of first 24 terms of an A.P. whose nth term given by $a_n = 3 + 2n$.

37. The angle of elevation of the top of a tower as observed from a point in a horizontal plane through the foot of the tower is 30°. When the observer moves towards the tower a distance of 100 m, he finds the angle of elevation of the top to be 60°. Find the height of the tower and the distance of first position from the tower.

38. Equilateral triangles are drawn on the sides of a right triangle. Show that the area of the triangle on the hypotenuse is equal to the sum of the areas of triangles on the other two sides.

OR

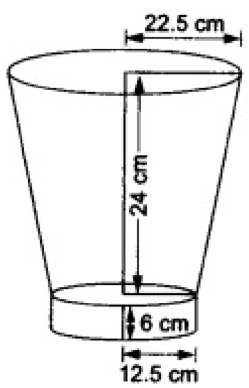
In the given figure, PA, QB, RC and SD are all perpendiculars to a line 'l', AB = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm. Find PQ, QR and RS.



39. The lengths of the sides of a triangle are in the ratio 3:4:5, and its perimeter is 144 cm. Find (i) the area of the triangle, and (ii) the height corresponding to the longest side.

OR

An open metallic bucket is in the shape of a frustum of a cone mounted on hollow cylindrical base made of metallic sheet. If the diameters of the two circular ends of the bucket are 45 cm and 25 cm, the total vertical height of the bucket is 30 cm and that of the cylindrical portion is 6 cm, find the area of the metallic sheet used to make the bucket. Also, find the volume of water it can hold. [Take $\pi = 22/7$]



40. The following distribution gives the daily income of 50 workers of a factory :

Daily income (in Rs)	200-250	250-300	300 - 350	350-400	400-450	450-500
Number of workers	10	5	11	8	6	10

Convert the distribution to a 'less than type' cumulative frequency distribution and draw its ogive. Hence obtain the median of daily income.

Solution

Section A

1. (a) –19

   Explanation:

   First, find the HCF of 210 and 55 by Euclid's Division Algorithm

   210 = 55 $\times$ 3 + 45

   55 = 45 $\times$ 1 + 10

   45 = 10 $\times$ 4 + 5

   10 = 5 $\times$ 2 + 0 (zero remainder)

   therefore, HCF (210 , 55) = 5

   Now,

   $\therefore 5 = 210 \times 5 + 55y$

   $\Rightarrow 5 - 1050 = 55y$

   $\Rightarrow -1045 = 55y$

   $\Rightarrow y = -19$

2. (c) $0 \leqslant r \ < \ 3$

   Explanation:

   Since a is a positive integer, therefore, r = 0, 1, 2 only.
   So, that a = 3q, 3q + 1, 3q + 2.

3. (b) 3 decimal places

   Explanation:

   $\frac{21}{24}$ = $\frac{7}{8}$ = $\frac{7}{2^3}$ Here, in the denominator of the given fraction the highest power of prime factor 2 is 3, therefore, the decimal expansion of the rational number $\frac{7}{2^3}$ will terminate after 3 decimal places.

4. (c) 5 cm

   Explanation:

   Since Tangents from an external point to a circle are equal.

   ∴ PE = EC = 3 cm and AB = AE = 8 cm

   Therefore, AE = AC - EC = 8 - 3 = 5 cm

5. (d) 46

   Explanation:

   Median = $l+\frac{\frac{n}{2}-cf}{f}\times h$

   = $40+\frac{\frac{230}{2}-76}{65}\times 10$

   = $40+\frac{115-76}{65}\times 10$

   = $40+\frac{39}{65}\times 10$

   =40 + $\frac{390}{65}$

   = 40 + 6

   = 46

6. (c) $\frac{21}{26}$

   Explanation:

   We have,

   Number of vowels = 5 (a e i o u)

   Number of consonants = 21 (26 - 5 = 21)

Number of possible outcomes = 21

Number of total outcomes = 26

$\therefore$ Required Probability = $\frac{21}{26}$

7. (b) 3

   Explanation:

   The degree of the polynomial $5x^3 - 3x^2 - x + \sqrt{2}$ is 3. The degree of a polynomial is the highest power of that polynomial.

8. (b) – 4 and – 5

   Explanation:

   ($x^2 + 9x + 20$) =0 Splitting the middle term, we get
   $x^2 + 5x + 4x + 20$=0
   = $x\left(x+5\right)+4\left(x+5\right)$=0
   = $\left(x+5\right)\left(x+4\right)$=0
   $\therefore x + 5 = 0$ and $x + 4 = 0$
   $\Rightarrow x = -5$ and $x = -4$

9. (c) right angled isosceles

   Explanation:

   Let A $\left(-3, 0\right)$, B$\left(1, -3\right)$ and C (4, 1) are the vertices of a triangle ABC.
   $\therefore$ AB = $\sqrt{\left(1+3\right)^2 + \left(-3-0\right)^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ = 5 units
   BC = $\sqrt{\left(4-1\right)^2 + \left(1+3\right)^2}$ = $\sqrt{9+16}$ = $\sqrt{25}$ = 5 units
   CA = $\sqrt{\left(-3-4\right)^2 + \left(0-1\right)^2}$ = $\sqrt{49+1}$ = $\sqrt{50}$ = $5\sqrt{2}$ units
   Now, check if $\mathrm{AC}^2 = \mathrm{AB}^2 + \mathrm{BC}^2$
   $\Rightarrow \left(5\sqrt{2}\right)^2 = (5)^2 + (5)^2$
   $\Rightarrow 50 = 50$
   Therefore, $\Delta\mathrm{ABC}$ is a right-angled triangle.and also AB = BC = 5 units

   Therefore triangle ABC is a right-angled isosceles triangle

10. (c) x – axis

    Explanation:

Since the ordinates of given points are 0. Therefore, points lie on $x-$axis.

11. 5 units

12. unique

    OR

    infinite

13. -1

14. $\frac{3}{2}$

15. 1.8cm

16. We know that, $\cos 30^\circ = \frac{\sqrt{3}}{2}$ and $\cos 45^\circ = \frac{1}{\sqrt{2}}$, $\sin 30^\circ = \frac{1}{2}$ and $\sin 45^\circ = \frac{1}{\sqrt{2}}$

    Now we have,
    $\cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$
    $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

    OR

    Given ,
    $\frac{\sin 41^\circ}{\cos 49^\circ}$
    $= \frac{\sin(90^\circ - 49^\circ)}{\cos 49^\circ}$
    $= \frac{\cos 49^\circ}{\cos 49^\circ}$ [Since, Sin(90° - A) = Cos A] .
    = 1.

17. Area of circle = π m^2
    New diameter = 2 m + $\frac{100}{100} \times 2\text{m}$= 4 m
    New radius = 2 m

    New area = 4π m^2

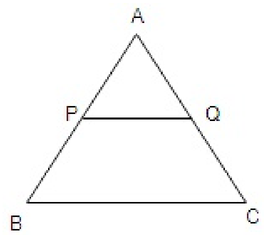
    Increase in area = 4π - π = 3π m^2
    % increase in area = $\frac{3\pi}{\pi} \times 100$= 300%

18. At least one head is obtained if any one of the following elementary events happens:

HH, HT, TH

Favourable number of elementary events = 3

Hence, required probability$= \frac{3}{4}$

19. 

$\frac{AP}{AB} = \frac{3}{9} = \frac{1}{3}$ ....(i)

$\frac{AQ}{AC} = \frac{5}{15} = \frac{1}{3}$ ....(ii)

from eqn (i) and (2) we have

$\frac{AP}{AB} = \frac{AQ}{AC}$

Hence, PQ | | BC [Converse of basic proportionality theorem.]

So $\triangle ABC \sim \triangle APQ$

$\frac{AP}{AB} = \frac{AQ}{BC} = \frac{PQ}{BC}$

$\frac{3}{9} = \frac{5}{15} = \frac{PQ}{BC}$

$\frac{PQ}{BC} = \frac{1}{3}$

BC = 3 PQ

Hence proved.

20. Given AP = $-5, -\frac{5}{2}, 0, \frac{5}{2}\ldots$

$\therefore a = -5$

$d = (-\frac{5}{2}) - (-5) = -\frac{5}{2} + 5 = \frac{-5+10}{2} = \frac{5}{2}$

General term of AP, $a_n = a + (n-1)d$

$\therefore$25th term of the AP , i.e. n=25

$a_{25} = -5 + (25-1) \times \frac{5}{2} = 55$

Section B

21. Out of 600 electric bulbs one bulb can be chosen in 600 ways.

Total number of elementary events = 600

There are 588 (= 600 - 12) non-defective bulbs out of which one bulb can be chosen in 588 ways.

Favourable number of elementary events = 588

Hence, P (Getting a non-defective bulb) $= \frac{588}{600} = \frac{49}{50} = 0.98$

22. According to the question,we are given that,

    Total cards = 48

    {13,14,15...60}

    i. Favourable outcomes of a card divisible by 5 = 10

       {15, 20, 25, 30, 35, 40, 45, 50, 55, 60}

       Therefore,Probability of card divisible by 5 = $\frac{10}{48}$=$\frac{5}{24}$

    ii. Favourable outcomes of a card which is a perfect sqaure = 4

       {16,25,36,49}

       Therefore, Probability of a card which is a perfect square = $\frac{4}{48}$ = $\frac{1}{12}$.

23. Let O be the centre of the given circle.

    AB is the tangent drawn touching the circle at A.

    Draw AC $\perp$ AB at point A, such that point C lies on the given circle.

    $\angle$OAB = 90° (Radius of the circle is perpendicular to the tangent)

    Given $\angle$CAB = 90°

    $\therefore$ $\angle$OAB = $\angle$CAB

    This is possible only when centre O lies on the line AC.

    Hence, perpendicular at the point of contact to the tangent to a circle passes through the centre of the circle.

OR

Perimeter of $\triangle$APQ = AP + AQ + PQ

= AP + AQ + PR + RQ

= AP + AQ + PB + CQ

= (AP + PB) + (AQ + QC)

= AB + AC

= 2AB = 2 $\times$ 25 = 50 cm

24. According to the question, $\sin\theta = \frac{a}{b}$
Perpendicular = a,
hypotenuse = b
So, base $= \sqrt{b^2 - a^2}$
$\sec\theta = \frac{b}{\sqrt{b^2-a^2}}$
$\tan\theta = \frac{a}{\sqrt{b^2-a^2}}$
Now, $\sec\theta + \tan\theta = \frac{b}{\sqrt{b^2-a^2}} + \frac{a}{\sqrt{b^2-a^2}}$
$= \frac{a+b}{\sqrt{b^2-a^2}}$
$= \frac{a+b}{\sqrt{(b+a)(b-a)}}$
$\sec\theta + \tan\theta = \sqrt{\frac{a+b}{b-a}}$

OR

We have,
LHS = $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$
$\Rightarrow$ LHS = $(\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta$
$\Rightarrow$ LHS = $(\sin^2\theta+\cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta) + 3\sin^2\theta\cos^2\theta$ [$\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)$]
$\Rightarrow$ LHS = $1 - 3\sin^2\theta\cos^2\theta + 3\sin^2\theta\cos^2\theta = 1$ = RHS

25. Area of square PQRS = $14 \times 14 = 196$ cm^2
The area of region A = $\frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77$ cm^2
The area of region B = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2}$ cm^2
The area of region C = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2}$ cm^2
Area of shaded part = area of square - area of region A - area of region B - area of region C.
$= \left(196 - 77 - \frac{77}{2} - \frac{77}{2}\right)$
= 42 cm^2

26. (a) Let the number of rides that Kavya had be x and the number of times she played hoopla be y.

Now, the situation can be represented by the two equations as:

3x + 4y = 20------(i)

y = $\frac{1}{2}$ x ------(ii)

(b) By substitution method, put the value of y in eq. (i), we get

3x + 4($\frac{1}{2}$ x) = 20

3x + 2x = 20

5x = 20

x = 4

Now put the value of x in (ii) we get,

y = $\frac{1}{2} \times 4$ = 2

i.e. Number of rides Kavya takes is 4 and the number of times she played hoopla is 2.

Section C

27. Let $p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$

Here $\sqrt{2}$ and $-\sqrt{2}$ are zeroes of p(x)

$(x - \sqrt{2})(x + \sqrt{2}) = \left(x^2 - 2\right)$ is a factor of p(x)

$2x^2 - 3x + 1$

$x^2 - 2 \overline{\big) \, 2x^4 - 3x^3 - 3x^2 + 6x - 2}$

$2x^4 \quad \mp 4x^2$

$-3x^3 + x^2 + 6x - 2$

$\mp 3x^3 \quad \pm 6x$

$x^2 - 2$

$x^2 \mp 2$

0

$q(x) = 2x^2 - 3x + 1$

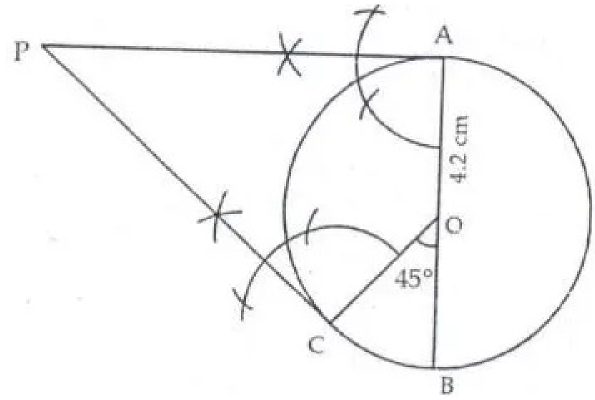
$= 2x^2 - 2x - x + 1$

$= 2x(x - 1) - 1(x - 1)$

$= (2x - 1)(x - 1)$

$\therefore$ other two zero's are

x = 1 and $x = \frac{1}{2}$

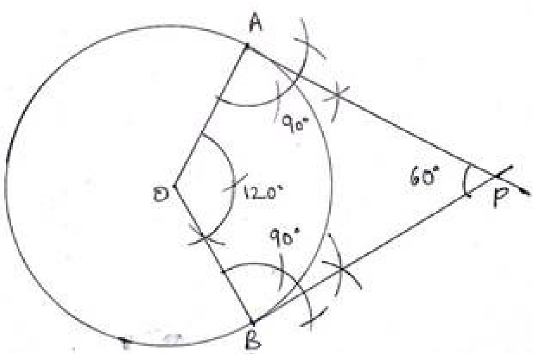
28. 

Steps of construction:

1. Draw a circle with centre O and radius 4.2 cm
2. Draw diameter AB
3. With OB as a base, draw BOC = 45°
4. At A, draw a line perpendicular to OA.
5. At C, draw a line perpendicular to OC.

These lines intersect each other at P.
Thus, PA and PC are the required tangents.

OR



Steps of construction:

i. Draw a circle of radius 3 cm with center O.
ii. Take a point A on the circumference of the circle and join OA.
iii. Draw a perpendicular to OA at point A.
iv. Draw a radius OB, making an angle of 120° (180° -60°) i.e. $\angle AOB = 120^\circ$
v. Draw a perpendicular to OB at point B.
vi. Let the two perpendiculars intersect each other at P. Then, PA and PC are required

tangents.

Justification:
Since OA is the radius, so PA has to be a tangent to the circle. Similarly, PC is also tangent to the circle.

$\angle APC = 360^\circ - (\angle OAP + \angle OCP + \angle AOC)$

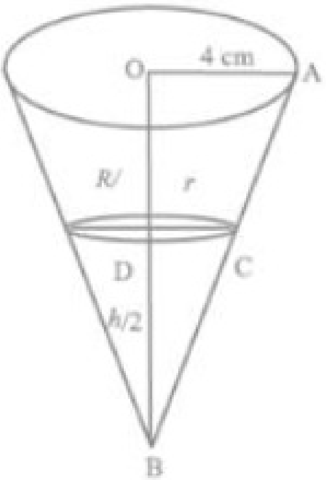
= 360° - (90° + 90° + 120°)

= 360° - 300°

= 60°

Hence, tangents PA and PC are inclined to each other at an angle of 60°.

29. Let h be the height of the given cone.
On dividing the cone through the midpoint of its axis and parallel to its base into two parts, we get the following figure:



In Δ's OAB and DCB,we have,

$$\angle BOA = \angle BDC = 90^\circ$$
$$\angle OBA = \angle DBC \ (same\ angle)$$

Therefore,by AA criteria of similarity of two triangles,we have,

$$\triangle OAB \sim \ \triangle DCB$$

Since sides of two similar triangles are proportional,therefore,

$$\frac{OA}{CD} = \frac{OB}{BD}$$
$$\Rightarrow \frac{4}{r} = \frac{h}{h/2}$$

$\Rightarrow$ r = 2

Now, $\frac{\text{Volume of the smaller cone}}{\text{Volume of the frustum of the cone}}$

$= \frac{\frac{1}{3}\pi(2)^2 \times \left(\frac{h}{2}\right)}{\frac{1}{3}\pi\times\left(\frac{h}{2}\right)\left[4^2+2^2+4\times2\right]}$

= $\frac{4}{28}$

= $\frac{1}{7}$

Therefore, the ratio of volume of the similar cone to the volume of the frustum of the cone is 1:7.

30. We know that,

cos50° = cos (90°-40)° = sin 40°

cosec^2 59° = cosec^2 (90° - 31°) = sec^2 31°

and tan 78° = tan (90° -12°) = cot 12°

Now,

$= \frac{\cos 50^\circ}{2\sin 40^\circ} + \frac{4\left(\text{cosec}^2 59^\circ - \tan^2 31^\circ\right)}{3\tan^2 45^\circ} - \frac{2}{3} tan12^o tan78^o sin90^o$

= $\frac{\sin 40^\circ}{2\sin 40^\circ} + \frac{4\left(\sec^2 31^\circ - \tan^2 31^\circ\right)}{3\times(1)^2} - \frac{2}{3}\tan 12^\circ \cot 12^\circ \times 1$

$= \frac{1}{2} + \frac{4}{3} - \frac{2}{3} = \frac{7}{6}$

OR

Given, $\frac{\sec 39^\circ}{\text{cosec } 51^\circ} + \frac{2}{\sqrt{3}}$

$tan17^\circ tan38^\circ tan60^\circ tan52^\circ tan73^\circ - 3(sin^2 31^\circ + sin^2 59^\circ)$

= $\frac{\sec 39^\circ}{\text{cosec}(90^\circ - 39^\circ)} + \frac{2}{\sqrt{3}}$ tan17°tan 38° tan 60°tan (90°-38°) tan (90°-17°) -3(sin^{2}31° + sin^2(90° - 31°))

$= \frac{\sec 39^\circ}{\sec 39^\circ} + \frac{2}{\sqrt{3}}$ tan 17° tan 38° $\times$ $\sqrt{3}$ $\times$ cot 38° $\times$ cot 17° -3(sin^{2}31° + cos^{2}31°)

= 1 + $\frac{2}{\sqrt{3}}$ $\times$ $\sqrt{3}$ $\times$ 1 $\times$ 1 - 3 [$\because tan\theta . cot\theta = 1$]

$= 1 + 2 - 3$

$= 0$

31. Since, the three persons start walking together.

$\therefore$ The minimum distance each should walk so that all can cover the same distance in complete steps will be equal to LCM of 80,85 and 90.

Prime factors of 80,85 and are as following

$$80 = 16 \times 5 = 2^4 \times 5$$

$$85 = 5 \times 17$$

$$90 = 2 \times 9 \times 5 = 2 \times 3^2 \times 5$$

So LCM of 80,85 and 90 $= 2^4 \times 3^2 \times 5 \times 17$

=16×9×5×17=12240

∴ Each person should walk the minimum distance

=12240 cm =122 meter 40 cm

Value of morning walk :

"An early morning walk is a blessing for the whole day."

OR

Given: Size of bathroom = 10 ft by 8 ft.

= (10 $\times$ 12) inch by (8 × 12) inch

= 120 inch by 96 inch

Area of bathroom = 120 inch by 96 inch

To find the largest size of tile required , we find HCF of 120 and 96.

By applying Euclid's division lemma

120 = 96 $\times$ 1 + 24

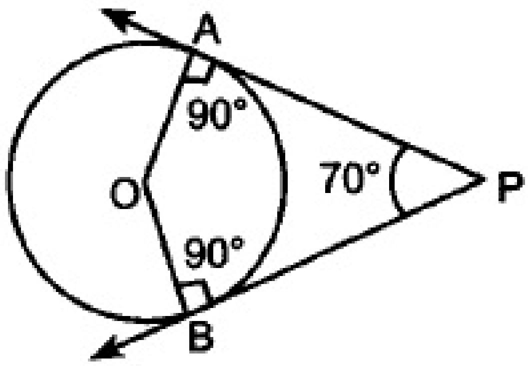
96 = 24 $\times$ 4 + 0

Therefore, HCF = 24

Therefore, Largest size of tile required = 24 inches

no.of tiles required $= \frac{\text{area of bathroom}}{\text{area of 2 tile}} = \frac{120 \times 96}{24 \times 24} = 5 \times 4$ = 20 tiles

Hence number of tiles required is 20 and size of tiles is 24 inches.

32. 

PA and PB are tangents to the circle.

$\angle A = \angle B = 90^\circ$

In quadrilateral OAPB

$\angle AOB + \angle A + \angle P + \angle B = 360^\circ$ [Angle sum property of a quadrilateral],

$\Rightarrow \angle AOB + 90^\circ + 70^\circ + 90^\circ = 360^\circ$

$\Rightarrow \angle AOB = 360^\circ - 250^\circ = 110^\circ$

33. i. Distance between house and bank $= \sqrt{(5-2)^2 + (8-4)^2}$
$= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

ii. Distance between bank and daughter's school $= \sqrt{(13-5)^2 + (14-8)^2}$
$= \sqrt{(8)^2 + (6)^2} = \sqrt{64+36} = \sqrt{100} = 10$

iii. Distance between daughter's school and office $= \sqrt{(13-13)^2 + (26-14)^2}$
$= \sqrt{0 + (12)^2} = 12$

Total distance (House + Bank + School + Office) travelled = 5 + 10 + 12 = 27 km

Distance between house to office

$= \sqrt{(13-2)^2 + (26-4)^2} = \sqrt{(11)^2 + (22)^2}$

$= \sqrt{121 + 484} = \sqrt{605}$ = 24.59 = 24.6 km

So, Extra distance travelled by Ayush in reaching his office = 27 - 24.6 = 2.4 km

34. We can write the given system of equations as

$2x - 3y = 7$

$(k+2)x - (2k+1)y = 3(2k-1)$

The given system of equations are of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Where, $a_1 = 2,\ b_1 = -3$, $c_1 = -7$

a_2 = k+2, b_2 = -(2k + 1), c_2 = -3(2k - 1)

Since we know that for unique solution, we have

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\frac{2}{k+2} = \frac{-3}{(-2k+1)} = \frac{-7}{-3(2k-1)}$

$\frac{2}{k+2} = \frac{-3}{-(2k+1)}$

and $\frac{-3}{-(2k+1)} = \frac{-7}{-3(2k-1)}$

$\Rightarrow$ 2(2k + 1) = 3(k + 2) and 3 $\times$ 3(2k - 1) = 7(2k + 1)

$\Rightarrow$ 4k + 2 = 3k + 6 and 18k - 9 = 14k + 7

$\Rightarrow$ k = 4 and 4k = 16 $\Rightarrow$ k = 4

Thus, if k = 4, then the given system of equations will have infinitely many solutions

Section D

35. Let digit at unit's place be x and digit at ten's place be y

$\therefore$ Number = 10y + x....(i)

According to given condition,

$10y + x = 6(x + y)$

$\Rightarrow 10y + x = 6x + 6y$

$\Rightarrow 4y = 5x$

$\Rightarrow$y = $\frac{5}{4}$ x ...(ii)

Also $10y + x = 3xy - 6$

$\Rightarrow$10$\times$ $\frac{5}{4}$ x + x = 3x $\times$ $\frac{5}{4}$ x - 6 (using(ii))

$\Rightarrow$ $\frac{25}{2}$ x + x = $\frac{15}{4}$ x^2 - 6

$\Rightarrow$ $\frac{27x}{2}$ = $\frac{15x^2-24}{4}$

$\Rightarrow 15x^2 - 54x - 24 = 0$

$\Rightarrow 5x^2 - 18x - 8 = 0$

$\Rightarrow 5x^2 - 20x + 2x - 8 = 0$

$\Rightarrow 5x(x - 4) + 2(x - 4) = 0$

$\Rightarrow$(x - 4)(5x + 2) = 0

$\Rightarrow$x = 4 or x = $\frac{-2}{5}$

Rejecting x = $\frac{-2}{5}$, we have x =4 when

x = 4, y = $\frac{5}{4}$ $\times$ 4 = 5 [Using (i)]

$\therefore$Number = 10$\times$5 + 4 = 54.

36. The sequence formed by the given numbers is 103,107,111,115, ...,999.

This is an AP in which a = 103 and d = (107 - 103) = 4.

Let the total number of these terms be n. Then,

T_n = 999 $\Rightarrow$ a + (n-1)d = 999

$\Rightarrow$ 103 + (n -1) $\times$ 4 = 999

$\Rightarrow$ (n-1)$\times$4 = 896 $\Rightarrow$ (n-1) = 224 $\Rightarrow$ n = 225.

$\therefore$ middle term = ($\frac{n+1}{2}$)th term = ($\frac{225+1}{2}$)th term = 113th term.

T_{113} = (a + 112d) = (103 +112 $\times$ 4) = 551.

$\therefore$ T_{112} = (551 - 4) = 547.

So, we have to find S_{112} and (S_{225} - S_{113}).

Using the formula S_m = $\frac{m}{2}$ (a + l) for each sum, we get

s_{112} = $\frac{112}{2}$ (103+ 547) = (112$\times$325) = 36400

(S_{225} - S_{113}) = $\frac{225}{2}$ (103 + 999) - $\frac{113}{2}$ (103 + 551)

= (225 $\times$ 551)-(113 $\times$ 327)

= 123975 - 36951 =87024.

Sum of all numbers on LHS of the middle term is 36400.

Sum of all numbers on RHS of the middle term is 87024.

OR

Given, a_n = 3 + 2n

a_1 = 3 + 2$\times$1 = 5

a_2 = 3 + 2$\times$2 = 7

a_3 = 3 + 2$\times$3 = 9

Thus the series is 5, 7, 9,......in which a = 5 and d = 7 - 5 = 2

$\therefore \quad S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{24} = \frac{24}{2}(2 \times 5 + (24-1) \times 2)$

$S_{24} = \frac{24}{2}(2 \times 5 + 23 \times 2)$

$S_{24} = 12(10 + 46)$

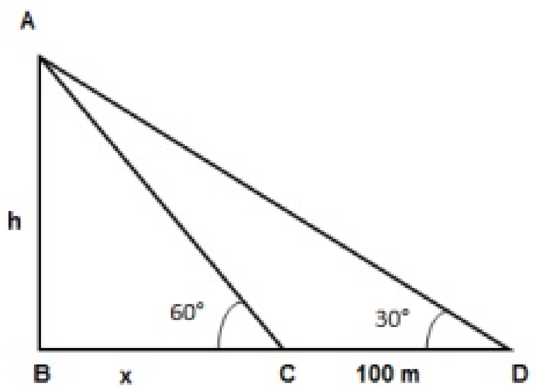
$S_{24} = 12(56)$

Hence, S_{24} = 672

37. As shown in figure AB is the tower.

Initial position of observer is at D and 2nd position is at C

Given that CD = 100 m.



Let h is the height of tower and BC = x

Now in ΔABC,

tan 60° = $\frac{\mathrm{AB}}{\mathrm{BC}}=\frac{\mathrm{h}}{\mathrm{x}}$

$\sqrt{3}=\frac{h}{x}$

$x=\frac{\mathbf{h}}{\sqrt{3}}$ .... (i)

Now in △ABD,

tan 30° = $\frac{\mathrm{AB}}{\mathrm{BD}}=\frac{\mathrm{h}}{\mathrm{x+100}}$

$\frac{1}{\sqrt{3}}=\frac{h}{x+100}$

x + 100 = $h\sqrt{3}$

$\frac{\mathrm{h}}{\sqrt{3}}+100=\mathrm{h}\sqrt{3}$

h + $100\sqrt{3}$ = 3h

2h = $100\sqrt{3}$

h = $50\sqrt{3}$ = 50 $\times$ 1.732 = 86.6 m

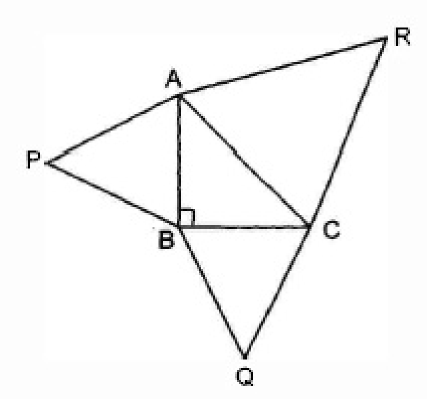
So, height of the tower is 86.6 m.

The distance of first position from the tower = BD = x + 100 = 50 + 100 = 150 m

38. **Given** A right-angled triangle ABC with right angle at B. Equilateral triangles PAB, QBC and RAC are described on sides AB, BC and CA respectively.

TO PROVE : The area of the triangle on the hypotenuse is equal to the sum of the areas of triangles on the other two sides i.e

Area (ΔPAB)+ Area (ΔQBC) = Area (ΔRAC)



PROOF Since triangles PAB, QBC and RAC are equilateral. Therefore, they are equiangular and hence similar.

$$\therefore \quad \frac{\text{Area }(\Delta PAB)}{\text{Area }(\Delta RAC)} + \frac{\text{Area }(\Delta QBC)}{\text{Area }(\Delta RAC)} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$\Rightarrow \quad \frac{\text{Area }(\Delta PAB)}{\text{Area }(\Delta RAC)} + \frac{\text{Area }(\Delta QBC)}{\text{Area }(\Delta RAC)} = \frac{AB^2+BC^2}{AC^2}$$

$$\Rightarrow \quad \frac{\text{Area}(\Delta PAB)}{\text{Area}(\Delta RAC)} + \frac{\text{Area}(\Delta QBC)}{\text{Area}(\Delta RAC)} = \frac{AC^2}{AC^2} = 1$$

$$\left[\begin{array}{l}\because \Delta ABC \text{ is a right angled triangle} \\ \text{with } \angle B = 90^\circ \therefore AC^2 = AB^2 + BC^2\end{array}\right]$$

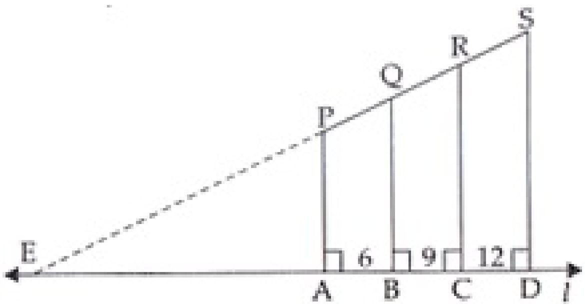
$$\Rightarrow \quad \frac{\text{Area }(\Delta PAB)+\text{Area}(\Delta QBC)}{\text{Area }(\Delta RAC)} = 1$$

$\Rightarrow$ Area (ΔPAB) + Area (ΔQBC) = Area (ΔRAC)

OR

Given: PA, QB, RC and SD are perpendicular on line l.

AB = 6 cm, BC = 9 cm, CD = 12 cm, SP=36 cm



To find: PQ, QR and RS.

Construction: we produce SP so that it joins l at E.

Proof: In ΔEDS,

AP || BQ || CR || SD [Given]

∴ PQ : QR : RS = AB : BC : CD

PQ : QR : RS = 6 : 9 : 12

Let PQ = 6x

then QR = 9x

and RS = 12x

Now, PQ + QR + RS = 36 cm (given)

⇒ 6x + 9x + 12x = 36

⇒ 27x = 36

$\Rightarrow x = \frac{36}{27} = \frac{4}{3}$

Therefore, PQ = 6 × $\frac{4}{3}$ = 8 cm

QR = 9 × $\frac{4}{3}$ = 12 cm

RS = 12 × $\frac{4}{3}$ = 16 cm

39. It is given that lengths of the sides of the triangle are in the ratio of 3 : 4 : 5 and also perimeter of a given triangle is 144 cm therefore on dividing 144 cm in the ratio 3:4:5, we get
$a = \left(144 \times \frac{3}{12}\right) \text{cm} = 36\text{cm}, b = \left(144 \times \frac{4}{12}\right) \text{cm} = 48\text{cm}$
and $c = \left(144 \times \frac{5}{12}\right) \text{cm} = 60\text{cm}$
∴ s = $\frac{1}{2}$ (36 + 48 + 60) cm = 72 cm.
(s - a) = (72 - 36) cm = 36 cm,
(s - b) = (72- 48) cm = 24 cm

and (s - c) = (72 - 60) cm = 12 cm.

i. Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$
   = $\sqrt{72 \times 36 \times 24 \times 12}\text{cm}^2$ = 72 × 12 cm^2 = 864 cm^2.
ii. Let base = 60 cm and the corresponding height = h cm.

Then, area of the triangle = $= \left(\frac{1}{2} \times 60 \times h\right) \mathrm{cm}^2 = (30h)\mathrm{cm}^2$.

$\therefore \quad 30h = 864 \Rightarrow h = \frac{864}{30} = 28.8.$

Longest side = 60 cm, corresponding height = 28.8 cm.

OR

Here $R = \frac{45}{2}\mathrm{cm}, r = \frac{25}{2}\mathrm{cm} = 12.5\mathrm{cm}$

Height of the frustum of the cone, h = (30 - 6) cm = 24 cm.

$\therefore$ h = 24 cm.

Slant height of the frustum of the cone,

$l = \sqrt{h^2 + (R - r)^2}$ units $= \sqrt{(24)^2 + (22.5 - 12.5)^2}\mathrm{cm}$

$= \sqrt{(24)^2 + (10)^2}\mathrm{cm} = \sqrt{576 + 100}\mathrm{cm}$

$= \sqrt{676}\mathrm{cm} = 26\mathrm{cm}$

Area of metallic sheet used

= (curved surface area of the frustum of the cone) + (area of the base) + (curved surface area of the cylinder)

= $\pi l(R + r) + \pi r^2 + 2\pi rH$,where H = 6 cm

= π.(l{R + r) + r^2 + 2rh}sq. units

$= \frac{22}{7} \cdot \left(26(22.5 + 12.5) + (12.5)^2 + 2 \times 12.5 \times 6\right) \mathrm{cm}^2$

$= \frac{22}{7} \cdot \{(26 \times 35) + (12.5 \times 12.5) + 150\}\mathrm{cm}^2$

$= \frac{22}{7} \cdot (910 + 156.25 + 150)\mathrm{cm}^2 = \left(\frac{22}{7} \times 1216.25\right) \mathrm{cm}^2$

= (22 $\times$ 173.75)cm^2 = 3822.5 cm^2

Volume of water which the bucket can hold

$= \frac{1}{3}\pi h \left[R^2 + r^2 + Rr\right] \mathrm{cm}^3$

$= \frac{1}{3} \times \frac{22}{7} \times 24 \times \left[\left(\frac{45}{2}\right)^2 + \left(\frac{25}{2}\right)^2 + \left(\frac{45}{2} \times \frac{25}{2}\right)\right] \mathrm{cm}^3$

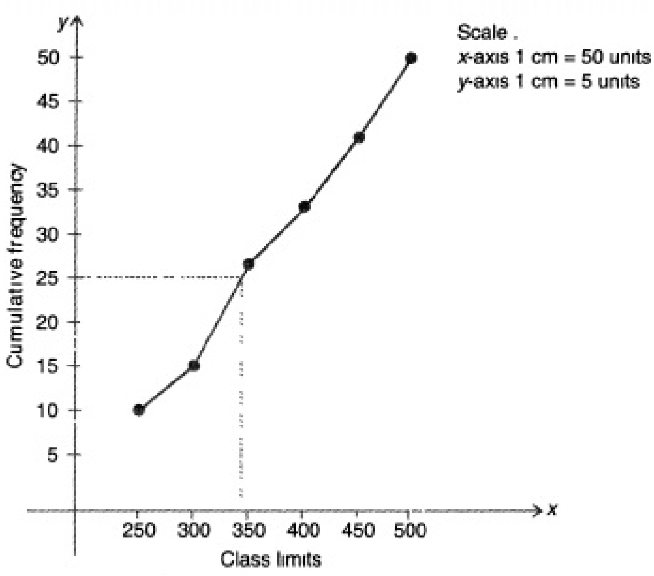
$= \frac{176}{7} \times \left(\frac{2025}{4} + \frac{625}{4} + \frac{1125}{4}\right) \mathrm{cm}^3 = \left(\frac{176}{7} \times \frac{3775}{4}\right) \mathrm{cm}^3$

$= \frac{166100}{7}\mathrm{cm}^3 = 23728.57\mathrm{cm}^3 = 23.73 \ \ litres.$

40.

Daily income (Classes)	No. of workers (c.f)
Less than 250	10
Less than 300	15

Less than 350	26
Less than 400	34
Less than 450	40
Less than 500	50



From graph, $\frac{N}{2} = \frac{50}{2}$ = 25

Hence, Median of daily income = Rs 345