Mathematics
Class XII
Sample Paper – 7

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. If A =[a_{ij}], such that $a_{ij} = \frac{i+2j}{2}$, find the value of element at 3rd column and 2nd row.

2. Find $\frac{dy}{dx}$, if y + sin y = cos x

3. Determine the order and degree of the following differential equation:

$$\left(\frac{dy}{dx}\right)^2 + \frac{1}{\frac{dy}{dx}} = 2$$

4. Find the vector equation of line through point (5, 2, -4) and which is parallel to the vector $3\hat{i} + 2\hat{j} - 8\hat{k}$.

OR

Find the vector equation of a line passing through the points (-1, 0, 2) and (3, 4, 6).

SECTION – B

5. Prove that the relation R in the set A = {1, 2, 3, 4, 5} given by R = {(a, b): |a – b| is even}, is an equivalence relation.

6. Find matrix A if

$$A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}.$$

7. $\int \sin x \sin 2x \sin 3x \, dx$

8. Evaluate: $\int \frac{2}{1-x \quad 1+x^2}\, dx$

OR

Evaluate:

$$\int \frac{\sin\ x-a}{\sin\ x+a} dx$$

9. Form a differential equation corresponding to $y^2 = a(b - x)(b + x)$, by eliminating parameters a and b

10. If $|\vec{a}|=5, |\vec{b}|=13, \vec{a}\cdot\vec{b}=60$, find $|\vec{a}\times\vec{b}|$

OR

Find the value of λ which makes the vectors $\vec{a}, \vec{b}$, and $\vec{c}$ coplanar, where $\vec{a}=2\hat{i}-\hat{j}+\hat{k}, \vec{b}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{c}=3\hat{i}-\lambda\hat{j}+5\hat{k}$.

11. Find the probability distribution of the number of Kings drawn when two cards are drawn one by one, without replacement, from a pack of 52 playing cards

12. Two bags I and II contain 4 red, 3 black balls and 2 red and 4 black balls respectively. One bag is selected at random and from the bag selected, a ball is drawn. Find the probability that the ball is red.

OR

A company has two factories to manufacture machinery. Factory I manufactures 70% of the machinery and factory II manufactures 30% of the machinery. At factory I, 80% of the machinery are rated to be of a standard quality and at factory II 90% of the machinery are rated to be of a standard quality. A machine is chosen at random and is found to be of a standard quality. What is the probability that it came from factory II?

SECTION – C

13. Let A = R × R and * be a binary operation on A defined by
(a, b) * (c, d) = (a + c, b + d) Show that * is commutative and associative.
Find the identity element for * on A. Also find the inverse of every element (a, b) $\in$ A.

OR

Consider f : R - $\left\{-\frac{4}{3}\right\} \rightarrow$ R - $\left\{\frac{4}{3}\right\}$ given by f(x) = $\frac{4x+3}{3x+4}$ Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x)=2$

14. Prove that $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$

15. If a, b, and c are in A.P., find the value of the determinant

$$\Delta = \begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$$

16. If $y=\tan^{-1}\left\{\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right\}, -1<x<1, x \neq 0$, find $\frac{dy}{dx}$.

OR

If $\log\left(x^2+y^2\right)=2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx}=\frac{x+y}{x-y}$

17. If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right)=\tan^{-1}a$, prove that $\frac{dy}{dx}=\frac{y}{x}$.

18. A balloon, which always remains spherical on inflation, is being by inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.

19. Evaluate:

$$\int \frac{x+2}{\sqrt{x^2+5x+6}}dx$$

20. Prove that, $\int_0^{\pi} \frac{x}{1+\sin x}\,dx = \pi$

21. Solve the differential equation:

$$\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$$

OR

Solve the differential equation:

$$\frac{dy}{dx}=(4x+y+1)^2$$

22. The scalar product of the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum of vectors $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b}+\vec{c}$.

23. Find the shortest distance between the lines given by $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$

SECTION – D

24. If $A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$, then prove that $A^n=\begin{bmatrix}3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\\3^{n-1}&3^{n-1}&3^{n-1}\end{bmatrix}$ for every positive integer n.

OR

Let $A=\begin{bmatrix}0&-\tan\left(\frac{x}{2}\right)\\\tan\left(\frac{x}{2}\right)&0\end{bmatrix}$ and I be the identity matrix of order 2.

Show that: $I + A = (I - A)\begin{bmatrix}\cos x&-\sin x\\\sin x&\cos x\end{bmatrix}$

25. A square piece of tin with side 18 cm is to be made into a box without a top by cutting a square piece from each corner and folding up the flaps. What should be the side of the square to be cut off, so that the volume of the box is the largest? Also find the maximum volume of the box.

26. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bonded by $x = 0$, $x = 4$, $y = 4$, and $y = 0$ into three equal parts.

OR

Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the straight line $\frac{x}{3}+\frac{y}{2}=1$.

27. Find the angle between the following pair of lines:

$$\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}$$

And check whether the lines are parallel or perpendicular.

OR

Write the vector equations of the following lines and hence determine the distance between them:

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6};\frac{x-3}{4}=\frac{y-3}{6}=\frac{z+5}{12}$$

28. A brick manufacturer has two depots, A and B, with stocks of 30,000 and 20,000 bricks respectively. He receives orders from three builders P, Q and R for 15,000, 20,000 and 15,000 bricks respectively. The cost in rupees for transporting 1000 bricks to the builders from the depots are given below:

From \ To	P	Q	R
A	40	20	30
B	20	60	40

How should the manufacturer fulfil the orders so as to keep the cost of transportation minimum?

29. A factory manufactures screws. Machines X, Y and Z manufacture respectively 1000, 2000, and 3000 screws, of which 1%, 1.5% and 2 % of their outputs are respectively defective. A screw is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by machine X?