Interference of Light (Part - 1)

Q.64. Demonstrate that when two harmonic oscillations are added, the time-averaged energy of the resultant oscillation is equal to the sum of the energies of the constituent oscillations, if both of them
(a) have the same direction and are incoherent, and all the values of the phase difference between the oscillations are equally probable;
(b) are mutually perpendicular, have the same frequency and an arbitrary phase difference.

Ans. (a) In this case the net vibration is given by

$$x = a_1 \cos \omega t + a_2 \cos(\omega t + \delta)$$

where δ is the phase difference between the two vibrations which varies rapidly and randomly in the interval $(0, 2\pi)$. (This is what is meant by incoherence.)

Then $$x = (a_1 + a_2 \cos \delta) \cos \omega t + a_2 \sin \delta \sin \omega t$$

The total energy will be taken to be proportional to the time average of the square of the displacement

Thus $$E = \langle (a_1 + a_2 \cos \delta)^2 + a_2^2 \sin^2 \delta \rangle = a_1^2 + a_2^2$$

as $\langle \cos \delta \rangle = 0$ and we have put $\langle \cos^2 \omega t \rangle = \langle \sin^2 \omega t \rangle = \frac{1}{2}$ and has been absorbed in the overall constant of proportionality.

In the same units the energies of the two oscillations are a_1^2 and a_2^2 respectively so the proposition is proved.

(b)

Here $$\vec{r} = a_1 \cos \omega t \,\hat{i} + a_2 \cos(\omega t + \delta)\,\hat{j}$$

and the mean square displacement is $\propto a_1^2 + a_2^2$

If 5 is fixed but arbitrary. Then as in (a) we see that $E = E_1 + E_2$.

Q.65. By means of plotting find the amplitude of the oscillation resulting from the addition of the following three oscillations of the same direction:

$\xi_1 = a \cos \omega t, \quad \xi_2 = 2a \sin \omega t, \quad \xi_3 = 1.5a \cos (\omega t + \pi/3).$

Ans. It is easier to do it analytically.

$$\xi_1 = a \cos \omega t, \; \xi_2 = 2a \sin \omega t$$

$$\xi_3 = \frac{3}{2} a \left(\cos \frac{\pi}{3} \cos \omega t - \sin \frac{\pi}{3} \sin \omega t \right)$$

Resultant vibration is

$$\xi = \frac{7a}{4} \cos \omega t + a \left(2 - \frac{3\sqrt{3}}{4} \right) \sin \omega t$$

This has an amplitude $= \frac{a}{4} \sqrt{49 + (8 - 3\sqrt{3})^2} = 1{\cdot}89\, a$

Q.66. A certain oscillation results from the addition of coherent oscillations of the same direction $\xi_k = a \cos [\omega t + (k - 1) \varphi]$, where k is the number of the oscillation $(k = 1, 2, \ldots, N)$, φ is the phase difference between the kth and (k — 1)th oscillations. Find the amplitude of the resultant oscillation.

Ans. We use the method of complex amplitudes. Then the amplitudes are

$$A_1 = a, \; A_2 = a e^{i\varphi}, \; \ldots \; A_N = a e^{i(N-1)\varphi}$$

and the resultant complex amplitude is

$$A = A_1 + A_2 + \ldots + A_N = a (1 + e^{i\varphi} + e^{2i\varphi} + \ldots + e^{i(N-1)\varphi})$$

$$= a \frac{1 - e^{iN\varphi}}{1 - e^{i\varphi}}$$

The corresponding ordinary amplitude is

$$|A| = a \left| \frac{1 - e^{iN\varphi}}{1 - e^{i\varphi}} \right| = a \left[\frac{1 - e^{iN\varphi}}{1 - e^{i\varphi}} \times \frac{1 - e^{-iN\varphi}}{1 - e^{-i\varphi}} \right]^{1/2}$$

$$= a \left[\frac{2 - 2\cos N\varphi}{2 - 2\cos \varphi} \right]^{1/2} = a \frac{\sin \frac{N\varphi}{2}}{\sin \frac{\varphi}{2}}.$$

Q.67. A system illustrated in Fig. 5.12 consists of two coherent point sources 1 and 2 located in a certain plane so that their dipole moments are oriented at right angles to that plane. The sources are separated by a distance d, the radiation wavelength is equal to λ. Taking into account that the oscillations of source 2 lag in phase behind the oscillations of source 1 by φ (φ < a), find:
(a) the angles θ at which the radiation intensity is maximum;

(b) the conditions under which the radiation intensity in the direction $\theta = \pi$ is maximum and in the opposite direction, minimum.

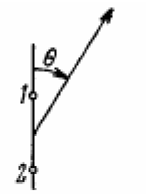
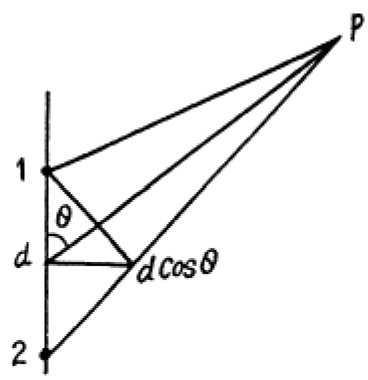


Fig. 5.12.

Ans.

(a) With dipole moment $\perp^r$ to plane there is no variation with θ of individual radiation amplitude. Then the intensity variation is due to interference only.

In the direction given by angle θ the phase difference is



$$\frac{2\pi}{\lambda}(d\cos\theta) + \varphi = 2k\pi \quad \text{for maxima}$$

Thus $\quad d\cos\theta = \left(k - \frac{\varphi}{2\pi}\right)\lambda$

$$k = 0, \pm 1, \pm 2, \ldots$$

We have added φ to $\frac{2\pi}{\lambda} d\cos\theta$ because the extra path that the wave from 2 has to travel K

in going to P (as compared to 1) makes it lag more than it already is (due to φ).

(b) Maximum for $\theta = \pi$ gives $-d = \left(k - \frac{\varphi}{2\pi}\right)\lambda$

Minimum for $\theta = 0$ gives $d = \left(k' - \frac{\varphi}{2\pi} + \frac{1}{2}\right)\lambda$

Adding we get $$\left(k + k' - \frac{\varphi}{\pi} + \frac{1}{2}\right)\lambda = 0$$

This can be true only if $k' = -k$, $\varphi = \frac{\pi}{2}$

since $0 < \varphi < \pi$

Then $$-d = \left(k - \frac{1}{4}\right)\lambda$$

Here $k = 0, -1, -2, -3, \ldots$

(Otherwise R.H.S. will become $+ve$).

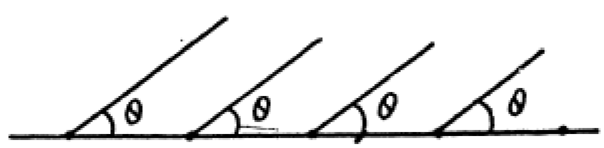
Putting $k = -\bar{k}$, $\bar{k} = 0, +1, +2, +3, \ldots$

$$d = \left(\bar{k} + \frac{1}{4}\right)\lambda.$$

Q.68. A stationary radiating system consists of a linear chain of parallel oscillators separated by a dis- tance d, with the oscillation, phase varying linearly along the chain. Find the time dependence of the phase difference Δφ between the neighbouring oscillators at which the principal radiation maximum of the system will be "scanning" the surroundings with the constant angular velocity ω.

Ans. If Δφ is the phase difference between neighbouring radiators then for a maximum in the direction θ we must have

$$\frac{2\pi}{\lambda} d \cos\theta + \Delta\varphi = 2\pi k$$



$\theta = \omega t + \beta$

Thus $$\frac{d}{\lambda}\cos(\omega t + \beta) + \frac{\Delta\varphi}{2\pi} = k$$

or $$\Delta\varphi = 2\pi\left[k - \frac{d}{\lambda}\cos(\omega t + \beta)\right]$$

To get the answer of the book, put $\beta = \alpha - \pi/2$.

Q.69. In Lloyd's mirror experiment (Fig. 5.13) a light wave emitted directly by the source S (narrow slit) interferes with the wave reflected from a mirror M. As a result, an interference fringe pattern is

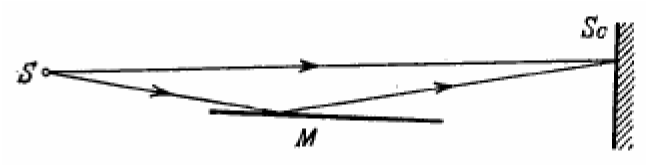


Fig. 5.13.

Formed on the screen Sc. The source and the mirror are separated by a distance l = 100 cm. At a certain position of the source the fringe width on the screen was equal to Δx= 0.25 mm, and after the source was moved away from the mirror plane by Δh = 0.60 mm, the fringe width decreased η = 1.5 times. Find the wavelength of light.

Ans. From the general formula

$$\Delta x = \frac{l\lambda}{d}$$

We find that

$$\frac{\Delta x}{\eta} = \frac{l\lambda}{d + 2\Delta h}$$

since d increases to $d + 2\Delta h$ when the source is moved away from the mirror plane by Δh.

Thus $\quad \eta d = d + 2\Delta h \quad$ or $\quad d = 2\Delta h/(\eta - 1)$

Finally $\quad \lambda = \dfrac{2\Delta h \Delta x}{(\eta - 1) l} = 0{\cdot}6\ \mu\text{m}.$

Q.70. Two coherent plane light waves propagating with a divergence angle ψ ≪ 1 fall almost normally on a screen. The amplitudes of the waves are equal. Demonstrate that the distance between the neighbouring maxima on the screen is equal to Δx = λ/ψ), where λ is the wavelength.

Ans. We can think of the two coherent plane waves as emitted from two coherent point sources very far away. Then

$$\Delta x = \frac{l\lambda}{d} = \frac{\lambda}{d/l}$$

But $\dfrac{d}{l} = \psi\ (\text{if } \psi \ll 1)$

So $\Delta x = \dfrac{\lambda}{\psi}.$

Q.71. Figure 5.14 illustrates the interference experiment with Fresnel mirrors. The angle between the mirrors is α = 12', the distances from the mirrors' intersection line to the narrow slit S and the screen Sc are equal to r = 10.0 cm and b = 130 cm respectively. The wavelength of light is λ = 0.55 p.m. Find:
(a) the width of a fringe on the screen and the number of possible maxima;
(b) the shift of the interference pattern on the screen when the slit is displaced by δl = 1.0 mm along the arc of radius r with centre at the point 0;
(c) at what maximum width δ_{max} of the slit the interference fringes on the screen are still observed sufficiently sharp.

$\psi \ll 1$

$\Delta x = \lambda/\psi,$

$\delta l = 1.0$

δ_{max}

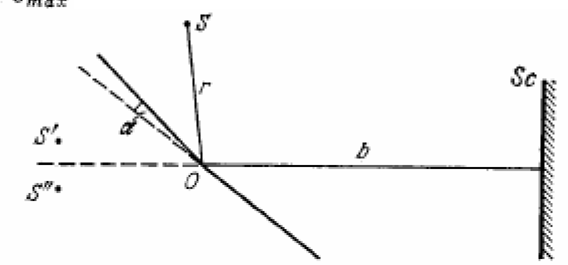


Fig. 5.14.

Ans.

(a) Here $S'S'' = d = 2r\alpha$

Then $\Delta x = \dfrac{(b+r)\lambda}{2\alpha}$

Putting $b = 1{\cdot}3$ metre, $r = {\cdot}1$ metre

$\lambda = 0{\cdot}55\ \mu\text{m}$, $\alpha = 12' = \dfrac{1}{5 \times 57}$ radian

we get $\Delta x = 1{\cdot}1$ mm

Number of possible maxima $= \dfrac{2b\alpha}{\Delta x} + 1 = 8{\cdot}3 + 1 = 9$

($2b\alpha$ is the length of the spot on the screen which gets light after reflection from both mirror. We add 1 above to take account of the fact that in a distance Δx there are two maxima).

(b) When the silt moves by δl along the arc of radius r the incident ray on the mirror rotates by $\dfrac{\delta l}{r}$; this is the also the rotation of the reflected ray. There is then a shift of the fringe of magnitude.

$$b\frac{\delta l}{r} = 13 \text{ mm.}$$

(c) If the width of the slit is δ then we can imagine the slit to consist of two narrow slits with separation δ. The fringe pattern due to the wide slit is the superposition of the pattern due to these two narrow slits. The full pattern will not be sharp at all if the pattern due to the two narrow slits are $\frac{1}{2}\Delta x$ apart because then the maxima due to one will fill the minima due to the other. Thus we demand that

$$\frac{b\,\delta_{max}}{r} = \frac{1}{2}\Delta x = \frac{(b+r)\lambda}{4r\alpha}$$

$$\delta_{max} = \left(1+\frac{r}{b}\right)\frac{\lambda}{4\alpha} = 42\ \mu\text{m}.$$

Q.72. A plane light wave falls on Fresnel mirrors with an angle α = 2.0' between them. Determine the wavelength of light if the width of the fringe on the screen Δx = 0.55 mm.

Ans. To get this case we must let $r \to \infty$ in the formula for Δx of the last e x a m p le .

So $\Delta x = \frac{(b+r)\lambda}{2\alpha r} \to \frac{\lambda}{2\alpha}$.

(A plane wave is like light emitted from a point source at ∞).
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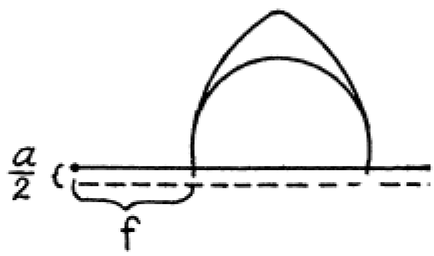
Then $\lambda = 2\alpha\Delta x = 0{\cdot}64\ \mu\text{m}.$

Q.73. A lens of diameter 5.0 cm and focal length f = 25.0 cm was cut along the diameter into two identical halves. In the process, the layer of the lens a = 1.00 mm in thickness was lost. Then the halves were put together to form a composite lens. In this focal plane a narrow slit was placed, emitting monochromatic light with wavelength λ = 0.60 µm. Behind the lens a screen was located at a distance b = 50 cm from it. Find:
(a) the width of a fringe on the screen and the number of possible maxima;
(b) the maximum width of the slit δ_{max} at which the fringes on the screen will be still observed sufficiently sharp.

Ans.



(a) We show the upper half oi the lens. The emergent light is at an angle $\frac{a}{2f}$ from the axis. Thus the divergence angle of the two incident light beams is

$$\psi = \frac{a}{f}$$

When they interfere the fringes produced have a width

$$\Delta x = \frac{\lambda}{\psi} = \frac{f\lambda}{a} = 0\cdot15 \text{ mm.}$$

The patch on the screen illuminated by both light has a width b ψ and this contains

$$\frac{b\psi}{\Delta x} = \frac{b\,a^2}{f^2\lambda} \text{ fringes} = 13 \text{ fringes}$$

(If we ignore 1 in compare is on to $\frac{b\psi}{\Delta x}$ (if 5.71 (a))

(b) We follow the logic of (5.71 c). From one edge of the slit to the other edge the

distance is of magnitude $\delta \left(\text{i.e. } \frac{a}{2} \text{ to } \frac{a}{2}+\delta\right).$

If we imagine the edge to shift by this distance, the angle $\psi/2$ will increase by $\frac{\Delta\psi}{2} = \frac{\delta}{2f}$

and the light will shift $\pm\, b\frac{\delta}{2f}$

The fringe pattern will therefore shift by $\frac{\delta \cdot b}{f}$

Equating this to $\frac{\Delta x}{2} = \frac{f\lambda}{2a}$ we get $\delta_{max} = \frac{f^2\lambda}{2ab} = 37\cdot5\ \mu\text{m}.$

Q.74. The distances from a Fresnel biprism to a narrow slit and a screen are equal to a = 25 cm and b = 100 cm respectively.

The refracting angle of the glass biprism is equal to θ = 20'. Find the wavelength of light if the width of the fringe on the screen is Δx = 0.55 mm.

Ans.

$$\Delta x = \frac{l\lambda}{d}$$

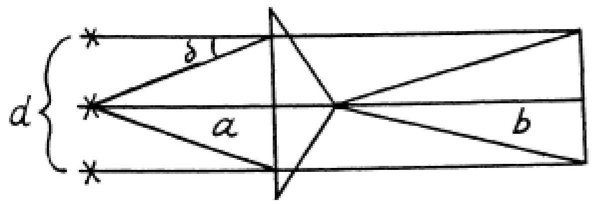
$$l = a + b$$

$$d = 2(n-1)\theta a$$

$$\delta = (n-1)\theta$$

$$d = 2\delta \cdot a$$

$n =$ R.I. of glass



Thus $$\lambda = \frac{2(n-1)\theta a \Delta x}{a+b} = 0{\cdot}64\,\mu\text{m}.$$

Q.75. A plane light wave with wavelength λ = 0.70 μm falls normally on the base of a biprism made of glass (n = 1.520) with refracting angle θ = = 5.0°. Behind the biprism (Fig. 5.15) there is a plane-parallel plate, with the space between them filled up with benzene (n' = 1.500).

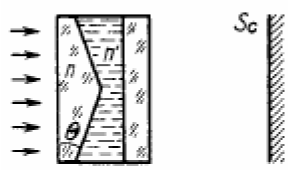
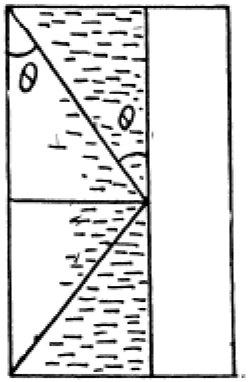


Fig. 5.15.

Find the width of a fringe on the screen Sc placed behind this system.

Ans. It will be assumed that the space between the biprism and the glass plate filled with benzene constitutes complementary prisms as shown.



Then the two prisms being oppositely placed, the net deviation produced by them is

$$\delta = (n-1)\theta - (n'-1)\theta = (n-n')\theta$$.

Hence as in the previous problem

$$d = 2a\delta = 2a\theta(n-n')$$

So $$\Delta x = \frac{(a+b)\lambda}{2a\theta(n-n')}$$

For plane incident wave we let $a \to \infty$

so $$\Delta x = \frac{\lambda}{2\theta(n-n')} = 0{\cdot}2\text{ mm}.$$

Q.76. A plane monochromatic light wave falls normally on a diaphragm with two narrow slits separated by a distance d = 2.5 mm.
A fringe pattern is formed on a screen placed at a distance / = = 100 cm behind the diaphragm. By what distance and in which direction will these fringes be displaced when one of the slits is covered by a glass plate of thickness h = 10 µm?

Ans. Extra phase difference introduced by the glass plate is

$$\frac{2\pi}{\lambda}(n-1)h$$

This will cause a shift equal to $(n-1)\frac{h}{\lambda}$ fringe widths

i.e. by $$(n-1)\frac{h}{\lambda} \times \frac{l\lambda}{d} = \frac{(n-1)hl}{d} = 2\text{ mm}.$$

The fringes move down if the lower slit is covered by the plate to compensate for the extra phase shift introduced by the plate.

Q.77. Figure 5.16 illustrates an interferometer used in measurements of refractive indices of transparent substances. Here S is

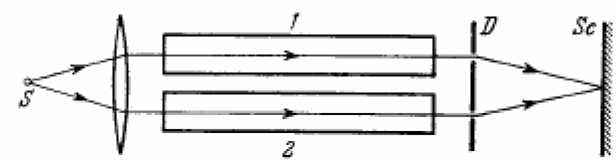


Fig. 5.16.

a narrow slit illuminated by monochromatic light with wavelength λ = 589 nm, 1 and 2 are identical tubes with air of length l = 10.0 cm each, D is a diaphragm with two slits. After the air in tube 1 was replaced with ammonia gas, the interference pattern on the screen Sc was displaced upward by N = 17 fringes. The refractive index of air is equal to n = 1.000277. Determine the refractive index of ammonia gas.

Ans. No. of fringes shifted $= (n' - n)\frac{l}{\lambda} = N$

So $n' = n + \frac{N\lambda}{l} = 1{\cdot}000377$.

Q.78. An electromagnetic wave falls normally on the boundary between two isotropic dielectrics with refractive indices n_1 and n_2. Making use of the continuity condition for the tangential components, E and H across the boundary, demonstrate that at the interface the electric field vector E
(a) of the transmitted wave experiences no phase jump;
(b) of the reflected wave is subjected to the phase jump equal to π if it is reflected from a medium of higher optical density.

Ans. (a) Suppose the vector $\vec{E}, \vec{E}', \vec{E}''$ correspond to the incident, reflected and the transmitted wave. Due to the continuity of the tangential component of the electric field across the interface, it follows that

$$E_\tau + E'_\tau = E''_\tau \qquad (1)$$

where the subscript τ means tangential.

The energy flux density is $\vec{E} \times \vec{H} = \vec{S}$.

Since $$H\sqrt{\mu \mu_0} = E\sqrt{\varepsilon \varepsilon_0}$$

$$H = E\sqrt{\frac{\varepsilon_0}{\mu_0}}\sqrt{\varepsilon/\mu} = n\sqrt{\frac{\varepsilon_0}{\mu_0}}E$$

Now $S \sim nE^2$ and since the light is incident normally

Or $n_1 E_\tau^2 = n_1 E_\tau'^2 + n_2 E_\tau''^2$ (2)

$$n_1(E_\tau^2 - E_\tau'^2) = n_2 E_\tau''^2$$

So $n_1(E_\tau - E_\tau') = n_2 E_\tau''$ (3)

So $$E_\tau'' = \frac{2n_1}{n_1 + n_2}E_\tau$$

Since E_τ'' and E_τ have the same sign , there is no phase change involved in this case,
(b) From (1) & (3)

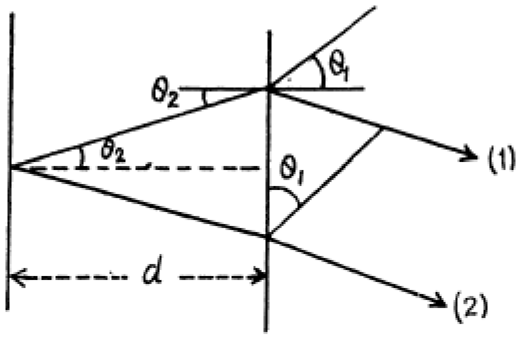
$$(n_2 + n_1)E_\tau' + (n_2 - n_1)E_\tau = 0$$

Or $$E_\tau' = \frac{n_1 - n_2}{n_1 + n_2}E_\tau.$$

If $n_2 > n_1$, then E_τ' & E_τ have opposite signs. Thus the reflected wave has an abrupt change of phase by π if $n_2 > n_1$ i.e. on reflection from the interface between two media when light is incident from the rarer to denser medium.

Q.79. A parallel beam of white light falls on a thin film whose refractive index is equal to n = 1.33. The angle of indices is θ_1 = = 52°. What must the film thickness be equal to for the reflected light to be coloured yellow (λ = 0.60 µm) most intensively?

Ans.



Path difference between (1) & (2) is

$$2\,n\,d \sec\theta_2 - 2\,d \tan\theta_2 \sin\theta_1$$

$$= 2\,d \frac{n - \frac{\sin^2\theta_1}{n}}{\sqrt{1 - \frac{\sin^2\theta_1}{n^2}}} = 2\,d\sqrt{n^2 - \sin^2\theta_1}$$

For bright fringes this must equal $\left(k + \frac{1}{2}\right)\lambda$ where $\frac{1}{2}$ comes from the phase change of π for (1).

Here $$k = 0, 1, 2, \ldots$$

Thus $$4\,d\sqrt{n^2 - \sin^2\theta_1} = (2\,k + 1)\,\lambda$$

or $$d = \frac{\lambda(1 + 2\,k)}{4\sqrt{n^2 - \sin^2\theta_1}} = 0{\cdot}14\,(1 + 2\,k)\,\mu\,\text{m} ..$$

Q.80. Find the minimum thickness of a film with refractive index 1.33 at which light with wavelength 0.64 µm experiences maximum reflection while light with wavelength 0.40 µm is not reflected at all. The incidence angle of light is equal to 30°.

Ans. Given

$$2\,d\sqrt{n^2 - 1/4} = \left(k + \frac{1}{2}\right) \times 0{\cdot}64\,\mu\,\text{m} \text{ (bright fringe)}$$

$$2\,d\sqrt{n^2 - 1/4} = k' \times 0{\cdot}40\,\mu\,\text{m} \qquad \text{(dark fringe)}$$

where k, k' are integers.

Thus $$64\left(k + \frac{1}{2}\right) = 40\,k' \quad \text{or} \quad 4\,(2\,k + 1) = 5\,k'$$

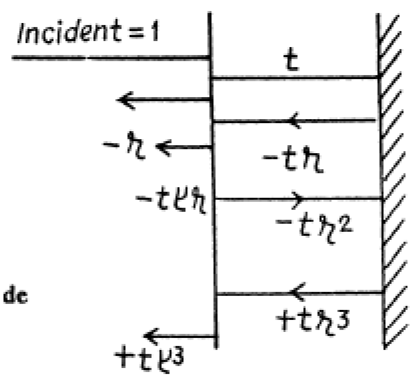
$$k = 2,\ k' = 4$$

Hence $$d = \frac{4 \times 0{\cdot}40}{2\sqrt{n^2 - 1/4}} = 0{\cdot}65\ \mu\text{m}.$$

Q.81. To decrease light losses due to reflection from the glass surface the latter is coated with a thin layer of substance whose refractive index $n' = \sqrt{n},$ where n is the refractive index of the glass. In this case the amplitudes of electromagnetic oscillations reflected from both coated surfaces are equal. At what thickness of that coating is the glass reflectivity in the direction of the normal equal to zerofor light with wavelength λ?

Ans. When the glass surface is coated with a material of R.I. $n' = \sqrt{n}$ (n = R.I. of glass) of appropriate thickness, reflection is zero because of interference between various multiply reflected waves. We show this below.
Let a wave of unit amplitude be normally incident from the left. The reflected amplitude is - r where

$$r = \frac{\sqrt{n} - 1}{\sqrt{n} + 1}$$



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Its phase is - ve so we write the reflected wave as - r. The transmitted wave has amplitude t

$$t = \frac{2}{1 + \sqrt{n}}$$

This wave is reflected at the second face and has amplitude

$$-tr$$
$$\left(\text{because } \frac{n - \sqrt{n}}{n + \sqrt{n}} = \frac{\sqrt{n} - 1}{\sqrt{n} + 1}.\right)$$

The emergent wave has amplitude $-tt'r$.

We prove below that $-tt' = 1-r^2$. There is also a reflected part of amplitude $trr' = -tr^2$, where r' is the reflection coefficient for a ray incident from the coating towards air. After reflection from the second face a wave of amplitude $+tt'r^3 = +(1-r^2)r^3$
Emerges. Let δ be the phase of the w a v e after traversing the coating both ways.

Then the complete reflected wave is

$$-r-(1-r^2)re^{i\delta}+(1-r^2)r^3e^{2i\delta}$$
$$-(1-r^2)r^5e^{3i\delta}\ldots\ldots$$

$$= -r-(1-r^2)re^{i\delta}\frac{1}{1+r^2e^{i\delta}}$$

$$= -r\left[1+r^2e^{i\delta}+(1-r^2)e^{i\delta}\right]\frac{1}{1+r^2e^{i\delta}}$$

$$= -r\frac{1+e^{i\delta}}{1+r^2e^{i\delta}}$$

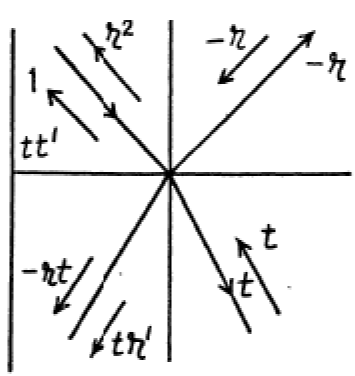
This vanishes if $\delta = (2k+1)\pi$. But

$$\delta = \frac{2\pi}{\lambda}2\sqrt{n}\,d \text{ so}$$

$$d = \frac{\lambda}{4\sqrt{n}}(2k+1)$$

We now deduce $tt' = 1-r^2$ and $r' = +r$. This follows from the principle of reversibility of light path as shown in the figure below.

$$tt'+r^2 = 1$$
$$-rt+r't = 0$$
$$\therefore\ tt' = 1-r^2$$
$$r' = +r.$$



(- r is the reflection ratio for the wave entering a denser medium)

Q.82. Diffused monochromatic light with wavelength $\lambda = 0.60\ \mu m$ falls on a thin film with refractive index n = 1.5. Determine the film thickness if the angular separation of neighbouring maxima observed in reflected light at the angles close to $\theta = 45°$ to the normal is equal to $\delta\theta = 3.00$.

Ans. We have the condition for maxima

$$2d\sqrt{n^2 - \sin^2\theta_1} = \left(k + \frac{1}{2}\right)\lambda$$

This must hold for angle $\theta \pm \frac{\delta\theta}{2}$ with successive values of k. Thus

$$2d\sqrt{n^2 - \sin^2\left(\theta + \frac{\delta\theta}{2}\right)} = \left(k - \frac{1}{2}\right)\lambda$$

$$2d\sqrt{n^2 - \sin^2\left(\theta - \frac{\delta\theta}{2}\right)} = \left(k + \frac{1}{2}\right)\lambda$$

Thus

$$\lambda = 2d\left\{\sqrt{n^2 - \sin^2\theta + \delta\theta\sin\theta\cos\theta} - \sqrt{n^2 - \sin^2\theta - \delta\theta\sin\theta\cos\theta}\right\}$$

$$= 2d\,\frac{\delta\theta\sin\theta\cos\theta}{\sqrt{n^2 - \sin^2\theta}}$$

Thus $$d = \frac{\sqrt{n^2 - \sin^2\theta}\,\lambda}{\sin 2\theta\,\delta\theta} = 15{\cdot}2\ \mu m$$

Interference of Light (Part - 2)

Q.86. The convex surface of a piano-convex glass lens comes into contact with a glass plate. The curvature radius of the lens's convex surface is R, the wavelength of light is equal to λ. Find the width Δr of a Newton ring as a function of its radius r in the region where Δr ≪ r.

Ans. We have

$$r^2 = \frac{1}{2} k \lambda R$$

So for k differing by 1 ($\Delta k = 1$)

$$2 r \Delta r = \frac{1}{2} \Delta k \lambda R = \frac{1}{2} \lambda R$$

or

$$\Delta r = \frac{\lambda R}{4 r}.$$

Q.87. The convex surface of a piano-convex glass lens with curvature radius R = 40 cm comes into contact with a glass plate. A certain ring observed in reflected light has a radius r = 2.5 mm. Watching the given ring, the lens was gradually removed from the plate by a distance Δh = 5.0 p,m. What has the radius of that ring become equal to?

Ans.

$$\frac{r^2}{R} = \frac{1}{2} k \lambda$$

$$\frac{r'^2}{R} + 2 \Delta h = \frac{1}{2} k \lambda$$

$$r' = \sqrt{r^2 - 2 R \Delta h} = 1{\cdot}5 \text{ mm}.$$

Q.88. At the crest of a spherical surface of a piano-convex lens there is a ground-off plane spot of radius r_0 = 3.0 mm through which the lens comes into contact with a glass plate. The curvature radius of the lens's convex surface is equal to R = 150 cm. Find the radius of the sixth bright ring when observed in reflected light with wavelength λ = 655 nm.

Ans.

In this case the path difference is $\frac{r^2 - r_0^2}{R}$ for $r > r_0$ and zero for $r \le r_0$.

This must equals $(k - 1/2)\lambda$ (where $k = 6$ for the six[th] bright ring.)

Thus $r = \sqrt{r_0^2 + \left(k - \frac{1}{2}\right)\lambda R} = 3{\cdot}8\text{ mm}$

Q.89. A piano-convex glass lens with curvature radius of spherical surface R = 12.5 cm is pressed against a glass plate. The diameters of the tenth and fifteenth dark Newton's rings in reflected light are equal to d_1 = 1.00 mm and d_2 = 1.50 mm. Find the wavelength of light.

Ans.

$$\frac{d_1^2}{4} = k_1 R\lambda, \quad \frac{d_2^2}{4} = k_2 R\lambda$$

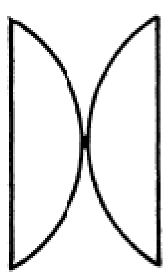
so
$$\frac{d_2^2 - d_1^2}{4(k_2 - k_1)R} = \lambda$$

Substituting the values, $\lambda = 0{\cdot}5\ \mu\text{m}$.

Q.90. Two piano-convex thin glass lenses are brought into contact with their spherical surfaces. Find the optical power of such a system if in reflected light with wavelength λ = 0.60 pm the diameter of the fifth bright ring is d = 1.50 mm.

Ans.

$r^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$



$$r^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{2k+1}{2}\lambda$$

Thus
$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{9}{2}\lambda \cdot \frac{4}{d^2} = \frac{18\lambda}{d^2}$$

Now
$$\frac{1}{f_1} = (n-1)\frac{1}{R_1}, \quad \frac{1}{f_2} = (n-1)\frac{1}{R_2}$$

so $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = (n-1)\frac{18\lambda}{d^2} = \Phi = 2{\cdot}40\,\text{D}$$

Here n = R.I. of glass = 1·5.

Q.91. Two thin symmetric glass lenses, one biconvex and the other biconcave, are brought into contact to make a system with optical power φ = 0.50 D. Newton's rings are observed in reflected light with wavelength λ = 0.61 µm. Determine:
(a) the radius of the tenth dark ring;
(b) how the radius of that ring will change when the space between the lenses is filled up with water.
Ans.

Here $\Phi = (n-1)\left(\frac{2}{R_1} - \frac{2}{R_2}\right)$

so $$\frac{1}{R_1} - \frac{1}{R_2} = \frac{\Phi}{2(n-1)}.$$

$$r_k^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{\Phi}{2(n-1)} r_k^2 = k\lambda$$

$k = 0$ is dark spot; excluding it, we take $k = 10$ hre.

Then $$r = \sqrt{\frac{20\lambda(n-1)}{\Phi}} = 3{\cdot}49\text{ mm}.$$

$$n_0 \bar{r}^2\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where $\bar{r}$ = new radius of the ring. Thus

$$n_0 \bar{r}^2 = r^2$$

or $$\bar{r} = r/\sqrt{n_0} = 3{\cdot}03\text{ mm}.$$

Where n_0 = R.I. of water = 1·33.

Q.92. The spherical surface of a piano-convex lens comes into contact with a glass plate. The space between the lens and the plate is filled up with carbon dioxide. The refractive indices of the lens, carbon dioxide, and the plate are equal to n_1 = 1.50, n_2 = 1.63, and n_3 = 1.70 respectively. The curvature radius of the spherical surface of the lens is equal to R = 100 cm. Determine the radius of the fifth dark Newton's ring in reflected light with wavelength λ = 0.50 µm.
Ans.

$$\frac{r^2}{R} n_2 = \left(k + \frac{1}{2}\right)\lambda,$$

$$r = \sqrt{(2K-1)\lambda R/2n_2}\,,\ K = 5$$

Q.93. In a two-beam interferometer the orange mercury line composed of two wavelengths λ_1 = 576.97 nm and λ_2 = 579.03 nm is employed. What is the least order of interference at which the sharpness of the fringe pattern is the worst?

Ans.

$$n_1 \lambda_1 = \left(n_1 - \frac{1}{2}\right) \lambda_2$$

or putting $\qquad \lambda_1 = \lambda, \; \lambda_2 = \lambda + \Delta\lambda$

we get $\qquad n_1 \Delta\lambda = \frac{\lambda}{2}$

or $\qquad n_1 = \frac{\lambda}{2\Delta\lambda} \approx \frac{\lambda_1}{2(\lambda_2 - \lambda_1)} = 140\,.$

Q.94. In Michelson's interferometer the yellow sodium line composed of two wavelengths $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm was used. In the process of translational displacement of one of the mirrors the interference pattern vanished periodically (why?). Find the displacement of the mirror between two successive appearances of the sharpest pattern.

Ans.

$$2\Delta h = k\lambda_2 = (k+1)\lambda_1$$

$$k(\lambda_2 - \lambda_1) = \lambda_1$$

$$k = \frac{\lambda_1}{\lambda_2 - \lambda_1}$$

$$\Delta h = \frac{\lambda_1 \lambda_2}{2(\lambda_2 - \lambda_1)} = \frac{\lambda^2}{2\Delta\lambda} \approx \cdot 29 \text{ mm}\,.$$

Q.95. When a Fabry-Perot &talon is illuminated by monochromatic light with wavelength λ an interference pattern, the system of con-

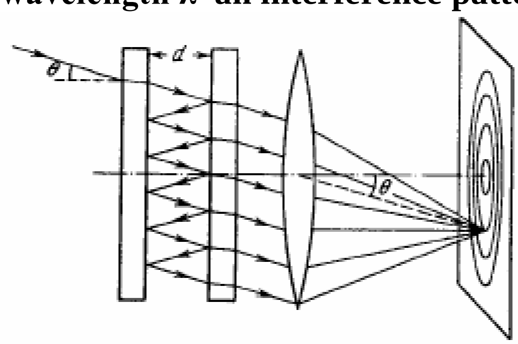
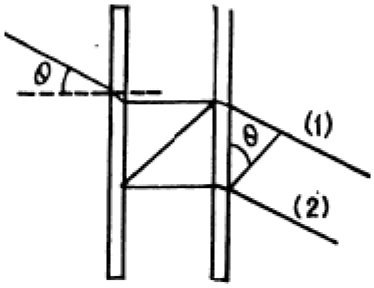


Fig. 5.18.

centric rings, appears in the focal plane of a lens (Fig. 5.18). The thickness of the &talon is equal to d. Determine how (a) the position of rings; (b) the angular width of fringes depends on the order of interference.

Ans.

$$\Delta = 2d \sec\theta - 2d\tan\theta\sin\theta$$
$$= 2d\cos\theta = k\lambda$$



$$2d\sin\theta\,\delta\theta = \lambda$$

on putting $\delta k = -1$. Thus

$$\delta\theta = \frac{\lambda}{2d\sin\theta}$$

$\delta\theta$ decreases as θ increases.

Q.96. For the Fabry-Perot etalon of thickness d = 2.5 cm find:
(a) the highest order of interference of light with wavelength λ = 0.50 p,m;
(b) the dispersion region Δλ , i.e. the spectral interval of wavelengths, within which there is still no overlap with other orders of interference if the observation is carried out approximately at wavelength λ = 0.50 pm.

Ans.

(a) We have $k_{max} = \frac{2d}{\lambda}\cdot$ for $\theta = 0 .= 10^5$.

$$2d\cos\theta = k\lambda = (k-1)(\lambda + \Delta\lambda)$$

Thus $\frac{1}{k} = \frac{\lambda}{2d}\cdot$ and $\Delta\lambda = \frac{\lambda}{k} = \frac{\lambda^2}{2d} = 5$ pm . on putting the values.

Q.83. Monochromatic light passes through an orifice in a screen Sc (Fig. 5.17) and being reflected from a thin transparent plate P produces fringes of equal inclination on the screen.

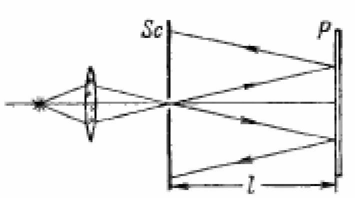


Fig. 5.17.

The thickness of the plate is equal to d, the distance between the plate and the screen is l, the radii of the ith and kth dark rings are r_i and r_k. Find the wavelength of light taking into account that $r_{i,k} \ll l$.

Ans. For small angles θ we write for dark fringes

$$2d\sqrt{n^2-\sin^2\theta} = 2d\left(n-\frac{\sin^2\theta}{2n}\right) = (k+0)\lambda$$

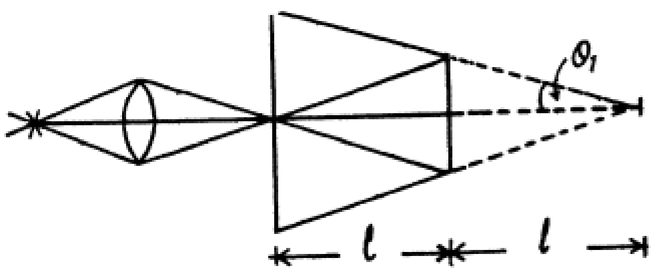
For the first dark fringe $\theta = 0$ and

$$2dn = (k_0+0)\lambda$$

For the i^{th} dark fringe

$$2d\left(n-\frac{\sin^2\theta_i}{2n}\right) = (k_0-i+1)\lambda$$

$$\sin^2\theta_i = \frac{n\lambda}{d}(i-1) = \frac{r_i^2}{4l^2}$$



Finally
$$\frac{n\lambda}{d}(i-k) = \frac{r_i^2-r_k^2}{4l^2}$$

so
$$\lambda = \frac{d(r_i^2-r_k^2)}{4l^2 n(i-k)}$$

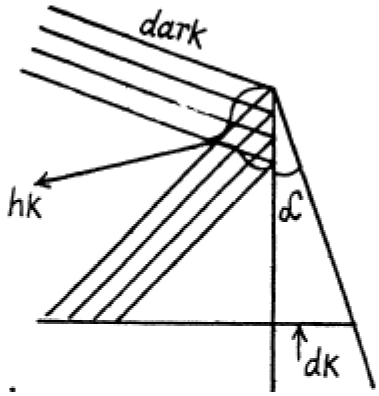
Q.84. A plane monochromatic light wave with wavelength λ falls on the surface of a glass wedge whose faces form an angle $\alpha \ll 1$. The plane of incidence is perpendicular to the edge, the angle of incidence is θ_1. Find the distance between the neighbouring fringe maxima on the screen placed at right angles to reflected light.

Ans. We have the usual equation for maxima

$$2h_k\alpha\sqrt{n^2-\sin^2\theta_1} = \left(k+\frac{1}{2}\right)\lambda$$

Here h_k = distance of the fringe from top

$h_k\alpha = d_k$ = thickness of the film



Thus on the screen placed at right angles to the reflected light

$$\Delta x = (h_k - h_{k-1}) \cos \theta_1$$
$$= \frac{\lambda \cos \theta_1}{2 \alpha \sqrt{n^2 - \sin^2 \theta_1}}$$

Q.85. Light with wavelength A. = 0.55 pm from a distant point source falls normally on the surface of a glass wedge. A fringe pattern whose neighbouring maxima on the surface of the wedge are separated by a distance Δx = 0.21 mm is observed in reflected light. Find:
(a) the angle between the wedge faces;
(b) the degree of light monochromatism (Δλ /λ) if the fringes disappear at a distance $l \simeq 1.5$ cm from the wedge's edge

Ans.

(a) For normal incidence we have using the above formula

$$\Delta x = \frac{\lambda}{2 n \alpha}$$

so $$\alpha = \frac{\lambda}{2 n \Delta x} = 3' \text{ on putting the values}$$

(b) In a distance / on the wedge there are $N = \frac{l}{\Delta x}$ fringes.
If the fringes disappear there, it must be due to the fact that the maxima due to the component of wavelength λ coincide with the minima due to the component of

wavelength $\lambda + \Delta \lambda$. Thus

$$N \lambda = \left(N - \frac{1}{2}\right)(\lambda + \Delta \lambda) \quad \text{or} \quad \Delta \lambda = \frac{\lambda}{2N}$$

so $$\frac{\Delta \lambda}{\lambda} = \frac{1}{2N} = \frac{\Delta x}{2l} = \frac{0{\cdot}21}{30} = 0{\cdot}007.$$

The answer given in the book is off by a factor 2.