Signals and Systems Test 2

Number of Questions: 35 **Section Marks: 90**

Directions for questions 1 to 35: Select the correct alternative from the given choices.

1. The final value of $X(t)$ if $X(s) = \frac{2}{s^2+3s+3}$ is ____

(A) 2 (B) zero
(C) 1 (D) ∞

2. If the system is causal and stable, the system poles must lie
(A) on the $j\omega$ axis
(B) on the left half of s – plane, with negative real part
(C) on the right half of s – plane, with positive real part
(D) both (a) & (b)

3. If the step response of a causal, LTI system is $s(t)$. Then the output of the system is, if the input is step function
(A) $\int_0^{\tau} \frac{ds(\tau)}{dt} u(t+\tau)\,d\tau$
(B) $\int_0^{t} \frac{ds(\tau)}{dt} u(t+\tau)\,d\tau + u(0)s(t)$
(C) $s(t)$
(D) (a) & (c)

4. The signal $x(t) = t\,u(t)$ is
(A) power signal
(B) energy signal
(C) neither energy nor power signal
(D) None of these

5. Invertible systems are those systems where
(A) input signal can be uniquely determined by observing output signal.
(B) system output is always constant
(C) output can be uniquely obtained from the knowledge of input
(D) A & C both

6. Z and laplace transforms are related by
(A) $s = \ln z$ (B) $s = \frac{\ln z}{T}$
(C) $s = z$ (D) $s = T \ln z$

7. If $X(\omega)$ is fourier transform of a real signal $x(t)$, then
(A) $\phi(-\omega) = -\phi(\omega)$ (B) $|X(-\omega)| = |X(\omega)|$
(C) $\phi(-\omega) = \phi(\omega)$ (D) both (a) & (b)

8. The inverse fourier transform of
$X(\omega) = \frac{1}{2-\omega^2+j3\omega}$
(A) $\left[e^{-t}-e^{2t}\right]u(t)$ (B) $\left[e^{t}-e^{2t}\right]u(t)$
(C) $\left[e^{-t}-e^{-2t}\right]u(t)$ (D) None of these

9. If $x[n]$ is real and odd then its discrete Fourier series coefficient C_k will be
(A) real (B) even
(C) imaginary (D) B & C

10. If the continuous time signal $X_a(t) = \sin(2500\pi t)$ is sampled at sampling frequency 10Hz, then the discrete time sequence $x[n]$ is
(A) $\sin(500\,\pi n)$ (B) $\cos(5000\pi n)$
(C) $\sin(250\pi n)$ (D) $\cos(250\pi n)$

11. Nyquist rate for the signal $x(t) = \frac{\sin 160\pi t}{\pi t}$ is _____,
(A) 160 Hz (B) 320 Hz
(C) 640 Hz (D) None of these

12. If the lower limit of ROC is less than upper limit of ROC for $X(z) = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$, then series
(A) diverges (B) converges
(C) can't say (D) zero

13. Fundamental period of the signal $x(t) = \cos(2t) + \sin\left(2\sqrt{2}t\right)$ is
(A) π (B) $\frac{\pi}{2}$
(C) Not periodic (D) $\sqrt{2}\pi$

14. For a finite sequence $x[n] = [3, 7, -1, 0, 3, 2]$
The z – transform & ROC is
(A) $3z^{-2} + 7z^{-1} - 1 + 3z^2 + 2z^3, 0 < |z| < \infty$
(B) $3z^2 + 7z - 1 + 3z^{-2} + 2z^{-3}, 0 < |z| < \infty$
(C) $3z^2 + 7z - 1 + 3z^{-2} + 2z^{-3}, |z| < \infty$
(D) $3z^{-2} + 7z^{-1} - 1 + 3z^2 + 2z^3, |z| > 0$

15. If $x(z)$ has poles at $z = \frac{-1}{3}$ and $z = 1$. Also given that $x[1] = 1$ & $x[-1] = \frac{1}{2}$ & ROC includes the point $z = \frac{2}{3}$, then the time signal $x[n]$ is ________,
(A) $u[n] + \frac{1}{2}\left(\frac{-1}{3}\right)^{n+1} u[-n-1]$
(B) $u[n] + \frac{1}{6}\left(\frac{-1}{3}\right)^{n} u[-n-1]$
(C) $u[n] - \frac{1}{2}\left(\frac{-1}{3}\right)^{n+1} u[-n-1]$
(D) None of these

16. If the impulse response of the system is given by $h(t) = e^{-3|t|}$. Then the system is _____

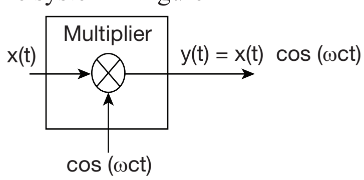
(A) causal only
(B) Not causal but not "BIBO stable "
(C) causal but unstable
(D) None of these

17. An LTI system is described by $H(s) = \dfrac{1}{s^2+3s+1}$.
The system response $y(t)$ is, if input
$x(t) = 2\cos(3t + 25°)$
(A) $0.17\cos(3t + 73.4°)$ (B) $2\cos(3t + 48.4°)$
(C) $0.17\cos(3t + 25°)$ (D) None of these

18. Wrong relationship is
(A) $h[n]\ x\ u[n] = \sum\limits_{k=-\infty}^{+\infty} h(k)u(n-k)$
(B) $h[n] * x[n] = \sum\limits_{k=-\infty}^{+\infty} h(k)x(n-k)$
(C) $\{x[n] * h_1[n] * h_2[n] = x[n] * \{h_1[n] * h_2[n]\}$
(D) $\sum\limits_{k=-\infty}^{\infty} h(k)x(n-k) = \sum\limits_{k=-\infty}^{\infty} x(k)h(n-k)$

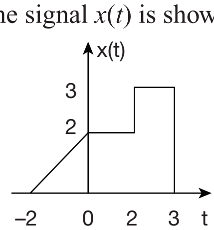
19. The impulse response of a discrete time LTI system is given by $h[n] = \left(\dfrac{1}{3}\right)^n u[n]$.
If $y[n]$ is the output of the system with input $x[n] = [n] + 3\delta[n-4]$ then $y[2]$ & $y[5]$ are
(A) $\dfrac{244}{243}$ and $\dfrac{1}{9}$ (B) $\dfrac{1}{3}$ and zero
(C) $\dfrac{244}{9}$ and $\dfrac{244}{243}$ (D) $\dfrac{1}{9}$ and $\dfrac{244}{243}$

20. Consider the system in figure

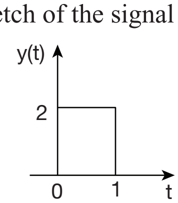


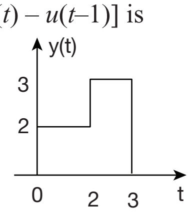
The system is
(A) memoryless & causal (B) LTI
(C) Stabel (D) A & C

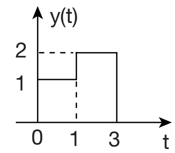
21. A continuous time signal $x(t)$ is shown in figure

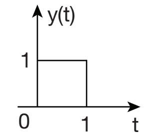


Sketch of the signal $y(t) = x(t)[u(t) - u(t-1)]$ is

(A)


(B)


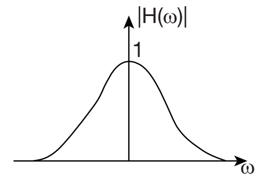
(C)


(D)


22. The equivalent bandwidth of a filter with frequency response $H(\omega)$ is defined by 'ω'

$$eq = \frac{1}{|H(\omega)|^2_{\max}} \int_0^\infty |H(\omega)|^2\, d\omega$$

If the filter is low pass RC filter and magnitude spectrum is given as



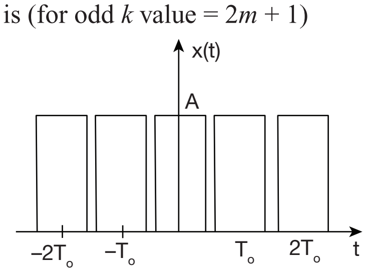
Then the equivalent bandwidth 'ω' eq is
(A) $\dfrac{\pi}{2RC}$ (B) $\dfrac{\pi}{RC}$
(C) $\dfrac{\pi}{2(RC)^3}$ (D) None of these

23. In a definition it is given that the bandwidth for a signal $x(t)$ is the 80 % energy containment bandwidth W_{85}, defined by

$$\frac{1}{2\Pi}\int_{-W_{80}}^{+W_{80}} |x(W_{80})|^2\, d\omega = \frac{1}{\pi}\int_0^{W_{80}} |x(\omega)|^2\, d\omega = 0.8E_x$$

Where E_x is the normalized energy content of the signal $x(t) = e^{-2at}u(t)$, $a > 0$
The W_{80} of the signal $x(t)$ is
(A) $3.077a$ (B) $6.15a$
(C) $2.33a$ (D) None of these

24. The Fourier series coefficient C_K of the periodic square wave $x(t)$ is (for odd k value = $2m + 1$)



(A) $(-1)^m \dfrac{A}{k\pi}$ (B) $\dfrac{A}{K\pi}$
(C) $(-1)^m \dfrac{A}{(2m+1)\pi}$ (D) None of these

25. The discrete fourier series coefficient, C_K for the given signal $x[n] = \cos\dfrac{\pi}{4}n$ is

(A) $C_1 = \frac{1}{2}, C_{-1} = -\frac{1}{2}$ and all other $C_K = 0$

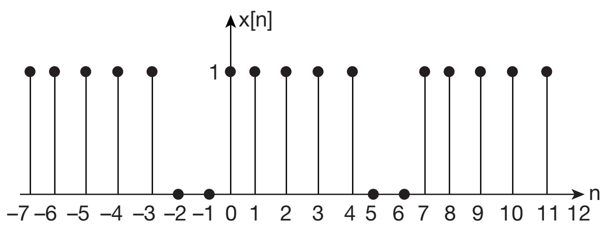
(B) $C_1 = \frac{1}{2}, C_7 = \frac{1}{2}$ and all other $C_K = 0$

(C) $C_{-1} = \frac{1}{2}, C_8 = \frac{1}{2}$ and all other $C_K \neq 0$

(D) None of these

26. If a discrete time LTI system with impulse response

$$h[n] = \frac{\sin\left(\frac{\pi}{8}n\right)}{\pi n}.$$

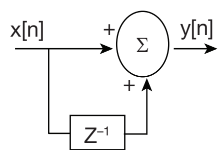


is given then what is output $y[n]$?if the input $x[n]$ is a periodic sequence with fundamental period N_o=7 as shown in figure

(A) 1 (B) $\frac{3}{5}$

(C) $\frac{5}{7}$ (D) $\frac{2}{7}$

27. A discrete time LTI System is given in figure.



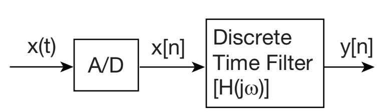
The frequency response $H(\Omega)$ of the system is _______

(A) $2e^{\frac{-j\Omega}{2}}\cos\left(\frac{\Omega}{2}\right)$ (B) $e^{\frac{-j\Omega}{2}}\sin\left(\frac{\Omega}{2}\right)$

(C) $1+e^{j\Omega}$] (D) Both A & C

28. An A/D converter is cascaded with the discrete time filter having frequency response

$$H(j\omega) = \frac{\frac{1}{2} - e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}}$$



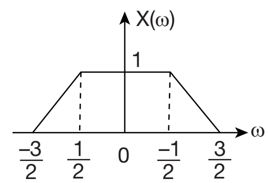
If the input $x(t)$ = sin ($125\pi t$) and sampling frequency is 500 Hz. Then output $y[n]$ of discrete time filter is ________

(A) $\sin\left(\frac{n\pi}{4} - 102.34°\right)$ (B) $\sin\left(\frac{n\pi}{4} - 40°\right)$

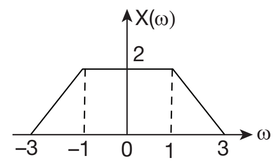
(C) $\cos\left(\frac{n\pi}{4} - 77.64\right)$ (D) $\cos\left(\frac{n\pi}{4} - 40°\right)$

29. Fourier transform of the signal $x(t) = \frac{\sin 2t . \sin t}{4\pi^2 t^2}$ is

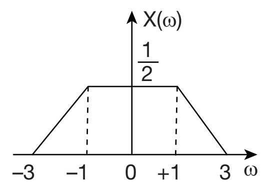
(A)



(B)



(C)



(D) None of these

30. A discrete time signal is given as $x[n] = u[n] - u[n-5]$ The even part of the signal is __________

(A) $\frac{u[n]+u[-n-1]-u[n-5]-u[-n+5]}{2}$

(B) $\frac{u[n]+u[-n]-u[n-5]-u[-n-5]}{2}$

(C) $\frac{u[n+4]-u[n-5]+\delta[n]}{2}$

(D) $\frac{1+\delta[n]-u[n-5]+u[n+4]}{2}$

31. The final value of the signal $x[n]$ is; whose z – transform is given as $X(z) = \frac{z^{-1}\left(1-z^{-4}\right)}{\left(1-z^{-1}\right)^2\left(1+z^{-1}\right)}$

(A) zero (B) 1

(C) 2 (D) infinite

Common data for Questions 32 and 33:

A real and odd periodic signal $x(t)$ with period 2 has Fourier series coefficients $X[k]$ =0 for $|k| > 1$ and $\frac{1}{2}\int_0^2 |x(t)|^2 \, dt = 1$

32. Then the Fourier series coefficient at $k = 0$ is ________

(A) zero (B) 1

(C) $\frac{1}{2}$ (D) $\frac{1}{3}$

33. The signal $x(t)$ is

(A) $-\sqrt{2}\sin(\pi t)$ (B) $+\sqrt{2}\sin(\pi t)$

(C) $-\frac{1}{\sqrt{2}}\cos(\pi t)$ (D) $+\frac{1}{\sqrt{2}}\sin(\pi t)$

Statement for linked Answer Questions 34 and 35:

The system formed by connecting two systems in cascade. The impulse response of the systems are given by $h_1(t) = e^{-3t}u(t)$ and $h_2(t) = 3e^{-t}u(t)$

34. The impulse response $h(t)$ of the over all system is given by ________

(A) $\left[e^{-3t} + 3e^{-t}\right]u(t)$ (B) $\frac{3}{2}\left[e^{-3t} - e^{-t}\right]u(t)$

(C) $\frac{3}{2}\left[e^{-t} - e^{-3t}\right]u(t)$ (D) None of these

35. The over all system is

(A) stable (B) unstable

(C) causal (D) Both (A) & (C)

Answer Keys

1. B	**2.** B	**3.** C	**4.** C	**5.** D	**6.** B	**7.** D	**8.** C	**9.** C	**10.** C
11. A	**12.** B	**13.** C	**14.** B	**15.** A	**16.** D	**17.** A	**18.** A	**19.** D	**20.** D
21. A	**22.** A	**23.** B	**24.** A	**25.** B	**26.** C	**27.** D	**28.** A	**29.** C	**30.** C
31. C	**32.** A	**33.** A	**34.** C	**35.** D					

Hints and Explanations

1. As we know that final value theorem is

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} s\,X(s)$$

$$= \lim_{s\to 0} s.\frac{2}{s^2+3s+3} = \lim_{s\to 0}\frac{2s}{s^2+3s+3} = 0$$ Choice (B)

2. If the system is causal and stable, both then the poles will lie only on the left half of the s – plane & for stability ROC must contain the $j\omega$. axis that is ROC is of the form $Re(s) > \sigma_{max}$ Choice (B)

3. If the step response of a causal, LTI system is $s(t)$.then

Impulse response $h(t) = \frac{ds(t)}{dt}$

And input $x(t) = u(t)$

So output response $y(t) = \frac{ds(t)}{dt} * u(t)$

$$= \int_{-\infty}^{t} \frac{ds(\tau)}{dt} u(t-\tau)\,d\tau$$

$$= \int_{0}^{t} \frac{ds(\tau)}{dt} d\tau = s(t)$$ Choice (C)

4. Energy $= E = \lim_{T\to\infty} \int_{-\frac{T}{2}}^{+\frac{T}{2}} |x(t)|^2\,dt$

$$= \lim_{T\to\infty} \int_{0}^{+\frac{T}{2}} t^2\,dt = \lim_{T\to\infty} \frac{\left(\frac{T}{2}\right)^3}{3} = \infty$$

Power $P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{+\frac{T}{2}} |x(t)|^2\,dt = \lim_{T\to\infty} \frac{1}{T}\frac{\left(\frac{T}{2}\right)^3}{3} = \infty$

So $x(t)$ is neither energy non power signal Choice (C)

5. Choice (D)

6. If $x(t)$ is sampled at a rate $\frac{1}{T}$ then $x(t)$ can be represent as $x(t) = \sum_{k=-\infty}^{+\infty} x(KT)\delta(t-KT)---(1)$

By taking Laplace Transform of equation"(1)

$$X(s) = \sum_{k=-\infty}^{\infty} r(KT)e^{-KsT}$$

If $e^{+sT} = Z$ then $X(s) = \sum_{k=-\infty}^{\infty} r(K)z^{-K} = X(z)$

So $Z = e^{ST}$

$\ln z = sT$ $s = \frac{\ell nz}{T}$ Choice (B)

7. Since $x(t)$ is real function so

By duality property

If $x(t) \leftrightarrow X(\omega)$

$X(t) \leftrightarrow 2\pi\, x(-\omega)$

and the magnitude of $|X(-\omega)| = |X(\omega)|$

and phase $\phi(\omega) = \tan^{-1}$ (function of ω)

so $\phi(\omega) = -\phi(-\omega)$

or $\phi(-\omega) = -\phi(\omega)$ Choice (D)

8. $X(\omega) = \frac{1}{2-\omega^2+j3\omega} = \frac{1}{2+(j\omega)^2+j3\omega}$

$$= \frac{1}{(1+j\omega)\;(2+j\omega)} = \frac{1}{(1+j\omega)} - \frac{1}{(2+j\omega)}$$

By taking inverse fourier transform of $x(\omega)$

$x(t) = \left[e^{-t} - e^{-2t}\right]u(t)$ Choice (C)

9. If $x[n]$ is real & even then its discrete Fourier series coefficient is real

$\Rightarrow$ If $x[n]$ is real and odd then its discrete Fourier series coefficient is imaginary Choice (C)

10. $x(t) = \sin(2500\,\pi t)$

$$x[n] = x[nT_s] = \sin\left(\frac{2500\pi n}{f_s}\right)$$

Where $f_s = \dfrac{1}{T_S} = 10\text{Hz}$

So $x[n] = \sin\left(\dfrac{2500\pi n}{10}\right) = \sin(250\pi n)$

Choice (C)

11. $\omega_m = 160\pi$

So $\omega_s = 2 \, x \, 160\pi = 320\pi = 320\pi$

$f_s = 160$ Hz   Choice (A)

12. ROC is defined as range of values for which Z – transform converges

Choice (B)

13. $x(t) = \cos(2t) + \sin\left(2\sqrt{2}t\right)$

Here $x_1(t) = \cos 2t$, is periodic with period

$$T_1 = \frac{2\pi}{2} = \pi$$

And $x_2(t) = \sin\left(2\sqrt{2}t\right)$ is periodic with period

$$T_2 = \frac{2\pi}{2\sqrt{2}} = \frac{\pi}{\sqrt{2}}$$

Since $\dfrac{T_1}{T_2} = \dfrac{\pi}{\pi\sqrt{2}} = \sqrt{2}$ is not a rational number

So $x(t)$ is not a periodic signal   Choice (C)

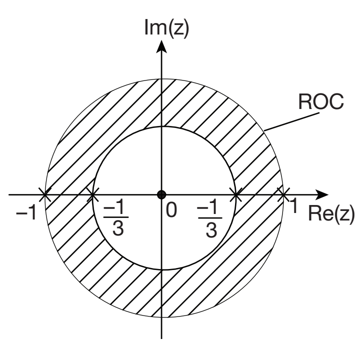
14. $X[z] = \sum_{n=-\infty}^{+\infty} x[n]z^{-n} = \sum_{n=-2}^{+3} x[n]z^{-n}$

$= 3z^{+2} + 7z^{+1} - 1 + 3z^{-2} + 2z^{-3}$

Since both positive & negative powers of z are included so ROC of $X(Z)$ will be $0 < |z| < \infty$ Choice (B)

15. Since ROC includes the point $z = \dfrac{2}{3}$

So ROC will be



So ROC will be $\dfrac{1}{3} < |z| < 1$

and $X(z)$ has two poles at $z = -\dfrac{1}{3}$ and $z = 1$

$$\text{so } X(z) = \left[\begin{array}{cc} \dfrac{A}{\left(1+\frac{1}{3}z^{-1}\right)} \quad + & \dfrac{B}{\left(1-z^{-1}\right)} \\ |z| > \dfrac{1}{3} & |z| < 1 \\ \text{Left sided signal} & \text{Right sided signal} \end{array}\right]$$

$$\text{so } x[n] = -A\left(\frac{-1}{3}\right)^n u[-n-1] + B[1]^n u[n]$$

As given that

$x[1] = 1 \Rightarrow 1 = B$

$$\& \; x[-1] = -A\left(\frac{-1}{3}\right)^{-1}.1$$

$$\frac{1}{2} = 3A \Rightarrow A = \frac{1}{6}$$

$$\& \; x[n] = u[n] - \frac{1}{6}\left(\frac{-1}{3}\right)^n u[-n-1]$$

$$= u[n] + \frac{1}{2}\left(\frac{-1}{3}\right)^{n+1} u[-n-1]$$

Choice (A)

16. $h(t) = e^{-3|t|}$

$h(t) = e^{-3t}u(t) + e^{3t}u(-t)$

we can see that $e^{3t}u(-t) \neq 0$ for $t < 0$, so given system is not causal.

For checking the system to be "BIBO stable"

We know that

$$\int_{-\infty}^{+\infty} h(t)\,dt < \infty$$

$$\text{So } \int_{-\infty}^{+\infty} h(t)\,dt = \int_0^{\infty} e^{-3t}u(t)\,dt + \int_{-\infty}^{0} e^{3t}\,u(t)\,dt$$

$$= -\frac{1}{3}\left[e^{-3t}\right]_0^{\infty} + \frac{1}{3}\left[e^{3t}\right]_{-\infty}^{0} = \frac{1}{3}[0-1] + \frac{1}{3} = \frac{2}{3} < \infty$$

So the system is "BIBO Stable"   Choice (D)

17. $x(t) = 2\cos(3t + 25°)$ can be expressed as real part of complex exponential with

$S = 3j$

$x(t) = 2Re\left(e^{j(3t+25°)}\right)$

$$\text{So } H(3j) = \frac{1}{(3j)^2 + (3\times 3j) + 1}$$

$$= \frac{1}{-9+9j+1} = \frac{1}{-8+9j}$$

so $= 0.083\, e^{j48.4°} = 0.083e^{j48.4°}$

so $Y_{ss}(t) = 2|H(3j)| \cos(3t + 25° + \angle H(3j))$

$= 2 \, x \, 0.083 \cos(3t + 25° + 48.4°)$

$= 0.17 \cos(3t + 73.4°)$

Choice (A)

18. $h[n] \times u[n] = \sum_{k=-\infty}^{+\infty} h(k)u(n-k)$ Choice (A)

19. $y[n] = x\,[n] * h[n]$

$$= \left(\delta[n] + 3\delta[n-4]\right) * \left(\frac{1}{3}\right)^n u[n]$$

$$y[n] = \left(\frac{1}{3}\right)^n u[n] + 3\left(\frac{1}{3}\right)^{n-4} u(n-4)$$

So $y[2] = \left(\frac{1}{3}\right)^2 u[2] + 3\left(\frac{1}{3}\right)^{-2} u(-2)$ &

$$y[5] = \left(\frac{1}{3}\right)^5 u[5] + 3\left(\frac{1}{3}\right)^1 u[1]$$

$$= \left(\frac{1}{3}\right)^5 + 1 = \frac{244}{243}$$

Choice (D)

20. For memory less

$y(t) = T\{x(t)\} = x(t)\cos \omega_c t$

Since the value of output $y(t)$ depends on only the present values of the input $x(t)$., The system is memory less

$\Rightarrow$ For causality

Since $y(t)$ does not depend upon future values of the input $x(t)$, so system is causal

$\Rightarrow$ for LTI

$Y_1(t)$ be output produced by the shifted input $x_1(t) = x(t-t_0)$.

Then $y_1(t) = T\,\{x(t-t_0)\}$

$= x\,(t-t_0)\cos \omega_c t$

But $y(t-t_0) = x\,(t-t_0)\cos w_c(t-t_0) \neq y_1(t)$

So system is not LTI

$\Rightarrow$ for stability

Since $\cos \omega_c t \leq 1$

$$|y(t)| = |x(t)\cos\omega_c t| \leq |x(t)|$$

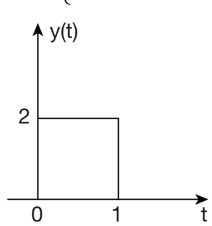
Thus if the input $x(t)$ is bounded then output $y(t)$ is also bounded. So system is BIBO stable

Choice (D)

21. As we know that

$$u(t) - u(t-1) = \begin{cases} 1 & 0 < t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

So $x(t)[u(t) - u(t-1)] = \begin{cases} 2 & 0 < t \leq 1 \\ 0 & \text{otherwise} \end{cases}$



Choice (A)

22. for a low pass RC filter $H(\omega) = \frac{1}{1 + j\,\omega RC}$

$$= \frac{1}{RC}\frac{1}{\left(\frac{1}{RC} + j\omega\right)}$$

As we can see that maximum value as $|H(\omega)|_{\max} = 1$ at $\omega = 0$

So $Weq = \frac{1}{1}\int_0^\infty \frac{1}{(RC)^2}\frac{1}{\left(\frac{R}{C}\right)^2 + (\omega)^2} d\omega$

$$Weq = \frac{1}{(RC)^2}\int_0^\infty \frac{d\omega}{\left(\frac{1}{RC}\right)^2 + \omega^2}$$

$$= \frac{RC}{(RC)^2}\left[\tan^{-1}\omega RC\right]_0^\infty = \frac{\pi}{2RC}$$

Choice (A)

23. $x(t) = e^{-2at}\,u(t)$

So $X(\omega) = \frac{1}{2a + j\omega}$

Energy $E = \int_{-\infty}^{+\infty} |X(\omega)|^2 dt$

$$\int_0^\infty e^{-4at}\quad dt = \frac{1}{4a}$$

Now by the given equation

$$\frac{1}{\pi}\int_0^{W_{80}} |x(\omega)|^2\, d\omega = \frac{1}{\pi}\int_0^{W_{80}} \frac{d\omega}{4a^2 + \omega^2}$$

$$\frac{1}{2\pi a}\tan^{-1}\left[\frac{W_{80}}{2a}\right] = 0.8Ex$$

$$\Rightarrow \quad \tan^{-1}\left[\frac{W_{80}}{2a}\right] = \frac{0.8 \times 2\pi a}{4a} = 0.40\pi$$

So $W_{80} = 2a\tan(0.4\pi) = 2a\ x\ 3.077$

$= 6.15a$ rad/s

Choice (B)

24. $x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_o} \quad \omega_o = \frac{2\pi}{T_o}$

$$C_k = \frac{1}{T_o}\int_{-T_o/2}^{T_o 2} x(t)e^{-jk\omega_o t}\ dt$$

$$= \frac{A}{-jk\omega_o T_o} \times -\left(e^{-jk\frac{\omega_0 T_0}{4}} - e^{jk\omega_o\frac{T_o}{4}}\right)$$

$$= \frac{A}{-jk2\pi} \times -\left[e^{\frac{-jk\pi}{2}} - e^{\frac{jk\pi}{2}}\right] = \frac{A}{k\pi}\sin\left(\frac{k\pi}{2}\right)$$

Thus $C_K = 0,\ k = 2m \neq 0$

$$C_k = (-1)^m \frac{A}{K\pi} \quad k = 2m+1$$

$$C_o = \frac{1}{T_o}\int_0^{T_o} x(t)dt = \frac{1}{T_o}\int_0^{\frac{T_o}{2}} Adt = \frac{A}{2}$$

Hence $C_o = \dfrac{A}{2}$, $C_{2m} = 0$, $m \neq 0$

$$C_{2m+1} = (-1)^m \frac{A}{(2m+1)\pi}$$

$$x(t) = \frac{A}{2} + \frac{A}{\pi}\sum_{m=-\infty}^{+\infty} \frac{(-1)^m}{2m+1} e^{j(2m+1)\omega_o}$$ Choice (A)

25. The fundamental period of $x[n]$ is No

So, $\Omega_o = \dfrac{2\pi}{N_o} = \dfrac{\pi}{4}$

So No = 8

By using Euler's formula

$$\cos\frac{\pi}{4}n = \frac{1}{2}\left(e^{j\left(\frac{\pi}{4}\right)} + e^{j\left(\frac{\pi}{4}\right)n}\right) = \frac{1}{2}e^{+j\Omega on} + \frac{1}{2}e^{-j\Omega on}$$

So fourier coefficients for $x[n]$ are

$$C_1 = \frac{1}{2}, C_{-1} = C_{-1+8} = C_7 = \frac{1}{2}$$

And all other $C_k = 0$ Choice (B)

26. As we know that Fourier transform of $h[n] = \dfrac{\sin\frac{\pi n}{8}}{\pi n}$ is

$$H(\Omega) = \begin{cases} 1 & |\Omega| \leq \pi/8 \\ 0 & \frac{\pi}{8} \leq |\Omega| \leq \pi \end{cases}$$

And $\Omega o = \dfrac{2\pi}{N_o} = \dfrac{2\pi}{7}$ and filter passes only frequencies in the range $|\Omega| \leq \dfrac{\pi}{8}$

So only D.C term will pass and discrete fourier series coefficient $C_o = \dfrac{1}{N_o}\displaystyle\sum_{n=0}^{No-1} x[n]$

$$= \frac{1}{7}\sum_{n=0}^{6} 1 = \frac{5}{7}$$ Choice (C)

27. From the figure $y[n] = x[n] + x[n-1]$

By taking Fourier transform

$Y(\Omega) = X(\Omega) + e^{-j\Omega} X(\Omega)$

So $H(\Omega) = \dfrac{Y(\Omega)}{X(\Omega)} = 1 + e^{-j\Omega}$

$$e^{\frac{-j\Omega}{2}}\left(e^{\frac{j\Omega}{2}} + e^{\frac{-j\Omega}{2}}\right)$$

$$= 2e^{\frac{-j\Omega}{2}}\cos\left(\frac{\Omega}{2}\right), |\Omega| \leq \pi$$ Choice (D)

28. $x(t) = \sin(125\pi t)$

$$x[n] = \sin\left(\frac{125\pi n}{500}\right) = \sin\left(\frac{n\pi}{4}\right)$$

$$\& \ H[\omega] = \frac{\frac{1}{2} - e^{-j\omega}}{1 - \frac{1}{2}e^{-j\omega}}$$

$$\text{Magnitude } |H(j\omega)|_{\omega=\frac{\pi}{4}} = \frac{\left(\frac{1}{2} - \frac{1}{\sqrt{2}}\right) + \left(\frac{j}{\sqrt{2}}\right)}{1 - \frac{1}{2\sqrt{2}} + \frac{j}{2\sqrt{2}}}$$

$$= \frac{(\sqrt{2} - 2 + 2j)}{(2\sqrt{2} - 1) + j} \approx 1$$

$$< H(j\omega) = \tan^{-1}\left(\frac{2}{-2+\sqrt{2}}\right) - \tan^{-1}\left(\frac{1}{2\sqrt{2}-1}\right)$$

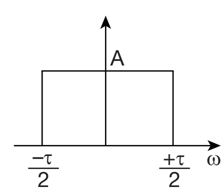
So, output of discrete filter $H(j\omega)$ is

$$y[n] = \sin\left(n\frac{\pi}{4} - 102.34°\right) = -73.67 - 28.67 = -102.34°$$

Choice (A)

29. $x(t) = \dfrac{\sin 2t}{2\pi t} \cdot \dfrac{\sin t}{2\pi t}$

& As we know that $\dfrac{A\tau}{2\pi}\dfrac{\sin\left(\frac{\tau}{2}t\right)}{\left(\frac{\tau}{2}t\right)} \xrightarrow{F.T}$



So by comparing I function $\dfrac{\sin 2t}{2\pi t} = \dfrac{A\tau \ \sin\frac{\tau}{2}.t}{\left(2\pi\frac{\tau}{2}.t\right)}$

$2t = \dfrac{\tau}{2}.t$

$\Rightarrow \dfrac{\tau}{2} = 2$

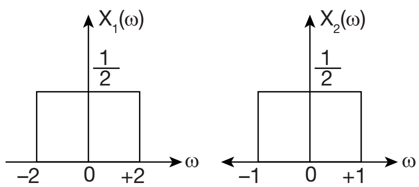
$\tau = 4$ and $\dfrac{A\tau}{2\pi} = \dfrac{1}{\pi}$

$\Rightarrow A = \dfrac{1}{2}$

By comparing II function $\dfrac{\sin t}{2\pi t} = \dfrac{A\tau}{2\pi}\cdot\dfrac{\sin\frac{\tau}{2}t}{\left(\frac{\tau}{2}t\right)}$

$\dfrac{\tau}{2} = 1 \Rightarrow \tau = 2$ & $\dfrac{A\tau}{2\pi} = \dfrac{1}{2\pi} \Rightarrow A = \dfrac{1}{2}$

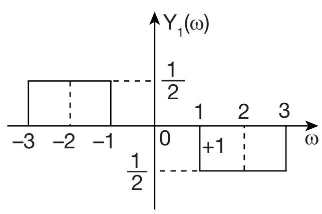
So $X(\omega)$



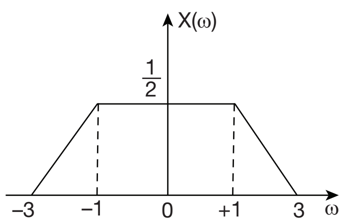
Let $Y_1(\omega) = \dfrac{dX_1(\omega)}{d\omega} X_2(\omega)$

$= [\delta(\omega + 2) - \delta(\omega - 2)] \,.\, X_2(\omega)$

$Y_1(\omega) = [X_2(\omega + 2) - X_2(\omega - 2)]$



$X(\omega) = \int y_1(\omega).d\omega.$



Choice (C)

30. Given that $x[n] = u[n] - u[n-5]$

Even part of signal $x[n] = x_e[n]$

So $x_e[n] = \dfrac{x[n]+x[-n]}{2}$

$x[-n] = u[-n] - u[-n+5]$

$$xe[n] = \frac{u[n]+u[-n]-u[n-5]-u[-n+5]}{2}$$

$$= \frac{1+\delta[n]-u[n-5]-u[-n+5]}{2}$$

$$= \frac{1-u[n-5]-u[-n+5]+\delta[n]}{2}$$

$$= \frac{\{1-u[-n+5]\}-u[n-5]+\delta[n]}{2}$$

$$= \frac{u[n+4]-u[n-5]+\delta[n]}{2}$$

Choice (C)

31. $x(z) = \dfrac{z^{-1}\left(1-z^{-4}\right)}{\left(1-z^{-1}\right)^2\left(1+z^{-1}\right)}$

$$= \frac{z^{-5}\left(z^4-1\right)}{z^{-3}\left(1-z\right)^2\left(z+1\right)}$$

$$= \frac{1\left(z^2+1\right)\left(z-1\right)z+1)}{z^2\left(z-1\right)^2\left(z+1\right)} = \frac{\left(z^2+1\right)}{z^2\left(z-1\right)}$$

By Appling final value theorem________

$\lim\limits_{n\to\infty} x[n] = \lim\limits_{z\to 1} (z-1)X(z)$

$$= \lim_{z\to 1}\frac{(z-1)\left(z^2+1\right)}{z^2\left(z-1\right)}$$

$$= \frac{(1+1)}{1} = 2$$

Choice (C)

32. Since $x(t)$ is real & odd, so $x[k]$ will be always odd imaginary.

Since $x(t)$ is real $x^*(t) = x(t)$

Again if $x(t)$ is an odd signal

So $x(t) = -x(-t)$

$X[k] = -X[-k]$

$X^*[-k] = -X[-k]$

for real & odd signal & also $x[0] = 0$ for an odd signal because in odd signal there must be zero at $k = 0$

Choice (A)

33. Since $X[k] = 0$ for $|k| > 1$

& $X[0] = 0$

So Fourier series coefficient will present only at $k = 1$ & -1

& By parseval's theorem

$$\frac{1}{T}\int_0^T |x(t)|^2\, dt = \sum_{k=-\infty}^{+\infty} |X[k]|^2$$

$$\frac{1}{2}\int_0^2 |x(t)|^2\, dt = \sum_{k=-1}^{+1} |X[k]|^2 = 1$$

Since Time period = 2

So $|X[-1]|^2 + |X[0]|^2 + |X[1]|^2 = 1$

$2|X[-1]|^2 = 1$

$|X[-1]|^2 = \dfrac{1}{2}$ $\quad \left(\begin{matrix} \because |X[1]| = |X[-1]| \\ X(o) = 0 \end{matrix}\right)$

fourier series coefficient is imaginary & odd

So $X[1] = \dfrac{j}{\sqrt{2}}$

$X[-1] = \dfrac{-j}{\sqrt{2}}$

so $x(t) = \sum\limits_{k=-1}^{+1} X[k]e^{jk\omega ot}$ $\quad \left(\because \omega_0 = \dfrac{2\pi}{2} = \pi\right)$

$$= \frac{j}{\sqrt{2}}\left[e^{j\pi t} - e^{-j\pi t}\right]$$

$$= \frac{2j.j}{\sqrt{2}}\left[\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right] = -\sqrt{2}\sin(\pi t)$$

Choice (A)

34. Since two systems are cascade so over all transfor $H(s) = H_1(s)\, H_2\,(s)$

$$= \frac{1}{s+3}.\frac{3}{(s+1)} = \frac{3}{2(s+1)} - \frac{3}{2(s+3)}$$

So $h(t) = \frac{3}{2}\left[e^{-t} - e^{-3t}\right]\, u(t)$ Choice (C)

35. for stability $\int_{-\infty}^{+\infty} h(t)dt < \infty$

$$so \int_{0}^{+\infty} \frac{3}{2}\left(e^{-t} - e^{-3t}\right)dt$$

$$= \frac{3}{2}\left[\frac{e^{-t}}{-1} - \frac{e^{-3t}}{-3}\right]_0^{\infty}$$

$$= \frac{3}{2}\left[(0-0) - \left(-1 + \frac{1}{3}\right)\right] = 1$$

So overall system is stable and causal also because it depends upon past & present values. Choice (D)