Exercise 14.1

1. On which axis do the following points lie?
   (i) P(5, 0)
   (ii) Q(0 – 2)
   (iii) R(−4, 0)
   (iv) S(0, 5)

   Sol:

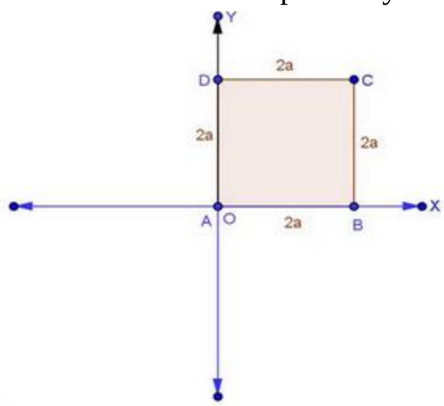
   (i) $P(5,0)$ lies on $x-axis$

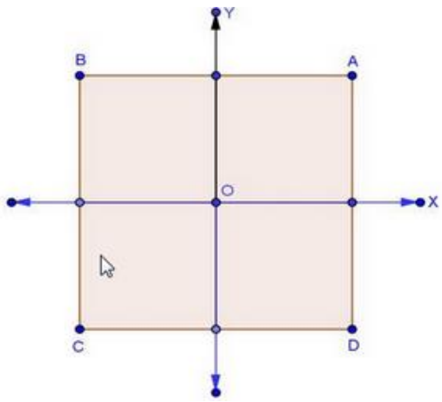
   (ii) $Q(0,-2)$ lies on $y-axis$

   (iii) $R(-4,0)$ lies on $x-axis$

   (iv) $S(0,5)$ lies on $y-axis$

2. Let ABCD be a square of side 2a. Find the coordinates of the vertices of this square when
   (i) A coincides with the origin and AB and AB and coordinate axes are parallel to the sides AB and AD respectively.
   (ii) The center of the square is at the origin and coordinate axes are parallel to the sides AB and AD respectively.





   Sol:

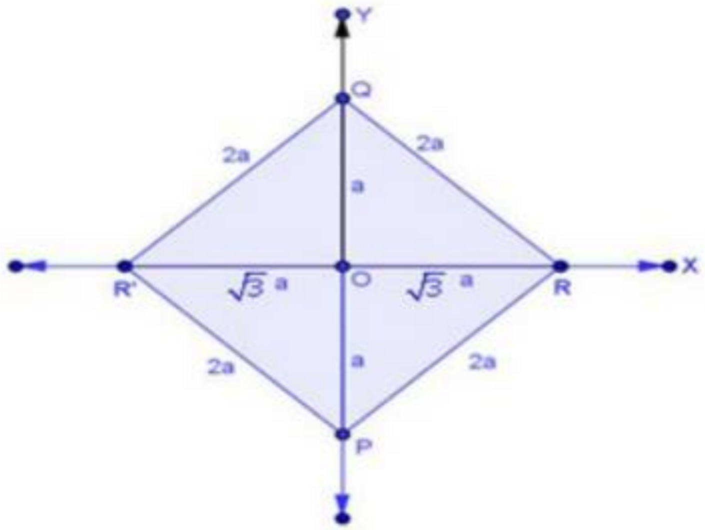
   (i) Coordinate of the vertices of the square of side $2a$ are:

   $A(0,0), B(2a,0), C(2a,2a)$ *and* $D(0,2a)$

   (ii) Coordinate of the vertices of the square of side $2a$ are:

   $A(a,a), B(-a,a), C(-a,-a)$ *and* $(a,-a)$

3. The base PQ of two equilateral triangles PQR and PQR' with side 2a lies along y-axis such that the mid-point of PQ is at the origin. Find the coordinates of the vertices R and R' of the triangles.



Sol:

We have two equilateral triangle PQR and PQR' with side $2a$.

O is the mid-point of PQ.

In ΔQOR, $\angle QOR = 90°$

Hence, by Pythagoras theorem

$OR^2 + OQ^2 = QR^2$

$OR^2 = (2a)^2 - (a)^2$

$OR^2 = 3a^2$

$OR = \sqrt{(3)}a$

Coordinates of vertex R is $(\sqrt{3}a, 0)$ and coordinate of vertex R' is $(-\sqrt{3}a, 0)$

Exercise 14.2

1. Find the distance between the following pair of points:

(i) $(-6, 7)$ and $(-1, -5)$

(ii) $(a+b, b+c)$ and $(a-b, c-b)$

(iii) $(a\sin\alpha, -b\cos\alpha)$ and $(-a\cos\alpha, b\sin\alpha)$

(iv) $(a, 0)$ and $(0, b)$

Sol:

(i) We have $P(-6,7)$ and $Q(-1,-5)$

Here,

$x_1 = -6, y_1 = 7$ and

$x_2 = -1, y_2 = -5$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{[-1-(-6)]^2 + (-5-7)^2}$$

$$PQ = \sqrt{(-1+6)^2 + (-5-7)^2}$$

$$PQ = \sqrt{(5)^2 + (-12)^2}$$

$$PQ = \sqrt{25+144}$$

$$PQ = \sqrt{169}$$

$$PQ = 13$$

(ii) we have $P(a+b, b+c)$ and $Q(a-b, c-b)$ here,

$x_1 = a+b, y_1 = b+c$ and $x_2 = a-b, y_2 = c-b$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{[a-b-(a+b)]^2 + (c-b-(b+c))^2}$$

$$PQ = \sqrt{(a-b-a-b)^2 + (c-b-b-c)^2}$$

$$PQ = \sqrt{(-2b)^2 + (-2b)^2}$$

$$PQ = \sqrt{4b^2 + 4b^2}$$

$$PQ = \sqrt{8b^2}$$

$$PQ = \sqrt{4 \times 2b^2}$$

$$PQ = 2\sqrt{2}b$$

(iii) we have $P(a\sin\alpha, -b\cos\alpha)$ and $Q(-a\cos\alpha, b\sin\alpha)$ here

$x_1 = a\sin\alpha, y_1 = -b\cos\alpha$ and

$x_2 = -a\cos\alpha, y_2 = b\sin\alpha$

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{(-a\cos\alpha - a\sin\alpha)^2 + [-b\sin\alpha - (-b\cos\alpha)]^2}$$

$$PQ=\sqrt{(-a\cos\alpha)^2+(-a\sin\alpha)^2+2(-a\cos\alpha)(-a\sin\alpha)+(b\sin\alpha)^2+(-b\cos\alpha)^2-2(b\sin\alpha)(-b\cos\alpha)}$$

$$PQ=\sqrt{a^2\cos^2\alpha+a^2\sin^2\alpha+2a^2\cos\alpha\sin\alpha+b^2\sin^2\alpha+b^2\cos^2\alpha+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{a^2\left(\cos^2\alpha+\sin^2\alpha\right)+2a^2\cos\alpha\sin\alpha+b^2\left(\sin^2\alpha+\cos^2\alpha\right)+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{a^2\times1+2a^2\cos\alpha\sin\alpha+b^2\times1+2b^2\sin\alpha\cos\alpha} \qquad \left[\because \sin^2\alpha+\cos^2\alpha=1\right]$$

$$PQ=\sqrt{a^2+b^2+2a^2\cos\alpha\sin\alpha+2b^2\sin\alpha\cos\alpha}$$

$$PQ=\sqrt{\left(a^2+b^2\right)+2\cos\alpha\sin\alpha\left(a^2+b^2\right)}$$

$$PQ=\sqrt{\left(a^2+b^2\right)(1+2\cos\alpha\sin\alpha)}$$

(iv) We have $P(a,0)$ and $Q(0,b)$

Here,

$x_1=a, y_1=0, x_2=0, y_2=b,$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$PQ=\sqrt{(0-a)^2+(b-0)^2}$$

$$PQ=\sqrt{(-a)^2+(b)^2}$$

$$PQ=\sqrt{a^2+b^2}$$

2. Find the value of a when the distance between the points (3, a) and (4, 1) is $\sqrt{10}$.

Sol:

We have $P(3,a)$ and $Q(4,1)$

Here,

$x_1=3, y_1=a$

$x_2=4, y_2=1$

$PQ=\sqrt{10}$

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$\Rightarrow \sqrt{10}=\sqrt{(4-3)^2+(1-a)^2}$$

$$\Rightarrow \sqrt{10}=\sqrt{(1)^2+(1-a)^2}$$

$$\Rightarrow \sqrt{10}=\sqrt{1+1+a^2-2a} \qquad \left[\because (a-b)^2=a^2+b^2-2ab\right]$$

$$\Rightarrow \sqrt{10}=\sqrt{2+a^2-2a}$$

Squaring both sides

$\Rightarrow \left(\sqrt{10}\right)^2 = \left(\sqrt{2+a^2-2a}\right)^2$

$\Rightarrow 10 = 2+a^2-2a$

$\Rightarrow a^2-2a+2-10=0$

$\Rightarrow a^2-2a-8=0$

Splitting the middle team.

$\Rightarrow a^2-4a+2a-8=0$

$\Rightarrow a(a-4)+2(a-4)=0$

$\Rightarrow (a-4)(a+2)=0$

$\Rightarrow a=4, a=-2$

3. If the points (2, 1) and (1, -2) are equidistant from the point (x, y) from (-3, 0) as well as from (3, 0) are 4.

Sol:

We have $P(2,1)$ and $Q(1,-2)$ and $R(X,Y)$

Also, $PR = QR$

$PR = \sqrt{(x-2)^2+(y-1)^2}$

$\Rightarrow PR = \sqrt{x^2+(2)^2-2xx\times 2+y^2+(1)^2-2\times y\times 1}$

$\Rightarrow PR = \sqrt{x^2+4-4x+y^2+1-2y}$

$\Rightarrow PR = \sqrt{x^2+5-4x+y^2-2y}$

$QR = \sqrt{(x-1)^2+(y+2)^2}$

$\Rightarrow PR = \sqrt{x^2+1-2x+y^2+4+4y}$

$\Rightarrow PR = \sqrt{x^2+5-2x+y^2+4y}$

$\because PR = QR$

$\Rightarrow \sqrt{x^2+5-4x+y^2-2y} = \sqrt{x^2+5-2x+y^2+4y}$

$\Rightarrow x^2+5-4x+y^2-2y = x^2+5-2x+y^2+4y$

$\Rightarrow x^2+5-4x+y^2-2y = x^2+5-2x+y^2+4y$

$\Rightarrow -4x+2x-2y-4y=0$

$\Rightarrow -2x-6y=0$

$\Rightarrow -2(x+3y)=0$

$\Rightarrow x+3y = \frac{0}{-2}$

$\Rightarrow x+3y=0$

Hence proved.

4. Find the values of x, y if the distances of the point (x, y) from (−3, 0) as well as from (3,0) are 4.

Sol:

We have $P(x,y), Q(-3,0)$ and $R(3,0)$

$PQ=\sqrt{(x+3)^2+(y-0)^2}$

$\Rightarrow 4=\sqrt{x^2+9+6x+y^2}$

Squaring both sides

$\Rightarrow (4)^2=\left(\sqrt{x^2+9+6x+y^2}\right)^2$

$\Rightarrow 16=x^2+9+6x+y^2$

$\Rightarrow x^2+y^2=16-9-6x$

$\Rightarrow x^2+y^2=7-6x$ ………….(1)

$PR=\left(\sqrt{(x-3)^2+(y-0)^2}\right)$

$\Rightarrow 4=\sqrt{x^2+9-6x+y^2}$

Squaring both sides

$(4)^2=\left(\sqrt{x^2+9-6x+y^2}\right)^2$

$\Rightarrow 16=x^2+9-6x+y^2$

$\Rightarrow x^2+y^2=16-9+6x$

$\Rightarrow x^2+y^2=7+6x$ …………..(2)

Equating (1) and (2)

$7-6x=7+6x$

$\Rightarrow 7-7=6x+6x$

$\Rightarrow 0=12x$

$\Rightarrow x=12$

Equating (1) and (2)

$7-6x=7+6x$

$\Rightarrow 7-7=6x+6x$

$\Rightarrow 0=12x$

$\Rightarrow x=12$

Substituting the value of $x=0$ in (2)

$x^2+y^2=7+6x$

$0+y^2=7+6\times 0$

$y^2=7$

$y=\pm\sqrt{7}$

5. The length of a line segment is of 10 units and the coordinates of one end-point are (2,−3). If the abscissa of the other end is 10, find the ordinate of the other end.

Sol:

Let two ordinate of the other end R be Y

$\therefore$ Coordinates of other end R are $(10, y)$ i.e., $R(10, y)$

Distance $PR=10$ [given]

$PR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\Rightarrow 10=\sqrt{(10-2)^2+(y+3)^2}$

$\Rightarrow 10=\sqrt{8^2+y^2+9+6y}$

$\Rightarrow 10=\sqrt{64+y^2+9+6y}$

$=10=\sqrt{73+y^2+6y}$

Squaring both sides

$(10)^2=\left(\sqrt{73+y^2+6y}\right)^2$

$\Rightarrow 100=73+y^2+6y$

$\Rightarrow y^2+6y+73-100=0$

$\Rightarrow y^2+6y-27=0$

Splitting the middle term

$y^2+9y-3y-27=0$

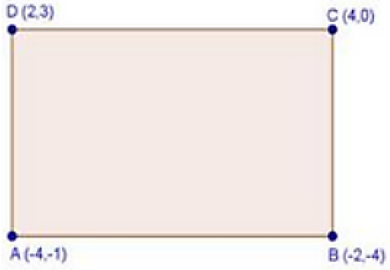
$\Rightarrow y^2+9y-3y-27=0$

$\Rightarrow y(y+9)-3(y+9)=0$

$\Rightarrow (y+9)(y-3)=0$

$\Rightarrow y=-9, y=3$

6. Show that the points (−2, − 4), (4, 0) and (2, 3) are the vertices points of are the vertices points of a rectangle.



Sol:

Let $A(-4,-1), B(-2,-4), C(4,0)$ *and* $D(2,3)$ be the given points

Now,

$AB = \sqrt{(-2+4)^2 + (-4+1)^2}$

$\Rightarrow AB = \sqrt{(2)^2 + (-3)^2}$

$\Rightarrow AB = \sqrt{4+9}$

$\Rightarrow AB = \sqrt{13}$

$CD = \sqrt{(4-2)^2 + (0-3)^2}$

$\Rightarrow CD = \sqrt{(2)^2 + (-3)^2}$

$\Rightarrow CD = \sqrt{4+9}$

$\Rightarrow CD = \sqrt{13}$

$BC = \sqrt{(4+2)^2 + (0+4)^2}$

$\Rightarrow BC = \sqrt{(6)^2 + (4)^2}$

$\Rightarrow BC = \sqrt{36+16}$

$\Rightarrow BC + \sqrt{52}$

$AD = \sqrt{(-4-2)^2 + (-1-3)^2}$

$\Rightarrow AD = \sqrt{(-6)^2 + (-4)^2}$

$\Rightarrow AD = \sqrt{36+16}$

$\Rightarrow AD = \sqrt{52}$

$\therefore$ $AB = CD$ and $AD = BC \Rightarrow ABCD$ is a parallelogram

Now,

$AC = \sqrt{(4+4)^2 + (0+1)^2}$

$\Rightarrow AC = \sqrt{(8)^2 + (1)^2}$

$\Rightarrow AC = \sqrt{64+1}$

$\Rightarrow AC = \sqrt{65}$

$BD = \sqrt{(2+2)^2 + (3+4)^2}$

$\Rightarrow BD = \sqrt{(4)^2 + (7)^2}$

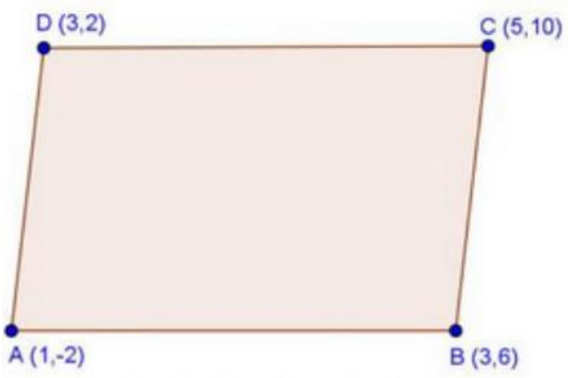
$\Rightarrow BD = \sqrt{16+49}$

$\Rightarrow BD = \sqrt{65}$

Since the diagonals of parallelogram $ABCD$ are equal i.e., $AC = BD$

Hence, $ABCD$ is a rectangle

7. Show that the points A(1, −2), B(3, 6), C(5, 10) and D(3, 2) are the vertices of a parallelogram



Sol:

Let $A(1,-2), B(3,6), C(5,10), D(3,2)$ be the given points

$AB = \sqrt{(3-1)^2 + (6+2)^2}$

$\Rightarrow AB = \sqrt{(2)^2 + (8)^2}$

$\Rightarrow AB = \sqrt{4+64}$

$\Rightarrow AB = \sqrt{68}$

$CD = \sqrt{(5-3)^2 + (10-2)^2}$

$\Rightarrow CD = \sqrt{(2)^2 + (8)^2}$

$\Rightarrow CD = \sqrt{4+64}$

$\Rightarrow CD = \sqrt{68}$

$AD = \sqrt{(3-1)^2 + (2+2)^2}$

$\Rightarrow AD = \sqrt{(2)^2 + (4)^2}$

$\Rightarrow AD = \sqrt{4+16}$

$\Rightarrow AD = \sqrt{20}$

$BC = \sqrt{(5-3)^2 + (10-6)^2}$

$\Rightarrow BC = \sqrt{(2)^2 + (4)^2}$

$\Rightarrow BC = \sqrt{4+16}$

$\Rightarrow BC = \sqrt{20}$

$\therefore \quad AB = CD$ and $AD = BC$

Since opposite sides of a parallelogram are equal

Hence, $ABCD$ is a parallelogram

8. Prove that the points A (1, 7), B (4, 2), C (−1, −1) and D (−4, 4) are the vertices of a square.

Sol:

Let $A(1,7), B(4,2), C(-1,-1)$ and $D(-4,4)$ be the given point. One way of showing that $ABCD$ is a square is to use the property that all its sides should be equal and both its diagonals should also be equal

Now,

$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$

$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$

$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$

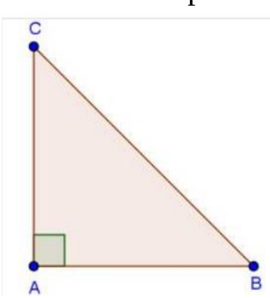
$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$

$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$

$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$

Since, $AB = BC = CD = DA$ and $AC = BD$, all the four sides of the quadrilateral $ABCD$ are equal and its diagonals AC and BD are also equal. Therefore, $ABCD$ is a square

9. Prove that the points (3, 0) (6, 4) and (-1, 3) are vertices of a right angled isosceles triangle.



Sol:

Let $A(3,0), B(6,4)$ and $C(-1,3)$ be the given points

$AB = \sqrt{(6-3)^2 + (4-0)^2}$

$\Rightarrow AB = \sqrt{(3)^2 + (4)^2}$

$\Rightarrow AB = \sqrt{9+16}$

$\Rightarrow AB = \sqrt{25}$

$BC = \sqrt{(-1-6)^2 + (3-4)^2}$

$\Rightarrow BC = \sqrt{(-7)^2 + (-1)^2}$

$\Rightarrow BC = \sqrt{49+1}$

$\Rightarrow BC = \sqrt{50}$

$AC = \sqrt{(-1-3)^2 + (3-0)^2}$

$\Rightarrow AC = \sqrt{(-4)^2 + (3)^2}$

$\Rightarrow A\sqrt{16+9}$

$\Rightarrow AC = \sqrt{25}$

$AB^2 = \left(\sqrt{25}\right)^2$

$\Rightarrow AB^2 = 25$

$AC^2 = 25$

$BC^2 = \left(\sqrt{50}\right)^2$

$BC^2 = 50$

Since $AB^2 + AC^2 = BC^2$ and $AB = AC$

$\therefore ABC$ is a right angled isosceles triangle

10. Prove that (2, —2), (—2, 1) and (5, 2) are the vertices of a right angled triangle. Find the area of the triangle and the length of the hypotenuse.

Sol:

Let $A(2,-2), B(-2,1)$ and $C(5,2)$ be the given points

$AB = \sqrt{(-2-2)^2 + (1+2)^2}$

$\Rightarrow AB = \sqrt{(-4)^2 + (3)^2}$

$\Rightarrow AB + \sqrt{16-9}$

$\Rightarrow AB = \sqrt{25}$

$BC = \sqrt{(5+2)^3 + (2-1)^2}$

$\Rightarrow B\sqrt{(7)^2 + (1)^2}$

$\Rightarrow BC = \sqrt{49+1}$

$\Rightarrow BC = \sqrt{50}$

$AC = \sqrt{(5-2)^2 + (2+2)^2}$

$\Rightarrow AC = \sqrt{(3)^2 + (4)^2}$

$\Rightarrow AC = \sqrt{9+16}$

$\Rightarrow AC = \sqrt{25}$

$AB^2 = \left(\sqrt{25}\right)^2$

$\Rightarrow AB^2 = 25$

$BC^2 = \left(\sqrt{50}\right)^2$

$\Rightarrow BC^2 = 50$

Since, $AB^2 + AC^2 = BC^2$

$\therefore ABC$ is a right angled triangle.

Length of the hypotenuse $BC = \sqrt{50} = 5\sqrt{2}$

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$= \frac{1}{2} \times \sqrt{25} \times \sqrt{25}$

$= \frac{25}{2}$ square units.

11. Prove that the points (2 a, 4 a), (2 a, 6 a) arid (2a + $\sqrt{3}a$, 5a) are the vertices of an equilateral triangle.

Sol:

Let $A(2a, 4a), B(2a, 6a)$ and $C\left(2a + \sqrt{3}a, 5a\right)$ be the given points

$AB = \sqrt{(2a-2a)^2 + (6a-4a)^2}$

$\Rightarrow AB = \sqrt{(0)^2 + (2a)^2}$

$\Rightarrow AB = \sqrt{4a^2}$

$\Rightarrow AB = 2a$

$BC = \sqrt{\left(2a + \sqrt{3}a - 2a\right)^2 + \left(5a - 6a\right)^2}$

$\Rightarrow BC = \sqrt{\left(\sqrt{3}a\right)^2 + \left(-a\right)^2}$

$\Rightarrow BC = \sqrt{3a^2 + a^2}$

$\Rightarrow BC = \sqrt{4a^2}$

$\Rightarrow BC = 2a$

$AC = \sqrt{\left(2a + \sqrt{3}a - 2a\right)^2 + \left(5a - 4a\right)^2}$

$\Rightarrow AC = \sqrt{\left(\sqrt{3}a\right)^2 + \left(a\right)^2}$

$\Rightarrow AC = \sqrt{3a^2 + a^2}$

$\Rightarrow AC = \sqrt{4a^2}$

$\Rightarrow AC = 2a$

Since, $AB = BC = AC$

$\therefore$ ABC is an equilateral triangle

12. Prove that the points (2, 3), (-4, -6) and (1, 3/2) do not form a triangle.

Sol:

Let $A(2,3), B(-4,-6)$ and $C(1,3/2)$ be the given points

$AB = \sqrt{(-4-2)^2 + (-6-3)^2}$

$\Rightarrow AB = \sqrt{(-6)^2 + (-9)^2}$

$\Rightarrow AB = \sqrt{36+81}$

$\Rightarrow AB = \sqrt{117}$

$BC = \sqrt{(1+4)^2 + \left(\frac{3}{2}+6\right)^2}$

$\Rightarrow BC = \sqrt{(5)^2 + \left(\frac{15}{2}\right)^2}$

$\Rightarrow BC = \sqrt{25 + \frac{225}{4}}$

$\Rightarrow BC = \sqrt{\frac{325}{4}}$

$\Rightarrow BC = \sqrt{8125}$

$$AC = \sqrt{(2-1)^2 + \left(3 - \frac{3}{2}\right)^2}$$

$$\Rightarrow AC = \sqrt{(1)^2 + \left(\frac{3}{2}\right)^2}$$

$$\Rightarrow AC = \sqrt{1 + \frac{9}{4}}$$

$$\Rightarrow AC = \sqrt{\frac{13}{4}}$$

$$\Rightarrow AC = \sqrt{3.25}$$

We know that for a triangle sum of two sides is greater than the third side

Here $AC + BC$ is not greater than AB.

$\therefore ABC$ is not triangle

13. An equilateral triangle has two vertices are (2, -1), (3, 4), (-2, 3) and (-3, -2), find the coordinates of the third vertex.

Sol:

Let $A(3,4), B(-2,3)$ and $C(x, y)$ be the three vertices of the equilateral triangle then,

$$AB^2 = BC^2 = CA^2$$

$$AB = \sqrt{(-2-3)^2 + (3-4)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$$

$$BC = \sqrt{(x+2)^2 + (y-3)^2} = \sqrt{x^2 + 4 + 4x + y^2 + 9 - 6y} = \sqrt{x^2 + y^2 - 6x - 8y + 25}$$

$$CA = \sqrt{(x-3)^2 + (y-4)^2} = \sqrt{x^2 + 9 - 6x + y^2 + 16 - 8y} = \sqrt{x^2 + y^2 - 6x - 8y + 25}$$

Now, $AB^2 = BC^2$

$$\Rightarrow x^2 + y^2 + 4x - 6y + 13 = 26$$

$$\Rightarrow x^2 + y^2 + 4x - 6y - 13 = 0 \quad \text{.......(i)}$$

$$AB^2 = CA^2$$

$$\Rightarrow 26 - x^2 + y^2 - 6x - 8y + 25$$

$$\Rightarrow x^2 + y^2 - 6x - 8y - 1 = 0 \quad \text{.......(ii)}$$

Subtracting (ii) from (i) we get,

$$10x + 2y - 12 = 0$$

$$\Rightarrow 5x + y = 6 \quad \text{.......(iii)}$$

$$\Rightarrow 5x = 6 - y$$

$$\Rightarrow x = \frac{6-y}{5}$$

Subtracting $x=\frac{6-y}{5}$ in (i) we get

$$\left(\frac{6-y}{5}\right)^2+y^2+4\left(\frac{6-y}{5}\right)-6y-13=0$$

$$\Rightarrow \frac{(6-y)^2}{25}+y^2+\frac{24-4y}{5}-6y-13=0$$

$$\Rightarrow \frac{36+y^2-12y}{25}+y^2+\frac{24-4y}{5}-6y-13=0$$

$$\Rightarrow \frac{36+y^2-12y+25y^2+120-20y-150-13\times 25}{25}=0$$

$$\Rightarrow 26y^2-32y+6-325=0$$

$$\Rightarrow 26y^2-32y-319=0$$

$$D=b^2-4ac$$

$$D=(-32)^2-4\times 26\times(-319)=1024+33176=34200$$

$$\therefore \quad y=\frac{-b\pm\sqrt{D}}{2a}=\frac{-(-32)\pm\sqrt{34200}}{2\times 26}$$

$$\therefore y=\frac{32+185}{52}=\frac{217}{52} \text{ or } y=\frac{32-185}{52}=\frac{-153}{52}$$

Substituting $y=\frac{217}{52}$ in (iii)

$$5x+\frac{217}{52}=6$$

$$5x=6-\frac{217}{52}=\frac{95}{52}$$

$$x=\frac{19}{52}$$

Again substituting $y=\frac{-153}{52}$ in (iii)

$$5x-\frac{153}{52}=6$$

$$5x=6+\frac{153}{52}=\frac{465}{52}$$

$$x=\frac{93}{52}$$

Therefore, the coordinates of the third vertex are $\left(\frac{19}{52},\frac{217}{52}\right)$ or $\left(\frac{93}{52},\frac{-153}{52}\right)$

14. Show that the quadrilateral whose vertices are (2, -1) (3, 4), (-2, 3) and (-3, -2) is a rhombus.

Sol:

Let $A(2,-1), B(3,4), C(-2,3)$ and $D(-3,-2)$

$$AB=\sqrt{(3-2)^2+(4+1)^2}=\sqrt{(1)^2+(5)^2}=\sqrt{1+25}=\sqrt{26}$$

$$BC=\sqrt{(-2-3)^2+(3-4)^2}=\sqrt{(-5)^2+(-1)^2}=\sqrt{25+1}=\sqrt{26}$$

$$CD=\sqrt{(-3+2)^2+(-2-3)^2}=\sqrt{(-1)^2+(-5)^2}=\sqrt{1+25}=\sqrt{26}$$

$$AD=\sqrt{(-3-2)^2+(-2+1)^2}=\sqrt{(-5)^2+(-1)^2}=\sqrt{25+1}=\sqrt{26}$$

Since $AB=BC=CD=AD$

$\therefore$ $ABCD$ is a rhombus

15. Two vertices of an isosceles triangle are (2, 0) and (2, 5). Find the third vertex if the length of the equal sides is 3.

Sol:

Two vertices of an isosceles triangle are $A(2,0)$ and $B(2,5)$. Let $C(x,y)$ be the third vertex

$$AB=\sqrt{(2-2)^2+(5-0)^2}=\sqrt{(0)^2+(5)^2}=\sqrt{25}=5$$

$$BC=\sqrt{(x-2)^2+(y-5)^2}=\sqrt{x^2+4-4x+y^2+25-10y}=\sqrt{x^2-4x+y^2-10y+29}$$

$$AC=\sqrt{(x-2)^2+(y-0)^2}=\sqrt{x^2+4-4x+y^2}$$

Also we are given that

$AC-BC=3$

$\Rightarrow AC^2=BC^2=9$

$\Rightarrow x^2+4-4x+y^2=x^2-4x+y^2-10y+29$

$\Rightarrow 10y=25$

$\Rightarrow y=\frac{25}{10}=\frac{5}{2}$

$AC^2=9$

$x^2+4-4x+y^2=9$

$x^2+4-4x+(2.5)^2=9$

$x^2+4-4x+6.25=9$

$x^2-4x+1.25=0$

$D=(-4)^2-4\times1\times1.25$

$D = 16 - 5$
$D = 11$
$x = \dfrac{-(-4) + \sqrt{11}}{2 \times 1} = \dfrac{4 + 3.31}{2} = \dfrac{7.31}{2} = 3.65$
Or $x = \dfrac{-(-4) - \sqrt{11}}{2} = \dfrac{4 - \sqrt{11}}{2} = \dfrac{4 - 3.31}{2} = 0.35$
The third vertex is $(3.65, 2.5)$ or $(0.35, 2.5)$

16. Which point on x-axis is equidistant from (5, 9) and (-4, 6)?
Sol:
Let $A(5,9)$ and $B(-4,6)$ be the given points.
Let $C(x,0)$ be the point on $x - axis$
Now,
$AC = \sqrt{(x-5)^2 + (0-9)^2}$
$\Rightarrow AC = \sqrt{x^2 + 25 - 10x + (-9)^2}$
$\Rightarrow AC = \sqrt{x^2 - 10x + 25 + 81}$
$\Rightarrow AC = \sqrt{x^2 - 10x + 106}$
$BC = \sqrt{(x+4)^2 + (0-6)^2}$
$\Rightarrow BC = \sqrt{x^2 + 16 + 8d + (-6)^2}$
$\Rightarrow BC = \sqrt{x^2 + 8x + 16 + 36}$
$\Rightarrow BC = \sqrt{x^2 + 8x + 5x}$
Since $AC = BC$
Or, $AC^2 = BC^2$
$x^2 - 10x + 106 = x^2 + 8x + 52$
$\Rightarrow -10x + 106 = 8x + 52$
$\Rightarrow -10x - 8x = 52 - 106$
$\Rightarrow -18x = -54$
$\Rightarrow x = \dfrac{54}{18}$
$\Rightarrow x = 3$
Hence the points on x-axis is $(3,0)$.

17. Prove that the points (— 2, 5), (0, 1) and (2, — 3) are collinear.

Sol:

Let $A(-2,5), B(0,1)$ and $C(2,-3)$ be the given points

$AB=\sqrt{(0+2)^2+(1-5)^2}$

$\Rightarrow AB=\sqrt{4+(-4)^2}$

$\Rightarrow AB=\sqrt{4+16}$

$\Rightarrow AB=\sqrt{20}$

$\Rightarrow AB=2\sqrt{5}$

$BC=\sqrt{(2-0)^2+(-3-1)^2}$

$\Rightarrow BC=\sqrt{(2)^2+(-4)^2}$

$\Rightarrow BC=\sqrt{4+16}$

$\Rightarrow BC=\sqrt{20}$

$\Rightarrow BX=2\sqrt{5}$

$AC=\sqrt{(2+2)^2+(-3-5)^2}$

$\Rightarrow AC=\sqrt{(4)^2+(-8)^2}$

$\Rightarrow AC=\sqrt{16+64}$

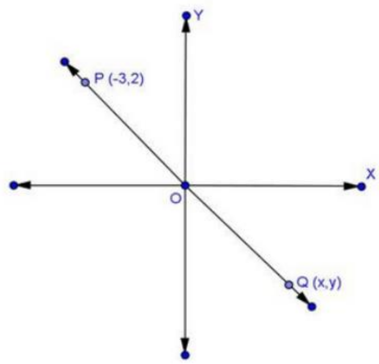
$\Rightarrow AC=\sqrt{80}$

$\Rightarrow AC=4\sqrt{5}$

Since $AB+BC=AC$

Hence $A(-2,5), B(0,1)$, and $C(2,-3)$ are collinear

18. The coordinates of the point P are (—3, 2). Find the coordinates of the point Q which lies on the line joining P and origin such that OP = OQ.



Sol:

Let the coordinates of Q be (x, y)

Since Q lies on the line joining P and O (origin) and $OP = OQ$

By mid-point theorem

$\frac{(x-3)}{2} = 0$ and $\frac{(y+2)}{2} = 0$

$\therefore x = 3, y = -2$

Hence coordinates of points Q are $(3, -2)$

19. Which point on y-axis is equidistant from (2, 3) and (—4, 1)?

Sol:

$A(2,3)$ and $B(-4,1)$ are the given points.

Let $C(0, y)$ be the points are $y-axis$

$AC = \sqrt{(0-2)^2 + (y-3)^2}$

$\Rightarrow AC = \sqrt{4 + y^2 + 9 - 6y}$

$\Rightarrow AC = \sqrt{y^2 - 6y + 13}$

$BC = \sqrt{(0+4)^2 + (y-1)^2}$

$\Rightarrow BC = \sqrt{16 + y^2 + 1 - 2y}$

$\Rightarrow BC = \sqrt{y^2 - 2y + 17}$

Since $AC = BC$

$AC^2 = BC^2$

$y^2 - 6y + 13 = y^2 - 2y + 17$

$\Rightarrow -6y + 2y = 17 - 13$

$\Rightarrow -4y = 4$

$\Rightarrow y = -1$

$\therefore$ The point on $y-axis$ is $(0, -1)$

20. The three vertices of a parallelogram are (3, 4), (3, 8) and (9, 8). Find the fourth vertex.

Sol:

Let $A(3,4), B(3,8)$ and $C(9,8)$ be the given points

Let the forth vertex be $D(x, y)$

$AB = \sqrt{(3-3)^2 + (8-4)^2}$

$\Rightarrow AB = \sqrt{0 + (4)^2}$

$\Rightarrow AB=\sqrt{16}$
$\Rightarrow AB=4$
$BC=\sqrt{(9-3)^2+(8-8)^2}$
$\Rightarrow BC=\sqrt{(6)^2+0}$
$\Rightarrow BC=\sqrt{36}$
$\Rightarrow BC=B$
$CD=\sqrt{(x-9)^2+(y-8)^2}$
$\Rightarrow CD=\sqrt{x^2+(9^2)-18x+y^2+(8^2)-16y}$
$\Rightarrow CD=\sqrt{x^2+81-18x+y^2+64-16y}$
$\Rightarrow CD=\sqrt{x^2-18x+y^2-16y+145}$
$AD=\sqrt{(x-3)^2+(y-4)^2}$
$\Rightarrow AD=\sqrt{x^2+9-6x+y^2+16-8y}$
$\Rightarrow AD=\sqrt{x^2-6x+y^2-8y+25}$
Since ABCD is a parallelogram and opposite sides of a parallelogram are equal
$AB=CD$ and $AD=BC$
$AB=CD$
$AB^2=CD^2$
$\Rightarrow x^2-18x+y^2-16y+145=16$
$\Rightarrow x^2-18x+y^2-16y+145-16=0$
$\Rightarrow x^2-18x+y^2-16y+129=0$ ………..(1)
$BC=AD$
$BC^2=AD^2$
$x^2-6x+y^2-8y+25=36$
$\Rightarrow x^2-6x+y^2-8y+25-36=0$
$\Rightarrow x^2-6x+y^2-8y-11=0$ ………..(2)
$x=9, y=4$
The fourth vertex is $D(9,4)$

21. Find the circumcenter of the triangle whose vertices are (—2, —3), (— 1, 0), (7, — 6).
Sol:

Circumcenter of a triangle is the point of intersection of all the three perpendicular bisectors of the sides of triangle. So, the vertices of the triangle lie on the circumference of the circle.

Let the coordinates of the circumcenter of the triangle be (x, y)

$\therefore (x, y)$ will the equidistant from the vertices of the triangle.

Using distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, it is obtained:

$D_1 = \sqrt{(x+2)^2 + (y+3)^2}$

$\Rightarrow D_1 = \sqrt{x^2 + 4 + 4x + y^2 + 9 + 6y}$ (Taking points (x, y) and $(-2, -3)$)

$\Rightarrow D_1 = \sqrt{x^2 + y^2 + 4x + 6y + 13}$

$D^2 = \sqrt{(x+1)^2 + (y-0)^2}$ (Taking points (x, y) and $(-1, 0)$)

$\Rightarrow D_2 = \sqrt{x^2 + 1 + 2x + y^2}$

$D_3 = \sqrt{(x-7)^2 + (y+6)^2}$ (Taking points (x, y) and $(7, -6)$)

$\Rightarrow D_3 = \sqrt{x^2 + 49 - 14x + y^2 + 36 + 12y}$

$\Rightarrow D_3 = \sqrt{x^2 + y^2 - 14x + 12y + 85}$

As (x, y) is equidistant from all the three vertices

So, $D_1 = D_2 = D_3$

$D_1 = D_2$

$\therefore \sqrt{x^2 + y^2 + 4x + 6y + 13} = \sqrt{x^2 + 1 + 2x + y^2}$

$\Rightarrow x^2 + y^2 + 4x + 6y + 13 = x^2 + 1 + 2x + y^2$

$\Rightarrow 4x + 6y - 2x = 1 - 13$

$\Rightarrow 2x + 6y = -12$

$\Rightarrow x + 3y = -6$ ……….(1)

$D_2 = D_3$

$\therefore \sqrt{x^2 + 1 + 2x + y^2} = \sqrt{x^2 + y^2 - 14x + 12y + 85}$

$\Rightarrow x^2 + 1 + 2x + y^2 = x^2 + y^2 - 14x + 12y + 85$

$\Rightarrow 2x + 14x - 12y = 85 - 1$

$\Rightarrow 16x - 12y = 84$

$\Rightarrow 4x - 3y = 21$ ……….(2)

Adding equations (1) and (2):

$x + 3y + 4x - 3y = -6 + 21$

$\therefore 5x = 15$

$\Rightarrow x = \frac{15}{3}$

$\Rightarrow x = 3$

When $x = 3$, we get

$y = \frac{4(3) - 21}{3}$ [Using (2)]

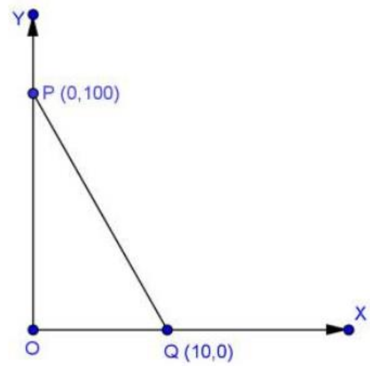
$\Rightarrow y = \frac{12 - 21}{3}$

$\Rightarrow y = -\frac{9}{3}$

$\Rightarrow y = -3$

$\therefore (3, -3)$ are the coordinates of the circumcenter of the triangle

22. Find the angle subtended at the origin by the line segment whose end points are (0, 100) and (10, 0).



Sol:

Let the point $P(0,100)$ and $Q(10,0)$ be the given points.

$\therefore$ The angle subtended by the line segment PQ at the origin O is $90°$.

23. Find the centre of the circle passing through (5, — 8), (2, — 9) and (2, 1).

Sol:

Let the center of the circle be $O(x, y)$

Since radii of the circle is constant

Hence, distance of O from $A(5, -8), B(2, -9)$ and $C\ (2,1)$ will be constant and equal

$\therefore OA^2 = OB^2 = OC^2$

$(x-5)^2 + (y+B)^2 = (x-2)^2 + (y+9)^2$

$x^2 + 25 - 10x + y^2 + 64 + 16y = x^2 + 4 - 4x + y^2 + 81 + 18y$

$-6x-2y+4=0$

$3x+y-2=0$

$y=2-3x$ …………(i)

Also, $OB^2=OC^2$

$(x-2)^2+(y+9)^2+(y-1)^2$

$y^2+81+18y=y^2+1-2y$

$80+20y=0$

$y=-4$

Substituting y in (i)

$-4=2-3x$

$3x=6$

$x=2$

Hence center of circle $(2,-4)$

24. Find the value of k, if the point P (0, 2) is equidistant from (3, k) and (k, 5).

Sol:

Let the point $P(0,2)$ is equidistant from $A(3,k)$ and $(k,5)$

$PA=PB$

$PA^2=PB^2$

$(3-0)^2+(k-2)^2=(k-0)^2+(5-2)^2$

$\Rightarrow 9+k^2+4-4k=k^2+9.$

$\Rightarrow 9+k^2+4-4k-k^2-9=0$

$\Rightarrow 4-4k=0$

$\Rightarrow -4k=-4$

$\Rightarrow k=1$

25. If two opposite vertices of a square are (5, 4) and (1, —6), find the coordinates of its remaining two vertices.

Sol:

Let $ABCD$ be a square and let $A(5,4)$ and $C(1,-6)$ be the given points.

Let (x,y) be the coordinates of B.

$AB=BC$

$AB^2=BC^2$

$(x-5)^2+(y-4)^2=(x-1)^2+(y+6)^2$

$\Rightarrow x^2+25-10x+y^2+16-8y=x^2+1-2x+y^2+36+12y$

$\Rightarrow x^2 - 10x + y^2 - 8y - x^2 + 2x - y^2 - 12y = 1 + 36 - 25 - 16$

$\Rightarrow -8x - 20y = -4$

$\Rightarrow -8x = 20y - 4$

$\Rightarrow x = \dfrac{20y - 4}{-8}$

$\Rightarrow x = \dfrac{4(5y-1)}{-8}$

$\Rightarrow x = \dfrac{5y-1}{-2}$

$\Rightarrow x = \dfrac{1-5y}{2}$ ………..(1)

In right triangle ABC

$AB^2 + BC^2 = AC^2$

$(x-5)^2 + (y-4)^2 + (x-1)^2 + (y+6)^2 = (5-1)^2 + (4+6)^2$

$\Rightarrow x^2 + 25 - 10x + y^2 + 16 - 8y + x^2 + 1 - 2x + y^2 + 36 + 12y = 16 + 100$

$\Rightarrow 2x^2 + 2y^2 - 12x + 4y = 116 - 78$

$\Rightarrow 2x^2 + 2y^2 - 12 + 4y = 38$

$\Rightarrow x^2 + y^2 - 6x + 2y = 19$

$\Rightarrow x^2 + y^2 - 6x + 2y - 19 = 0$ ………..(2)

Substituting the value of x form (1) in (2), we get

$\left(\dfrac{1-5y}{2}\right)^2 + y^2 - 6\left(\dfrac{1-5y}{2}\right) + 2y - 19 = 0$

$\Rightarrow \dfrac{(1-5y)^2}{4} + y^2 - 3(1-5y) + 2y - 19 = 0$

$\Rightarrow \dfrac{1 + 25y^2 - 10y}{4} + y^2 - 3 + 15y + 2y - 19 = 0$

$\Rightarrow \dfrac{1 + 25y^2 - 10y + 4y^2 - 12 + 60y + 8y - 76}{4} = 0$

$\Rightarrow 29y^2 + 58y - 87 = 0$

$\Rightarrow y^2 + 2y - 3 = 0$

$\Rightarrow y^2 + 3y - y - 3 = 0$

$\Rightarrow y(y+3) - 1(y+3) = 0$

$\Rightarrow (y+3)(y-1) = 0$

$\Rightarrow y = -3, y = 1$

Substituting $y = -3$ and $y = 1$ in equation (1), we get

$$x = \frac{1-5(-3)}{2}$$

$$\Rightarrow x = \frac{1+15}{2}$$

$$\Rightarrow x = 8$$

$$x = \frac{1-5(1)}{2}$$

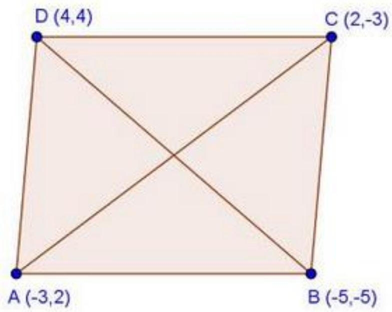
$$\Rightarrow x = \frac{1-5}{2}$$

$$\Rightarrow x = \frac{-4}{2}$$

$$\Rightarrow x = -2$$

Hence, the required vertices of the square are $(-2,1)$ and $(8,-3)$.

26. Show that the points (—3, 2), (—5, —5), (2, —3) and (4, 4) are the vertices of a rhombus. Find the area of this rhombus.



Sol:

$A(-3,2), B(-5,-5), C(2,-3)$ and $D(4,4)$ be the given points.

$$AB = \sqrt{(-5+3)^2 + (-5-2)^2}$$

$$\Rightarrow AB = \sqrt{(2)^2 + (-7)^2}$$

$$\Rightarrow AB = \sqrt{4+49}$$

$$\Rightarrow AB = \sqrt{53}$$

$$BC = \sqrt{(2+5)^2 + (-5-2)^2}$$

$$\Rightarrow BC = \sqrt{(7)^2 + (2)^2}$$

$$\Rightarrow BC = \sqrt{49+4}$$

$$\Rightarrow BC = \sqrt{53}$$

$CD=\sqrt{(4-2)^2+(4+3)^2}$

$\Rightarrow CD=\sqrt{(2)^2+(7)^2}$

$\Rightarrow CD=\sqrt{4+49}$

$\Rightarrow CD=\sqrt{53}$

$AD=\sqrt{(4+3)^2+(4-2)^2}$

$\Rightarrow AD=\sqrt{(7)^2+(2)^2}$

$\Rightarrow AD=\sqrt{49+4}$

$\Rightarrow AD=\sqrt{53}$

$AC=\sqrt{(2+3)^2+(-3-2)^2}$

$\Rightarrow AC=\sqrt{(5)^2+(-5)^2}$

$\Rightarrow AC=\sqrt{25+25}$

$\Rightarrow AC=\sqrt{50}$

$BD=\sqrt{(4+5)^2+(4+5)^2}$

$\Rightarrow BD=\sqrt{(9)^2+(9)^2}$

$\Rightarrow BD=\sqrt{81+81}$

$\Rightarrow BD=\sqrt{162}$

Since $AB=BC=CD=AD$ and diagonals $AC\neq BD$

$\therefore ABCD$ is a rhombus

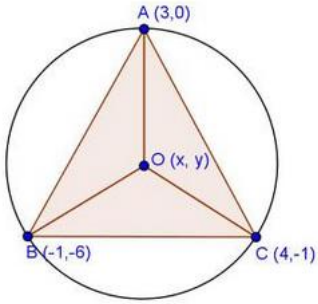
Area of rhombus $ABCD=\frac{1}{2}\times AC\times BD$

$=\frac{1}{2}\times\sqrt{50}\times\sqrt{162}$

$=\frac{1}{2}\times 90$

$=45$ sq. units

27. Find the coordinates of the circumcenter of the triangle whose vertices are (3, 0), (—1, —6) and (4,—1). Also, find its circumradius.



Sol:

Let $A(3,0), B(-1,-6)$ and $C(4,-1)$ be the given points.

Let $O(x,y)$ be the circumcenter of the triangle

$OA = OB = OC$

$OA^2 = OB^2$

$(x-3)^2 + (y-0)^2 = (x+1)^2 + (y+6)^2$

$\Rightarrow x^2 + 9 - 6x + y^2 = x^2 + 1 + 2x + y^2 + 36 + 12y$

$\Rightarrow x^2 - 6x + y^2 - x^2 - 2x - y^2 - 12y = 1 + 36 - 9$

$\Rightarrow -8x - 12y = 28$

$\Rightarrow -2x - 3y = 7$

$\Rightarrow 2x + 3y = -7$ ....(1)

Again

$OB^2 = OC^2$

$(x+1)^2 + (y+6)^2 = (x-4)^2 + (y+1)^2$

$\Rightarrow x^2 + 1 + 2x + y^2 + 36 + 12y = x^2 + 16 - 8x + y^2 + 1 + 2y$

$\Rightarrow x^2 + 2x + y^2 + 12y - x^2 + 8x - y^2 - 2y = 16 + 1 - 1 - 36$

$\Rightarrow 10x + 10y = -20$

$\Rightarrow x + y = -2$ ....(2)

Solving (1) and (2), we get

$x = 1, y = -3$

Hence circumcenter of the triangle is $(1,-3)$

Circum radius $= \sqrt{(1+1)^2 + (-3+6)^2}$

$= \sqrt{(2)^2 + (3)^2}$

$= \sqrt{4+9}$

$\sqrt{13}$ units

28. Find a point on the x-axis which is equidistant from the points (7, 6) and (—3, 4).

Sol:

Let $A(7,6)$ and $B(-3,4)$ be the given points.

Let $P(x.0)$ be the point on $x-axis$ such that $PA = PB$

$PA = PB$

$PA^2 = PB^2$

$(x-7)^2 + (0-6)^2 = (x+3)^2 + (0-4)^2$

$\Rightarrow x^2 + 49 - 14x + 36 = x^2 + 9 + 6x + 16$

$\Rightarrow x^2 - 14 - x^2 - 6x = 9 + 16 - 36 - 49$

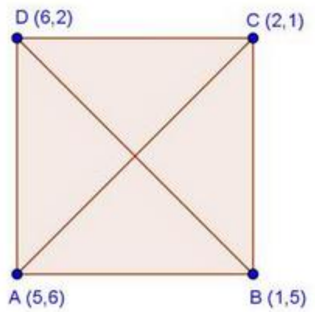
$\Rightarrow -20x = -60$

$\Rightarrow x = 3$

$\therefore$ The point on $x-axis$ is $(3,0)$.

29. (i) Show that the points A(5, 6), B (1, 5), C(2, 1)and D(6, 2) are the vertices of a square.

(ii) Prove that the points A (2, 3), B (—2, 2), C (—1, —2), and D (3, —1) are the vertices of a square ABCD.



Sol:

$A(5,6), B(1,5), C(2,1)$ and $D(6,2)$ are the given points

$AB = \sqrt{(5-1)^2 + (6-5)^2}$

$\Rightarrow AB = \sqrt{(4)^2 + (1)^2}$

$\Rightarrow AB = \sqrt{16+1}$

$\Rightarrow AB = \sqrt{17}$

$BC = \sqrt{(1-2)^2 + (5-1)^2}$

$\Rightarrow BC = \sqrt{(-1)^2 + (4)^2}$

$\Rightarrow BC = \sqrt{1+16}$

$\Rightarrow BC = \sqrt{17}$

$CD=\sqrt{(6-2)^2+(2-1)^2}$

$\Rightarrow CD=\sqrt{(4)^2+(1)^2}$

$\Rightarrow CD=\sqrt{16+1}$

$\Rightarrow CD=\sqrt{17}$

$AD=\sqrt{(6-5)^2+(2-6)^2}$

$\Rightarrow AD=\sqrt{(1)^2+(-4)^2}$

$\Rightarrow AD=\sqrt{1+16}$

$\Rightarrow AD=\sqrt{17}$

$AC=\sqrt{(5-2)^2+(6-1)^2}$

$\Rightarrow AC=\sqrt{(3)^2+(5)^2}$

$\Rightarrow AC=\sqrt{9+25}$

$\Rightarrow AC=\sqrt{34}$

$BD=\sqrt{(6-1)^2+(2-5)^2}$

$\Rightarrow BD=\sqrt{(5)^2+(-3)^2}$

$\Rightarrow BD=\sqrt{25+9}$

$\Rightarrow BD=\sqrt{34}$

Since $AB=BC=CD=AD$ and diagonals $AC=BD$

$\therefore ABCD$ is a square

30. Find the point on x-axis which is equidistant from the points (—2, 5) and (2,—3).

Sol:

Let $A(-2,5)$ and $(2,-3)$ be the given points.

Let $(x,0)$ be the point on $x-axis$

Such that $PA=PB$

$PA=PB$

$PA^2=PB^2$

$(x+2)^2+(0-5)^2=(x-2)^2+(0+3)^2$

$\Rightarrow x^2+4+4x+25=x^2+4-4x+9$

$\Rightarrow x^2+4x+x^2+4x=4+9-4-25$

$\Rightarrow 8x=-16$

$\Rightarrow x=-2$

$\therefore$ The point on $x-axis$ is $(-2,0)$

31. Find the value of x such that PQ = QR where the coordinates of P, Q and R are (6,—1), (1, 3) and (x, 8) respectively.

Sol:

$P(6,-1), Q(1,3)$ and $R(x,8)$ are the given points.

$PQ = QR$

$PQ^2 = QR^2$

$\Rightarrow (6-1)^2 + (-1-3)^2 = (x-1)^2 + (8-3)^2$

$\Rightarrow (5)^2 + (-4)^2 = x^2 + 1 - 2x + (5)^2$

$\Rightarrow 25 + 16 = x^2 + 1 - 2x + 25$

$\Rightarrow 41 = x^2 - 2x + 26$

$\Rightarrow x^2 - 2x + 26 - 41 = 0$

$\Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow x^2 - 5x + 3x - 15 = 0$

$\Rightarrow x(x-5) + 3(x-5) = 0$

$\Rightarrow (x+3)(x-5) = 0$

$\Rightarrow x = -3$ or $x = 5$

32. Prove that the points (0, 0), (5, 5) and (—5, 5) are the vertices of a right isosceles triangle.

Sol:

Let $A(0,0), B(5,5)$ and $C(-5,5)$ be the given points

$AB = \sqrt{(5-0)^2 + (5-0)^2}$

$\Rightarrow AB = \sqrt{25+25}$

$\Rightarrow AB = \sqrt{50}$

$BC = \sqrt{(5+5)^2 + (5-5)^2}$

$\Rightarrow BC = \sqrt{(10)^2 + 0}$

$\Rightarrow BC = \sqrt{100}$

$AC = \sqrt{(0+5)^2 + (0-5)^2}$

$\Rightarrow AC = \sqrt{25+25}$

$\Rightarrow AC = \sqrt{50}$

$AB^2 = 50$

$BC^2 = 100$
$AC^2 = 50$
$\Rightarrow AB^2 + AC^2 = BC^2$
Since, $AB = AC$ and $AB^2 + AC^2 = BC^2$
$\therefore ABC$ is a right isosceles triangle

33. If the point P(x, y) is equidistant from the points A(5, 1) and B (1, 5), prove that x = y.
Sol:
Since $P(x, y)$ is equidistant from $A(5,1)$ and $B(1,5)$
$AP = BP$
Hence, $AP^2 = BP^2$
$(x-5)^2 + (y-1)^2 = (x-1)^2 + (y-5)^2$
$x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 1 - 2x + y^2 + 25 - 10y$
$-10x + 2x = -10y + 2y$
$-8x = -8y$
$x = y$
Hence, proved.

34. If Q (0, 1) is equidistant from P (5, —3) and R (x, 6), find the values of x. Also, find the distances QR and PR
Sol:
Given $Q(0,1)$ is equidistant from $P(-5,-3)$ and $R(x,6)$ so $PQ = QR$
$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(0-x)^2 + (1-6)^2}$
$\sqrt{(5)^2 + (-4)^2} = \sqrt{(-x)^2 + (-5)^2}$
$\sqrt{25+16} = \sqrt{x^2 + 25}$
$41 = x^2 + 25$
$16 = x^2$
$x = \pm 4$
So, point R is $(4,6)$ or $(-4,6)$
When point R is $(4,6)$
$PR = \sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{1^2 + (-9)} = \sqrt{1+81} = \sqrt{82}$
$QR = \sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$
When point R is $(-4,6)$

$$PR=\sqrt{\left(5-\left(-4\right)\right)^2+\left(-3-6\right)^2}=\sqrt{\left(9\right)^2+\left(-9\right)^2}=\sqrt{81+81}=9\sqrt{2}$$

$$QR=\sqrt{\left(0\left(-4\right)\right)^2+\left(1-6\right)^2}=\sqrt{\left(4\right)^2+\left(-5\right)^2}=\sqrt{16+25}=\sqrt{41}$$

35. Find the values of y for which the distance between the points P (2, —3) and Q (10, y) is 10 units

Sol:

Given that distance between $(2,-3)$ and $(10,y)$ is 10

Therefore using distance formula $\sqrt{\left(2-10\right)^2+\left(-3-y\right)^2}=10$

$\sqrt{\left(-8\right)^2+\left(3+y\right)^2}=10$

$64+\left(y-3\right)^2=100$

$\left(y+3\right)^2=36$

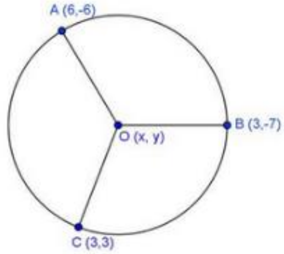
$y+3=\pm 6$

$y+3=6$ or $y+3=-6$

Therefore $y=3$ or -9

36. Find the centre of the circle passing through (6, —6), (3, —7) and (3, 3)

Sol:



Let $O(x,y)$ be the center of the circle passing through $A(6,-6), B(3,-7)$ and $C(3,3)$

$OA=OB=OC$

$OA^2=OB^2$

$\left(x-6\right)^2+\left(y+6\right)^2=\left(x-3\right)^2+\left(y-7\right)^2$

$\Rightarrow x^2+36-12x+y^2+36+12y=x^2+9-6x+y^2+49+14y$

$\Rightarrow x^2+36-12x+y^2+36+12y-x^2-9+6x-y^2-49-14y=0$

$\Rightarrow -6x-2y=-36-36+9+49$

$\Rightarrow -6x-2y=-14$ ....(1)

$OB^2=OC^2$

$(x-3)^2+(y+7)^2=(x-3)^2+(y-3)^2$

$\Rightarrow x^2+9-6x+y^2+49+14y=x^2+9-6x+y^2+9-6y$

$\Rightarrow x^2-6x+y^2+14y-x^2+6x-y^2+6y=9+9-9-49$

$\Rightarrow 20y=-40$

$\Rightarrow y=-2$

Substituting $y=-2$ in (1)

$-6x-2(-2)=-14$

$\Rightarrow -6x+4=-14$

$\Rightarrow -6x=-14-4$

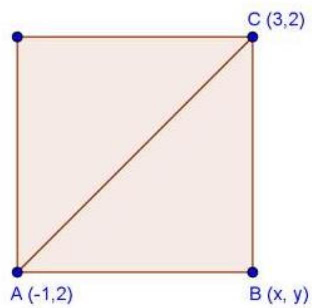
$\Rightarrow -6x=-18$

$\Rightarrow x=3$

$\therefore$ The centre of the circle is $(3,-2)$

37. Two opposite vertices of a square are (—1, 2) and (3, 2). Find the coordinates of other two vertices.

Sol:



Let $ABCD$ be a square and let $A(-1,2)$ and $(3,2)$ be the opposite vertices and let $B(x,y)$ be the unknown vertex.

$AB=BC$

$AB^2=BC^2$

$(x+1)^2+(y-2)^2=(x-3)^2+(y-2)^2$

$\Rightarrow x^2+1+2x+y^2+4-4y=x^2+9-6x+y^2+4-4y$

$\Rightarrow x^2+2x+y^2-4y-x^2+6x-y^2+4y=9+4-1-4$

$\Rightarrow 8x=8$

$\Rightarrow x=1$ ....(1)

In right triangle ABC

$AB^2+BC^2=AC^2$

$\Rightarrow (x+1)^2 + (y-2)^2 + (x-3)^2 + (y-2)^2 = (3+1)^2 + (2-2)^2$

$\Rightarrow x^2 + 1 + 2x + y^2 + 4 - 4y + x^2 + 9 - 6x + y^2 + 4 - 4y = 16$

$\Rightarrow 2x^2 + 2y^2 - 4x - 8y = 16 - 1 - 4 - 9 - 4$

$\Rightarrow 2x^2 + 2y^2 - 4x - 8y = -2$ ....(2)

Substituting $x=1$ from (1) and (2)

$2(1)^2 + 2y^2 - 4(1) - 8y = -2$

$\Rightarrow 2 + 2y^2 - 4 - 8y = -2$

$\Rightarrow 2y^2 - 8y - 2 + 2 = 0$

$\Rightarrow 2y^2 - 8y = 0$

$\Rightarrow 2y(y-4) = 0$

$\Rightarrow y = 0,$ or $y = 4$

Hence the required vertices of the square are $(1,0)$ and $(1,4)$

38. Name the quadrilateral formed, if any, by the following points, and give reasons for your answers:

(i) A (—1, — 2), B (1, 0), C (—1, 2), D (—3, 0)

(ii) A (—3, 5), B (3, 1), C (0, 3), D (—1, — 4)

(iii) A (4, 5), B (7, 6), C (4, 3), D (1, 2)

Sol:

(i) Let, $A=(-1,-2), B=(1,0), C=(-1,2,), D(-3,0)$

$AB = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$CD = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$AD = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

Diagonal $AC - \sqrt{(-1-(-1))^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$

Diagonal $BD - \sqrt{(1-(-3))^2 + (0-0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$

Here, all sides of this quadrilateral are of same length and also diagonals are of same length. So, given points are vertices of a square

(ii) Let, $A=(-3,5), B=(3,1), C=(0,3), D=(-1,-4)$

$AB = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$BC=\sqrt{(3-0)^2+(1-3)^2}=\sqrt{(3)^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$

$CD=\sqrt{(0-(-1))^2+(3-(-4))^2}=\sqrt{(1)^2+(7)^2}=\sqrt{1+49}=\sqrt{50}=5\sqrt{2}$

$AD=\sqrt{(-3-(-1))^2+(5-(-4))^2}=\sqrt{(-2)^2+(9)^2}=\sqrt{4+81}=\sqrt{85}$

Here, all sides of this quadrilateral are of different length . So, we can say that it is only a general quadrilateral not specific like square, rectangle etc.

(iii) Let, $A=(4,5),B=(7,6),C=(4,3),D=(1,4)$

$AB=\sqrt{(4-7)^2+(5-6)^2}=\sqrt{(-3)^2+(-1)^2}=\sqrt{9+1}=\sqrt{10}$

$BC=\sqrt{(7-4)^2+(6-3)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}$

$CD=\sqrt{(4-1)^2+(3-2)^2}=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}$

$AD=\sqrt{(4-1)^2+(5-2)^2}=\sqrt{(3)^2+(3)^2}=\sqrt{9+9}=\sqrt{18}$

Diagonal $AC-\sqrt{(4-4)^2+(5-3)^2}=\sqrt{(0)^2+(2)^2}=\sqrt{0+4}=2$

Diagonal $BD-\sqrt{(7-1)^2+(6-2)^2}=\sqrt{(6)^2+(4)^2}=\sqrt{36+16}=\sqrt{52}=13\sqrt{2}$

Here, opposite sides of this quadrilateral are of same length but diagonals are different length . So, given points are vertices of a parallelogram.

39. Find the equation of the perpendicular bisector of the line segment joining points (7,1) and (3, 5).

Sol:

Bisector passes through midpoint

Midpoint of $(7,1)$ and $(3,5)=\left[\frac{(7+3)}{2},\frac{(1+5)}{2}\right]=(5,3)$

Perpendicular bisector has slope that is negative reciprocal of line segment joining points $(7,1)$ and $(3,5)$

Slope of line segment $=\left(\frac{5-1}{3-7}\right)=\frac{4}{(-4)}=-1$

Perpendicular bisector has slope = 1 and passes through point $(4,4)$

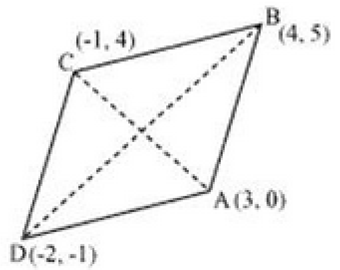
Use point slope form

$y-3=1(x-5)$

$y=x\cdot 2$

40. Prove that the points (3, 0), (4, 5), (—1, 4) and (—2, —1), taken in order, form a rhombus. Also, find its area.

Sol:



Let the given vertices be $A(3,0), B(4,5), C(-1,4)$ and $D(-2,-1)$

Length of $AB = \sqrt{(4-3)^2 + (5-0)^2} = \sqrt{1+25} = \sqrt{26}$

Length of $BC = \sqrt{(-1-4)^2 + (4-5)^2} = \sqrt{25+1} = \sqrt{26}$

Length of $CD = \sqrt{(-2+1)^2 + (-1-4)^2} = \sqrt{1+25} = \sqrt{26}$

Length of $DA = \sqrt{(3+2)^2 + (0+1)^2} = \sqrt{25+1} = \sqrt{26}$

Length of diagonal $AC = \sqrt{\left[3-(-1)^2 + (0-4)^2\right]}$

$= \sqrt{16+16} = 4\sqrt{2}$

Length of diagonal $BD = \sqrt{\left[4-(-2)^2\right] + \left[5-(-1)\right]^2}$

$= \sqrt{36+36} = 6\sqrt{2}$

Here all sides of the quadrilateral ABCD are off same lengths but the diagonals are of different lengths

So, ABCD is a rhombus.

Therefore area of rhombus $ABCD = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$

$= 24$ square units

41. In the seating arrangement of desks in a classroom three students Rohini, Sandhya and Bina are seated at A (3, 1), B (6, 4) and C (8, 6). Do you think they are seated in a line?

Sol:

Let $A(3,1), B(6,4)$ and $C(8,6)$ be thee given points

$AB = \sqrt{(6-3)^2 + (4-1)^2}$

$\Rightarrow AB = \sqrt{(3)^2 + (3)^2}$

$\Rightarrow AB = \sqrt{9+9}$

$\Rightarrow AB = \sqrt{18}$

$\Rightarrow AB = 3\sqrt{2}$

$BC = \sqrt{(8-6)^2 + (6-4)^2}$

$\Rightarrow BC = \sqrt{(2)^2 + (2)^2}$

$\Rightarrow BC = \sqrt{4+4}$

$\Rightarrow BC = \sqrt{8}$

$\Rightarrow BC = 2\sqrt{2}$

$AC = \sqrt{(8-3)^2 + (6-1)^2}$

$\Rightarrow AC = \sqrt{(5)^2 + (5)^2}$

$\Rightarrow AC = \sqrt{25+25}$

$\Rightarrow AC = \sqrt{50}$

$\Rightarrow AC = 5\sqrt{2}$

Since, $AB + BC = AC$

Points A, B, C are collinear

Hence, Rohini, Sandhya and Bina are seated in a line

42. Find a point on y-axis which is equidistant from the points (5, — 2) and (— 3, 2).

Sol:

Let $A(5,-2)$ and $B(-3,2)$ be the given points,

Let $P(0,y)$ be the point on $y-axis$

$PA = PB$

$PA^2 = PB^2$

$(0-5)^2 + (y+2)^2 = (0+3)^2 + (y-2)^2$

$\Rightarrow 25 + y^2 + 4 + 4y = 9 + y^2 + 4 - 4y$

$\Rightarrow y^2 + 4y - y^2 + 4y = 9 + 4 - 4 - 25$

$\Rightarrow 8y = -16$

$\Rightarrow y = -2$

43. Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (—3, 4).

Sol:

Point (x, y) is equidistant form $(3,6)$ and $(-3,4)$

Therefore $\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x-(-3))^2+(y-4)^2}$

$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x+3)^2+(y-4)^2}$

$(x-3)^2+(y-6)^2=(x+3)^2+(y-4)^2$

$x^2+9-6x+y^2+36-12y=x^2+9+6x+y^2+16-8y$

$36-16=6x+6x+12y-8y$

$20=12x+4y$

$3x+y=5$

44. If a point A (0, 2) is equidistant from the points B (3, p) and C (p, 5), then find the value of p.

Sol:

$A(0,2), B(3,P)$ and $C(p,5)$ are given points

It is given that $AB=AC$

$\therefore AB^2=AC^2$

$(3-0)^2+(p-2)^2=(p-0)^2+(5-2)^2$

$9+p^2+4-4p=p^2+9$

$4-4p=0$

$p=1$

Exercise 14.3

1. Find the coordinates of the point which divides the line segment joining (— 1, 3) and (4, — 7) internally in the ratio 3 : 4.

Sol:

Let $P(x, y)$ be the required point.

$x=\frac{mx_2+nx_1}{m+n}$

$y=\frac{my_2+ny_1}{m+n}$

Here, $x_1=-1$

$y_1=3$

$x_2=4$

$y_2=-7$

$m:n=3:4$

$x = \frac{3 \times 4 + 4 \times (-1)}{3+4} 3$

$x = \frac{12-4}{7}$

$x = \frac{8}{7}$

$y = \frac{3 \times (-7) + 4 \times 3}{3+4}$

$y = \frac{-21+12}{7}$

$y = \frac{-9}{7}$

$\therefore$ The coordinates of P are $\left(\frac{8}{7}, \frac{-9}{7}\right)$

2. Find the points of trisection of the line segment joining the points:

(i) (5, —6) and (— 7, 5),

(ii) (3, —2) and (—3, —4)

(iii) (2, —2) and (—7, 4).

Sol:

(i) Let P and Q be the point of trisection of AB i.e., $AP = PQ = QB$

$(5, -6)$ $(-7, 5)$

Therefore, P divides AB internally in the ratio of 1:2, thereby applying section formula, the coordinates of P will be

$\left(\frac{1(-7)+2(5)}{1+2}\right), \left(\frac{1(5)+2(-6)}{1+2}\right) i.e., \left(1, \frac{-7}{3}\right)$

Now, Q also divides AB internally in the ratio of 2:1 there its coordinates are

$\left(\frac{2(-7)+1(5)}{2+1}\right), \frac{2(5)+1(-6)}{2+1} i.e., \left(-3, \frac{4}{3}\right)$

(ii)

Let P, Q be the point of tri section of AB i.e.,

$AP = PQ = QB$

$(3,-2)$ $(-3,-4)$

Therefore, P divides AB internally in the ratio of 1:2

Hence by applying section formula, Coordinates of P are

$$\left(\left(\frac{1(-3)+2(3)}{1+2}\right),\frac{1(-4)+1(-2)}{1+2}\right) i.e., \left(1,\frac{-8}{3}\right)$$

Now, Q also divides as internally in the ratio of 2:1

So, the coordinates of Q are

$$\left(\left(\frac{2(-3)+1(3)}{2+1}\right),\frac{2(-4)+1(-2)}{2+1}\right) i.e., \left(-1,\frac{-10}{3}\right)$$

Let P and Q be the points of trisection of AB i.e., $AP = PQ = OQ$

Therefore, P divides AB internally in the ratio 1 : 2. Therefore, the coordinates of P, by applying the section formula, are

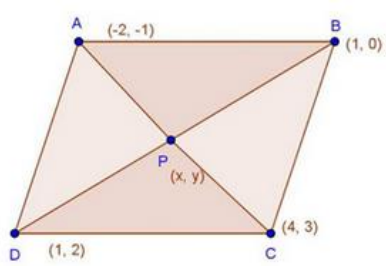
$$\left(\left(\frac{1(-7)+2(2)}{(1+2)}\right),\left(\frac{1(4)+2(-2)}{(1+2)}\right)\right), i.e., (-1,0)$$

Now, Q also divides AB internally in the ration 2 : 1. So, the coordinates of Q are

$$\left(\frac{2(-7)+1(2)}{2+1},\frac{2(4)+1(2)}{2+1}\right), i.e., (-4,2)$$

3. Find the coordinates of the point where the diagonals of the parallelogram formed by joining the points (—2, —1), (1, 0), (4, 3) and (1, 2) meet.

Sol:



Let P(x, y) be the given points.

We know that diagonals of a parallelogram bisect each other.

$x=\frac{-2+4}{2}$

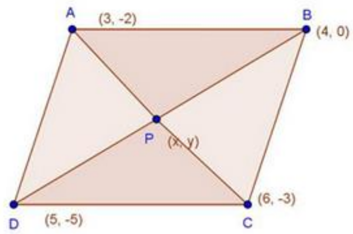
$\Rightarrow x=\frac{2}{2}=1$

$y=\frac{-1+3}{2}=\frac{2}{2}=1$

$\therefore$ Coordinates of P are $(1,1)$

4. Prove that the points (3, —2), (4, 0), (6, —3) and (5, —5) are the vertices of a parallelogram.

Sol:



Let $P(x, y)$ be the point of intersection of diagonals AC and 80 of $ABCD$.

$x=\frac{3+6}{2}=\frac{9}{2}$

$y=\frac{-2-3}{2}=\frac{-5}{2}$

Mid – point of $AC=\left(\frac{9}{5},\frac{-5}{2}\right)$

Again,

$x=\frac{5+4}{2}=\frac{9}{2}$

$y=\frac{-5+0}{2}=\frac{-5}{2}$

Mid – point of $BD=\left(\frac{9}{2},-\frac{5}{2}\right)$

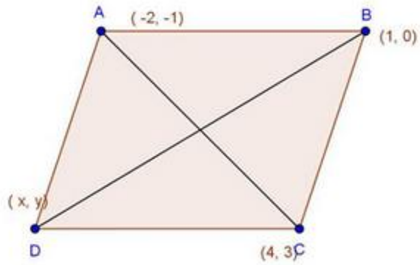
Here mid-point of AC – Mid - point of BD i.e, diagonals AC and BD bisect each other.

We know that diagonals of a parallelogram bisect each other

$\therefore$ $ABCD$ is a parallelogram.

5. Three consecutive vertices of a parallelogram are (—2, —1), (1, 0) and (4, 3). Find the fourth vertex.

Sol:



Let $A(-2,-1), B(1,0), C(4,3)$ and $D(x,y)$ be the vertices of a parallelogram *ABCD* taken in order.

Since the diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of the mid - point of *AC* = Coordinates of the mid-point of *BD.*

$\Rightarrow \frac{-2+4}{2} = \frac{1+x}{2}$

$\Rightarrow \frac{2}{2} = \frac{x+1}{2}$

$\Rightarrow 1 = \frac{x+1}{2}$

$\Rightarrow x+1 = 2$

$\Rightarrow x = 1$

And, $\frac{-1+3}{2} = \frac{y+0}{2}$

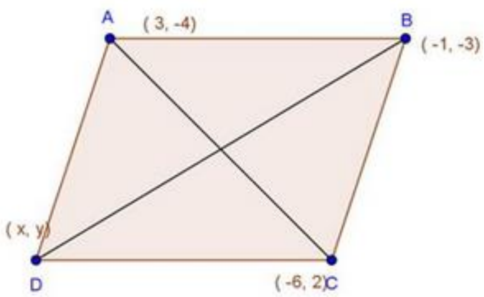
$\Rightarrow \frac{2}{2} = \frac{y}{2}$

$\Rightarrow y = 2$

Hence, fourth vertex of the parallelogram is $(1,2)$

6. The points (3, —4) and (—6, 2) are the extremities of a diagonal of a parallelogram. If the third vertex is (—1, —3). Find the coordinates of the fourth vertex.

Sol:



Let $A(3,-4)$ and $C(-6,-2)$ be the extremities of diagonal *AC* and $B(-1,-3)$, $D(x,y)$ be the extremities of diagonal *BD.*

Since the diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of mid-point of *AC* = Coordinates of mid point of *BD.*

$\Rightarrow \frac{3-6}{2} = \frac{x-1}{2}$

$\Rightarrow \frac{-3}{2} = \frac{x-1}{2}$

$\Rightarrow x = -2$

And, $\frac{-4+2}{2} = \frac{y-3}{2}$

$\Rightarrow \frac{-2}{2} = \frac{y-3}{2}$

$\Rightarrow y = 1$

Hence, fourth vertex of parallelogram is $(-2,1)$

7. Find the ratio in which the point (2, y) divides the line segment joining the points A (— 2, 2) and B (3, 7). Also, find the value of y.

Sol:

Let the point $P(2,y)$ divide the line segment joining the points $A(-2,2)$ and $B(3,7)$ in the ratio $K:1$

Then, the coordinates of *P* are

$\left[\frac{3k+(-2)\times 1}{k+1}, \frac{7k+2\times 1}{k1}\right]$

$= \left[\frac{3k-2}{k+1}, \frac{7k+2}{k+1}\right]$

But the coordinates of *P* are given as $(2,y)$

$\therefore \frac{3k-2}{k+1} = 2$

$\Rightarrow 3k - 2 = 2k + 2$

$\Rightarrow 3k - 2k = 2 + 2$

$\Rightarrow k = 4$

$\frac{7k+2}{k+1} = y$

Putting the value of k, we get

$\frac{7 \times 4 + 2}{4+1} = y$

$\frac{30}{5} = y$

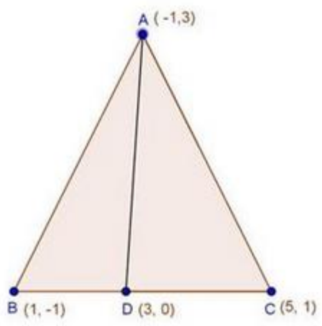
$6 = y$

i.e., $y = 6$

Hence the ratio is $4:1$ and $y = 6$.

8. If A (—1, 3), B (1, —1) and C (5, 1) are the vertices of a triangle ABC, find the length of the median through A.

Sol:



Let $A(1,3), B(1,-1)$ and $C(5,1)$ be the vertices of triangle ABC and let AD be the median through A.

Since, AD is the median, D is the mid-point of BC

$\therefore$ Coordinates of D are $\left(\frac{1+5}{2}, \frac{-1+1}{2}\right) = (3,0)$

Length of median $AD = \sqrt{(3+1)^2 + (0-3)^2}$

$= \sqrt{(4)^2 + (-3)^2}$

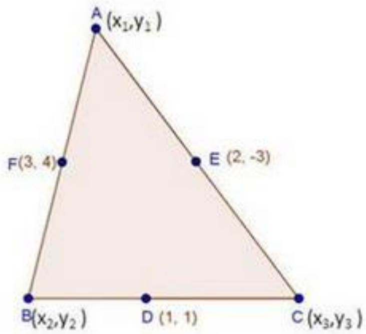
$=\sqrt{16+9}$

$=\sqrt{25}$

$=5$ units.

9. If the coordinates of the mid-points of the sides of a triangle are (1, 1), (2, —3) and (3, 4), find the vertices of the triangle.

Sol:



Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC.

Let $D(1,1), E(2,-3)$ and $F(3,4)$ be the mid-points of sides BC, CA and AB respectively.

Since, D is the mid-point of BC.

$\therefore \frac{x_2+x_3}{2}=1$ and $\frac{y_2+y_3}{2}=1$

$\Rightarrow x_2+x_3=2$ and $y_2+y_3=2$ ....(i)

Similarly E and F are the mid-points of CA and AB respectively.

$\therefore \frac{x_1+x_3}{2}=2$ and $\frac{y_1+y_3}{2}=-3$

$\Rightarrow x_1+x_3=4$ and $y_1+y_3=6$ ....(ii)

And, $\frac{x_1+x_2}{2}=3$ and $\frac{y_1+y_2}{2}=4$

$\Rightarrow x_1+x_2=6$ and $y_1+y_2=8$ ....(iii)

From (i), (ii) and (iii) we get

$x_2+x_3+x_1+x_3+x_1+x_2=2+4+6$ and

$y_2+y_3+y_1+y_3+y_1+y_2=2+(-6)+8$

$\Rightarrow 2(x_1+x_2+x_3)=12$ and $2(y_1+y_2+y_3)=4$

$x_1+x_2+x_3=6$ and $y_1+y_2+y_3=2$ ....(iv)

From (i) and (iv) we get

$x_1+2=6$ and $y_1+2=2$

$\Rightarrow x_1 = 6 - 2$ and $\Rightarrow y_2 = 2 - 2\,2$

$\Rightarrow x_1 = 4 \Rightarrow y_1 = 0$

So the coordinates of A are $(4, 0)$

From (ii) and (iv) we get

$x_2 + 4 = 6$ and $y_2 + (-6) = 2$

$\Rightarrow x_2 = 2 \Rightarrow y_2 - 6 = 2 \Rightarrow y_2 = 8$

So the coordinates of B are $(2, 8)$

From (iii) and (iv) we get

$6 + x_3 = 6$ and $8 + y_3 = 2$

$\Rightarrow x_3 = 6 - 6 \Rightarrow y_3 = 2 - 8$

$\Rightarrow x_3 = 0$ and $y_3 = -6$

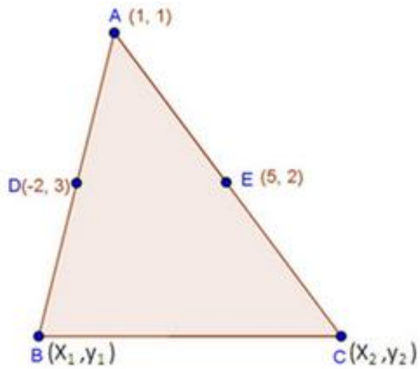
So the coordinates of C are $(0, -6)$

Hence, the vertices of triangle ABC are:

$A(4, 0), B(2, 8)$ and $C(0, -6)$

10. If a vertex of a triangle be (1, 1) and the middle points of the sides through it be (—2, 3) and (5, 2), find the other vertices.

Sol:



Let $A(1,1)$, be the given vertex

And, $D(-2,3), E(5,2)$ be the mid-point of AB and AC respectively,

Now, since D and E are the midpoints of AB and AC

$\frac{x_1 + 1}{2} = -2, \frac{y_1 + 1}{2} = 3$

$\Rightarrow x_1 + 1 = -4 \Rightarrow y_1 + 1 = 6$

$\Rightarrow x_1 = -5 \Rightarrow y_1 = 5$

So, the coordinates of B are $(-5,5)$

And, $\frac{x_2+1}{2}=5, \quad \frac{y_2+1}{2}=2$

$\Rightarrow x_2+1=10 \Rightarrow y_2+1=4$

$\Rightarrow x_2=9 \Rightarrow y_2=3$

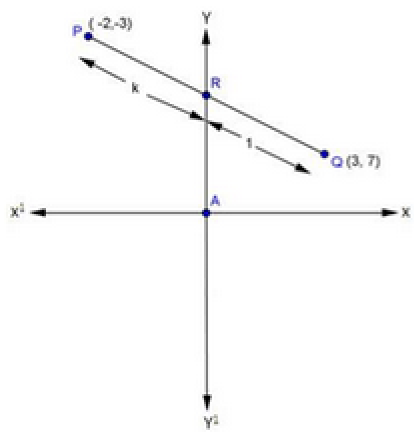
So the coordinates of C are $(9,3)$

Hence, the over vertices are $B(-5,5)$ and $C(9,3)$

11. (i) In what ratio is the line segment joining the points (—2, —3) and (3, 7) divided by the y-axis? Also, find the coordinates of the point of division.

(ii) In what ratio is the line segment joining (—3, —1) and (—8, —9) divided at the point $(-5, -\frac{21}{5})$?

Sol:



Suppose $y-axis$ divides PQ in the ration $K:1$ at R

Then, the coordinates of the point of division are:

$$R\left[\frac{3k+(-2)\times1}{k+1}, \frac{7k+(-3)\times1}{k+1}\right]$$

$$=R\left[\frac{3k-2}{k+1}, \frac{7k-3}{k+1}\right]$$

Since, R lies on $y-axis$ and $x-$coordinate of every point on y-axis is zero

$$\therefore \frac{3k-2}{k+1}=0$$

$$\Rightarrow 3k-2=0$$

$$\Rightarrow 3k=2$$

$$\Rightarrow k=\frac{2}{3}$$

Hence, the required ratio is $\frac{2}{3}:1$

i.e., 2 : 3

Putting $k=\frac{2}{3}$ in the coordinates of R

We get, $(0,1)$

Let the point P divide AB in the ratio $K:1$

Then, the coordinates of P are $\left[\frac{-8k-3}{k+1},\frac{-9k-1}{k+1}\right]$

But the coordinates of P are given as $\left(-5,\frac{-21}{5}\right)$

$\therefore \frac{-8k-3}{k+1}=-5$

$\Rightarrow -8k-3=-5k-5$

$\Rightarrow -8k+5k=-5+3$

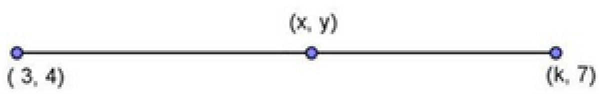
$\Rightarrow -3k=-2$

$\Rightarrow k=\frac{2}{3}$

Hence, the point P divides AB in the ratio $\frac{2}{3}:1\Rightarrow 2:3$

12. If the mid-point of the line joining (3, 4) and (k, 7) is (x, y) and 2 x + 2 y + 1 = 0. find the value of k.

Sol:



Since, (x,y) is the mid-point

$x=\frac{3+k}{2},y=\frac{4+7}{2}=\frac{11}{2}$

Again,

$2x+2y+1=0$

$\Rightarrow 2\times\frac{(3\times k)}{2}+2\times\frac{11}{2}+1=0$

$\Rightarrow 3+k+11+1=0$

$\Rightarrow 3+k+12=0$
$\Rightarrow k+15=0$
$\Rightarrow k=-15$

13. Determine the ratio in which the straight line x — y — 2 = 0 divides the line segment joining (3, —1) and (8, 9).

Sol:

Suppose the line $x-y-2=0$ divides the line segment joining $A(3,-1)$ and $B(8,9)$ in the ratio $K:1$ at point P. Then the coordinates of P are

$\left(\frac{8k+3}{k+1},\frac{9k-1}{k+1}\right)$

But P lies on $x-y-2=0$

$\therefore \frac{8k+3}{k+1}-\frac{9k-1}{k+1}-2=0$

$\Rightarrow \frac{8k+3}{k+1}-\frac{9k-1}{k+1}=2$

$\Rightarrow \frac{8k+3-9k+1}{k+1}=2$

$\Rightarrow -k+4=2k+2$

$\Rightarrow -k-2k=2-4$

$\Rightarrow -3k=-2 \Rightarrow k=\frac{2}{3}$

So, the required ratio is $2:3$

14. Find the ratio in which the line segment joining (—2, —3) and (5, 6) is divided by

(i) x-axis

(ii) y-axis.

Also, find the coordinates of the point of division in each case.

Sol:

(i) Suppose $x-axis$ divides AB in the ratio $K:1$ at point P

Then, the coordinates of the point of division of division are

$P\left[\frac{5k-2}{k+1},\frac{6k-3}{k+1}\right]$

Since, P lies on x-axis, and y-coordinates of every point on x-axis is zero.

$\therefore \frac{6k-3}{k+1}=0$

$\Rightarrow 6k-3=0$

$\Rightarrow 6k=3$

$\Rightarrow k = \frac{3}{6} \Rightarrow k = \frac{1}{2}$

Hence, the required ratio is $1:2$

Putting $k = \frac{1}{2}$ in the coordinates of P

We find that its coordinates are $\left(\frac{1}{3}, 0\right)$.

(ii) Suppose y-axis divides AB in the ratio $k:1$ at point Q.

Then, the coordinates of the point of division are

$Q\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$

Since, Q lies on $y-axis$ and x-coordinates of every point on y-axis is zero.

$\therefore \frac{5k-2}{k+1} = 0$

$\Rightarrow 5k - 2 = 0$

$\Rightarrow k = \frac{2}{5}$

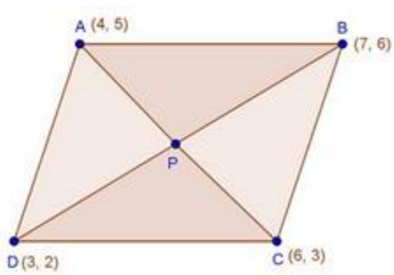
Hence, the required ratio is $\frac{2}{5}:1 = 2:5$

Putting $k = \frac{2}{5}$ in thee coordinates of Q.

We find that the coordinates are $\left(0, \frac{-3}{7}\right)$

15. Prove that the points (4, 5), (7, 6), (6, 3), (3, 2) are the vertices of a parallelogram. Is it a rectangle.

Sol:



Let $A(4,5), B(7,6), C(6,3)$ and $D(3,2)$ be the given points.

And, P the points of intersection of AC and BD.

Coordinates of the mid-point of AC are $\left(\frac{4+6}{2}, \frac{5+3}{2}\right) = (5,4)$

Coordinates of the mid-point of BD are $\left(\frac{7+3}{2}, \frac{6+2}{2}\right) = (5,4)$

Thus, AC and BD have the same mid-point.

Hence, $ABCD$ is a parallelogram

Now, we shall see whether $ABCD$ is a rectangle.

We have,

$AC = \sqrt{(6-4)^2 + (3-5)^2}$

$\Rightarrow AC = \sqrt{4+4}$

$\Rightarrow AC = \sqrt{8}$

And, $BD + \sqrt{(7-3)^2 + (6-2)^2}$

$\Rightarrow BD = \sqrt{16+16}$

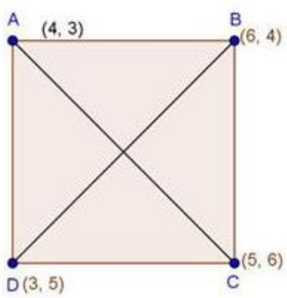
$\Rightarrow BD = \sqrt{32}$

Since, $AC \neq BD$

So, $ABCD$ is not a rectangle

16. Prove that (4, 3), (6, 4), (5, 6) and (3, 5) are the angular points of a square.

Sol:



Let $A(4,3), B(6,4), C(5,6)$ and $D(3,5)$ be the given points.

Coordinates of the mid-point of AC are $\left(\frac{4+5}{2}, \frac{3+6}{2}\right) = \left(\frac{9}{2}, \frac{9}{2}\right)$

Coordinates of the mid-point of BD are $\left(\frac{6+3}{2}, \frac{4+5}{2}\right) = \left(\frac{9}{2}, \frac{9}{2}\right)$

AC and BD have the same mid-point

$\therefore$ $ABCD$ is a parallelogram

Now,

$AB = \sqrt{(6-4)^2 + (4-3)^2}$

$\Rightarrow AB = \sqrt{4+1}$

$\Rightarrow AB = \sqrt{5}$

And, $BC = \sqrt{(6-5)^2 + (4-6)^2}$

$\Rightarrow BC = \sqrt{1+4}$

$\Rightarrow BC = \sqrt{5}$

$\therefore \quad AB = BC$

So, ABCD is a parallelogram whose adjacent sides are equal

$\therefore \quad ABCD$ is a rhombus

We have,

$AC = \sqrt{(5-4)^2 + (6-3)^2}$

$\Rightarrow AC = \sqrt{10}$

$BD = \sqrt{(6-3)^2 + (4-5)^2}$

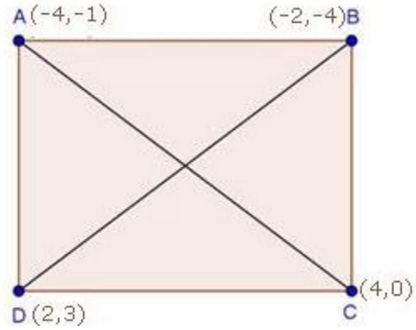
$\Rightarrow BD = \sqrt{10}$

$AC = BD$

Hence, $ABCD$ is a square

17. Prove that the points (—4, —1), (—2, —4), (4, 0) and (2, 3) are the vertices of a rectangle.

Sol:



Let $A(-4,-1), B(-2,-4), C(4,0)$ and $D(2,3)$ be the given points

Coordinates of the mid-point of AC are $\left(\frac{-4+4}{2}, \frac{-1+0}{2}\right) = \left(0, \frac{-1}{2}\right)$

Coordinates of the mid-point of BD are $\left(\frac{-2+2}{2}, \frac{-4+3}{2}\right) = \left(0, \frac{-1}{2}\right)$

Thus AC and BD have the same mid-point

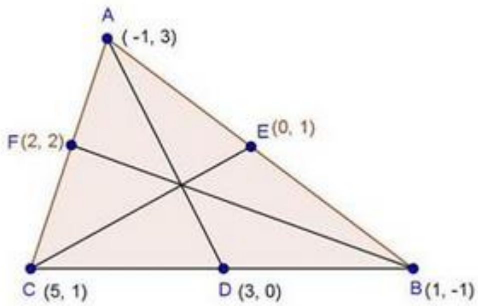
$AC = \sqrt{(4+4)^2 + (0+1)^2} = \sqrt{65}$

$BD = \sqrt{(-2-2)^2 + (-4-3)^2} = \sqrt{65}$

Hence $ABCD$ is a rectangle

18. Find the lengths of the medians of a triangle whose vertices are A (−1, 3), B (1, −1) and C (5,1).

Sol:



Let AD, BF and CE be the medians of $\Delta ABCD$

Coordinates of D are $\left(\frac{5+1}{2}, \frac{1-1}{2}\right) = (3, 0)$

Coordinates of E are $\left(\frac{-1+1}{2}, \frac{3-1}{2}\right) = (0, 1)$

Coordinates of F are $\left(\frac{5-1}{2}, \frac{1+3}{2}\right) = (2, 2)$

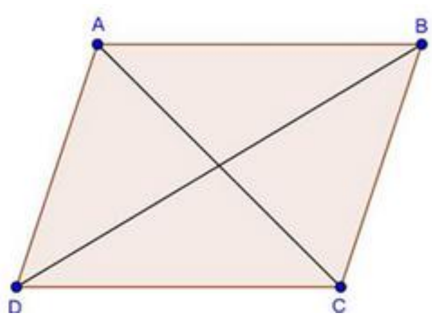
Length of $AD = \sqrt{(-1-3)^2 + (3-0)^2} = 5$ units

Length of $BF = \sqrt{(2-1)^2 + (2+1)^2} = \sqrt{10}$ units

Length of $CE = \sqrt{(5-0)^2 + (1-1)^2} = 5$ units

19. Three vertices of a parallelogram are (a + b, a — b), (2a + b, 2a — b), (a — b, a + b). Find the fourth vertex.

Sol:



Let $A(a+b, a-b), B(2a+b, 2a-b), C(a-b, a+b)$ and (x, y) be the given points

Since, the diagonals of a parallelogram bisect each other

$\therefore$ Coordinates of the midpoint of $AC =$ Coordinates of the midpoint of BD

$$\left(\frac{a+b+a-b}{2},\frac{a-b+a+b}{2}\right)=\left(\frac{2a+b+x}{2},\frac{2a-b+y}{2}\right)$$

$$\Rightarrow (a,a)=\left(\frac{2a+b+x}{2},\frac{2a-b+y}{2}\right)$$

$$\Rightarrow \frac{2a+b+x}{2}=a \text{ and } \frac{2a-b+y}{2}=a$$

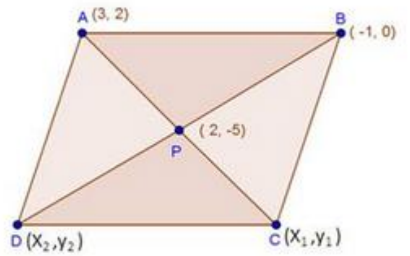
$$\Rightarrow 2a+b+x=2a \Rightarrow 2a-b+y=2a$$

$$\Rightarrow x=-b \Rightarrow y=b$$

Hence, the fourth vertex is $(-b,b)$.

20. If two vertices of a parallelogram are (3, 2), (—1, 0) and the diagonals cut at (2, —5), find the other vertices of the parallelogram.

Sol:



Let $A(3,2), B(-1,0), C(x_1,y_1)$ and $D(x_2,y_2)$ be the given points.

Since, the diagonals of parallelogram bisect each other.

Coordinates of the midpoint of $AC =$ Coordinates of the midpoint of BD

$$\left(\frac{x_1+3}{2},\frac{y_1+2}{2}\right)=\left(\frac{x_2-1}{2},\frac{y_2+0}{2}\right)$$

But $\frac{x_1+3}{2}=2, \frac{y_1+2}{2}=-5$

$$\Rightarrow x_1+3=4 \Rightarrow y_1=-10-2$$

$$\Rightarrow x_1=1 \Rightarrow y_1=-12$$

And, $\frac{x_2-1}{2}=2$

$$\Rightarrow x_2-1=4$$

$$\Rightarrow x_2=5$$

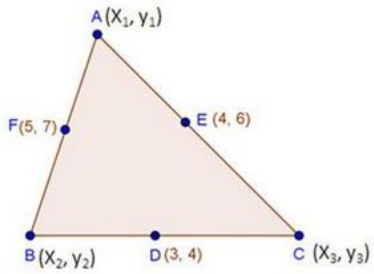
$$\frac{y_2+0}{2}=-5$$

$$y_2=-10$$

Hence, the other vertices of parallelogram are $(1,-12)$ and $(5,-10)$.

21. If the coordinates of the mid-points of the sides of a triangle are (3, 4), (4, 6) and (5, 7), find its vertices

Sol:



Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC

Let $D(3,4), E(4,6)$ and $F(5,7)$ be the midpoints of BC, CA and AB.

Since, D is the midpoint of BC

$\therefore \frac{x_2 + x_3}{2} = 3$ and $\frac{y_2 + y_3}{2} = 4$

$\Rightarrow x_2 + x_3 = 6$ and $y_2 + y_3 = 8$ ……….(i)

Since, E is the midpoint of CA

$\therefore \frac{x_1 + x_3}{2} = 4$ and $\frac{y_1 + y_3}{2} = 6$

$\therefore x_1 + x_3 = 8$ and $y_1 + y_3 = 12$ ……….(ii)

Since F is the mid-point of AB

$\frac{x_1 + x_2}{2} = 5$ and $\frac{y_1 + y_2}{2} = 7$

$\Rightarrow x_1 + x_2 = 10$ and $y_1 + y_3 = 14$ ……….(iii)

From (i), (ii) and (iii), we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 6 + 8 + 10$

$x_1 + x_2 + x_3 = 12$ ……….(iv)

And $y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 8 + 12 + 14$

$y_1 + y_2 + y_3 = 17$ ……….(iv)

From (i) and (iv)

$x_1 + 6 = 12, y_1 + 8 = 17$

$x_1 = 6, y_1 = 9$

From (ii) and (iv)

$x_2 + 8 = 12, y_2 + 12 = 17$

$x_2 = 4, y_2 = 5$

From (iii) and (iv)

$x_3 + 10 = 12, y_3 + 14 = 17$

$x_3 = 2, y_3 = 3$

Hence the vertices of triangle ABC are $(6,9);(4,5);(2,3)$

22. The line segment joining the points P (3, 3) and Q (6, — 6) is bisected at the points A and B such that A is nearer to P. If A also lies on the line given by 2x + y + k = 0, find the value of k.

Sol:

We are given PQ is the line segment, A and B are the points of trisection of PQ.

We know that $PA : QA = \ 1 : 2$

So, the coordinates of A are

$\left(\frac{6\times1+3\times2}{2+1}, \frac{-6\times1+3\times2}{2+1}\right)$

$= \frac{12}{3}, 0$

$= (4,0)$

Since, A lies on the line

$2x + y + k = 0$

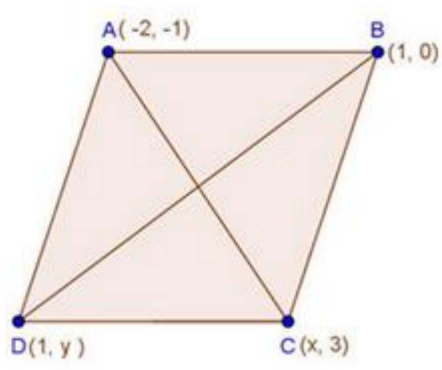
$\Rightarrow 2\times4+0+k=0$

$\Rightarrow 8+k=0$

$\Rightarrow 8+k=-8$

23. If the points (—2 , —1), (1, 0), (x, 3) and (1, y) form a parallelogram, find the values of x and y.

Sol:



Let $A(-2,-1), B(1,0), C(x,3)$ and $D(1,y)$ be the given points.

We know that diagonals of a parallelogram bisect each other

$\therefore$ Coordinates of the mid-point of $AC =$ Coordinates of the mid-point of BD

$$\left(\frac{x-2}{2},\frac{3-1}{2}\right)=\left(\frac{1+1}{2},\frac{y+0}{2}\right)$$

$$\Rightarrow\left(\frac{x-2}{2},1\right)=\left(1,\frac{y}{2}\right)$$

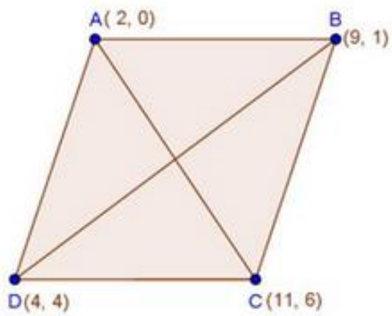
$$\Rightarrow\frac{x-2}{2}=1 \text{ and } \frac{y}{2}=1$$

$$\Rightarrow x-2=2 \Rightarrow y=2$$

$$\Rightarrow x=4 \Rightarrow y=2$$

24. The points A (2, 0), B (9, 1), C (11, 6) and D (4, 4) are the vertices of a quadrilateral ABCD. Determine whether ABCD is a rhombus or not.

Sol:



Let $A(2,0), B(9,1), C(11,6)$ and $D(4,4)$ be the given points.

Coordinates of midpoint AC are $\left(\frac{11+2}{2},\frac{6+0}{2}\right)=\left(\frac{13}{2},3\right)$

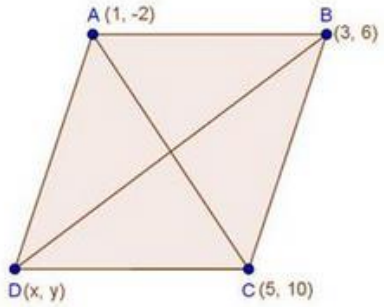
Coordinates of midpoint BD are $\left(\frac{9+4}{2},\frac{1+4}{2}\right)=\left(\frac{13}{2},\frac{5}{2}\right)$

Since, coordinates of mid-point of $AC \neq$ coordinates of mid-point of BD.

So, $ABCD$, is not a parallelogram. Hence, it is not a rhombus.

25. If three consecutive vertices of a parallelogram are (1, —2), (3, 6) and (5, 10), find its fourth vertex.

Sol:



Let $A(1,-2), B(3,6), C(5,10)$ and $D(x,y)$ be the given points taken in order.

Since, diagonals of parallelogram bisect each other

Coordinates of mid-point of $AC =$ Coordinates of midpoint of BD

$$\left(\frac{5+1}{2},\frac{10-2}{2}\right)=\left(\frac{x+3}{2},\frac{y+6}{2}\right)$$

$$\Rightarrow (3,4)=\frac{x+3}{2},\frac{y+6}{2}$$

$$\Rightarrow \frac{x+3}{2}=3 \text{ and } \frac{y+6}{2}=4$$

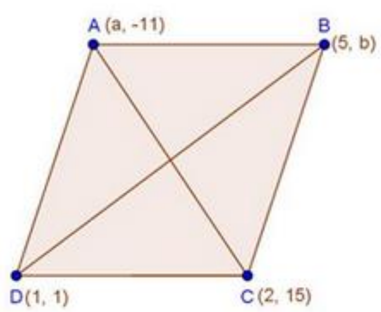
$$\Rightarrow x+3=6 \qquad \Rightarrow y+6=8$$

$$\Rightarrow x=3 \qquad \Rightarrow y=2$$

Hence, the fourth vertex is $(3,2)$.

26. If the points A (a, —11), B (5, b), C (2, 15) and D (1, 1) are the vertices of a parallelogram ABCD, find the values of a and b.

Sol:



Let $A(a,-11), B(5,b), C(2,15)$ and $D(1,1)$ be the given points.

We know that diagonals of parallelogram bisect each other.

$\therefore$ Coordinates of mid-point of $AC =$ Coordinates of mid-point of BD

$$\left(\frac{a+2}{2},\frac{15-11}{2}\right)=\left(\frac{5+1}{2},\frac{b+1}{2}\right)$$

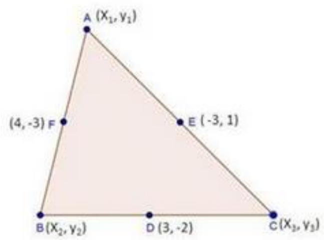
$\Rightarrow \frac{a+2}{2} = 3$ and $\frac{b+1}{2} = 2$

$\Rightarrow a+2=6 \quad \Rightarrow b+1=4$

$\Rightarrow a=4 \quad \Rightarrow b=3$

27. If the coordinates of the mid-points of the sides of a triangle be (3, —2), (—3, 1) and (4, —3), then find the coordinates of its vertices.

Sol:



Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC

Let $D(3,-2), E(-3,1)$ and $F(4,-3)$ be the midpoint of sides BC, CA and AB respectively

Since, D is the midpoint of BC

$\therefore \frac{x_2+x_3}{2} = 3$ and $\frac{y_2+y_3}{2} = -2$

$\Rightarrow x_2+x_3 = 6$ and $y_2+y_3 = -4$ …….(i)

Similarly, E and F are the midpoint of CA and AB respectively.

$\therefore \frac{x_1+x_3}{2} = -3$ and $\frac{y_1+y_3}{2} = 1$

$\Rightarrow x_1+x_3 = -6$ and $y_1+y_3 = 2$ …….(ii)

And,

$\therefore \frac{x_1+x_2}{2} = 4$ and $\frac{y_1+y_2}{2} = -3$

$\Rightarrow x_1+x_2 = 8$ and $y_1+y_2 = -6$ …….(iii)

From (i), (ii) and (iii), we have

$x_2+x_3+x_1+x_3+x_1+x_2 = 6+(-6)+8$ and

$y_2+y_3+y_1+y_3+y_1+y_2 = -4+2-6$

$\Rightarrow 2(x_1+x_2+x_3) = 8$ and $2(y_1+y_2+y_3) = -8$

$\Rightarrow x_1+x_2+x_3 = 4$ and $y_1+y_2+y_3 = -4$ …….(iv)

From (i) and (iv)

$x_1+6=4$ and $y_1-4=-4$

$\Rightarrow x_1 = -2 \qquad \Rightarrow y_1 = 0$

So, the coordinates of A are $(-2,0)$

From (ii) and (iv)

$x_2 - 6 = 4$ and $y_2 + 2 = -4$

$\Rightarrow x_2 = 10 \Rightarrow y_2 = -6$

So, the coordinates of B are $(10,-6)$

From (iii) and (iv)

$x_3 + 8 = 4$ and $y_3 - 6 = -4$

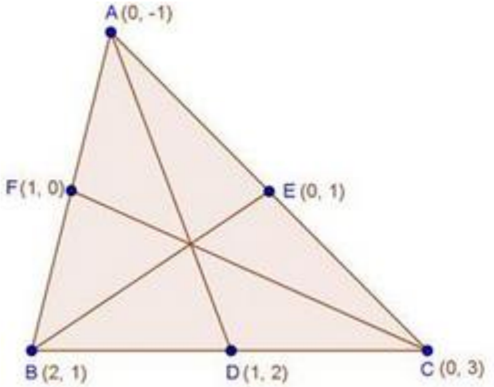
$\Rightarrow x_3 = -4 \qquad \Rightarrow y_3 = 2$

So, the coordinates of C are $(-4,2)$

Hence, the vertices of ΔABC are $A(-2,0), B(10,6)$ and $C(-4,2)$.

28. Find the lengths of the medians of a ΔABC having vertices at A (0,—1), B (2, 1) and C (0, 3).

Sol:



Let $A(0,1), B(2,1)$ and $C(0,3)$ be the given points

Let AD, BE and CF be the medians

Coordinates of D $are \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1,2)$

Coordinates of E $are \left(\frac{0}{2}, \frac{3-1}{2}\right) = (0,1)$

Coordinates of F $are \left(\frac{2+0}{2}, \frac{1-1}{2}\right) = (1,0)$

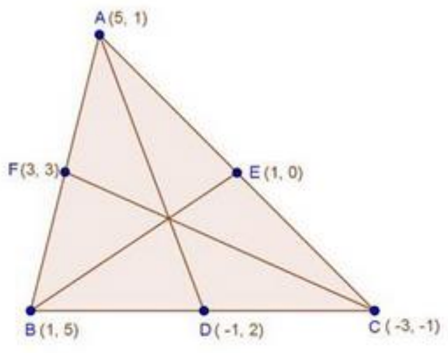
Length of median $AD = \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{10}$ units

Length of median $BE = \sqrt{(2-0)^2 + (1-1)^2} = 2$ units

Length of median $CF=\sqrt{(1-0)^2+(0-3)^2}=\sqrt{10}$ units

29. Find the lengths of the medians of a ΔABC having vertices at A (5, 1), B (1, 5), and C (−3,-1).

Sol:



Let $A(5,1), B(1,5)$ and $C(-3,-1)$ be vertices of ΔABC

Let AD, BE and CF be the medians

Coordinates of D $are \left(\frac{1-3}{2},\frac{5-1}{2}\right)=(-1,2)$

Coordinates of E $are \left(\frac{5-3}{2},\frac{1-1}{2}\right)=(1,0)$

Coordinates of F $are \left(\frac{5+1}{2},\frac{1+5}{2}\right)=(3,3)$

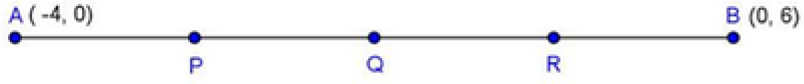
Length of median $AD=\sqrt{(5+1)^2+(1-2)^2}=\sqrt{37}$ units

Length of median $BE=\sqrt{(1-1)^2+(5-0)^2}=5$ units

Length of median $CF=\sqrt{(3+3)^2+(3+1)^2}=2\sqrt{13}=\sqrt{52}$ units

30. Find the coordinates of the points which divide the line segment joining the points (−4, 0) and (0, 6) in four equal parts.

Sol:



Let $A(-4,0)$ and $B(0,6)$ be the given points.

And, Let P,Q,R be the points which divide AB in four equal points..

We know $AP:PB=1:3$

$\therefore$Coordinates of P are

$\left(\frac{1\times0+3(-4)}{1+3},\frac{1\times6+3\times0}{1+3}\right)$

$=\left(-3,\frac{3}{2}\right)$

We know that Q is midpoint of AB

$\therefore$ Coordinates of Q are

$\left(\frac{3\times0+1\times(-4)}{3+1},\frac{3\times6+1\times0}{3+1}\right)$

$=\left(-1,\frac{9}{2}\right)$

31. Show. that the mid-point of the line segment joining the points (5, 7) and (3, 9) is also the mid-point of the line segment joining the points (8, 6) and (0, 10).

Sol:

Let $A(5,7),B(3,9),C(8,6)$ and $D(0,,10)$ be the given points

Coordinates of the mid-point of $AB\ are\left(\frac{5+3}{2},\frac{7+9}{2}\right)=(4,8)$

Coordinates of the mid-point of $CD\ are\left(\frac{8+0}{2},\frac{6+10}{2}\right)=(4,8)$

Hence, the midpoints of AB = midpoint of CD.

32. Find the distance of the point (1, 2) from the mid-point of the line segment joining the points (6, 8) and (2, 4).

Sol:

Let $P(1,2),A(6,8)$ and $B(2,4)$ be thee given points.

Coordinates of midpoint of the line segment joining $A(6,8)$ and $B(2,4)$ are

$Q\left(\frac{6+2}{2},\frac{8+4}{2}\right)=Q(4,6)$

Now, distance $PQ=\sqrt{(4-1)^2+(6-2)^2}$

$\Rightarrow PQ=\sqrt{9+16}$

$\Rightarrow PQ=\sqrt{25}$

$\Rightarrow PQ=5$

Hence, the distant $=5$ units

33. If A and B are (1, 4) and (5, 2) respectively, find the coordinates of P when $\frac{AP}{BP} = \frac{3}{4}$

Sol:

Let $A(1,4)$ and $B(5,2)$ be the given points.

We know that $\frac{AP}{BP} = \frac{3}{4}$

Or, $AP : BP = 3 : 4$

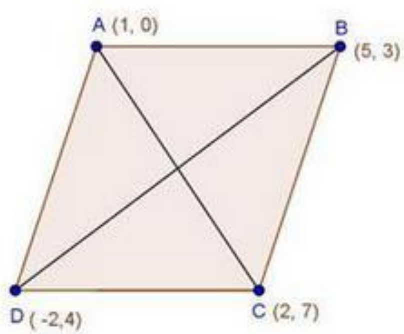
Coordinates of P are

$$\left(\frac{3\times5+4\times1}{3+4}, \frac{3\times2+4\times4}{3+4}\right)$$

$$= \left(\frac{19}{7}, \frac{22}{7}\right)$$

34. Show that the points A (1, 0), B (5, 3), C (2, 7) and D (—2, 4) are the vertices of a parallelogram.

Sol:



Let $A(1,0), B(5,3), C(2,7)$ and $D(-2,4)$ be the given points

Coordinates of the midpoint of AC are $\left(\frac{1+2}{2}, \frac{0+7}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

Coordinates of the midpoint of BD are $\left(\frac{5-2}{2}, \frac{3+4}{2}\right) = \left(\frac{3}{2}, \frac{7}{2}\right)$

Since, coordinates of midpoint of $AC =$ coordinates off midpoint of BD

$\therefore ABCD$ is a parallelogram as we know diagonals of parallelogram bisect each other.

35. Determine the ratio in which the point P (m, 6) divides the join of A(—4, 3) and B(2, 8). Also, find the value of m.

Sol:

Let $P(m,6)$ divides the join of $A(-4,3)$ and $B(2,8)$ in the ratio $K:1$

Then, the coordinates of P are

$$\left(\frac{2k+1\times(-4)}{k+1},\frac{8k+1\times 3}{k+1}\right)$$

$$=\left(\frac{2k-4}{k+1},\frac{8k+3}{k+1}\right)$$

But, $\frac{8k+3}{k+1}=6$

$\Rightarrow 8k+3=6k+6$

$\Rightarrow 8k-6k=3$

$\Rightarrow k=\frac{3}{2}$

Hence, P divides AB in the ratio $3:2$

Again,

$$\frac{2k-4}{k+1}=m$$

Substituting $k=\frac{3}{2}$, we get

$$\frac{2\times\frac{3}{2}-4}{\frac{3}{2}+1}=m$$

$$\Rightarrow \frac{-1}{\frac{5}{2}}=m$$

$$\Rightarrow \frac{-2}{5}=m$$

$$\therefore m=\frac{-2}{5}$$

36. Determine the ratio in which the point (—6, a) divides the join of A(—3, 1) and B(—8, 9). Also find the value of a.

Sol:

Let $P(-6,a)$ divides the join of $A(-3,1)$ and $B(-8,9)$ in the ratio $k:1$

Then, the coordinates of P are

$$\left(\frac{-8k-3}{k+1},\frac{9k+1}{k+1}\right)$$

But, $\frac{-8k-3}{k+1}=-6$

$\Rightarrow -8k-3=-6k-6$

$\Rightarrow -8k+6k=-6+3$

$\Rightarrow -2k=-3$

$\Rightarrow k=\frac{3}{2}$

Hence, P divides AB in the ratio $3:2$

Again

$\frac{9k+1}{k+1}=a$

Substituting $k=\frac{3}{2}$

We get,

$$\frac{9\times\frac{3}{2}+1}{\frac{3}{2}+1}=a$$

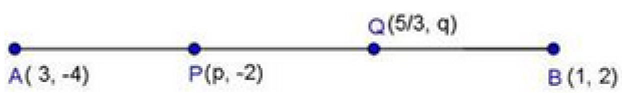
$$\Rightarrow \frac{\frac{29}{2}}{\frac{5}{2}}=a$$

$$\Rightarrow \frac{29}{5}=a$$

$$\therefore a=\frac{29}{5}$$

37. The line segment joining the points (3, —4) and (1, 2) is trisected at the points P and Q . If the coordinates of P and Q are (p, —2) and ($\frac{5}{3}$, q) respectively. Find the values of p and q.

Sol:



We have $P(p,-2)$ and $Q\left(\frac{5}{3},q\right)$ are the points of trisection of the line segment joining $A(3,-4)$ *and* $B(1,2)$

We know $AP:PB=1:2$

$\therefore$ Coordinates of P are

$$\left(\frac{1\times1+2\times3}{1+2},\frac{1\times2+2\times(-4)}{1+2}\right)$$

$$=\left(\frac{7}{3},-2\right)$$

Hence, $P=\frac{7}{3}$

Again we know that $AQ:QB=2:1$

$\therefore$ Coordinates of Q are

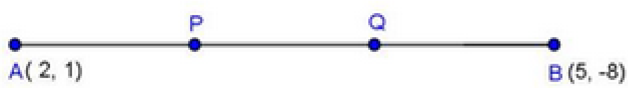
$$\left(\frac{2\times1+1\times3}{2+1},\frac{2\times2+1\times(-4)}{2+1}\right)$$

$$=\left(\frac{5}{3},0\right)$$

Hence, $q=0$

38. The line joining the points (2, 1) and (5,—8) is trisected at the points P and Q. If point P lies on the line 2x — y + k = 0. Find the value of k.

Sol:



Since, P is the point of trisection of the line segment joining the point $A(2,1)$ and $B(5,-8)$

We have $AP:PB=1:2$

$\therefore$ Coordinates of the point P are

$$\left(\frac{1\times5+2\times2}{1+2},\frac{1\times(-8)+2\times1}{1+2}\right)$$

$$=(3,-2)$$

But, P lies on the line

$$2x-y+k=0$$

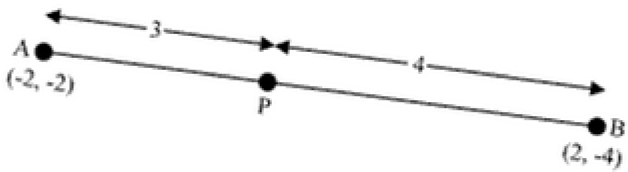
$$\Rightarrow 2\times3-(-2)+k=0$$

$$\Rightarrow 6+2+k=0$$

$\Rightarrow 8+k=0$
$\Rightarrow k=-8$

39. If A and B are two points having coordinates (—2, —2) and (2, —4) respectively, find the coordinates of P such that AP = AB.

Sol:



The Coordinates of point A and B are $(-2,-2)$ and $(2,-4)$ respectively

Since $AP=\frac{3}{7}AB$

Therefore $AP:PB=3:4$

So, point P divides the line segment AB in a ratio $3:4$.

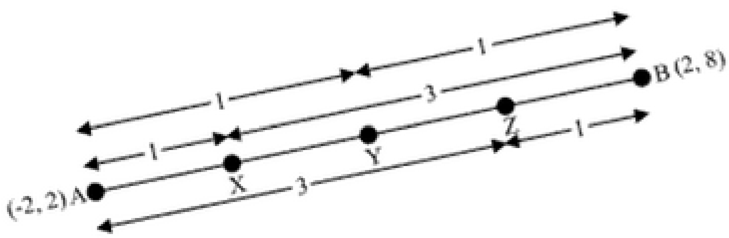
Coordinates of $P=\left(\frac{3\times 2+4\times(-2)}{3+4},\frac{3\times(-4)+4\times(-2)}{3+4}\right)$

$=\left(\frac{6-8}{7},\frac{-12-8}{7}\right)$

$=\left(\frac{-2}{7},\frac{20}{7}\right)$

40. Find the coordinates of the points which divide the line segment joining A (—2, 2) and B (2, 8) into four equal parts.

Sol:



From the figure we have points X, Y, Z are dividing the line segment in a ratio $1:3, 1:1, 3:1$ respectively.

Coordinates of $X=\left(\frac{1+2+3\times(-2)}{1+3},\frac{1\times 8+3\times 2}{1+3}\right)$

$=\left(-1,\frac{7}{2}\right)$

Coordinates of $Y=\left(\frac{2+(-2)}{2},\frac{2+8}{2}\right)$

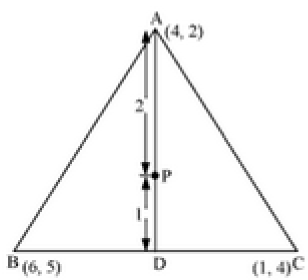
$=(0,5)$

Coordinates of $Z=\left(\frac{3\times 2+1\times(-2)}{3+1},\frac{3\times 8+1\times 2}{3+1}\right)$

$=\left(1,\frac{13}{2}\right)$

41. A (4, 2), B (6, 5) and C (1, 4) are the vertices of ΔABC.
 (i) The median from A meets BC in D. Find the coordinates of the point D.
 (ii) Find the coordinates of point P on AD such that AP : PD = 2 : 1.
 (iii) Find the coordinates of the points Q and R on medians BE and CF respectively such that BQ : QE = 2 : l and CR : RF 2 : 1.
 (iv) What do you observe?

Sol:



(i) Median AD of the triangle will divide the side BC in two equals parts. So D is the midpoint of side BC.

Coordinates of D $=\left(\frac{6+1}{2},\frac{5+4}{2}\right)=\left(\frac{7}{2},\frac{9}{2}\right)$

(ii) Point P divides the side AD in a ratio 2:1

Coordinates of P $=\left(\frac{2\times\frac{7}{6}+1\times 4}{2+1},\frac{2\times\frac{9}{2}+1\times 2}{2+1}\right)$

$=\left(\frac{11}{3},\frac{11}{3}\right)$

(iii) Median BE of the triangle will divide the side AC in two equal parts. So E is the midpoint of side AC.

Coordinates of $E = \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$

Point Q divides the side BE in a ratio 2:1

Coordinates of $Q = \left(\frac{2\times\frac{5}{2}+1\times 6}{2+1}, \frac{2\times 3+1\times 5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

Median CF of the triangle will divide the side AB in two equal parts. So F is the midpoint of side AB.

Coordinates of $F = \left(\frac{4+6}{2}, \frac{2+1}{2}\right) = \left(5, \frac{7}{2}\right)$

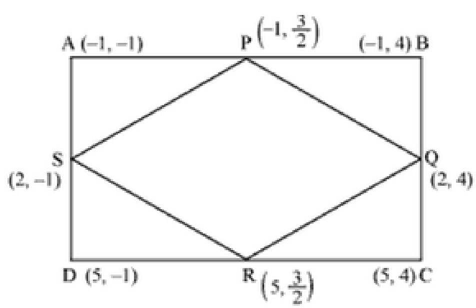
Point R divides the side CF in a ratio 2:1.

Coordinates of $R = \left(\frac{2\times 5+1\times 1}{2+1}, \frac{2\times\frac{7}{2}+1\times 4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$

(iv) Now we may observe that coordinates of point P, Q, R are same. So, all these are representing same point on the plane i.e. centroid of the triangle.

42. ABCD is a rectangle formed by joining the points A (—1, —1), B (—1, 4), C (5, 4) and D (5, −1). P, Q, R and S are the mid-points of sides AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.

Sol:



Length of $PQ = \sqrt{(-1-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of $QR = \sqrt{(2-5)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of $RS = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

Length of $SP = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$

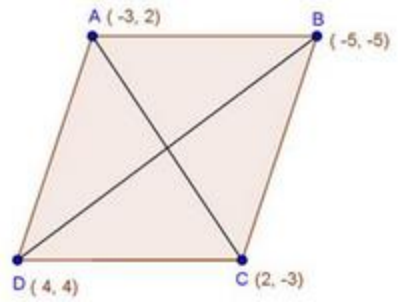
Length of $PR\sqrt{(-1-5)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = 6$

Length of $QS\sqrt{(2-2)^2 + (4+1)^2} = 5$

Here all sides of given quadrilateral is of same measure but the diagonals are of different lengths. So, PQRS is a rhombus.

43. Show that A(—3, 2), B(—5,—5), C(2,—3) and D(4,4)are the vertices of a rhombus.

Sol:



Let $A(-3,2), B(-5,-5), C(2,-3)$ and $D(4,4)$ be the given points

Coordinates of the midpoint of $AC\ are \left(\frac{-3+2}{2}, \frac{2-3}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$

Coordinates of the midpoint of $BD\ are \left(\frac{-5+4}{2}, \frac{-5+4}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$

Thus, AC and BD have the same midpoint

Hence, $ABCD$ is a parallelogram

Now, $AB = \sqrt{(-5+3)^2 + (-5-2)^2}$

$\Rightarrow AB = \sqrt{4+49}$

$\Rightarrow AB = \sqrt{53}$

Now, $BC = \sqrt{(-5-2)^2 + (-5+3)^2}$

$\Rightarrow BC = \sqrt{49+4}$

$\Rightarrow BC = \sqrt{53}$

$\therefore AB = BC$

So, $ABCD$ is a parallelogram whose adjacent sides are equal.

Hence, $ABCD$ is a rhombus.

44. Find the ratio in which the y-axis divides the line segment joining the points (5, —6) and (—1, —4). Also, find the coordinates of the point of division.

Sol:

Let $P(5,-6)$ and $Q(-1,-4)$ be the given points.

Let y-axis divide PQ in the ratio $k:1$

Then, the coordinates of the point of division are

$$R\left[\frac{-k+5}{k+1},\frac{-4k-6}{k+1}\right]$$

Since, R lies on y-axis and x-coordinates of every point on y-axis is zero

$$\therefore \frac{-k+5}{k+1}=0$$

$$\Rightarrow -k+5=0$$

$$\Rightarrow k=5$$

Hence, the required ratio is $5:1$

Putting $k=5$ in the coordinates of R, we get

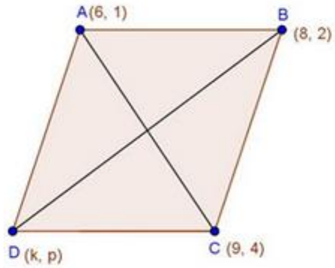
$$\left(\frac{-5+5}{5+1},\frac{-4\times5-6}{5+1}\right)$$

$$=\left(0,\frac{-13}{3}\right)$$

Hence, the coordinates of the point of division are $\left(0,-\frac{13}{3}\right)$.

45. If the points A (6, 1), B (8, 2), C (9, 4) and D (k, p) are the vertices of a parallelogram taken in order, then find the values of k and p.

Sol:



Let $A(6,1),B(8,2),C(9,4)$ and $D(k,p)$ be the given points.

Since, $ABCD$ is a parallelogram

Coordinates of midpoint of $AC=$Coordinates of the midpoints of BD

$\Rightarrow \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+k}{2}, \frac{2+p}{2}\right)$

$\Rightarrow \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+k}{2}, \frac{2+p}{2}\right)$

$\Rightarrow \frac{8+k}{2} = \frac{15}{2}$ and $\frac{2+p}{2} = \frac{5}{2}$

$\Rightarrow 8+k = 15 \quad \Rightarrow 2+p = 5$

$\Rightarrow k = 7 \quad \Rightarrow p = 3$

46. In what ratio does the point (—4, 6) divide the line segment joining the points A (--6, 10) and B (3, -8)?

Sol:

Let $(-4,6)$

Divide AB internally in the ratio $k:1$ using thee section formula, we get

$(-4,6) = \left(\frac{3k-6}{k+1}, \frac{-8k+10}{k+1}\right)$ ……..(2)

So, $-4 = \frac{3k-6}{k+1}$

i.e., $-4k-4 = 3k-6$

i.e., $7k = 2$

i.e., $k:1 = 2:7$

You c check for the y-coordinate also. So, the point $(-4,6)$ divides the line segment j joining the points $A(-6,10)$ and $B(3,-8)$ in the ratio 2: 7

47. Find the coordinates of a point A, where AB is a diameter of the circle whose centre is (2, —3) and B is (1, 4).

Sol:

Let coordinates of point A be (x, y)

Mid-point of diameter AB is center of circle $(2,-3)$

$(2,-3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$

$\frac{x+1}{2} = 2$ *and* $\frac{y+4}{2} = -3$

$x+1 = 4$ and $y+4 = -6$

$x = 3$ and $y = -10$

Therefore coordinates of A are $(3,10)$

48. A point P divides the line segment joining the points A (3, — 5) and B (— 4, 8) such that $\frac{AP}{PB} = \frac{k}{1}$. If P lies on the line x + y = 0, then find the value of y.

Sol:

Given points are $A(3,-5)$ and $B(-4,8)$

P divides AB in the ratio $k:1$,

Using the section formula, we have:

Coordinate of point P are $\{(-4k+3/k+1)(8k-5/k+1)\}$

Now it is given, that P lies on the line $x+y=0$

Therefore

$-4k+3/k+1+8k-5/k+1=0$

$\Rightarrow -4k+3+8k-5=0$

$\Rightarrow 4k-2=0$

$\Rightarrow k=2/4$

$\Rightarrow k=1/2$

Thus, the value of k is $\frac{1}{2}$.

Exercise 14.4

1. Find the centroid of the triangle whose vertices are:

(i) $(1,4),(-1,-1)$ *and* $(3,-2)$

Sol:

We know that the coordinates of the centroid of a triangle whose vertices are $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ are

$$\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

So, the coordinates of the centroid of a triangle whose vertices are

$(1,4),(-1,-1)$ *and* $(3,-2)$ are $\left(\frac{1-1+3}{3},\frac{4-1-2}{3}\right)$

$$=\left(1,\frac{1}{3}\right)$$

2. Two vertices of a triangle are (1, 2), (3, 5) and its centroid is at the origin. Find the coordinates of the third vertex.

Sol:

Let the coordinates of the third vertex bee (x, y), Then

Coordinates of centroid of triangle are

$$\left(\frac{x+1+3}{3}, \frac{y+2+5}{3}\right)$$

We have centroid is at origin $(0, 0)$

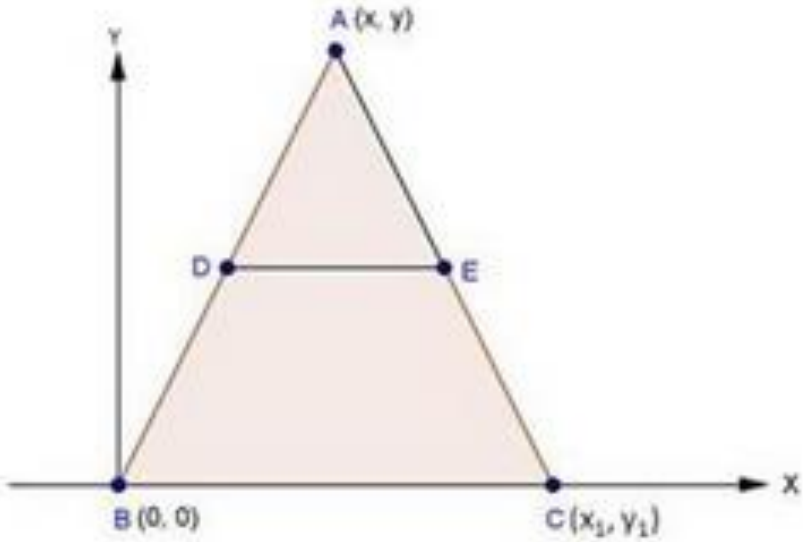
$$\therefore \frac{x+1+3}{3} = 0 \ and \ \frac{y+2+5}{3} = 0$$

$\Rightarrow x+4=0 \quad \Rightarrow y+7=0$

$\Rightarrow x=-4 \quad \Rightarrow y=-7$

3. Prove analytically that the line segment joining the middle points of two sides of a triangle is equal to half of the third side.

Sol:



Let ABC be a triangle such that BC is along x-axis

Coordinates of A, B and C are $(x, y), (0, 0)$ and (x_1, y_1)

D and E are the mid-points of AB and AC respectively

Coordinates of D are $\left(\frac{x+0}{2}, \frac{y+0}{2}\right)$

$$= \left(\frac{x}{2}, \frac{y}{2}\right)$$

Coordinates of E are $\left(\frac{x+x_1}{2}, \frac{y+y_1}{2}\right)$

Length of $BC = \sqrt{x_1^2 + y_1^2}$

Length of DE $= \sqrt{\left(\frac{x+x_1}{2}-\frac{x}{2}\right)^2+\left(\frac{x+y_1}{2}-\frac{y}{2}\right)^2}$

$= \sqrt{\left(\frac{x_1}{2}\right)^2+\left(\frac{y_1}{2}\right)^2}$

$= \sqrt{\frac{x_1^2}{4}+\frac{y_1^2}{4}}$

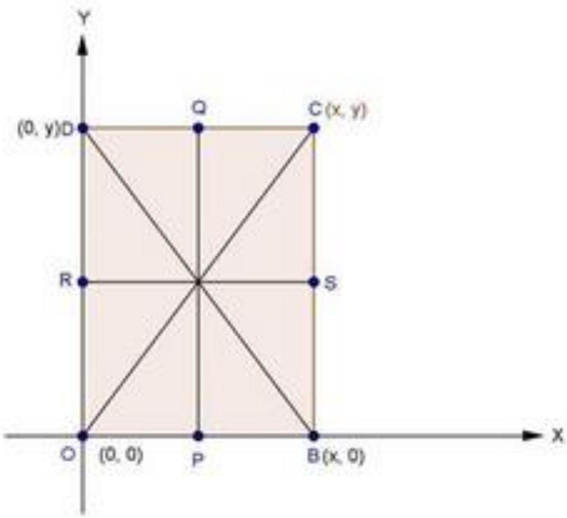
$= \sqrt{\frac{1}{4}\left(x_1^2+y_1^2\right)}$

$= \frac{1}{2}\sqrt{x_1^2+y_1^2}$

$= \frac{1}{2}BC$

4. Prove that the lines joining the middle points of the opposite sides of a quadrilateral and the join of the middle points of its diagonals meet in a point and bisect one another.

Sol:



Let $OBCD$ be the quadrilateral P,Q,R,S be the midpoint off OB,CD,OD and BC.

Let the coordinates of O,B,C,D are $(0,0),(x,0),(x,y)$ and $(0,y)$

Coordinates of P are $\left(\frac{x}{2},0\right)$

Coordinates of Q are $\left(\frac{x}{2},y\right)$

Coordinates of R are $\left(0,\frac{y}{2}\right)$

Coordinates of S are $\left(x, \frac{y}{2}\right)$

Coordinates of midpoint of PQ are

$$\left[\frac{\frac{x}{2}+\frac{x}{2}}{2}, \frac{0+y}{2}\right] = \left(\frac{x}{2}, \frac{y}{2}\right)$$

Coordinates of midpoint of RS are $\left[\frac{(0+x)}{2}, \frac{\frac{y}{2}+\frac{y}{2}}{2}\right] = \left[\frac{x}{2}, \frac{y}{2}\right]$

Since, the coordinates of the mid-point of $PQ =$ coordinates of mid-point of RS

$\therefore PQ$ and RS bisect each other

5. If G be the centroid of a triangle ABC and P be any other point in the plane, prove that $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$.

Sol:

Let $A(0,0), B(a,0)$, and $C(c,d)$ are the o-ordinates of triangle ABC

Hence, $G\left[\frac{c+0+a}{3}, \frac{d}{3}\right]$

i.e., $G\left[\frac{a+c}{3}, \frac{d}{3}\right]$

let $P(x, y)$

To prove:

$$PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$$

Or, $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + GP^2 + GP^2 + GP^2$

Or, $PA^2 - GP^2 + PB^2 - GP^2 + PC^2 + GP^2 = GA^2 + GB^2 + GC^2$

Proof:

$$PA^2 = x^2 + y^2$$

$$GP^2 = \left(x - \frac{a+c}{3}\right)^2 + \left(y - \frac{d}{3}\right)^2$$

$$PB^2 = (x-a)^2 + y^2$$

$$PC^2 = (x-c)^2 + (y-d)^2$$

L.H.S

$$= x^2 + y^2 - \left[x^2 + \left(\frac{a+c}{3}\right)^2 + 2x\frac{(a+c)}{3} + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right] + (x-a)^2 + y^2$$

$$- \left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right] + (x-c)^2 + (y-d)^2$$

$$- \left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right]$$

$$= x^2 + y + x^2 + x^2 + a^2 - 2ax + y^2 + x^2 + c^2 - 2xc + y^2 + d^2 - 2yd - 3$$

$$\left[x^2 + \left(\frac{a+c}{3}\right)^2 - 2x\left(\frac{a+c}{3}\right) + y^2 + \frac{d^2}{9} - \frac{2yd}{3}\right]$$

$$= \cancel{3x^2} + \cancel{3y^2} + a^2 + c^2 + d^2 - 2ax - 2xc - 2yd - \cancel{3x^2} - \frac{(a+c)^2}{3} + 2x(a+c) - \cancel{3y^2} - \frac{d^2}{3} + 2yd$$

$$= a^2 + c^2 + d^2 - \cancel{2ax} - \cancel{2xc} - \cancel{2yd} - \frac{a^2 + c^2 + 2ac}{3} + \cancel{2ax} + \cancel{2cx} - \frac{d^2}{3} + \cancel{2yd}$$

$$= \frac{3a^2 + 3c^2 + 3d^2 - a^2 - c^2 - 2ac - d^2}{3} = \frac{2a^2 + 2c^2 + 2d^2 - 2ac}{3} = L.H.S$$

Solving R.H.S

$$GA^2 + GB^2 + GC^2$$

$$GA^2 = \left(\frac{a+c}{3}\right)^2 + \left(\frac{d}{3}\right)^2 = \frac{a^2 + c^2 + 2ac}{9} + \frac{d^2}{9}$$

$$GC^2 = \left(\frac{a+c}{3} = a\right)^2 + \left(\frac{d}{3}\right)^2 = \left(\frac{c-2a}{3}\right)^2 + \left(\frac{d}{3}\right)^2$$

$$= \frac{a^2 + 4c^2 - 4ca}{9} + \frac{4d^2}{9}$$

$$GB^2 = \left(\frac{a+c}{3} - a\right)^2 + \left(\frac{d}{3}\right)^2 = \left(\frac{c-2a}{3}\right)^2 + \left(\frac{d}{3}\right)^2$$

$$= \frac{c^2 + 4a^2 - 4ac}{9} + \frac{d^2}{9}$$

$$GA^2 + GB^2 + GC^2 = \frac{a^2 + c^2 + 2ac}{9} + \frac{d^2}{9} + \frac{a^2 + 4c^2 - 4ac}{9} + \frac{4d^2}{9} + \frac{c^2 + 4a^2 - 4ac}{9} + \frac{d^2}{9}$$

$$= \frac{a^2 + c^2 + 2ac + d^2 + a^2 + 4c^2 - 4ac + 4d^2 + c^2 + 4a^2 - 4ac + d^2}{9}$$

$$= \frac{6a^2 + 6c^2 + 6d^2 + 6ac}{9} = \frac{2a^2 + 2c^2 + 2d^2 + 2ac}{3}$$

$$\therefore L.H.S = R.H.S$$

6. If G be the centroid of a triangle ABC, prove that:
$AB^2 + BC^2 + CA^2 = 3\left(GA^2 + GB^2 + GC^2\right)$

Sol:

Let $A(b,c), B(0,0)$ and $C(a,0)$ be the coordinates of ΔABC

Then coordinates of centroid are $G\left[\frac{a+b}{3}, \frac{c}{3}\right]$

To prove:

$AB^2 + BC^2 + CA^2 = 3\left(GA^2 + GB^2 + GC^2\right)$

Solving L.H.S

$AB^2 + BC^2 + CA^2$

$= b^2 + c^2 + a^2 \left(a-b\right)^2 + c^2$

$= b^2 + c^2 + a^2 + a^2 + b^2 - 2ab + c^2$

$= 2a^2 + 2b^2 + 2c^2 - 2ab$

Solving R.H.S

$$3\left[\left(\frac{a+b}{3} - b\right)^2 + \left(c - \frac{c}{3}\right)^2 + \left(\frac{a+b}{3}\right)^2 + \left(\frac{c}{3}\right)^2 + \left(\frac{a+b}{2} - a\right)^2 + \left(\frac{c}{3}\right)^2\right]$$

$$= 3\left[\left(\frac{a-2b}{3}\right)^2 + \left(\frac{2c}{3}\right)^2 + \left(\frac{a+b}{3}\right)^2 + \left(\frac{c}{3}\right)^2 + \left(\frac{b-2a}{3}\right)^2 + \left(\frac{c}{3}\right)^2\right]$$

$$= 3\left[\frac{a^2 + 4b^2 - 4ab}{9} + \frac{4c^2}{9} + \frac{a^2 + b^2 + 2ab}{9} + \frac{c^2}{9} + \frac{b^2 + 4a^2 - 4ab}{9} + \frac{c^2}{9}\right]$$

$$= 3\left[\frac{a^2 + 4b^2 - 4ab + 4c^2 + a^2 + b^2 + 2ab + c^2 + b^2 + 4a^2 - 4ab + c^2}{9}\right]$$

$$= 3\left[\frac{6a^2 + 6b^2 + 6c^2 - 6ab}{9}\right]$$

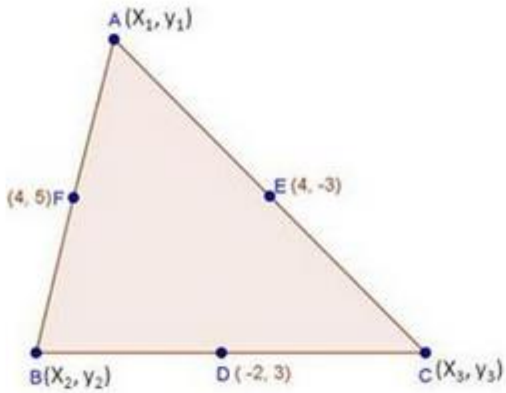
$$= \cancel{3} \times 3\left[\frac{2a^2 + 2b^2 + 2c^2 - 2ab}{\cancel{9}}\right]$$

$= 2a^2 + 2b^2 + 2c^2 - 2ab$

$\therefore$ L.H.S = R.H.S proved

7. If (—2, 3), (4, —3) and (4, 5) are the mid-points of the sides of a triangle, find the coordinates of its centroid.

Sol:



Let $A(x_1, y_2), B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC

Let $D(-2,3), E(4,-3)$ and $F(4,5)$ be the midpoints of sides BC, CA *and* AB respectively

Since, D is the midpoint of BC

$\frac{x_2 + x_3}{2} = -2$ *and* $\frac{y_2 + y_3}{2} = 3$

$\Rightarrow x_2 + x_3 = -4$ *and* $y_2 + y_3 = 6$ ....(i)

And, $\frac{x_1 + x_3}{2} = 4$ *and* $\frac{y_1 + y_3}{2} = -3$

$\Rightarrow x_1 + x_2 = 8$ *and* $y_1 + y_3 = -6$ ....(ii)

And, $\frac{x_1 + x_2}{2} = 4$ *and* $\frac{y_1 + y_2}{2} = 5$

$\Rightarrow x_1 + x_2 = 8$ *and* $y_1 + y_3 = 10$ ....(iii)

From (i), (ii) and (iii), we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = -4 + 8 + 8$ and

$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 6 - 6 + 10$

$\Rightarrow 2(x_1 + x_2 + x_3) = 12$ and $2(y_1 + y_2 + y_3) = 10$

$\Rightarrow x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 5$ ....(iv)

From (i) and (iv), we get

$x_1 - 4 = 6$ *and* $y_1 + 6 = 5$

$\Rightarrow x_1 = 10$ $\Rightarrow y_1 = -1$

So, the coordinates of A are $(10, -1)$

From (ii) and (iv)

$x_2 + 8 = 6$ *and* $y_2 - 6 = 5$

$\Rightarrow x_2 = -2$ $\Rightarrow y_2 = 11$

So, the coordinates of B are $(-2,11)$

From (iii) and (iv)

$x_3+8=6$ *and* $y_3+10=5$

$\Rightarrow x_3=-2 \qquad \Rightarrow y_3=-5$

So, the coordinates of C are $(-2,-5)$

$\therefore$ The vertices of ΔABC are $A(10,-1), B(-2,11)$ and $C(-2,-5)$

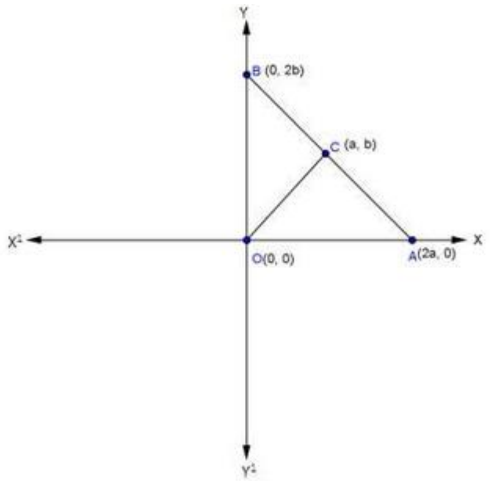
Hence, coordinates of the centroid of ΔABC are

$$\left(\frac{10-2-2}{3},\frac{-1+11-5}{3}\right)$$

$$=\left(2,\frac{5}{3}\right)$$

8. In below Fig. a right triangle BOA is given. C is the mid-point of the hypotenuse AB. Show that it is equidistant from the vertices O, A and B.

Sol:



Given a right triangle *BOA* with vertices $B(0,2b), 0(0,0)$ and $A(2a,0)$

Since, C is the midpoint of AB

$\therefore$ coordinates of C are $\left(\frac{2a+0}{2},\frac{0+2b}{2}\right)$

$=(a,b)$

Now, $CO=\sqrt{(a-0)^2+(b-0)^2}=\sqrt{a^2+b^2}$

$CA = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{a^2+b^2}$

$CB = \sqrt{(a-0)^2 + (b-2b)^2} = \sqrt{a^2+b^2}$

Since, $CO = CA = CB.$

$\therefore C$ is equidistant from $O, A \text{ and } B.$

9. Find the third vertex of a triangle, if two of its vertices are at (-3, 1) and (0, -2) and the centroid is at the origin

Sol:

Let the coordinates of the third vertex be (x, y), Then

Coordinates of centroid of triangle are

$$\left(\frac{x-3+0}{3}, \frac{y+1-2}{3}\right) = \left(\frac{x-3}{2}, \frac{y-1}{3}\right)$$

We have centroid is at origin $(0,0)$

$\therefore \frac{x-3}{3} = 0 \text{ and } \frac{y-1}{3} = 0$

$\Rightarrow x-3=0 \qquad \Rightarrow y-1=0$

$\Rightarrow x=3 \qquad \Rightarrow y=1$

Hence, the coordinates of the third vertex are $(3,1)$.

10. A (3, 2) and B (−2, 1) are two vertices of a triangle ABC whose centroid G has the coordinates $\left(\frac{5}{11}, \frac{1}{3}\right)$. Find the coordinates of the third vertex C of the triangle.

Sol:

Let the third vertex be $C(x, y)$

Two vertices $A(3,2)$ and $B(-2,1)$

Coordinates of centroid of triangle are

$$\left(\frac{x+3-2}{3}, \frac{y+2+1}{3}\right)$$

But the centroid of the triangle are $\left(\frac{5}{3}, -\frac{1}{3}\right)$

$\therefore \frac{x+3-2}{3} = \frac{5}{3} \text{ and } \frac{y+2+1}{3} = -\frac{1}{3}$

$\Rightarrow \frac{x+1}{3} = \frac{5}{3} \qquad \Rightarrow \frac{y+3}{3} = -\frac{1}{3}$

$\Rightarrow x+1=5 \qquad \Rightarrow y+3=-1$

$\Rightarrow x=4 \qquad \Rightarrow y=-4$

Hence, the third vertex of the triangle is $C(4,-4)$

Exercise 14.5

1. Find the area of a triangle whose vertices are

(i) $(6,3),(-3,5)$ and $(4,-2)$

(ii) $\left[\left(at_1^2, 2at_1\right),\left(at_2^2, 2at_2\right)\left(at_3^2, 2at_3\right)\right]$

(iii) $(a, c+a),(a, c)\ and\ (-a, c-a)$

Sol:

(i) Area of a triangle is given by

$$\frac{1}{2}\left[x_1\left(y_2-y_3\right)+x_2\left(y_3-y_1\right)+x_3\left(y_1+y_2\right)\right]$$

Here, $x_1=6, y_1=3, x_2=-3, y_2=5, x_3=4, y_3=-2$

Let $A(6,3), B(-3,5)$ and $C(4,-2)$ be the given points

Area of $\Delta ABC=\frac{1}{2}[6(5+2)+(-3)(-2-3)+4(3-5)]$

$$=\frac{1}{2}[6\times 7-3\times(-5)+4(-2)]$$

$$=\frac{1}{2}[42+15-8]$$

$$=\frac{49}{2}\ sq.units$$

(ii) Let $A=\left(x_1, y_1\right)=\left(at_1^2, 2at_1\right)$

$B=\left(x_2, y_2\right)=\left(at_2^2, 2at_2\right)$

$=\left(x_3, y_3\right)=\left(at_3^2, 2at_3\right)$ be the given points.

The area of ΔABC

$$=\frac{1}{2}\left[at_1^2\left(2at_2-2at_3\right)+at_2^2\left(2at_3-2at_1\right)+at_3^2\left(2at_1-2at_2\right)\right]$$

$$=\frac{1}{2}\left[2a^2t_1^2t_2-2a^2t_1^2t_3+2a^2t_2^2t_3-2a^2t_2^2t_1+2a^2t_3^2t_1-2a^2t_3^2t_2\right]$$

$$=\frac{1}{2}\times 2\left[a^2t_1^2\left(t_2-t_3\right)+a^2t_2^2\left(t_3-t_1\right)+a^2+t_3^2\left(t_1-t_2\right)\right]$$

$$=a^2\left[t_1^2\left(t_2-t_3\right)+t_2^2\left(t_3-t_1\right)+t_3^2\left(t_1-t_2\right)\right]$$

(iii) Let $A=\left(x_1, y_1\right)=(a, c+a)$

$B=\left(x_2, y_2\right)=(a, c)$

$C=(x_3,y_3)=(-a,c-a)$ be the given points

The area of ΔABC

$=\frac{1}{2}\left[a\left(c-\{c-a\}\right)+a\left(c-a-(c+a)\right)+(-a)(c+a-a)\right]$

$=\frac{1}{2}\left[a(c-c+a)+a(c-a-c-a)-a(c+a-c)\right]$

$=\frac{1}{2}\left[a\times a+ax(-2a)-a\times a\right]$

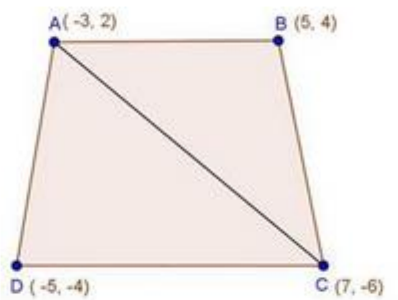
$=\frac{1}{2}\left[a^2-2a^2-a^2\right]$

$=\frac{1}{2}\times(-2a)^2$

$=-a^2$

2. Find the area of the quadrilaterals, the coordinates of whose vertices are
(i) (—3, 2), (5, 4), (7, — 6) and (—5, — 4)
(ii) (1, 2), (6, 2), (5, 3) and (3, 4)
(iii) (—4, — 2), (—3, — 5), (3, — 2), (2, 3)

Sol:



Let $A(-3,2),B(5,4),C(7,-6)$ and $D(-5,-4)$ be the given points.

Area of ΔABC

$=\frac{1}{2}\left[-3(4+6)+5(-6-2)+7(2-4)\right]$

$=\frac{1}{2}\left[-3\times10+5\times(-8)+7(-2)\right]$

$=\frac{1}{2}[-30-40-14]$

$=-42$

But area cannot be negative

$\therefore$ Area of $\Delta ADC=42$ square units

Area of ΔADC

$=\frac{1}{2}\left[-3(-6+4)+7(-4-2)+(-5)(2+6)\right]$

$=\frac{1}{2}\left[-3(-2)+7(-6)-5\times 8\right]$

$=\frac{1}{2}[6-42-40]$

$=\frac{1}{2}\times -76$

$=-38$

But area cannot be negative

$\therefore$ Area of $\Delta ADC=38$ square units

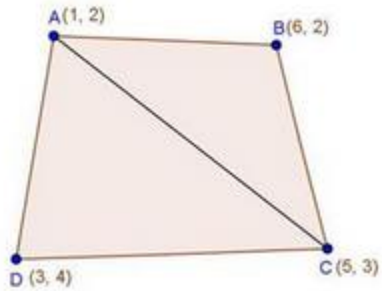
Now, area of quadrilateral $ABCD$

$=Ar.\ of\ ABC+Ar\ of\ ADC$

$=(42+38)$

$=80$ square. units

(i)



Let $A(1,2),B(6,2),C(5,3)$ and $(3,4)$ be the given points

Area of ΔABC

$=\frac{1}{2}\left[1(2-3)+6(3-2)+5(2-2)\right]$

$=\frac{1}{2}\left[-1+6\times(1)+0\right]$

$=\frac{1}{2}[-1+6]$

$=\frac{5}{2}$

Area of ΔADC

$=\frac{1}{2}\left[1(3-4)+5(4-2)+3(2-3)\right]$

$=\frac{1}{2}\left[-1\times 5\times 2+3(-1)\right]$

$=\frac{1}{2}[-1+10-3]$

$=\frac{1}{2}[6]$

$=3$

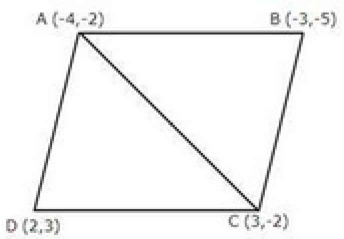
Now, Area of quadrilateral $ABCD$

$=$Area of $ABC+$Area of ADC

$=\left(\frac{5}{2}+3\right) sq.\ units$

$=\frac{11}{2}\ sq.\ units$

(ii)



Let $A(-4,-2), B(-3,-5), C(3,-2)$ and $D(2,3)$ be the given points

Area of $\Delta ABC=\frac{1}{2}\left|(-4)(-5+2)-3(-2+2)+3(-2+5)\right|$

$=\frac{1}{2}\left|(-4)(-3)-3(0)+3(3)\right|$

$=\frac{21}{2}$

Area of $\Delta ACD=\frac{1}{2}\left|(-4)(3+2)+2(-2+2)+3(-2-3)\right|$

$=\frac{1}{2}\left|-4(5)+2(0)+3(-5)\right|=\frac{-35}{2}$

But area can't be negative, hence area of $\Delta ADC=\frac{35}{2}$

Now, area of quadrilateral $(ABCD)=ar(\Delta ABC)+ar(\Delta ADC)$

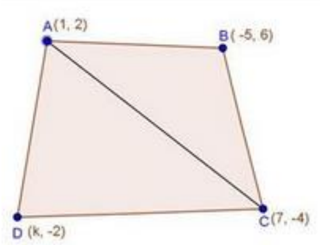
Area (quadrilateral $ABCD$) $=\frac{21}{2}+\frac{35}{2}$

Area (quadrilateral $ABCD$) $=\frac{56}{2}$

Area (quadrilateral $ABCD$) $=28$ square. Units

3. The four vertices of a quadrilateral are (1, 2), (−5, 6), (7, −4) and (k, −2) taken in order. If the area of the quadrilateral is zero, find the value of k.

Sol:



Let $A(1,2), B(-5,6), C(7,-4)$ and $(k,-2)$ be the given points.

Area of ΔABC

$= \frac{1}{2}\left[1(6+4)+(-5)(-4-2)+7(2-6)\right]$

$= \frac{1}{2}[10+30-28]$

$= \frac{1}{2}\times 12$

$= 6$

Area of ΔADC

$= \frac{1}{2}\left[1(-4+2)+7(-2-2)+k(2+4)\right]$

$= \frac{1}{2}\left[-2+7\times(-4)+k\times 6\right]$

$= \frac{1}{2}[-2-28+6k]$

$= \frac{1}{2}[-30+6k]$

$= -15+3k$

$= 3k-15$

Area of quadrilateral $ABCD$

= Area of $ABC +$ Area of ADC

$= (6+3k-15)$

But are of quadrilateral $= 0$ (given)

$\therefore 6+3k-15=0$

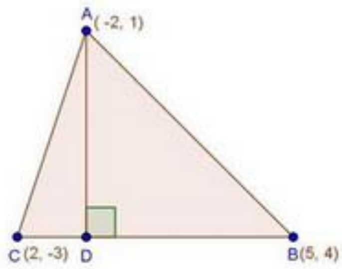
$\Rightarrow 3k = 15-6$

$\Rightarrow 3k = 9$

$\Rightarrow k = 3$

4. The vertices of ΔABC are (—2, 1), (5, 4) and (2, —3) respectively. Find the area of the triangle and the length of the altitude through A.

Sol:



Let $A(-2,1), B(5,4)$ and $C(2,-3)$ be the vertices of ΔABC.

Let AD be the altitude through A.

Area of ΔABC

$$= \frac{1}{2}[-2(4+3)+5(-3-1)+2(1-4)]$$

$$= \frac{1}{2}[-14-20-6]$$

$$= \frac{1}{2} \times -40$$

$$= -20$$

But area cannot be negative

$\therefore$ Area of $\Delta ABC = 20$ square units

Now, $BC = \sqrt{(5-2)^2 + (4+3)^2}$

$$\Rightarrow BC = \sqrt{(3)^2 + (7)^2}$$

$$\Rightarrow BC = \sqrt{58}$$

We know that area of Δ

$$= \frac{1}{2} \times Base \times Altitude$$

$$\therefore 20 = \frac{1}{2} \times \sqrt{58} \times AD$$

$$\Rightarrow AD = \frac{40}{\sqrt{58}}$$

$\therefore$ Length of the altitude $AD = \frac{40}{\sqrt{58}}$

5. Show that the following sets of points are collinear.
(a) (2, 5), (4, 6) and (8, 8) (b) (1, —1), (2, 1) and (4, 5)

Sol:

(a) Let $A(2,5), B(4,6)$ and $C(8,8)$ be the given points

Area of ΔABC

$=\frac{1}{2}[2(6-8)+4(8-5)+8(5-6)]$

$=\frac{1}{2}[2\times(-2)+4\times3+8\times(-1)]$

$=\frac{1}{2}[-4+12-8]$

$=\frac{1}{2}\times0$

$=0$

Since, area of $\Delta ABC=0$

$\therefore (2,5),(4,6)$ and $(8,8)$ are collinear.

(b) Let $A(1,-1), B(2,1)$ and $C(4,5)$ be the given points

Area of ΔABC

$=\frac{1}{2}[1(1-5)+2(5+1)+4(-1-1)]$

$=\frac{1}{2}[-4+12-8]$

$=\frac{1}{2}\times0$

$=0$

Since, area of $\Delta ABC=0$

$\therefore$ The points $(1,-1),(2,1)$ and $(4,5)$ are collinear

6. Prove that the points (a, 0), (0, b) and (1, 1) are collinear if, $\frac{1}{a}+\frac{1}{b}=1$.

Sol:

Let $A(a,0), B(0,b)$ and $C(1,1)$ be the given points

Area of ΔABC

$=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$

$=\frac{1}{2}\{a(b-1)+0(1-0)+1(0-b)\}$

$=\frac{1}{2}\{ab-a+0-b\}$

$=\frac{1}{2}\{ab-a-b\}$

$=\frac{1}{2}\{ab-(a+b)\}$

$=\frac{1}{2}\{ab-ab\}$ $\left[\because \frac{1}{a}+\frac{1}{b}=1\right]$

$\Rightarrow \frac{a+b}{ab}=1$

$\Rightarrow a+b=ab$

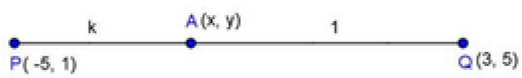
$=\frac{1}{2}\times 0$

$=0$

Hence, $A(a,0), B(0,b)$ and $(1,1)$ are collinear if $\frac{1}{a}+\frac{1}{b}=1.$

7. The point A divides the join of P (—5, 1) and Q (3, 5) in the ratio k : 1. Find the two values of k for which the area of ΔABC where B is (1, 5) and C (7, —2) is equal to 2 units.

Sol:



Let $A(x,y)$ divides the join of $P(-5,1)$ and $(3,5)$ in the ratio $k:1$

$x=\frac{3k-5}{k+1}, y=\frac{5k+1}{k+1}$

Area of ΔABC with $A\left(\frac{3k-5}{k+1},\frac{5k+1}{k+1}\right)B(1,5)$ and $C(7,-2)$

$=\frac{1}{2}\left\{\frac{3k-5}{k+1}(5-2)+1\left(2-\frac{5k+1}{k+1}\right)+7\left(\frac{5k+1}{k+1}=5\right)\right\}$

$=\frac{1}{2}\left\{\frac{3k-5}{k+1}\times 7+\frac{-7k-3}{k+1}+\frac{-4}{k+1}\right\}$

$=\frac{1}{2}\left\{\frac{21k-35}{k+1}+\frac{-7k-3}{k+1}+\frac{-4}{k+1}\right\}$

$=\frac{1}{2}\left\{\frac{21k-35-7k-3-4}{k+1}\right\}$

$=\frac{1}{2}\left\{\frac{14k-42}{k+1}\right\}$

$= \frac{14k-42}{2(k+1)}$

But area of $\Delta ABC = 2$ given,

$\Rightarrow \frac{14k-42}{2(k+1)} = 2$

$\Rightarrow 14k - 42 = 4(k+1)$

$\Rightarrow 14k - 42 = 4k + 4$

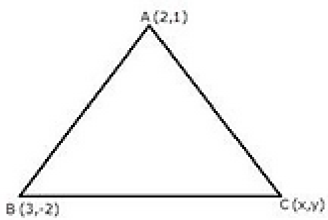
$\Rightarrow 14k - 4k = 4 + 42$

$\Rightarrow 10k = 46$

$\Rightarrow k = \frac{46}{10} = \frac{23}{5}$

8. The area of a triangle is 5. Two of its vertices are (2, 1) and (3, -2). Third vertex lies on y = x + 3. Find the third vertex.

Sol:



Let $A(2,1), B(3,-2)$ be the vertices of Δ

And $C(x,y)$ be the third vertex

Area of $\Delta ABC = \frac{1}{2}\left|2(-2-y)+3(y-1)+x(1+2)\right|$

$= \frac{1}{2}\left|-4-2y+3y-3+3x\right|$

$= \frac{1}{2}\left|3x+y-7\right|$

But it is given that area of $\Delta ABC = 5$

$\therefore 5 = \frac{\pm 1}{2}\left[3x+y-7\right]$

$\pm 10 = 3x + y - 7$

$3x + y = 17$ or $3x + y = -3$ (i)

But it is given that third vertices lies on $y = x + 3$

Hence subsisting value of y in (i)

$3x + x + 3 = 17 \ \ or \ \ 3x + x + 3 = -3$

$4x = 14 \quad or \quad 4x = -6$

$x = \frac{7}{2} \quad or \quad x = \frac{-3}{2}$

$y = \frac{7}{2} + 3 \quad or \quad y = \frac{-3}{2} + 3$

$y = \frac{13}{2} \quad or \quad y = \frac{3}{2}$

Hence coordinates of c will be $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

9. If $a \neq b \neq c$, prove that the points $(a, a^2), (b, b^2), (c, c^2)$ can never be collinear.

Sol:

Let $A(a, a^2)\, B(b, b^2)$ and (c, c^2) be the given points.

$\therefore$ Area of ΔABC

$= \frac{1}{2}\left\{a(b^2 - c^2) + b(c^2 - a^2) + c(a^2 - b^2)\right\}$

$= \frac{1}{2}\left\{ab^2 - ac^2 + bc^2 - ba^2 + ca^2 - cb^2\right\}$

$= \frac{1}{2} \times 0$

$= 0 \qquad [if \;\; a = b = c]$

i.e., the points are collinear if $a = b = c$

Hence, the points can never be collinear if $a \neq b \neq c$.

10. Four points A(6, 3), B (-3, 5), C(4, -2) and D(x, 3x) are given in such a way that $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$, find *x*.

Sol:

Area of $\Delta DBC = \frac{1}{2}\left\{x(5+2) + (-3)(-2-3x) + 4(3x-5)\right\}$

$= \frac{1}{2}\left\{7\times + (6+9x) + 12x - 20\right\}$

$= \frac{1}{2}\left\{28x - 14\right\}$

Area of $\Delta ABC = \frac{1}{2}\left\{6(5+2) + (-3)(-2-3) + 4(3-5)\right\}$

$= \frac{1}{2}\left\{42 + 15 - 8\right\}$

$=\frac{1}{2}\times 49$

Given

$\frac{\Delta DBC}{\Delta ABC}=\frac{1}{2}$

$\Rightarrow \frac{\frac{1}{2}(28x-14)}{\frac{1}{2}\times 49}=\frac{1}{2}$

$\Rightarrow \frac{28x-14}{49}=\frac{1}{2}$

$\Rightarrow 2(28x-14)=49$

$\Rightarrow 56x-28=49$

$\Rightarrow 56x=77$

$\Rightarrow x=\frac{77}{56}$

$\Rightarrow x=\frac{11}{8}$

11. For what value of a point (a, 1), (1, -1) and (11, 4) are collinear?

Sol:

Let $A(a,1), B(1,-1)$ and $C(11,4)$ be the given points

Area of ΔABC

$=\frac{1}{2}\{a(-1-4)+1(4-1)+11(1+1)\}$

$=\frac{1}{2}\{-5+3+22\}$

$=\frac{1}{2}\{-5a+25\}$

For the points to be collinear

Area of $\Delta ABC=0$

$=\frac{1}{2}\{-5a+25\}=0$

$\Rightarrow -5a+25=0$

$\Rightarrow -5a=-25$

$\Rightarrow a=5$

12. Prove that the points (a, b), (a_1, b_1) and $(a-a_1, b-b_1)$ are collinear if $ab_1=a_1b$

Sol:

Let $A(a,b), B(a_1,b_1)$ and $C(a-a_1, b-b_1)$ be the given points.

Area of ΔABC

$$=\frac{1}{2}\left\{a\left[b_1-(b-b_1)+a_1(b-b_1-b)+(a-a_1)(b-b_1)\right]\right\}$$

$$=\frac{1}{2}\left\{a(b_1-b+b_1)+a_1(-b)_1+ab-ab_1-a_1b+a_1b_1\right\}$$

$$=\frac{1}{2}\left\{ab_1-ab+ab_1-a_1b_1+ab-ab_1-a_1b+a_1b_1\right\}$$

$$=\frac{1}{2}\left\{ab_1=a_1b\right\}$$

$$=\frac{1}{2}\times 0=0 \qquad [if\ \ ab_1=a_1b]$$

Hence, the points are collinear if $ab_1=a_1b$.