Chapter 7: Probability Distributions

EXERCISE 7.1 [PAGES 232 - 233]

Exercise 7.1 | Q 1 | Page 232

Let X represent the difference between the number of heads and the number of tails when a coin is tossed 6 times. What are the possible values of X?

SOLUTION 1

Given: X = Number of heads − Number of tails

Number of heads	Number of heads	Number of heads − Number of tails
0	6	−6
1	5	−4
2	4	−2
3	3	0
4	2	2
5	1	4
6	0	6

Therefore, the possible values of X are :
−6, −4, −2, 0, 2, 4, 6

SOLUTION 2

When a coin is tossed 6 times, the number of heads can be 0, 1, 2, 3, 4, 5, 6.

The corresponding number of tails will be 6, 5, 4, 3, 2, 1, 0.

∴ X can take values 0 – 6, 1 – 5, 2 – 4, 3 – 3, 4 – 2, 5 – 1, 6 – 0

i.e. -6, -4, -2, 0, 2, 4, 6.

∴ X = { -6, -4, -2, 0, 2, 4, 6 }.

Exercise 7.1 | Q 2 | Page 232

An urn contains 5 red and 2 black balls. Two balls are drawn at random. X denotes number of black balls drawn. What are possible values of X?

SOLUTION

The urn contains 5 red and 2 black balls. If two balls are drawn from the urn, it contains either 0 or 1 or 2 black balls.

X can take values 0, 1, 2.

∴ X = {0, 1, 2}.

Exercise 7.1 | Q 3.1 | Page 232

State if the following is not the probability mass function of a random variable. Give reasons for your answer.

X	0	1	2
P(X)	0.4	0.4	0.2

SOLUTION

P.m.f. of random variable should satisfy the following conditions:

(a) $0 \leq p_i \leq 1$

(b) $\sum p_i = 1$

X	0	1	
P(X)	0.4	0.4	0.2

(a) Here $0 \leq p_i \leq 1$

(b) $\sum p_i = 0.4 + 0.4 + 0.2 = 1$

Hence, P(X) can be regarded as p.m.f. of the random variable X.

Exercise 7.1 | Q 3.2 | Page 232

State if the following is not the probability mass function of a random variable. Give reasons for your answer.

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	-0.1	0.2

SOLUTION

P.m.f. of random variable should satisfy the following conditions:

(a) $0 \leq p_i \leq 1$

(b) $\sum p_i = 1$

X	0	1			
P(X)	0.1	0.5	0.2	-0.1	0.2

P(X = 3) = -0.1, i.e. $p_i < 0$ which does not satisfy $0 \le p_i \le 1$

Hence, P(X) cannot be regarded as p.m.f. of the random variable X.

Exercise 7.1 | Q 3.3 | Page 232

State if the following is not the probability mass function of a random variable. Give reasons for your answer.

X	0	1	2
P(X)	0.1	0.6	0.3

SOLUTION

P.m.f. of random variable should satisfy the following conditions :

(a) $0 \le p_i \le 1$

(b) ∑pi = 1

X	0	1	
P(X)	0.1	0.6	0.3

(a) Here $0 \le p_i \le 1$

(b) ∑pi = 0.1 + 0.6 + 0.3 = 1

Hence, P(X) can be regarded as p.m.f. of the random variable X.

Exercise 7.1 | Q 3.4 | Page 232

State if the following is not the probability mass function of a random variable. Give reasons for your answer

Z	3	2	1	0	-1
P(Z)	0.3	0.2	0.4	0	0.05

SOLUTION

P.m.f. of random variable should satisfy the following conditions :

(a) $0 \le p_i \le 1$

(b) $\sum p_i = 1$.

Z	3	2	1	0	-1
P(Z)	0.3	0.2	0.4	0	0.05

Here $\sum p_i = 0.3 + 0.2 + 0.4 + 0 + 0.05 = 0.95 \ne 1$

Hence, P(Z) cannot be regarded as p.m.f. of the random variable Z.

Exercise 7.1 | Q 3.5 | Page 232

State if the following is not the probability mass function of a random variable. Give reasons for your answer.

Y	-1	0	1
P(Y)	0.6	0.1	0.2

SOLUTION

P.m.f. of random variable should satisfy the following conditions :

(a) $0 \leq pi \leq 1$

(b) $\sum pi = 1$

Y	-1	0	1
P(Y)	0.6	0.1	0.2

Here $\sum pi = 0.6 + 0.1 + 0.2 = 0.9 \neq 1$

Hence, P(Y) cannot be regarded as p.m.f. of the random variable Y.

Exercise 7.1 | Q 3.6 | Page 232

State if the following is not the probability mass function of a random variable. Give reasons for your answer.

X	0	-1	-2
P(X)	0.3	0.4	0.3

SOLUTION

P.m.f. of random variable should satisfy the following conditions :

(a) $0 \leq p_i \leq 1$

(b) $\sum p_i = 1$.

X	0	-1	-2
P(X)	0.3	0.4	0.3

(a) Here $0 \leq p_i \leq 1$

(b) $\sum p_i = 0.3 + 0.4 + 0.3 = 1$

Hence, P(X) can be regarded as p.m.f. of the random variable X.

Exercise 7.1 | Q 4.1 | Page 232

Find the probability distribution of number of heads in two tosses of a coin.

SOLUTION

When one coin is tossed twice, the sample space is

{HH, HT, TH, TT}

Let X represent the number of heads s in two tosses of a coin.

∴ X (HH) = 2, X (HT) = 1, X (TH) = 1, X (TT) = 0

Therefore, X can take the value of 0, 1, or 2.

It is known that,

$$P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}$$

$$P(X = 0) = P(TT) = \frac{1}{4}$$

$$P(X = 1) = P(HT) + P(TH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(X = 2) = P(HH) = \frac{1}{4}$$

Thus, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Exercise 7.1 | Q 4.2 | Page 232

Find the probability distribution of number of tails in the simultaneous tosses of three coins.

SOLUTION

When three coins are tossed simultaneously, the sample space is

{HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}

Let X represent the number of tails.

It can be seen that X can take the value of 0, 1, 2, or 3.

$P(X = 0) = P(HHH) = \frac{1}{8}$

$P(X = 1) = P(HHT) + P(HTH) + P(THH) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

$P(X = 2) = P(HTT) + P(THT) + P(TTH) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

$P(X = 3) = P(TTT) = \frac{1}{8}$

Thus, the probability distribution is as follows.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Exercise 7.1 | Q 4.3 | Page 232

Find the probability distribution of number of heads in four tosses of a coin

SOLUTION

When a coin is tossed four times, the sample space is

S = {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT}

∴ n (S) = 16

Let X be the random variable, which represents the number of heads.

It can be seen that X can take the value of 0, 1, 2, 3, or 4.

When X = 0, then X = {TTTT}

∴ n (X) = 1

$\therefore P(X = 0) = \frac{n(X)}{n(S)} = \frac{1}{16}$

When X = 1, then

X = {HTTT, THTT, TTHT, TTTH}

∴ n (X) = 4

$\therefore$ P (X = 1) = $\frac{n(X)}{n(S)} = \frac{4}{16} = \frac{1}{4}$

When X = 2, then

X = {HHTT, HTHT, HTTH, THHT, THTH, TTHH}

$\therefore$ n (X) = 6

$\therefore$ P (X = 2) = $\frac{n(X)}{n(S)} = \frac{6}{16} = \frac{3}{8}$

When X = 3, then

X = {HHHT, HHTH, HTHH, THHH}

$\therefore$ n (X) = 4

$\therefore$ P (X = 3) = $\frac{n(X)}{n(S)} = \frac{4}{16} = \frac{1}{4}$

When X = 4, then

X = {HHHH}

$\therefore$ n (X) = 1

$\therefore$ P (X = 4) = $\frac{n(X)}{n(S)} = \frac{1}{16}$

$\therefore$ the probability distribution of X is as follows :

X	0	1	2	3	4
P (X)	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

Exercise 7.1 | Q 5 | Page 232

Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as number greater than 4 appears on at least one die.

SOLUTION

When a die is tossed twice, the sample space S has 6 × 6 = 36 sample points.

∴ n(S) = 36

Trial will be a success if the number on at least one die is 5 or 6.

Let X denote the number of dice on which 5 or 6 appears.

Then X can take values 0, 1, 2

When X = 0 i.e., 5 or 6 do not appear on any of the dice, then

X = {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)}.

∴ n(X) = 16.

∴ P(X = 0) = $\frac{n(X)}{n(S)} = \frac{16}{36} = \frac{4}{9}$

When X = 1, i.e. 5 or 6 appear on exactly one of the dice, then

X = {(1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (6, 1), (6, 2), (6, 3), (6, 4)}

∴ n(X) = 16

∴ P(X = 1) = $\frac{n(X)}{n(S)} = \frac{16}{36} = \frac{4}{9}$

When X = 2, i.e. 5 or 6 appear on both of the dice, then

X = {(5, 5), (5, 6), (6, 5), (6, 6)}

∴ n(X) = 4.

∴ P(X = 2) = $\frac{n(X)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

∴ the required probability distribution is

X = x	0	1	2
P(X = x)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

Exercise 7.1 | Q 6 | Page 232

From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

SOLUTION

It is given that out of 30 bulbs, 6 are defective.

⇒ Number of non-defective bulbs = 30 − 6 = 24

4 bulbs are drawn from the lot with replacement.

Let X be the random variable that denotes the number of defective bulbs in the selected bulbs.

∴ X can take the value 0, 1, 2, 3, 4.

∴ P (X = 0) = P (4 non-defective and 0 defective) = $^4C_0 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{256}{625}$

P (X = 1) = P (3 non-defective and 1defective) =$^4C_1 \times \left(\frac{1}{5}\right) \times \left(\frac{4}{5}\right)^3 = \frac{256}{625}$

P (X = 2) = P (2 non-defective and 2defective) =$^4C_2 \times \left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)^2 = \frac{96}{625}$

P (X = 3) = P (1 non-defective and 3defective) =$^4C_3 \times \left(\frac{1}{5}\right)^3 \times \left(\frac{4}{5}\right) = \frac{16}{625}$

P (X = 4) = P (0 non-defective and 4defective) =$^4C_4 \times \left(\frac{1}{5}\right)^4 \times \left(\frac{4}{5}\right)^0 = \frac{1}{625}$

Therefore, the required probability distribution is as follows.

X = x	0	1	2	3	4
P (X = x)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Exercise 7.1 | Q 7 | Page 232

A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

SOLUTION

Given a biased coin such that heads is 3 times as likely as tails.

$\therefore$ P(H) = $\frac{3}{4}$ and P(T) = $\frac{1}{4}$

The coin is tossed twice.

Let X can be the random variable for the number of tails.

Then X can take the value 0, 1, 2.

$\therefore$ P (X = 0) = P (HH) = $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

P (X = 1) = P (HT, TH) = $\frac{3}{4} \times \frac{1}{4} + \frac{1}{4} \times \frac{3}{4} = \frac{6}{16} = \frac{3}{8}$

P (X = 2) = P (TT) = $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{9}{16}$	$\frac{3}{8}$	$\frac{1}{16}$

Exercise 7.1 | Q 8 | Page 232

A random variable X has the following probability distribution :

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine :
(i) k
(ii) P (X < 3)
(iii) P (X > 4)

SOLUTION

(i) Since P (x) is a probability distribution of x,

$\Sigma_{x=0}^{7} P(x) = 1$

∴ P (0) + P (1) + P(2) + P(3) + P(4) + P(5) + P (6) + P (7) = 1

∴ 0 + k + 2k + 2k + 3k + k^2 + $2k^2$ + $7k^2$ + k = 1

∴ $10k^2$ + 9k - 1 = 0

∴ $10k^2$ + 10k - k - 1 = 0

∴ 10k (k + 1) - 1 (k + 1) = 0

∴ (k + 1) (10k - 1) = 0

∴ 10k - 1 = 0

∴ 10k - 1 = 0...........(k ≠ - 1)

∴ k = $\frac{1}{10}$

(ii) P (X < 3) = P (0) + P(1) + P(2)

= 0 + k + 2k = 3k

= $3\left(\frac{1}{10}\right) = \frac{3}{10}$.

(ii) P (0 < X < 3) = + P(1) + P(2)

= k + 2k = 3k

= $3\left(\frac{1}{10}\right) = \frac{3}{10}$.

Exercise 7.1 | Q 9 | Page 232

Find expected value and variance of X for the following p.m.f.

x	-2	-1	0	1	2

P(X)	0.2	0.3	0.1	0.15	0.25

SOLUTION

We construct the following table to calculate E (X) and V (X) :

$X = x_i$	**$p_i = P[X = x_i]$**	**$x_i \cdot p_i$**	**$x_i^2 \cdot p_i = x_i \times x_i \cdot p_i$**
-2	0.2	-0.4	0.8
-1	0.3	-0.3	0.3
0	0.1	0	0
1	0.15	0.15	0.15
2	0.25	0.5	1
Total	1	-0.05	2.25

From the table, $\Sigma x_i \cdot p_i = -0.05$ and $\Sigma x_i^2 \cdot p_i = 2.25$

$\therefore E(X) = \Sigma x_i \cdot p_i = -0.05$

and $V(X) = \Sigma x_i^2 \cdot p_i - (\Sigma x_i \cdot p_i)^2$

$= 2.25 - (-0.05)^2$

$= 2.25 - 0.0025 = 2.2475$

Hence, E (X)= -0.05 and V (X) = 2.2475.

Exercise 7.1 | Q 10 | Page 233

Find expected value and variance of X, where X is number obtained on uppermost face when a fair die is thrown.

SOLUTION

If a die is tossed, then the sample space for the random variable X is

S = {1, 2, 3, 4, 5, 6}

$\therefore$ P(X) = $\frac{1}{6}$; X = 1, 2, 3, 4, 5, 6.

$\therefore$ E(X) = $\Sigma_{X \in S} X . P(X)$

= $1\left(\frac{1}{6}\right) + 2\left(\frac{1}{6}\right) + 3\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 5\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right)$

= $\frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6)$

= $\frac{21}{6} = \frac{7}{2} = 3.5$

V (X) = E(X^2) -[E(X)]^2

$\Sigma_{X \in S} X^2 \cdot P(X) - \left(\frac{7}{2}\right)^2$

= $\left[(1)^2\left(\frac{1}{6}\right) + (2)^2\left(\frac{1}{6}\right) + (3)^2\left(\frac{1}{6}\right) + (4)^2\left(\frac{1}{6}\right) + (5)^2\left(\frac{1}{6}\right) + (6)^2\left(\frac{1}{6}\right)\right] - \frac{49}{4}$

= $\frac{1}{6}(1 + 4 + 9 + 16 + 25 + 36) - \frac{49}{4}$

= $\frac{91}{6} - \frac{49}{4} = \frac{182 - 147}{12} = \frac{35}{12} = 2.9167$

Exercise 7.1 | Q 11 | Page 233

Find the mean number of heads in three tosses of a fair coin.

SOLUTION

Let X denote the success of getting heads.

Therefore, the sample space is

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

It can be seen that X can take the value of 0, 1, 2, or 3.

$\therefore$ P(X = 0) = P(TTT)

= P(T) . P(T) . P(T)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

∴ P(X = 1) = P(HHT) + P(HTH) + P(THH)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{8}$

∴ P(X = 2) = P(HHT) + P(HTH) + P(THH)

$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{8}$

∴ P(X = 3) = P(HHH)

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{8}$

Therefore, the required probability distribution is as follows.

X	0	1	2	3
P(X)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Mean of X E(X), μ = $\sum X_i P(X_i)$

$= 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8}$

$= 0 + \frac{3}{8} + \frac{3}{4} + \frac{3}{8}$

$= \frac{3}{12}$

= 1.5

Exercise 7.1 | Q 12 | Page 233

Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

SOLUTION

Here, X represents the number of sixes obtained when two dice are thrown simultaneously. Therefore, X can take the value of 0, 1, or 2.

∴ P (X = 0) = P (not getting six on any of the dice) = $\frac{25}{36}$

P (X = 1) = P (six on first die and no six on second die) + P (no six on first die and six on second die)

$2\left(\frac{1}{6} \times \frac{5}{6}\right) = \frac{10}{36}$

P (X = 2) = P (six on both the dice) = $\frac{1}{36}$

Therefore, the required probability distribution is as follows.

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Then, expectation of X = E(X) = $\sum X_i P(X_i)$

$= 0 \times \frac{25}{36} + 1 \times \frac{10}{36} + 2 \times \frac{1}{36}$

$= \frac{1}{3}$

Exercise 7.1 | Q 13 | Page 233

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find E(X).

SOLUTION

The two positive integers can be selected from the first six positive integers without replacement in 6 × 5 = 30 ways

X represents the larger of the two numbers obtained. Therefore, X can take the value of 2, 3, 4, 5, or 6.

For X = 2, the possible observations are (1, 2) and (2, 1).

$$\therefore P(X=2)=\frac{2}{30}=\frac{1}{15}$$

For X = 3, the possible observations are (1, 3), (2, 3), (3, 1), and (3, 2).

$$\therefore P(X=3)=\frac{4}{30}=\frac{2}{15}$$

For X = 4, the possible observations are (1, 4), (2, 4), (3, 4), (4, 3), (4, 2), and (4, 1).

$$\therefore P(X=4)=\frac{6}{30}=\frac{1}{5}$$

For X = 5, the possible observations are (1, 5), (2, 5), (3, 5), (4, 5), (5, 4), (5, 3), (5, 2), and (5, 1).

$$\therefore P(X=5)=\frac{8}{30}=\frac{4}{15}$$

For X = 6, the possible observations are (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6,5) , (6, 4), (6, 3), (6, 2), and (6, 1).

$$\therefore P(X=6)=\frac{10}{30}=\frac{1}{3}$$

Therefore, the required probability distribution is as follows.

X	2	3	4	5	6
P(X)	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{1}{3}$

Then, E(X) = $\Sigma x_i P(x_i)$

$$= 2\times\frac{1}{15}+3\times\frac{2}{15}+4\times\frac{3}{15}+5\times\frac{4}{15}+6\times\frac{5}{15}$$

$$=\frac{2+6+12+20+30}{15}$$

$$=\frac{70}{15}$$

$$=\frac{14}{3}$$

Exercise 7.1 | Q 14 | Page 233
Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the standard deviation of X.

SOLUTION

If two fair dice are rolled then the sample space S of this experiment is

S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,5),(1,6),(2,1),(2,2),(2,3),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

∴ n(S) = 36

Let X denote the sum of the numbers on uppermost faces.

Then X can take the values 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

sum of Nos. (x)	Favourable events	No of favourable	P (x)
2	(1,1)	1	$\frac{1}{36}$
3	(1, 2), (2, 1)	2	$\frac{2}{36}$
4	(1, 3), (2, 2), (3, 1)	3	$\frac{3}{36}$
5	(1, 4), (2, 3), (3, 2), (4, 1)	4	$\frac{4}{36}$
6	(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)	5	$\frac{5}{36}$
7	(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)	6	$\frac{6}{36}$
8	(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)	5	$\frac{5}{36}$
9	(3, 6), (4, 5), (5, 4), (6, 3)	4	$\frac{4}{36}$

10	(4, 6), (5, 5), (6, 4)	3	$\frac{3}{36}$
11	(5, 6), (6, 5)	2	$\frac{2}{36}$
12	(6,6)	1	$\frac{1}{36}$

∴ the probability distribution of X is given by

$X=x_i$	2	3	4	5	6	7	8	9	10	11	12
$P[X=x_i]$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Expected value = E (X) = $\Sigma x_i \cdot P(x_i)$

$$2\left(\frac{1}{36}\right)+3\left(\frac{2}{36}\right)+4\left(\frac{3}{36}\right)+5\left(\frac{4}{36}\right)+6\left(\frac{5}{36}\right)+7\left(\frac{6}{36}\right)+8\left(\frac{5}{36}\right)+9\left(\frac{4}{36}\right)+10\left(\frac{3}{36}\right)+11\left(\frac{2}{36}\right)+12\left(\frac{1}{36}\right)$$

$$=\frac{1}{36}(2+6+12+20+30+42+40+36+30+22+12)$$

$$\frac{1}{36}\times 252=7.$$

Also, $\Sigma x_i^2 \cdot P(x_i)$

$$=4\times\frac{1}{36}+9\times\frac{2}{36}+16\times\frac{3}{36}+25\times\frac{4}{36}+36\times\frac{5}{36}+49\times\frac{6}{36}+64\times\frac{5}{36}+81\times\frac{4}{36}+100\times\frac{3}{36}+121\times\frac{2}{36}+144\times\frac{1}{36}$$

$= \frac{1}{36}[4 + 18 + 48 + 100 + 180 + 294 + 320 + 324 + 300 + 242 + 144]$

$= \frac{1}{36}(1974) = 54.83$

$\therefore$ variance = $V(X) = \Sigma x_i^2 \cdot P(x_i) - [E(X)]^2$

$= 54.83 - 49$

$= 5.83$

$\therefore$ standard deviation = $\sqrt{V(X)}$

$= \sqrt{5.83} = 2.41$

Exercise 7.1 | Q 15 | Page 233

A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

SOLUTION

There are 15 students in the class. Each student has the same chance to be chosen. Therefore, the probability of each student to be selected is 1/15

The given information can be compiled in the frequency table as follows.

X	14	15	16	17	18	19	20	21
f	2	1	2	3	1	2	3	1

$P(X=14)=\frac{2}{15}, P(X=15)=\frac{1}{15}, P(X=16)=\frac{2}{15},$

$P(X=16)=\frac{3}{15}, P(X=18)=\frac{1}{15}, P(X=19)=\frac{2}{15}, P(X=20)=\frac{3}{15}, P(X=21)=\frac{1}{15}$

Therefore, the probability distribution of random variable X is as follows.

X	14	15	16	17	18	19	20	21
f	$\frac{2}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Then, mean of X = E(X)

$\Sigma X_i \ P(X_i)$

$= 14 \times \frac{2}{15} + 15 \times \frac{1}{15} + 16 \times \frac{2}{15} + 17 \times \frac{3}{15} + 18 \times \frac{1}{15} + 19 \times \frac{2}{15} + 20 \times \frac{3}{15} + 21 \times \frac{1}{15}$

= $\frac{1}{15}(28 + 15 + 32 + 51 + 18 + 38 + 60 + 21)$

= $\frac{263}{15}$

= 17.53

$E(X^2) = \Sigma X_i^2 \ P(X_i)$

$= (14)^2 \times \frac{2}{15} + (15)^2 \times \frac{1}{15} + (16)^2 \times \frac{2}{15} + (17)^2 \times \frac{3}{15} + (18)^2 \times \frac{1}{15} + (19)^2 \times \frac{2}{15} + (20)^2 \times \frac{3}{15} + (21)^2 \times \frac{1}{15}$

= $\frac{1}{15}(392 + 225 + 512 + 867 + 324 + 722 + 1200 + 441)$

= $\frac{4683}{15}$

= 312.2

Variance = V(X) = $\Sigma x_i^2 . P(x_i) - [E(X)]^2$

= $312.2 - (17.53)^2$

= 312.2 - 307.3 = 4.9

Standard deviation = $\sqrt{V(X)} = \sqrt{4.9}$ = 2.21

Hence, mean = 17.53, variance = 4.9 and standard deviation = 2.21.

Exercise 7.1 | Q 16 | Page 233

In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var(X).

SOLUTION

It is given that P(X = 0) = 30% = 30/100 = 0.3

$$P(X=1) = 70\% = \frac{70}{100} = 0.7$$

Therefore, the probability distribution is as follows.

X	0	1
P(X)	0.3	0.7

Then, E(X) = $\sum X_i P(x_i)$

= 0 × 0.3+1 × 0.7

= 0.7

E(X2) = $\sum X_i P(x_i)$

= 0^2 x 0.3 + (1) x 0.7

= 0.7

It is known that , var (x) = E (X^2) - $[E(X)]^2$

= 0.7 − $(0.7)^2$

= 0.7 − 0.49

= 0.21

Hence, E(X) = 0.7 and Var(X) = 0.21.

EXERCISE 7.2 [PAGES 238 - 239]

Exercise 7.2 | Q 1.1 | Page 238

Verify which of the following is p.d.f. of r.v. X:

f(x) = sin x, for $0 \le x \le \frac{\pi}{2}$

SOLUTION

f (x) is the p.d.f. of r.v. X if

(a) f (x) ≥ 0 for all x ∈ R and

(b) $\int_{-\infty}^{\infty} f(x)dx = 1$

(a) f (x) = sin x ≥ 0 if 0 ≤ x ≤ $\frac{\pi}{2}$

(b) $\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{-\infty}^{\frac{\pi}{2}} f(x)dx + \int_{\frac{\pi}{2}}^{\infty} f(x)dx$

= 0 + $\int_{0}^{\frac{\pi}{2}} \sin x \, dx + 0$

= $[-\cos x]_0^{\frac{\pi}{2}} = -\cos\left(\frac{\pi}{2}\right) + \cos 0 = 0 + 1 = 1$

Hence, f (x) is the p.d.f. of X.

Exercise 7.2 | Q 1.2 | Page 238

Verify which of the following is p.d.f. of r.v. X:
f(x) = x, for 0 ≤ x ≤ 1 and -2 -x for 1 < x < 2

SOLUTION

f (x) is the p.d.f. of r.v. X if

(a) f (x) ≥ 0 for all x ∈ R and

(b) $\int_{-\infty}^{\infty} f(x)dx = 1$

f (x) = x ≥ 0 if 0 ≤ x ≤ 1

For 1 < x < 2, < -2 < - x < -1

∴ - 2 - 2 < - 2 - x < - 2 - 1

i.e. - 4 < f(x) < - 3 if 1< x < 2

Hence, f (x) is not p.d.f. of X.

Exercise 7.2 | Q 1.3 | Page 238

Verify which of the following is p.d.f. of r.v. X:
f(x) = 2, for 0 ≤ x ≤ 1.

SOLUTION

f (x) is the p.d.f. of r.v. X if

(a) f (x) ≥ 0 for all x ∈ R and

(b) $\int_{-\infty}^{\infty} f(x)dx = 1$

(a) f(x) = 2 ≥ 0 for 0 ≤ x ≤ 1

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx$$

= 0 + $\int_{0}^{1} 2dx$+ 0

= $[2x]_0^1$

= 2 - 0

= 2 ≠ 1

Hence, f (x) is not p.d.f. of X.

Exercise 7.2 | Q 2.1 | Page 239

The following is the p.d.f. of r.v. X:
f(x) = $\frac{x}{8}$, for 0 < x < 4 and = 0 otherwise.
Find P (x < 1·5)

SOLUTION

P (x < 1·5)

$$= \int_0^{1.5} f(x)dx$$

$$= \int_0^{1.5} \frac{x}{8} dx$$

$$= \frac{1}{8}\left[\frac{x^2}{2}\right]_0^{1.5}$$

$$= \frac{(1.5)^2}{16} - 0$$

$$= \frac{\left(\frac{9}{4}\right)}{16}$$

$$= \frac{9}{64}.$$

Exercise 7.2 | Q 2.2 | Page 239

The following is the p.d.f. of r.v. X :

f(x) = $\frac{x}{8}$, for 0 < x < 4 and = 0 otherwise

P (1 < x < 2)

SOLUTION

P (1 < x < 2)

$$= \int_1^{2} f(x)dx$$

$$= \int_1^{2} \frac{x}{8} dx$$

$$\frac{1}{8}\left[\frac{x^2}{2}\right]_1^{2}$$

$= \frac{1}{8}\left[\frac{4}{2} - \frac{1}{2}\right]$

$= \frac{3}{16}$

Exercise 7.2 | Q 2.3 | Page 239

The following is the p.d.f. of r.v. X:

f(x) = $\frac{x}{8}$, for 0 < x < 4 and = 0 otherwise.

P(x > 2)

SOLUTION

P(x > 2)

$= \int_2^4 f(x)dx$

$= \int_2^4 \frac{x}{8}dx$

$= \frac{1}{8}\left[\frac{x^2}{2}\right]_2^4$

$= \frac{1}{8}\left[\frac{16}{2} - \frac{4}{2}\right]$

$= \frac{1}{8} \times 6$

$= \frac{3}{4}$

Exercise 7.2 | Q 3.1 | Page 239

It is known that error in measurement of reaction temperature (in 0° c) in a certain experiment is continuous r.v. given by f (x) = $\frac{x^2}{3}$, for −1 < x < 2 and = 0 otherwise

Verify whether f (x) is p.d.f. of r.v. X.

SOLUTION

f (x) = $\frac{x^2}{3} \geq \mathbf{0},$ for -1 < x < 2

Also, $\int_{-\infty}^{\infty} f(x)dx$

= $\int_{-\infty}^{-1} f(x)dx + \int_{-1}^{2} f(x)dx + \int_{2}^{\infty} f(x)dx$

= 0+ $\int_{-1}^{2} f\left(\frac{x^2}{3}\right)dx$ + 0 = $\frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{2}$

= $\frac{1}{3}\left[\frac{8}{3} - \frac{(-1)}{3}\right] = \frac{1}{3}\left[\frac{9}{3}\right] = 1$

∴ f (x) is the p.d.f. of X.

Exercise 7.2 | Q 3.2 | Page 239

It is known that error in measurement of reaction temperature (in 0° c) in a certain experiment is continuous r.v. given by

f (x) = $\frac{x^2}{3}$, for −1 < x < 2 and = 0 otherwise

SOLUTION

P(0 < x ≤1) = $\int_{0}^{1} f(x)dx$

= $\int_{0}^{1} \frac{x^2}{3}dx$

$= \frac{1}{3}\left[\frac{x^3}{3}\right]_0^1$

$= \frac{1}{3}\left[\frac{1}{3} - 0\right]$

$= \frac{1}{9}$

Exercise 7.2 | Q 3.3 | Page 239

It is known that error in measurement of reaction temperature (in 0° c) in a certain experiment is continuous r.v. given by

f (x) = $\frac{x^2}{3}$, for −1 < x < 2 and = 0 otherwise

Find probability that X is negative

SOLUTION

P(x is negative)

= P(0< x ≤ 1) = $\int_{-1}^{0} f(x)dx$

$= \int_{-1}^{0} \frac{x^2}{3} dx$

$= \frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^0$

$= \frac{1}{3}\left[0 - \left(-\frac{1}{3}\right)\right]$

$= \frac{1}{9}$

Exercise 7.2 | Q 4.1 | Page 239

Find k if the following function represent p.d.f. of r.v. X

f (x) = kx, for 0 < x < 2 and = 0 otherwise, Also find P $\left(\frac{1}{4} < x < \frac{3}{2}\right)$.

SOLUTION

Since, the function f is p.d.f. of X

$\therefore \int_{-\infty}^{\infty} f(x)dx = 1$

$\therefore \int_{-\infty}^{0} f(x)dx + \int_{0}^{2} f(x)dx + \int_{2}^{\infty} f(x)dx = 1$

$\therefore 0 + \int_{0}^{2} kxdx + 0 = 1$

$\therefore k\left[\frac{x^2}{2}\right]_0^2 = 1$

$\therefore k\left[\frac{4}{2} - 0\right] = 1$

$\therefore 2k = 1$

$\therefore k = \frac{1}{2}$

P $\left(\frac{1}{4} < x < \frac{3}{2}\right) = \int_{\frac{1}{4}}^{\frac{3}{2}} f(x)dx$

$\int_{\frac{1}{4}}^{\frac{3}{2}} kxdx,$

where k= $\frac{1}{2}$

$= \frac{1}{2} \int_{\frac{1}{4}}^{\frac{3}{2}} xdx$

$= \frac{1}{2}\left[\frac{x^2}{2}\right]_{\frac{1}{4}}^{\frac{3}{2}}$

$= \frac{1}{4}\left[\frac{9}{4} - \frac{1}{16}\right]$

$= \frac{1}{4}\left[\frac{36-1}{16}\right]$

$= \frac{35}{64}$

Exercise 7.2 | Q 4.2 | Page 239

Find k if the following function represent p.d.f. of r.v. X.

f (x) = kx (1 – x), for 0 < x < 1 and = 0 otherwise, Also find P $\left(\frac{1}{4} < x < \frac{1}{2}\right), P\left(x < \frac{1}{2}\right)$.

SOLUTION

Since, the function f is the p.d.f. of X,

$\int_{-\infty}^{\infty} f(x)dx = 1$

$\therefore \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx = 1$

$\therefore 0 + \int_{0}^{1} kx(1-x)dx + 0 = 1$

$\therefore k\int_{0}^{1} (x - x^2)dx = 1$

$\therefore k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$

$\therefore k\left(\frac{1}{2} - \frac{1}{3} - 0\right) = 1$

$\therefore \frac{k}{6} = 1$

$\therefore$ k = 6

P $\left(\frac{1}{4} < x < \frac{1}{2}\right) = \int_{\frac{1}{4}}^{\frac{1}{2}} f(x)dx$

$= \int_{\frac{1}{4}}^{\frac{1}{2}} kx(1-x)dx$

$= k\int_{\frac{1}{4}}^{\frac{1}{2}} (x-x^2)dx$

$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{\frac{1}{4}}^{\frac{1}{2}}$ ....[$\because$ k = 6]

$= 6\left[\left(\frac{1}{8} - \frac{1}{24}\right) - \left(\frac{1}{32} - \frac{1}{192}\right)\right]$

$= 6\left[\frac{2}{24} - \frac{5}{192}\right]$

$= 6\left(\frac{11}{192}\right)$

$\therefore$ P $\left(\frac{1}{4} < x < \frac{1}{2}\right) = \frac{11}{32}$

P $\left(x < \frac{1}{2}\right) = \int_{0}^{\frac{1}{2}} f(x)dx$

$= 0 + \int_0^{\frac{1}{2}} kx(1-x)dx$

$= k\int_0^{\frac{1}{2}} (x - x^2)dx$

$= k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{\frac{1}{2}}$

$= k\left[\frac{1}{8} - \frac{1}{24} - 0\right]$

$= k\left(\frac{2}{24}\right)$

$= 6\left(\frac{1}{12}\right)$............[$\because$ k = 6]

$\therefore$ P $\left(x < \frac{1}{2}\right)$

$= \frac{1}{2}$

Exercise 7.2 | Q 5.1 | Page 239

Let X be amount of time for which a book is taken out of library by randomly selected student and suppose X has p.d.f
f (x) = 0.5x, for 0 ≤ x ≤ 2 and = 0 otherwise.
Calculate: P(x≤1)

SOLUTION

f(x) = 0.5 x , 0 ≤ x ≤ 2

= 0 , otherwise

P (X ≤ 1) = $\int_0^1 0.5\text{ x dx}$

$= 0.5\int_0^1 \text{x dx}$

$$= 0.5\left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$\therefore$ P (X ≤ 1) = $\frac{1}{4}$

Exercise 7.2 | Q 5.2 | Page 239

Let X be amount of time for which a book is taken out of library by randomly selected student and suppose X has p.d.f
f (x) = 0.5x, for 0 ≤ x ≤ 2 and = 0 otherwise.
Calculate: P(0.5 ≤ x ≤ 1.5)

SOLUTION

f(x) = 0.5 x,   0 ≤ x ≤ 2

= 0,   otherwise

P (0.5 ≤ X ≤ 1.5)

$$= \int_{0.5}^{1.5} 0.5\, x$$

$$= 0.5 \int_{0.5}^{1.5} x\, dx$$

$$= \frac{1}{2} \times \left[\frac{x^2}{2}\right]_{0.5}^{1.5}$$

$$= \frac{1}{2} \times \frac{1}{2}\left[(1.5)^2 - (0.5)^2\right]$$

$$= \frac{1}{2} \times \frac{1}{2}[2.25 - 0.25]$$

$$= \frac{1}{4} \times 2 = \frac{1}{2}$$

P (0.5 ≤ X ≤ 1.5) = $\frac{1}{2}$

Exercise 7.2 | Q 5.3 | Page 239

Let X be amount of time for which a book is taken out of library by randomly selected student and suppose X has p.d.f
f (x) = 0.5x, for 0 ≤ x ≤ 2 and = 0 otherwise. Calculate: P(x ≥ 1.5)

SOLUTION

f(x) = 0.5 x , 0 ≤ x ≤ 2

= 0 , otherwise

P(x ≥ 1.5)

$$= \int_{1.5}^{2} f(x)dx$$

$$= \int_{1.5}^{2} 0.5x dx$$

$$= 0.5\left[\frac{x^2}{2}\right]_{1.5}^{2}$$

$$= 0.5\left[\frac{4}{2} - \frac{2.25}{2}\right]$$

$$= 0.5 \times \frac{1.75}{2}$$

$$= \frac{0.875}{2}$$

$$= \frac{0.875}{2} \times \frac{1000}{1000}$$

$$= \frac{875 \div 125}{2000 \div 125}$$

$$= \frac{7}{16}$$

Exercise 7.2 | Q 6.1 | Page 239

Suppose that X is waiting time in minutes for a bus and its p.d.f. is given by

f (x) = $\frac{1}{5}$, for $0 \le x \le 5$ and = 0 otherwise.

Find the probability that waiting time is between 1 and 3

SOLUTION

Required probabilityP(1 < X < 3)

= $\int_1^3 f(x)dx$

= $\int_1^3 \frac{1}{5}(x)dx$

= $\int_1^3 \frac{1}{5}\ 1dx$

= $\frac{1}{5}\ [x]_1^3$

= $\frac{1}{5}[3-1]$

= $\frac{2}{5}$

Exercise 7.2 | Q 6.2 | Page 239

Suppose that X is waiting time in minutes for a bus and its p.d.f. is given by

f (x) = $\frac{1}{5}$, for $0 \le x \le 5$ and = 0 otherwise.

Find the probability that waiting time is more than 4 minutes.

SOLUTION

Required probabilityP(X > 4)

$= \int_4^\infty f(x)dx$

$= \int_4^5 f(x)dx + \int_5^\infty f(x)dx$

$= \int_4^5 \frac{1}{5}dx + 0$

$= \frac{1}{5}\int_4^5 1dx$

$= \frac{1}{5}[x]_4^5$

$= \frac{1}{5}[5-4]$

$= \frac{1}{5}$

Exercise 7.2 | Q 6.2 | Page 239

Suppose that X is waiting time in minutes for a bus and its p.d.f. is given by $f(x) = \frac{1}{5}$, for $0 \le x \le 5$ and = 0 otherwise.

Find the probability that waiting time is more than 4 minutes.

SOLUTION

Required probabilityP(X > 4)

$= \int_4^\infty f(x)dx$

$= \int_4^5 f(x)dx + \int_5^\infty f(x)dx$

$= \int_4^5 \frac{1}{5} dx + 0$

$= \frac{1}{5} \int_4^5 1 dx$

$= \frac{1}{5} [x]_4^5$

$= \frac{1}{5} [5 - 4]$

$= \frac{1}{5}$

Exercise 7.2 | Q 7.1 | Page 239

Suppose error involved in making a certain measurement is continuous r.v. X with p.d.f.
f (x) = k $\left(4 - x^2\right)$, for −2 ≤ x ≤ 2 and = 0 otherwise.
P(x > 0)

SOLUTION

Since, f is the p.d.f. of X,

$\int_{-\infty}^{\infty} f(x) dx = 1$

$\therefore \int_{-\infty}^{-2} f(x) dx + \int_{-2}^{2} f(x) dx + \int_{2}^{\infty} f(x) dx = 1$

$\therefore 0 + \int_{-2}^{2} k\left(4 - x^2\right) dx = 1$

$\therefore k \int_{-2}^{2} \left(4 - x^2\right) dx = 1$

$\therefore k \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$

$\therefore$ k $\left[\left(8-\frac{8}{3}\right)-\left(-8+\frac{8}{3}\right)\right]$ = 1

$\therefore$ k $\left(\frac{16}{3}+\frac{16}{3}\right)$ = 1

$\therefore$ k $\left(\frac{32}{3}\right)$ = 1

$\therefore$ k = $\frac{3}{32}$

P(x > 0)

= $\int_0^{\infty} f(x)dx$

= $\int_0^{2} f(x)dx + \int_2^{\infty} f(x)dx$

= $\int_0^{2} k(4-x^2)dx + 0$

= k $\int_0^{2} (4-x^2)dx$

= $\frac{3}{32}\left[4x-\frac{x^3}{3}\right]_0^2$ ....[$\because$ k = $\frac{3}{32}$]

= $\frac{3}{32}\left[8-\frac{8}{3}\right] = \frac{3}{32} \times \frac{16}{3} = \frac{1}{2}$

Exercise 7.2 | Q 7.2 | Page 239

Suppose error involved in making a certain measurement is continuous r.v. X with p.d.f.
f (x) = k $(4-x^2)$, for –2 ≤ x ≤ 2 and = 0 otherwise
P(–1 < x < 1)

SOLUTION

Since, f is the p.d.f. of X,

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\therefore \int_{-\infty}^{-2} f(x)dx + \int_{-2}^{2} f(x)dx + \int_{2}^{\infty} f(x)dx = 1$$

$$\therefore 0 + \int_{-2}^{2} k(4 - x^2)dx = 1$$

$$\therefore k \int_{-2}^{2} (4 - x^2)dx = 1$$

$$\therefore k\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$$

$$\therefore k\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = 1$$

$$\therefore k\left(\frac{16}{3} + \frac{16}{3}\right) = 1$$

$$\therefore k\left(\frac{32}{3}\right) = 1$$

$$\therefore k = \frac{3}{32}$$

$P(-1 < x < 1)$

$$= \int_{-1}^{1} f(x)dx$$

$$= \int_{-1}^{1} k(4 - x^2)dx$$

$$= k \int_{-1}^{1} (4 - x^2)dx$$

$= \frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-1}^{1}$ ....$[\because k = \frac{3}{32}]$

$= \frac{3}{32}\left[\left(4 - \frac{1}{3}\right) - \left(-4 + \frac{1}{3}\right)\right]$

$= \frac{3}{32}\left(\frac{11}{3} + \frac{11}{3}\right)$

$= \frac{3}{32}\left(\frac{22}{3}\right)$

$= \frac{11}{6}$

Exercise 7.2 | Q 7.3 | Page 239

Suppose error involved in making a certain measurement is continuous r.v. X with p.d.f.
f (x) = k $(4 - x^2)$, for –2 ≤ x ≤ 2 and = 0 otherwise.
P (–0·5 < x or x > 0·5)

SOLUTION

Since, f is the p.d.f. of X,

$\int_{-\infty}^{\infty} f(x)dx = 1$

$\therefore \int_{-\infty}^{-2} f(x)dx + \int_{-2}^{2} f(x)dx + \int_{2}^{\infty} f(x)dx = 1$

$\therefore 0 + \int_{-2}^{2} k(4 - x^2)dx = 1$

$\therefore k \int_{-2}^{2} (4 - x^2)dx = 1$

$\therefore k\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$

$\therefore$ k $\left[\left(8-\frac{8}{3}\right)-\left(-8+\frac{8}{3}\right)\right]$ = 1

$\therefore$ k $\left(\frac{16}{3}+\frac{16}{3}\right)$ = 1

$\therefore$ k $\left(\frac{32}{3}\right)$ = 1

$\therefore$ k = $\frac{3}{32}$

P (–0·5 < x or x > 0·5)

= P (x < –0·5) + P (x > – 0·5)

= $\int_{-\infty}^{-0.5} f(x)dx + \int_{0.5}^{\infty} f(x)dx$

= $\int_{-\infty}^{-2} f(x)dx + \int_{-2}^{-0.5} f(x)dx + \int_{0.5}^{2} f(x)dx + \int_{2}^{\infty} f(x)dx$

= 0+ $\frac{\int_{-2}^{-1}}{2} k\left(4-x^2\right)dx + \int_{\frac{1}{2}}^{2} k\left(4-x^2\right)dx$ + 0

=k $\frac{\int_{-2}^{1}}{2}\left(4-x^2\right)dx + \int_{\frac{1}{2}}^{2}\left(4-x^2\right)dx$

= $\frac{3}{32}\left[4x-\frac{x^3}{3}\right]_{-2}^{-\frac{1}{2}} + \frac{3}{32}\left[4x-\frac{x^3}{3}\right]_{\frac{1}{2}}^{2}$ ....[$\because$ k = $\frac{3}{32}$]

= $\frac{3}{32}\left[\left(-2+\frac{1}{24}\right)-\left(-8+\frac{8}{3}\right)\right]+\frac{3}{32}\left[\left(8-\frac{8}{3}\right)-\left(2-\frac{1}{24}\right)\right]$

= $\frac{3}{32}\left(\frac{-47}{24}+\frac{16}{3}\right)+\frac{3}{32}\left(\frac{16}{3}-\frac{47}{24}\right)$

$= \frac{3}{32}\left(\frac{-47}{24} + \frac{16}{3} + \frac{16}{3} - \frac{47}{24}\right)$

$= \frac{3}{32}\left(\frac{-47 + 128 + 128 - 47}{24}\right)$

$= \frac{3}{32}\left(\frac{162}{24}\right) = \frac{81}{128}$

= 0.6328.

Alternative Method :

P (x< – 0·5 or x > 0·5)

= 1 -P(- 0.5 ≤ x ≤ 0.5)

= 1 - $\int_{-0.5}^{0.5} f(x)dx$

= 1 - $\int_{-\frac{1}{2}}^{\frac{1}{2}} k(4 - x^2)dx$

= 1 - k $\int_{-\frac{1}{2}}^{\frac{1}{2}} (4 - x^2)dx$

$= 1 - \frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-\frac{1}{2}}^{\frac{1}{2}}$ ......[∵ k = $\frac{3}{32}$]

$= 1 - \frac{3}{32}\left[\left(2 - \frac{1}{24}\right) - \left(-2 + \frac{1}{24}\right)\right]$

$= 1 - \frac{3}{32}\left(2 - \frac{1}{24} + 2 - \frac{1}{24}\right)$

$= 1 - \frac{3}{32}\left(4 - \frac{1}{12}\right)$

$= 1 - \frac{3}{32} \times \frac{47}{12} = 1 - \frac{47}{128}$

= $\dfrac{128-47}{128} = \dfrac{81}{128}$

= 0.6328

Exercise 7.2 | Q 8.1 | Page 239

The following is the p.d.f. of continuous r.v.

f (x) = $\dfrac{x}{8}$, for 0 < x < 4 and = 0 otherwise.

Find expression for c.d.f. of X

SOLUTION

Let F(x) be the c.d.f. of X

Then F(x) = $\displaystyle\int_{-\infty}^{x} f(x)dx$

= $\displaystyle\int_{0}^{-\infty} f(x)dx + \int_{0}^{x} f(x)dx$

= 0+ $\displaystyle\int_{0}^{x} \frac{x}{8}dx$

= $\dfrac{1}{8}\left[\dfrac{x^2}{2}\right]_0^x$

= $\dfrac{1}{8}\left[\dfrac{x^2}{2} - 0\right] = \dfrac{x^2}{16}$

∴ F (x) = $\dfrac{x^2}{16}$

Exercise 7.2 | Q 8.2 | Page 239

The following is the p.d.f. of continuous r.v.

f (x) = $\dfrac{x}{8}$, for 0 < x < 4 and = 0 otherwise.

Find F(x) at x = 0·5 , 1.7 and 5

SOLUTION

F(0.5) = $F\left(\frac{1}{2}\right) = \frac{\left(\frac{1}{2}\right)^2}{16} = \frac{1}{64}$

F(1.7) = $\frac{(1.7)^2}{16} = \frac{2.89}{16} = 0.18$

f (x) = $\frac{x}{8}$, for 0 < x < 4 and 5 > 4

∴ F(5) = 1.

Exercise 7.2 | Q 9.1 | Page 239

Given the p.d.f. of a continuous r.v. X , f (x) = $\frac{x^2}{3}$,for −1 < x < 2 and = 0 otherwise

Determine c.d.f. of X hence find

P(x < 1)

SOLUTION

Let F(x) be the c.d.f. of X

Then F(x) = $\int_{-\infty}^{x} f(x)dx$

= $\int_{-\infty}^{-1} f(x)dx + \int_{-1}^{x} f(x)dx$

= 0 + $\int_{-1}^{x} \frac{x^2}{3} dx = \frac{1}{3} \int_{-1}^{x} x^2 dx$

= $\frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{x}$

= $\frac{1}{3}\left[\frac{x^3}{3} - \left(-\frac{1}{3}\right)\right]$

$\therefore$ f(x) = $\frac{x^3+1}{9}$

P(x < 1) = $F(1) - F(-1) - \left[\frac{1^3+1}{9}\right] - \left[\frac{(-1)^3+1}{9}\right] - \frac{2-0}{9} - \frac{2}{9}$

Exercise 7.2 | Q 9.2 | Page 239

Given the p.d.f. of a continuous r.v. X ,

f (x) = $\frac{x^2}{3}$, for −1 < x < 2 and = 0 otherwise

Determine c.d.f. of X hence find P(x < −2)

SOLUTION

Then F(x) = $\int_{-\infty}^{x} f(x)dx$

= $\int_{-\infty}^{-1} f(x)dx + \int_{-1}^{x} f(x)dx$

= 0 + $\int_{-1}^{x} \frac{x^2}{3}dx = \frac{1}{3}\int_{-1}^{x} x^2 dx$

= $\frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{x}$

= $\frac{1}{3}\left[\frac{x^3}{3} - \left(-\frac{1}{3}\right)\right]$

$\therefore$ f(x) = $\frac{x^3+1}{9}$

f (x) = $\frac{x^2}{3}$, for −1 < x < 2 And -2 < -1

$\therefore$F (-2) = 0 i.e P (x < -2) = 0

Exercise 7.2 | Q 9.3 | Page 239

Given the p.d.f. of a continuous r.v. X ,

f (x) = $\frac{x^2}{3}$, for −1 < x < 2 and = 0 otherwise

Determine c.d.f. of X hence find P(X > 0)

SOLUTION

Then F(x) = $\int_{-\infty}^{x} f(x)dx$

= $\int_{-\infty}^{-1} f(x)dx + \int_{-1}^{x} f(x)dx$

= 0 + $\int_{-1}^{x} \frac{x^2}{3}dx = \frac{1}{3}\int_{-1}^{x} x^2 dx$

= $\frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{x}$

= $\frac{1}{3}\left[\frac{x^3}{3} - \left(-\frac{1}{3}\right)\right]$

∴ f(x) = $\frac{x^3+1}{9}$

P (X > 0) = 1 - P(X ≤ 0)

= 1 - [F(0) - F(-1)]

= 1 - $\left[\left(\frac{0^3+1}{9}\right) - \left(\frac{(-1)^3+1}{9}\right)\right]$

= 1 - $\left(\frac{1}{9} - 0\right) = 1 - \frac{1}{9} = \frac{8}{9}$

Exercise 7.2 | Q 9.4 | Page 239

Given the p.d.f. of a continuous r.v. X ,

f (x) = $\frac{x^2}{3}$, for −1 < x < 2 and = 0 otherwise

Determine c.d.f. of X hence find P(1 < x < 2)

SOLUTION

Then F(x) = $\int_{-\infty}^{x} f(x)dx$

= $\int_{-\infty}^{-1} f(x)dx + \int_{-1}^{x} f(x)dx$

= 0 + $\int_{-1}^{x} \frac{x^2}{3} dx = \frac{1}{3} \int_{-1}^{x} x^2 dx$

= $\frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{x}$

= $\frac{1}{3}\left[\frac{x^3}{3} - \left(-\frac{1}{3}\right)\right]$

∴ f(x) = $\frac{x^3+1}{9}$

P (1 < x < 2) = F (2) - F (1)

= $\left(\frac{2^3+1}{9}\right) - \left(\frac{1^3+1}{9}\right) = 1 - \frac{2}{9} = \frac{7}{9}$

Exercise 7.2 | Q 10 | Page 239

If a r.v. X has p.d.f.,

f (x) = $\frac{c}{x}$, for 1 < x < 3, c > 0, Find c, E(X) and Var (X).

SOLUTION

Since, f(x) is p.d.f. of r.v. X

$\therefore \int_{-\infty}^{\infty} f(x)dx$ = 1

$\therefore \int_{-\infty}^{1} f(x)dx + \int_{3}^{1} f(x)dx + \int_{3}^{\infty} f(x)dx = 1$

$\therefore 0 + \int_{1}^{3} f(x)dx + 0 = 1$

$\therefore \int_{1}^{3} \frac{c}{x} dx = 1$

$\therefore c \int_{1}^{3} \frac{1}{x} dx = 1$

$\therefore c[\log x]_{1}^{3} = 1$

∴ c [log 3 - log 1] = 1

$\therefore \frac{1}{\log 3}$ ....[∵ log 1 = 0]

E(X) = $\int_{-\infty}^{\infty} xf(x)dx = \int_{-\infty}^{1} xf(x)dx + \int_{1}^{3} xf(x)dx + \int_{3}^{\infty} xf(x)dx$

$= 0 + \int_{1}^{3} xf(x)dx + 0 = \int_{1}^{3} x.\frac{c}{x} dx$

= $c \int_{1}^{31} dx, where\ c = \frac{1}{\log 3}$

= $\frac{1}{\log 3}[x]_{1}^{3} = \frac{1}{\log 3}[3-1] = \frac{2}{\log 3}$

= consider, $\int_{-\infty}^{\infty} x^2 f(x)dx = \int_{-\infty}^{1} x^2 f(x)dx + \int_{1}^{3} x^2 f(x)dx + \int_{3}^{\infty} x^2 f(x)dx$

= 0 + $\int_{1}^{3} x^2 f(x)dx + 0 = \int_{-\infty}^{\infty} x^2.\frac{c}{x} dx$

= $\frac{1}{\log 3}\int_1^3 x\,dx = \frac{1}{\log 3}\left[\frac{x^2}{2}\right]_1^3$

= $\frac{1}{\log 3}\left[\frac{9}{2}-\frac{1}{2}\right] = \frac{4}{\log 3}$

Now, var (x) = $\int_{-\infty}^{\infty} x^2 f(x)dx - [E(x)]^2$

= $\frac{4}{\log 3} - \left(\frac{2}{\log 3}\right)^2$

= $\frac{4}{\log 3} - \frac{4}{(\log 3)^2}$

= $4(\log 3) - \frac{4}{(\log 3)^2} = \frac{4[\log 3 - 1]}{(\log 3)^2}$

Hence, $c = \frac{1}{\log 3}, E(x) = \frac{2}{\log 3}$ and $Var(x) = \frac{4[\log 3 - 1]}{(\log 3)^2}$

MISCELLANEOUS EXERCISE [PAGES 241 - 242]

Miscellaneous Exercise | Q 1 | Page 241

Choose the correct option from the given alternative :

P.d.f. of a.c.r.v X is f (x) = 6x (1 − x), for 0 ≤ x ≤ 1 and = 0, otherwise (elsewhere)

If P (X < a) = P (X > a), then a =

- 1
- $\frac{1}{2}$
- $\frac{1}{3}$
- $\frac{1}{4}$

SOLUTION

If P (X < a) = P (X > a), then a = 1/2

Miscellaneous Exercise | Q 2 | Page 242

Choose the correct option from the given alternative:
If the p.d.f of a.c.r.v. X is f (x) = 3 (1 − 2x2), for 0 < x < 1 and = 0, otherwise (elsewhere) then the c.d.f of X is F(x) =

1. $2x - 3x^2$
2. $3x - 4x^3$
3. **$3x - 2x^3$**
4. $2x^3 - 3x$

SOLUTION

If the p.d.f of a.c.r.v. X is f (x) = 3 (1 – 2 x 2), for 0 < x < 1 and = 0, otherwise (elsewhere) then the c.d.f of X is F(x) = **$3x - 2x^3$**

Miscellaneous Exercise | Q 3 | Page 242

Choose the correct option from the given alternative:

If the p.d.f of a.c.r.v. X is f (x) = x^ $\frac{2}{18}$, for −3 < x < 3 and = 0, otherwise then P (| X | < 1) =

$\frac{1}{27}$

$\frac{1}{28}$

$\frac{1}{29}$

$\frac{1}{26}$

SOLUTION

If the p.d.f of a.c.r.v. X is f (x) = x^ $\frac{2}{18}$, for −3 < x < 3 and = 0, otherwise then P (| X | < 1) = 1/27

Miscellaneous Exercise | Q 4 | Page 242

Choose the correct option from the given alternative:
If a d.r.v. X takes values 0, 1, 2, 3, . . . which probability P (X = x) = k (x + 1)·5^{-x} , where k is a constant, then P (X = 0) =

$\frac{7}{25}$

$\frac{16}{25}$

$\frac{18}{25}$

$\frac{19}{25}$

SOLUTION

If a d.r.v. X takes values 0, 1, 2, 3, . . . which probability P (X = x) = k (x + 1)·5 −x , where k is a constant, then P (X = 0) = 16/25

Hint : $k\left[\frac{1}{5^0}+\frac{2}{5^1}+\frac{3}{5^2}+\ldots.\right]=1$

$Lets=\frac{k}{5^0}+2\frac{k}{5^1}+3\frac{k}{5^2}+\ldots\ldots\ldots\ldots\ldots$

i.e $s=k+2\frac{k}{5}+3\frac{k}{5^2}+\ldots\ldots\ldots.$

$\therefore\frac{1}{5}s=\frac{k}{5}+\frac{2k}{5^2}+\frac{3k}{5^3}+\ldots\ldots.$

$\therefore s-\frac{1}{5}s=k+\frac{k}{5}+\frac{k}{5^2}+\frac{k}{5^3}+\ldots\ldots\ldots.$

$\therefore\frac{4}{5}s=k\left[1+\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\ldots.\right]$

$=k\left[\frac{1}{1-\frac{1}{5}}\right]=5\frac{k}{4}$

$\therefore s=\frac{25k}{16}=1$

$\therefore k=\frac{16}{25}$

$\therefore P(x=0)=k(0+1)5^0=k=\frac{16}{25}.$

Miscellaneous Exercise | Q 5 | Page 242

Choose the correct option from the given alternative:

If p.m.f. of a d.r.v. X is P (X = x) = $\frac{\left(c_x^5\right)}{2^5}$, for x = 0, 1, 2, 3, 4, 5 and = 0, otherwise If a = P (X ≤ 2) and b = P (X ≥ 3), then E (X) =

1. a < b
2. a > b
3. **a = b**
4. a + b

SOLUTION

If p.m.f. of a d.r.v. X is P (X = x) = $\frac{\left(c_x^5\right)}{2^5}$, for x = 0, 1, 2, 3, 4, 5 and = 0, otherwise If a = P (X ≤ 2) and b = P (X ≥ 3), then E (X) = **a = b**

Miscellaneous Exercise | Q 6 | Page 242

Choose the correct option from the given alternative:

If p.m.f. of a d.r.v. X is P (X = x) = $\frac{x^2}{n(n+1)}$, for x = 1, 2, 3, . . ., n and = 0, otherwise then E (X) =

$\frac{n}{1}+\frac{1}{2}$

$\frac{n}{3}+\frac{1}{6}$

$\frac{n}{2}+\frac{1}{5}$

$\frac{n}{1}+\frac{1}{3}$

SOLUTION

If p.m.f. of a d.r.v. X is P (X = x) = $\frac{x^2}{n(n+1)}$, for x = 1, 2, 3, . . ., n and = 0, otherwise

then E (X) = $\frac{n}{3}+\frac{1}{6}$

Miscellaneous Exercise | Q 7 | Page 242

Choose the correct option from the given alternative :

If p.m.f. of a d.r.v. X is P (x) = $\frac{c}{x^3}$, for x = 1, 2, 3 and = 0, otherwise (elsewhere) then E (X) =

- $\frac{343}{297}$
- $\frac{294}{251}$
- $\frac{297}{294}$
- $\frac{294}{297}$

SOLUTION

If p.m.f. of a d.r.v. X is P (x) = $\frac{c}{x^3}$, for x = 1, 2, 3 and = 0, otherwise (elsewhere) then E (X) = 294/251

Miscellaneous Exercise | Q 8 | Page 242

Choose the correct option from the given alternative:

If the a d.r.v. X has the following probability distribution :

x	-2	-1	0	1	2	3
p(X=x)	0.1	k	0.2	2k	0.3	k

then P (X = −1) =

- $\frac{1}{10}$
- $\frac{2}{10}$
- $\frac{3}{10}$
- $\frac{4}{10}$

SOLUTION

If the a d.r.v. X has the following probability distribution:

x	-2	-1	0	1	2	3
p(X=x)	0.1	k	0.2	2k	0.3	k

then P (X = −1) = 1/10

Miscellaneous Exercise | Q 9 | Page 242

Choose the correct option from the given alternative:
If the a d.r.v. X has the following probability distribution:

X	1	2	3	4	5	6	7
P(X=x)	k	2k	2k	3k	k2	2k2	7k2+k

k =

$\frac{1}{7}$

$\frac{1}{8}$

$\frac{1}{9}$

$\frac{1}{10}$

SOLUTION

If the a d.r.v. X has the following probability distribution:

X	1	2	3	4	5	6	7
P(X=x)	k	2k	2k	3k	k2	2k2	7k2+k

k = 1/10

Miscellaneous Exercise | Q 10 | Page 242

Choose the correct option from the given alternative:
Find expected value of and variance of X for the following p.m.f.

X	-2	-1	0	1	2
P(x)	0.3	0.4	0.2	0.15	0.25

1. 0·85
2. **– 0·35**
3. 0·15
4. – 0·15

SOLUTION

Find expected value of and variance of X for the following p.m.f.

X	-2	-1	0	1	2
P(x)	0.3	0.4	0.2	0.15	0.25

= **– 0·35**

MISCELLANEOUS EXERCISE [PAGES 242 - 244]

Miscellaneous Exercise | Q 1.1 | Page 242

Solve the following :
Identify the random variable as either discrete or continuous in each of the following. Write down the range of it.
An economist is interested the number of unemployed graduate in the town of population 1 lakh.

SOLUTION

Let X = number of unemployed graduates in a town.

Since, the population of the town is 1 lakh, X takes the finite values.

∴ random variable X is **discrete.**

Range = {0, 1, 2, …, 99999, 100000}.

Miscellaneous Exercise | Q 1.2 | Page 242

Solve the following :
Identify the random variable as either discrete or continuous in each of the following. Write down the range of it.
Amount of syrup prescribed by physician.

SOLUTION

Let X = amount of syrup prescribed by a physician.

Then X takes uncountable infinite values.

∴ random variable X is **continuous.**

Miscellaneous Exercise | Q 1.3 | Page 242

Solve the following :
Identify the random variable as either discrete or continuous in each of the following. Write down the range of it.
The person on the high protein diet is interested gain of weight in a week.

SOLUTION

Let X = gain of weight in a week

Then X takes uncountable infinite values

∴ random variable X is **continuous.**

Miscellaneous Exercise | Q 1.4 | Page 242

Solve the following :
Identify the random variable as either discrete or continuous in each of the following. Write down the range of it.

20 white rats are available for an experiment. Twelve rats are male. Scientist randomly selects 5 rats number of female rats selected on a specific day

SOLUTION

Let X = number of female rats selected on a specific day

Since the total number of rats is 20 which include 12 males and 8 females, X takes the finite values

∴ random variable X is discrete.

Range = {0, 1, 2, 3, 4, 5}

Miscellaneous Exercise | Q 1.5 | Page 242

Solve the following:
Identify the random variable as either discrete or continuous in each of the following. Write down the range of it.
A highway safety group is interested in studying the speed (km/hrs) of a car at a check point.

SOLUTION

Let X = speed of the car in km/hr

Then X takes uncountable infinite values

∴ random variable X is **continuous.**

Miscellaneous Exercise | Q 2 | Page 242
The probability distribution of discrete r.v. X is as follows :

x = x	1	2	3	4	5	6
P[x=x]	k	2k	3k	4k	5k	6k

(i) Determine the value of k.
(ii) Find $P(X \leq 4)$, $P(2 < X < 4)$, $P(X \geq 3)$.

SOLUTION

(i) Since P (x) is a probability distribution of x,

$$\Sigma_{x=0}^{6} P(x) = 1$$

∴ P (0) + P (1) + P(2) + P(3) + P(4) + P(5) + P (6) = 1

∴ k + 2k + 3k + 4k + 5k + 6k = 1

∴ 21k = 1

∴ k = $\frac{1}{21}$

(ii)

P(X ≤ 4)

= P (0) + P (1) + P(2) + P(3) + P(4)

=k + 2k + 3k + 4k

= 10k

= $10\left(\frac{1}{21}\right)$

= $\frac{10}{21}$

P(2< X < 4)

= P(3)

= 3k

= $3\left(\frac{1}{21}\right)$

= $\frac{1}{7}$

P(X ≥ 3)

= P (1) + P(2) + P(3)

$= k + 2k + 3k$

$= 6k$

$= 6\left(\frac{1}{21}\right)$

$= \frac{2}{7}$

Miscellaneous Exercise | Q 3.1 | Page 242

Solve the following :
The following probability distribution of r.v. X

X=x	-3	-2	-1	0	1	2	3
P(X=x)	0.05	0.1	0.15	0.20	0.25	0.15	0.1

Find the probability that
X is positive

SOLUTION

P (X is positive) = P(X = 1) + P (X = 2) + P(X = 3)

= 0.25 + 0.15 +0.1+ = **0.50**

Miscellaneous Exercise | Q 3.3 | Page 242

Solve the following :

The following probability distribution of r.v. X

X=x	-3	-2	-1	0	1	2	3
P(X=x)	0.05	0.10	0.15	0.20	0.25	0.15	0.1

Find the probability that
X is odd

SOLUTION

P (X is odd)

= P (X = -3) + P (X = -1) + P (x=1) + P (X=3)

= 0.05 + 0.15 + 0.25 + 0.1 = **0.55**

Miscellaneous Exercise | Q 3.4 | Page 242

Solve the following :

The following probability distribution of r.v. X

X=x	-3	-2	-1	0	1	2	3
P(X=x)	0.05	0.10	0.15	0.20	0.25	0.15	0.1

Find the probability that
X is even

SOLUTION

P (X is even)

= P (X = - 2) + P (X = 0) + P (X = 2)

= 0.10 + 0.20 + 0.15 = **0.45**

Miscellaneous Exercise | Q 4 | Page 242

The p.m.f. of a r.v. X is given by P (X = x) = $\frac{{}^5C_x}{2^5}$, for x = 0, 1, 2, 3, 4, 5 and = 0, otherwise.

Then show that P (X ≤ 2) = P (X ≥ 3).

SOLUTION

P (X ≤ 2) = P(X = 0) + P(X = 1)+P(X = 2)

$= \frac{{}^5C_0}{2^5} + \frac{{}^5C_1}{2^5} + \frac{{}^5C_2}{2^5}$

$= \frac{{}^5C_5}{2^5} + \frac{{}^5C_4}{2^5} + \frac{{}^5C_3}{2^5}$ ....$[{}^nC_r = {}^nC_n - r]$

= P (X = 5) + P (X = 4) + P (X = 3)

= P (X ≥ 3)

∴ P (X ≤ 2) = P (X ≥ 3).

Miscellaneous Exercise | Q 5 | Page 242

In the p.m.f. of r.v. X

x	1	2	3	4	5
P (X)	$\frac{1}{20}$	$\frac{3}{20}$	a	2a	$\frac{1}{20}$

Find a and obtain c.d.f. of X.

SOLUTION

For p.m.f. of a r.v. X

$\sum_{i=1}^{5}$ P(X=x) = 1

∴ P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)

$$\therefore \frac{1}{20} + \frac{3}{20} + a + 2a + \frac{1}{20} = 1$$

$$\therefore 3a = 1 - \frac{5}{20} = 1 - \frac{1}{4} = \frac{3}{4}$$

∴ a = $\frac{1}{4}$

∴ the p.m.f. of the r.v. X is

x	1	2	3	4	5
P (X = x)	$\frac{1}{20}$	$\frac{3}{20}$	$\frac{5}{20}$	$\frac{10}{20}$	$\frac{1}{20}$

Let F (x) be the c.d.f. of X.

Then F (x) = P(X ≤ x)

∴ F (1) = P (X≤ 1) = P(X = 1) = $\frac{1}{20}$

F (2) = P (X≤ 2) = P(X = 1) + P(X = 2)

$$= \frac{1}{20} + \frac{3}{20} = \frac{4}{20} = \frac{1}{5}$$

F (3) = P (X≤ 3) = P(X = 1) + P(X = 2) + P(X=3)

$$= \frac{1}{20} + \frac{3}{20} + \frac{5}{20} = \frac{9}{20}$$

F (4) = P (X≤ 4) = P(X = 1) + P(X = 2) + P(X=3) + P(X=4)

$$= \frac{1}{20} + \frac{3}{20} + \frac{5}{20} + \frac{10}{20} = \frac{19}{20}$$

F (5) = P (X≤ 5) = P(X = 1) + P(X = 2) + P(X=3) + P(X=4) + P(X=5)

$$= \frac{1}{20} + \frac{3}{20} + \frac{5}{20} + \frac{10}{20} + \frac{1}{20} = \frac{20}{20} = 1$$

Hence, the c.d.f. of the random variable X is as follows :

x_i	1	2	3	4	5
$F(x_i)$	$\frac{1}{20}$	$\frac{1}{5}$	$\frac{9}{20}$	$\frac{19}{20}$	1

Miscellaneous Exercise | Q 6 | Page 242

Solve the following problem :

A fair coin is tossed 4 times. Let X denote the number of heads obtained. Identify the probability distribution of X and state the formula for p. m. f. of X.

SOLUTION 1

When a fair coin is tossed 4 times then the sample space is

S = {HHHH,HHHT,HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT}

∴ n (S) = 16

X denotes the number of heads.

∴ X can take the value 0, 1, 2, 3, 4 When X = 0,

then X= {TTTT}

∴ n (X) = 1

∴ P (X=0) = $\frac{n(x)}{n(s)} = \frac{1}{16} = \frac{^4C_0}{16}$

When X = 1, then

X = {HTTT, THTT, TTHT, TTTH}

∴ n (X) = 4

∴ P (X=1) = $\frac{n(x)}{n(s)} = \frac{4}{16} = \frac{^4C_1}{16}$

When X = 2, then

X ={ HHTT, HTHT, HTTH, THHT, THTH, TTHH}

∴ n (X) = 6

∴ P (X=2) = $\frac{n(x)}{n(s)} = \frac{6}{16} = \frac{^4C_2}{16}$

When X = 3, then

X ={ HHHT, HHTH, HTHH, THHH}

∴ n (X) = 4

∴ P (X=3) = $\frac{n(x)}{n(s)} = \frac{4}{16} = \frac{^4C_3}{16}$

When X = 4, then

X = {HHHH}

∴ n (X) = 1

∴ P (X=4) = $\frac{n(x)}{n(s)} = \frac{1}{16} = \frac{^4C_4}{16}$

∴ the probability distribution of X is as follows :

x	0	1			
p(x)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Also, the formula for p.m.f. of X is

P (x) = $\frac{^4C_x}{16}$

x = 0,1,2,3,4

= 0 otherwise.

SOLUTION 2

A coin is tossed 4 times.

∴ n(S) = 2^4 = 16

Let X be the number of heads.

Thus, X can take values 0, 1, 2, 3, 4

When X = 0, i.e., all tails {TTTT},

n(X) = $^4C_0$ = 1

∴ P(X = 0) = $\frac{1}{16}$

When X = 1, i.e., only one head.

n(X) = $^4C_1$ = 4

∴ P(X = 1) = $\frac{4}{16}$

When X = 2, i.e., two heads.

n(X) = $^4C_2 = \frac{4!}{2!2!}$ = 6

∴ P(X = 2) = $\frac{6}{16}$

When X = 3, i.e., three heads.

$n(X) = {}^4C_3 = 4$

$\therefore P(X = 3) = \frac{4}{16} = \frac{1}{14}$

When X = 4, i.e., all heads ≅ {HHHH},

$n(X) = {}^4C_4 = 1$

$\therefore P(X = 4) = \frac{1}{16}$

Then,

X	0	1	2	3	4
P(X)	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

∴ Formula for p.m.f. of X is

$P(X) = \frac{\binom{4}{x}}{16}, x = 0, 1, 2, 3, 4$

= 0, otherwise.

Miscellaneous Exercise | Q 7 | Page 244

Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

(i) number greater than 4

(ii) six appears on at least one die

SOLUTION

When a die is tossed two times, we obtain (6 × 6) = 36 number of observations.

Let X be the random variable, which represents the number of successes.

Here, success refers to the number greater than 4.

P (X = 0) = P (number less than or equal to 4 on both the tosses) = $\frac{4}{6} \times \frac{4}{6} = \frac{16}{36} = \frac{4}{9}$

P (X = 1) = P (number less than or equal to 4 on first toss and greater than 4 on second toss) + P (number greater than 4 on first toss and less than or equal to 4 on second toss)

$$= \frac{4}{6} \times \frac{2}{6} + \frac{4}{6} \times \frac{2}{6} = \frac{8}{36} + \frac{8}{36} = \frac{16}{36} = \frac{4}{9}$$

P (X = 2) = P (number greater than 4 on both the tosses)

$$= \frac{2}{6} \times \frac{2}{6} = \frac{4}{36} = \frac{1}{9}$$

Thus, the probability distribution is as follows.

X	0	1	2
P(X)	$\frac{4}{9}$	$\frac{4}{9}$	$\frac{1}{9}$

(ii) Here, success means six appears on at least one die.

P (Y = 0) = P (six appears on none of the dice) = $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

P(Y = 1) = P (six appears on none of the dice x six appears on at least one of the dice) + P (six appears on none of the dice x six appears on at least one of the dice)

$$= \frac{1}{6} \times \frac{5}{6} + \frac{1}{6} \times \frac{5}{6} = \frac{5}{36} + \frac{5}{36} = \frac{10}{36}$$

P (Y = 2) = P (six appears on at least one of the dice) = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Thus, the required probability distribution is as follows

Y	0	1	2
P(Y)	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$

Miscellaneous Exercise | Q 8 | Page 244

A random variable X has the following probability distribution :

x = x	0	1	2	3				7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Determine (i) k

(ii) P(X> 6)

(iii) P(0<X<3).

SOLUTION

Refer to the solution of Q. 8 of Exercise 7.1.

Miscellaneous Exercise | Q 9.1 | Page 244

The following is the c.d.f. of r.v. X

x	-3	-2	-1	0	1	2	3	4
F(X)	0.1	0.3	0.5	0.65	0.75	0.85	0.9	1

Find p.m.f. of X.

SOLUTION

From the given table

F(-3)= 0.1, F(-2)=0.3,F(-1)=0.5

F(0) = 0.65, f(1) = 0.75, F(2) = 0.85

F(3) = 0.9, F(4) = 1

P(X= -3) = F(-3) = 0.1

P(X= - 2)= F(- 2)- F(-3)= 0.3 - 0.1 = 0.2

P(X= - 1)= F(-1)-F(-2) = 0.5 - 0.3 = 0.2

P(X = 0) = F(0) - F(- 1) = 0.65 - 0.5 = 0.15

P(X = 1) = F(1) - F(0) = 0.75 - 0.65 = 0.1

P(X = 2) = F(2) - F(1) = 0.85 - 0.75 = 0.1

P(X = 3) = F(3) - F(2) = 0.9 - 0.85 = 0.05

P(X = 4) = F(4) - F(3) = 1 - 0.9 = 0.1

∴ the p.m.f of X is as follows :

X = x	-3	-2	-1	0	1			4
P(X = x)	0.1	0.2	0.2	0.15	0.1	0.1	00.5	0.1

Miscellaneous Exercise | Q 9.2 | Page 244

The following is the c.d.f. of r.v. X

x	-3	-2	-1	0	1	2	3	4
F(X)	0.1	0.3	0.5	0.65	0.75	0.85	0.9	*1

P (–1 ≤ X ≤ 2)

SOLUTION

P (–1 ≤ X ≤ 2)

=P(X= -1) +P(X=0)+ P(X = 0)+P(X = 2)

= 0.2 + 0.15 + 0.1 + 0.1 = 0.55

Miscellaneous Exercise | Q 9.3 | Page 244

The following is the c.d.f. of r.v. X

x	-3	-2	-1	0	1	2	3	4
F(X)	0.1	0.3	0.5	0.65	0.75	0.85	0.9	1

P (X ≤ 3/ X > 0)

SOLUTION

(X ≤ 3) ∩ (X > 0)

= {-3, -2, -1, 0, 1, 2, 3} ∩ {1, 2, 3, 4}

= {1, 2, 3}

∴ P[(X ≤ 3) ∩ (X > 0)]

= P(X = 1)+ P(X = 2)+ P(X = 3)

∴ P[(X ≤ 3) / (X > 0)]

$$= \frac{P[(X \leq 3) \cap (X > 0)]}{P(X > 0)}$$

$$= \frac{P(X = 1) + P(X = 2) + P(X = 3)}{P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)}$$

$$= \frac{0.1 + 0.1 + 0.05}{0.1 + 0.1 + 0.05 + 0.1}$$

$= \frac{0.25}{0.35}$

$= \frac{5}{7}$

Miscellaneous Exercise | Q 10.1 | Page 244

Find the expected value, variance and standard deviation of the random variable whose p.m.f.'s are given below :

x = x	1	2	3
P (X = x)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

SOLUTION

We construct the following table to find the expected value, variance and standard deviation:

x_i	$P(x_i)$	$x_i \cdot P(x_i)$	$x_i^2 \cdot P(x_i) = x_i \times x_i \cdot P(x_i)$
1	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
2	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{8}{5}$
3	$\frac{2}{5}$	$\frac{6}{5}$	$\frac{18}{5}$
Total	1	$\frac{11}{5}$	$\frac{27}{5}$

From the table,

$\sum x_i \cdot P(x_i) = \frac{11}{5}$,

$\sum x_i^2 \cdot P(x_i) = \frac{27}{5}$

Expected value $= E(X) = \sum x_i \cdot P(x_i)$

$= \frac{11}{5} = 2.2$

Variance $= V(x) = \sum x_i^2 \cdot P(x_i) - [\sum x_i \cdot P(x_i)]^2$

$$= \frac{27}{5} - \left(\frac{11}{5}\right)^2$$

$$= \frac{27}{5} - \frac{121}{25}$$

$$= \frac{14}{25}$$

= 0.56

Standard deviation = $\sqrt{V(X)} = \sqrt{0.56}$

= 0.7483

Miscellaneous Exercise | Q 10.2 | Page 244

Find the expected value, variance and standard deviation of the random variable whose p.m.f.'s are given below :

x = x	-1	0	3
P (X = x)	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{2}{5}$

SOLUTION

We construct the following table to find the expected value, variance and standard deviation:

x_i	$P(x_i)$	$x_i \cdot P(x_i)$	$x_i{}^2 \cdot P(x_i) = x_i \times x_i \cdot P(x_i)$
-1	$\frac{1}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$
0	$\frac{2}{5}$	0	0
1	$\frac{2}{5}$	$\frac{2}{5}$	$\frac{2}{5}$
Total	1	$\frac{1}{5}$	$\frac{3}{5}$

From the table,

$\sum x_i \cdot P(x_i) = \frac{1}{5}$,

$\sum x_i^2 \cdot P(x_i) = \frac{3}{5}$

Expected value = E (X) = $\sum x_i \cdot P(x_i) = \frac{1}{5}$ = 0.2

Variance = V (x) = $\sum x_i^2 \cdot P(x_i) - [\sum x_i \cdot P(x_i)]^2$

$= \frac{3}{5} - \left(\frac{1}{5}\right)^2$

$= \frac{3}{5} - \frac{1}{25}$

$= \frac{15}{25} - \frac{1}{25}$

$= \frac{14}{25}$

= 0.56

Standard deviation = $\sqrt{V(X)}$

$= \sqrt{0.56}$

= 0.7483

Miscellaneous Exercise | Q 10.3 | Page 244

Find the expected value, variance and standard deviation of the random variable whose p.m.f.'s are given below :

x = x	1	2	3	...	n
P (X = x)	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	...	$\frac{1}{n}$

SOLUTION

We construct the following table to find the expected value, variance, and standard deviation:

x_i	$P(x_i)$	$x_i \cdot P(x_i)$	$x_i^2 \cdot P(x_i) = x_i \times x_i \cdot P(x_i)$
1	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$
2	$\frac{1}{n}$	$\frac{2}{n}$	$\frac{2^2}{n}$
3	$\frac{1}{n}$	$\frac{3}{n}$	$\frac{3^2}{n}$
...	...	...	...
n	$\frac{1}{n}$	$\frac{n}{n}$	$\frac{n^2}{n}$

From the table,

$$\sum x_i \cdot P(x_i) = \frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \ldots + \frac{n}{n}$$

$$= \frac{1}{n}(1 + 2 + 3 + \ldots + n)$$

$$= \frac{1}{n}\sum_{r=1}^{n} r = \frac{1}{n} \times \frac{n(n+1)}{2}$$

$$= \frac{n+1}{2}$$

$$\sum x_i^2 \cdot P(x_i) = \frac{1}{n} + \frac{2^2}{n} + \frac{3^2}{n} + \ldots + \frac{n^2}{n}$$

$$= \frac{1}{n}\left(1^2 + 2^2 + 3^2 + \ldots + n^2\right)$$

$$= \frac{1}{n}\sum_{r=1}^{n} r^2 = \frac{1}{n} \times \frac{n(n+1)(2n+1)}{6}$$

$= \dfrac{n+1}{2}$

Variance = V(X) = $\sum x_i^2 \cdot P(x_i) - [\sum x_i \cdot P(x_i)]^2$

$= \dfrac{(n+1)(2n+1)}{6} - \left(\dfrac{n+1}{2}\right)^2$

$= \dfrac{n+1}{2}\left[\dfrac{2n+1}{3} - \dfrac{n+1}{2}\right]$

$= \dfrac{n+1}{2}\left[\dfrac{4n+2-3n-3}{6}\right]$

$= \dfrac{(n+1)(n-1)}{12}$

$= \dfrac{n^2-1}{12}$

Standard deviation = $\sqrt{V(X)}$

$= \sqrt{\dfrac{n^2-1}{12}}$

$= \dfrac{\sqrt{n^2-1}}{2\sqrt{3}}$

Miscellaneous Exercise | Q 10.4 | Page 244

Find the expected value, variance, and standard deviation of the random variable whose p.m.f.'s are given below :

x = x	0	1	2	3	4	5
P (X = x)	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$

SOLUTION

We construct the following table to find the expected value, variance and standard deviation:

x_i	$P(x_i)$	$x_i \cdot P(x_i)$	$x_i^2 \cdot P(x_i) = x_i \times x_i \cdot P(x_i)$
0	$\frac{1}{32}$	0	0
1	$\frac{5}{32}$	$\frac{5}{32}$	$\frac{5}{32}$
2	$\frac{10}{32}$	$\frac{20}{32}$	$\frac{40}{32}$
3	$\frac{10}{32}$	$\frac{30}{32}$	$\frac{90}{32}$
4	$\frac{5}{32}$	$\frac{20}{32}$	$\frac{80}{32}$
5	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{25}{32}$
Total	$\frac{32}{32}$	$\frac{80}{32}$	$\frac{240}{32}$

From the table,

$\sum x_i \cdot P(x_i) = \frac{80}{32} = \frac{5}{2}$,

$\sum x_i^2 \cdot P(x_i) = \frac{240}{32} = \frac{15}{2}$

Expected value = $E(X) = \sum x_i \cdot P(x_i)$

$= \frac{5}{2} = 2.5$

Variance = $V(x) = \sum x_i^2 \cdot P(x_i) - [\sum x_i \cdot P(x_i)]^2$

$= \frac{15}{2} - \left(\frac{5}{2}\right)^2$

$= \frac{15}{2} - \frac{25}{4}$

$= \frac{30}{4} - \frac{25}{4}$

$= \frac{5}{4}$

$= 1.25$

Standard deviation = $\sqrt{V(X)} = \sqrt{1.25}$

$= 1.118$

Miscellaneous Exercise | Q 11 | Page 244

Solve the following problem :
A player tosses two coins. He wins ₹ 10 if 2 heads appear, ₹ 5 if 1 head appears, and ₹ 2 if no head appears. Find the expected value and variance of winning amount.

SOLUTION 1

When a coin is tossed twice, the sample space is

S = {HH, HT, TH, HH}

Let X denote the amount he wins.

Then X takes values 10, 5, 2.

P(X = 10) = P(2 heads appear)= $\frac{1}{4}$

P(X = 5) = P(1 head appears) = $\frac{2}{4} = \frac{1}{2}$

P(X = 2) = P(no head appears) = $\frac{1}{4}$

We construct the following table to calculate the mean and the variance of X :

x_i	P (x_i)	xi P (x_i)	x_i^2 P (x_i)
10	$\frac{1}{4}$	$\frac{5}{2}$	25
5	$\frac{1}{2}$	$\frac{5}{2}$	25/2
2	$\frac{1}{4}$	$\frac{1}{2}$*	1
Total	**1**	**5.5**	**38.5**

From the table $\sum x_i P(x_i) = 5.5$, $\sum x_i 2 \cdot P(x_i) = 38.5$

$E(X) = \sum x_i P(x_i) = 5.5$

$\text{Var}(X) = x_i^{\ 2} P(x_i) - [E(X)]^2$

$= 38.5 - (5.5)^2$

$= 38.5 - 30.25 = 8.25$

∴ Hence, expected winning amount ₹ 5.5 and variance of winning amount ₹8.25

SOLUTION 2

Let X denote the winning amount.
∴ Possible values of X are 2, 5, 10

Let P(getting head) = $p = \frac{1}{2}$

∴ $q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

∴ $P(X = 2) = P(\text{no head}) = qq = q^2 = \frac{1}{4}$

$P(X = 5) = P(\text{one head}) = pq + qp = 2pq$

$= 2 \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{2}{4}$

$P(X = 10) = P(\text{two heads}) = pp = p^2 = \frac{1}{4}$

∴ The probability distribution of X is as follows:

X = x	2	5	10
P(X = x)	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

Expected winning amount

$$= E(X) = \sum_{i=1}^{3} x_i P(x_i)$$

$= 2 \times \frac{1}{4} + 5 \times \frac{2}{4} + 10 \times \frac{1}{4}$

$= \frac{2 + 10 + 10}{4}$

$= \frac{22}{4}$

= ₹ 5.5

$E(X^2) = \sum_{i=1}^{3} x_i^2 P(x_i)$

$= (2)^2 \times \frac{1}{4} + (5)^2 \times \frac{2}{4} + (10)^2 \times \frac{1}{4}$

$= \frac{4 + 50 + 100}{4}$

$= \frac{154}{4}$

= 38.5

Variance of winning amount

= Var(X) = $E(X^2) - [E(X)]^2$

= $38.5 - (5.5)^2$

= 38.5 – 30.25

= ₹ 8.25

Miscellaneous Exercise | Q 12 | Page 244

Let the p.m.f. of r.v. X be

P (x) = $\frac{3 - x}{1} = 0$, for x = −1, 0, 1, 2 and = 0, otherwise

Calculate E(X) and Var (X).

SOLUTION

P(X)= $\frac{3-x}{10}$

X takes values -1, 0, 1, 2

P(X = -1)= P(-1) = $\frac{3+1}{10} = \frac{4}{10}$

P(X = 0)= P(0) = $\frac{3-0}{10} = \frac{3}{10}$

P(X = 1)= P(1) = $\frac{3-1}{10} = \frac{2}{10}$

P(X = 2)= P(2) = $\frac{3-2}{10} = \frac{1}{10}$

We construct the following table to calculate the mean and variance of X :

x_i	$P(x_i)$	$x_i P(x_i)$	$x_i^2 P(x_i)$
-1	$\frac{4}{10}$	$-\frac{4}{10}$	$\frac{4}{10}$
0	$\frac{3}{10}$	0	0
1	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{2}{10}$
2	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{4}{10}$
Total	**1**	**0**	**1**

From the table

$\sum x_i P(x_i) 0$ and $\sum x_i^2 \cdot P(x_i) = 1$

$E(X) = x_i P(x_i) = 0$

$Var(X) = \sum x_i^2 \cdot P(x_i) - [E(X)]^2$

= 1 - 0 = 1

Hence, E(X) = 0, Var (X) = 1.

Miscellaneous Exercise | Q 13.1 | Page 244

Suppose error involved in making a certain measurement is continuous r.v. X with p.d.f.
f (x) = k $\left(4 – x^2\right)$, for –2 ≤ x ≤ 2 and = 0 otherwise.
P(x > 0)

SOLUTION

Since, f is the p.d.f. of X,

$\int_{-\infty}^{\infty} f(x)dx$ = 1

∴ $\int_{-\infty}^{-2} f(x)dx$ + $\int_{-2}^{2} f(x)dx$ + $\int_{2}^{\infty} f(x)dx$= 1

∴ 0 + $\int_{-2}^{2} k\left(4 - x^2\right)dx$ = 1

∴ k $\int_{-2}^{2} \left(4 - x^2\right)dx$ = 1

∴ k $\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$ = 1

∴ k $\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right]$ = 1

∴ k $\left(\frac{16}{3} + \frac{16}{3}\right)$ = 1

∴ k $\left(\frac{32}{3}\right)$ = 1

$\therefore$ k = $\frac{3}{32}$

P(x > 0)

= $\int_0^{\infty} f(x)dx$

= $\int_0^2 f(x)dx + \int_2^{\infty} f(x)dx$

= $\int_0^2 k\left(4 - x^2\right)dx + 0$

= k $\int_0^2 \left(4 - x^2\right)dx$

= $\frac{3}{32}\left[4x - \frac{x^3}{3}\right]_0^2$ ....[$\because$ k = $\frac{3}{32}$]

= $\frac{3}{32}\left[8 - \frac{8}{3}\right] = \frac{3}{32} \times \frac{16}{3} = \frac{1}{2}$

Miscellaneous Exercise | Q 13.2 | Page 244

Suppose error involved in making a certain measurement is continuous r.v. X with p.d.f.
f (x) = k $\left(4 - x^2\right)$, for −2 ≤ x ≤ 2 and = 0 otherwise
P(−1 < x < 1)

SOLUTION

Since, f is the p.d.f. of X,

$\int_{-\infty}^{\infty} f(x)dx$ = 1

$\therefore \int_{-\infty}^{-2} f(x)dx + \int_{-2}^{2} f(x)dx + \int_2^{\infty} f(x)dx$ = 1

$\therefore$ 0 + $\int_{-2}^{2} k\left(4 - x^2\right)dx$ = 1

$\therefore$ k $\int_{-2}^{2} (4 - x^2)dx$ = 1

$\therefore$ k $\left[4x - \frac{x^3}{3}\right]_{-2}^{2}$ = 1

$\therefore$ k $\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right]$ = 1

$\therefore$ k$\left(\frac{16}{3} + \frac{16}{3}\right)$ = 1

$\therefore$ k$\left(\frac{32}{3}\right)$ = 1

$\therefore$ k = $\frac{3}{32}$

P(−1 < x < 1)

= $\int_{-1}^{1} f(x)dx$

= $\int_{-1}^{1} k(4 - x^2)dx$

= k $\int_{-1}^{1} (4 - x^2)dx$

= $\frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-1}^{1}$ ....[$\because$ k = $\frac{3}{32}$]

= $\frac{3}{32}\left[\left(4 - \frac{1}{3}\right) - \left(-4 + \frac{1}{3}\right)\right]$

= $\frac{3}{32}\left(\frac{11}{3} + \frac{11}{3}\right)$

= $\frac{3}{32}\left(\frac{22}{3}\right)$

$= \frac{11}{6}$

Miscellaneous Exercise | Q 13.3 | Page 244

Suppose error involved in making a certain measurement is continuous r.v. X with p.d.f.
f (x) = k $(4 - x^2)$, for –2 ≤ x ≤ 2 and = 0 otherwise.
P (–0·5 < x or x > 0·5)

SOLUTION

Since, f is the p.d.f. of X,

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\therefore \int_{-\infty}^{-2} f(x)dx + \int_{-2}^{2} f(x)dx + \int_{2}^{\infty} f(x)dx = 1$$

$$\therefore 0 + \int_{-2}^{2} k(4 - x^2)dx = 1$$

$$\therefore k \int_{-2}^{2} (4 - x^2)dx = 1$$

$$\therefore k\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 1$$

$$\therefore k\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = 1$$

$$\therefore k\left(\frac{16}{3} + \frac{16}{3}\right) = 1$$

$$\therefore k\left(\frac{32}{3}\right) = 1$$

$\therefore$ k = $\frac{3}{32}$

P (–0·5 < x or x > 0·5)

= P (x < –0·5) + P (x > – 0·5)

= $\int_{-\infty}^{-0.5} f(x)dx + \int_{0.5}^{\infty} f(x)dx$

= $\int_{-\infty}^{-2} f(x)dx + \int_{-2}^{-0.5} f(x)dx + \int_{0.5}^{2} f(x)dx + \int_{2}^{\infty} f(x)dx$

= 0+ $\frac{\int_{-2}^{-1}}{2} k\left(4 - x^2\right)dx + \int_{\frac{1}{2}}^{2} k\left(4 - x^2\right)dx$ + 0

=k $\frac{\int_{-2}^{1}}{2} \left(4 - x^2\right)dx + \int_{\frac{1}{2}}^{2} \left(4 - x^2\right)dx$

= $\frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-2}^{-\frac{1}{2}} + \frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{\frac{1}{2}}^{2}$ ....[$\because$ k = $\frac{3}{32}$]

= $\frac{3}{32}\left[\left(-2 + \frac{1}{24}\right) - \left(-8 + \frac{8}{3}\right)\right] + \frac{3}{32}\left[\left(8 - \frac{8}{3}\right) - \left(2 - \frac{1}{24}\right)\right]$

= $\frac{3}{32}\left(\frac{-47}{24} + \frac{16}{3}\right) + \frac{3}{32}\left(\frac{16}{3} - \frac{47}{24}\right)$

= $\frac{3}{32}\left(\frac{-47}{24} + \frac{16}{3} + \frac{16}{3} - \frac{47}{24}\right)$

= $\frac{3}{32}\left(\frac{-47 + 128 + 128 - 47}{24}\right)$

= $\frac{3}{32}\left(\frac{162}{24}\right) = \frac{81}{128}$

= 0.6328.

Alternative Method :

P (x< – 0·5 or x > 0·5)

= 1 -P(- 0.5 ≤ x ≤ 0.5)

= 1 - $\int_{-0.5}^{0.5} f(x)dx$

= 1 - $\int_{-\frac{1}{2}}^{\frac{1}{2}} k(4-x^2)dx$

= 1 - k $\int_{-\frac{1}{2}}^{\frac{1}{2}} (4-x^2)dx$

= $1 - \frac{3}{32}\left[4x - \frac{x^3}{3}\right]_{-\frac{1}{2}}^{\frac{1}{2}}$ ......[∵ k = $\frac{3}{32}$]

= $1 - \frac{3}{32}\left[\left(2 - \frac{1}{24}\right) - \left(-2 + \frac{1}{24}\right)\right]$

= $1 - \frac{3}{32}\left(2 - \frac{1}{24} + 2 - \frac{1}{24}\right)$

= $1 - \frac{3}{32}\left(4 - \frac{1}{12}\right)$

= $1 - \frac{3}{32} \times \frac{47}{12} = 1 - \frac{47}{128}$

= $\frac{128-47}{128} = \frac{81}{128}$

= 0.6328

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The p.d.f. of a continuous r.v. X is given by

f (x) = $\frac{1}{2a}$, for 0 < x < 2a and = 0, otherwise. Show that $P\left[X < \frac{a}{2}\right] = P\left[X > \frac{3a}{2}\right]$.

SOLUTION

P $(X < \frac{a}{2})$ = $\int_{-\infty}^{\frac{a}{2}} f(x)dx$

= $\int_{-\infty}^{0} f(x)dx + \int_{-\infty}^{\frac{a}{2}} f(x)dx$

= 0+ $\int_{0}^{\frac{a}{2}} \frac{1}{2a}dx$

= $\frac{1}{2a}\int_{0}^{\frac{a}{2}} 1dx = \frac{1}{2a}[x]_0^{\frac{a}{2}}$

= $\frac{1}{2a}\left[\frac{a}{2} - 0\right] = \frac{1}{4}$ ....(1)

P $\left[X > \frac{3a}{2}\right] = \int_{\frac{3a}{2}}^{\infty} f(x)dx$

= $\int_{\frac{3a}{2}}^{2a} f(x)dx + \int_{2a}^{\infty} f(x)dx$

= $\int_{\frac{3a}{2}}^{2a} \frac{1}{2a}dx + 0$

= $\frac{1}{2a}\int_{\frac{3a}{2}}^{2a} 1dx = \frac{1}{2a}[x]_{\frac{3a}{2}}^{2a}$

= $\frac{1}{2a}\left[2a - \frac{3a}{2}\right] = \frac{1}{2a}\left(\frac{a}{2}\right) = \frac{1}{4}$ ......(2)

From (1) and (2), we get

$P\left[X < \frac{a}{2}\right] = P\left[X > \frac{3a}{2}\right].$

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The p.d.f. of r.v. of X is given by

f (x) = $\frac{k}{\sqrt{x}}$, for 0 < x < 4 and = 0, otherwise. Determine k .

Determine c.d.f. of X and hence P (X ≤ 2) and P(X ≤ 1).

SOLUTION

Since, f is p.d.f. of the r.v. X,

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\therefore \int_{-\infty}^{\infty} f(x)dx + \int_{-0}^{4} f(x)dx + \int_{4}^{\infty} f(x)dx = 1$$

$$\therefore 0 + \int_{0}^{4} \frac{k}{\sqrt{x}}dx + 0 = 1$$

$$\therefore k \int_{0}^{4} x^{-\frac{1}{2}}dx = 1$$

$$\therefore k\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_{0}^{4} = 1$$

∴ 2k[2 - 0] = 1

∴ 4k = 1

∴ k = $\frac{1}{4}$

Let F(X) be the c.d.f. of X.

$\therefore$ F(X) = P(X ≤ x) = $\int_{-\infty}^{x} f(x)dx$

= $\int_{-\infty}^{0} f(x)dx + \int_{0}^{x} f(x)dx$

= $0 + \int_{0}^{x} \frac{k}{\sqrt{x}}dx$

= $k\int_{0}^{x} x^{-\frac{1}{2}}dx$

= $k\left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_{0}^{x}$

= $2k\sqrt{x} = 2\left(\frac{1}{4}\right)\sqrt{x}$ .....[$\because$ k = $\frac{1}{4}$]

$\therefore$ F(X) = $\frac{\sqrt{x}}{2}$

P(X ≤ 2) = F (2) = $\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

P(X ≤ 1) = F (1) = $\frac{\sqrt{1}}{2} = \frac{1}{2}$

Hence, $k\frac{1}{4}, P(X \leq 2) = \frac{1}{\sqrt{2}}, P(X \leq 1) = \frac{1}{2}$