Chapter 14. Locus and Constructions

Formulae

Theorems on Locus:
(a) The locus of a point equidistant from a fixed point is a circle with the fixed point as centre.
(b) The locus of a point equidistant from two interacting lines is the bisector of the angles between the lines.
(c) The locus of a point equidistant from two given points is the perpendicular bisector of the line joining the points.

Prove the Following

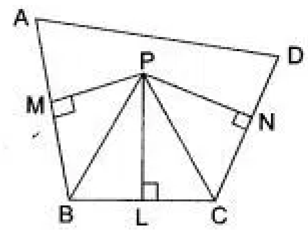
Question 1. The bisector of ∠ B and ∠C of a quadrilateral ABCD intersect in P, show that P is equidistant from the opposite sides AB and CD.

Given : A quadrilateral ABCD. Bisectors of $\angle B$ and $\angle C$ meet in P. PM $\perp$ AB and PN $\perp$ CD.

To prove that : $PM = PN$ ...(1)

Construction : Draw $PL \perp BC$

Proof : P lies on bisector or of $\angle B$



$\therefore$ $PM = PL$

P lies on bisector of $\angle C$

$PL = PN$ ...(2)

From (1) and (2), we have

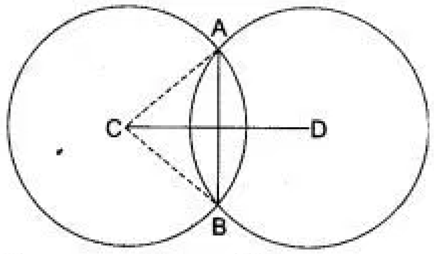
$PM = PN.$ Hence proved.

Question 2. Prove that the common chord of two intersecting circles is bisected at right angles by the line of centres.

Solution : Given : Two intersecting circles with centres C & D.

AB is their common chord.

To prove : AB bisected by CD at right angles.



Proof : CA = CB – (radii)

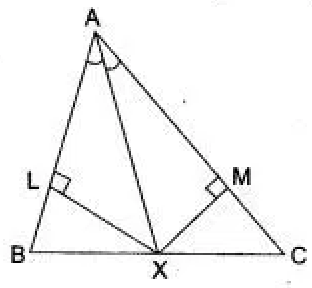
∴ C lies on the right bisector of AB.

Similarly, D lies on the right bisector of AB.

Therefore, CD is the right bisector of AB.

Hence proved.

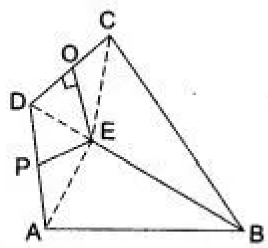
Question 3. In ΔABC, the bisector AX of $\angle A$ intersects BC at X. $XL \perp AB$ and $XM \perp AC$ are drawn (Fig.) is XL = XM ? Why or why not ?



Solution : Since, every point on the bisectors of the angles between two intersecting lines i s equidistant from the lines. Here, X lies on the bisector of $\angle BAC$. Therefore, X is equidistant from AB and AC. It is given that $XL \perp AB$ and $XM \perp AC$. Therefore, distance of X from AB and AC are XL and XM respectively.

Hence, XL = XM. Ans.

Question 4. In Fig. ABCD is a quadrilateral in which AB = BC. E is the point of intersection of the right bisectors of AD and CD. Prove that BE bisects ∠ABC.



Solution : Given : A quadrilateral ABCD in which AB = BC. PE and QE are right bisectors or AD and CD respectively such that they meet at E.

To prove : BE bisects $\angle ABC$.

Construction : Join AE, DE and CE.

Proof : Since, PE is the right bisector of AD and E lies on it.

∴ AE = ED ...(i)

[∵ Points on the right bisector of a line segment are equidistant from the ends of the segment]

Also, QE is the right bisector of CD and E lies on it.

$\therefore$ ED = EC ...(ii)

From (i) and (ii), we get

AE = EC ...(iii)

Now, in Δs ABE and CBE, we have

AB = BC [Given]

BE = BE [Common]

and AE = EC [From (iii)]

So, by SSS criterion of congruence

$\Delta ABE = \Delta ACE$

$\Rightarrow \angle ABE = \angle CBE$

$\Rightarrow$ BE bisects $\angle ABC$.

Hence, BE is the bisector of $\angle ABC$.

Hence proved.

Figure Based Questions

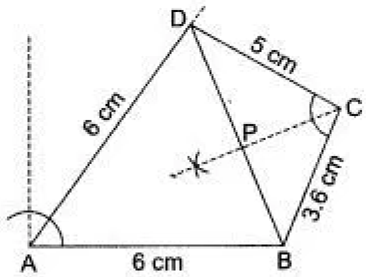
Question 1. Without using set squares and the protractor construct the quadrilateral ABCD in which $\angle BAD = 45°$, AD = AB = 6 cm, BC = 3·6 cm, CD = 5 cm.

(i) Measure $\angle BCD$

(ii) Locate point P on BD which is equidistant from BC and CD.

Solution : (i) $\angle BCD = 62°$. Ans.

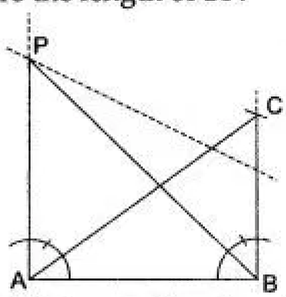
(ii) Draw angle bisector of $\angle BCD$. Join BD. The point on intersection of the bisector and BD is P. P is equidistant from BC and CD.



Question 2. Without using set squares or protractor, construct a triangle ABC in which AB = 4 cm, BC = 5 cm and $\angle ABC = 120°$.

(i) Locate the point P such that $\angle BAP = 90°$ and BP = CP.

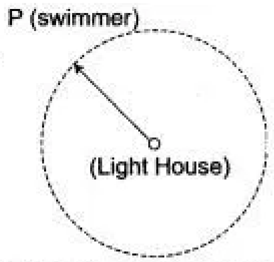
(ii) Measure the length of BP.



Solution : (i) Draw $\perp$ bisector of BC. Draw AP at A such that $\angle PAB = 90°$. The point of intersection P of bisector and AP is the required point. Ans.

(ii) BP = 6·5 cm. Ans.

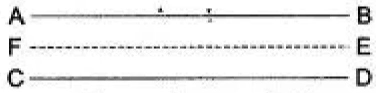
Question 3. State and draw the locus of a swimmer maintaining the same distance from a lighthouse.



Proof : The locus of the swimmer will be a circle with light house as the centre and the same distane between the light house and the swimmer as radius. Ans.

Question 4. State and draw the locus of a point equidistant from two given parallel lines.

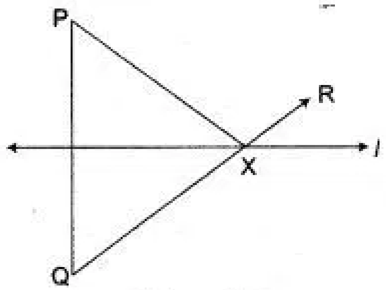
Solution :



The locus of a point equidistant from two given parallel lines AB and CD is the line EF parallel to AB or CD exactly mid-way between AB and CD. Ans.

Question 5. l is the perpendicular bisector of line segment PQ and R is a point on the same side of l as P. The segment QR intersects l at X. Prove that PX + XR = QR.

Solution : Since, line l is the perpendicular bisector of PQ and X lies on l. Therefore, X is equidistant from P and Q.



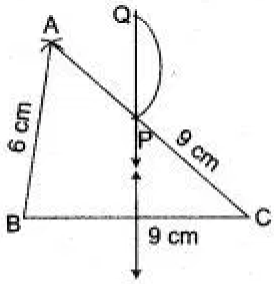
i.e., $PX = QX$

$\Rightarrow PX + XR = QX + XR$

$\Rightarrow PX + XR = QR.$ Hence proved.

Question 6. Construct a Δ ABC, with AB = 6 cm, AC = BC = 9 cm; find a point 4 cm from A and equidistant from B and C.

Solution : Construct the Δ ABC with given measurements. Draw perpendicular bisector of BC.



With A as centre and 4 cm as radius, draw an arc to intersect perpendicular bisector at P and Q.

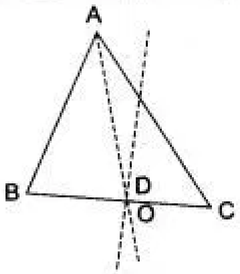
Then the points P and Q are the requisite points. Ans.

Question 7. Given a Δ ABC with unequal sides. Find a point which is equidistant from B and C as well as from AB and AC.

Solution : Draw the angular bisector of ∠ A and perpendicular bisector of side BC of Δ ABC. Let these two bisectors meet at point O. Hence 'O' is our required point.

Proof : Since, O lies on the right bisector of BC.

∴ O is equidistant from B and C.



Again, since O lies on the bisector of ∠ A, formed by AB and AC.

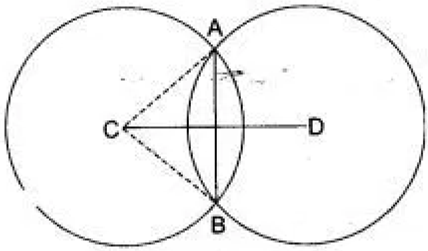
So O is equidistant from AB and AC.

Question 8. Prove that the common chord of two intersecting circles is bisected at right angles by the line of centres.

Solution : Given : Two intersecting circles with centres C & D.

AB is their common chord.

To prove : AB bisected by CD at right angles.



Proof : $CA = CB$ (radii)

$\therefore$ C lies on the right bisector of AB.

Similarly, D lies on the right bisector of AB.

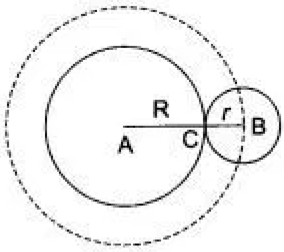
Therefore, CD is the right bisector of AB.

Hence proved.

Question 9. Find the locus of the centre of a circle of radius r touching externally a circle of radius R.

Solution : Let a circle of radius r (with centre B) touch a circle of radius R at C. Then ACB is a straight line and

$$AB = AC + CB = R + r$$



Thus, B moves such that its distance from fixed point A remains constant and is equal to R + r.

Hence, the locus of B is a circle whose centre is A and radius equal to R + r. Ans.

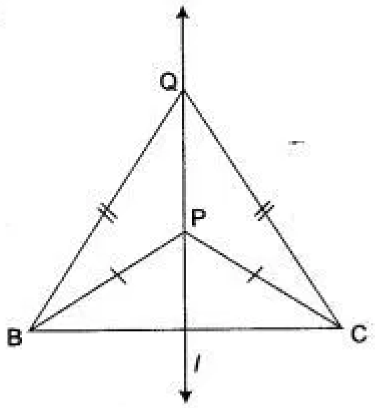
Question 10. Δ PBC and Δ QBC are two isosceles triangles on the same base. Show that the line PQ is bisector of BC and is perpendicular to BC.

Given : Δ PBC and Δ QBC are two isosceles triangles on the same base BC.

To prove : Line PQ is the perpendicular bisector of BC.

Proof : In Δ PBC, $PB = PC$

Since, the locus of a point equidistant from B and C is the perpendicular bisector of l of the line segment BC



$\therefore$ P lies on l

Similarly Q lies on l

Therefore, PQ is the perpendicular bisector of BC.

Hence proved.

Question 11. Using a ruler and compass only :

(i) Construct a triangle ABC with

BC = 6 cm, $\angle ABC = 120°$ and AB = 3.5 cm.

(ii) In the above figure, draw a circle with BC as diameter. Find a point 'P' on the circurference of the circle which is equidistant from AB and BC. Measure $\angle BCP$.

Solution : (i) Steps of construction :

(1) Draw BC = 6 cm.

(2) Draw $\angle$ ABC = 120°.

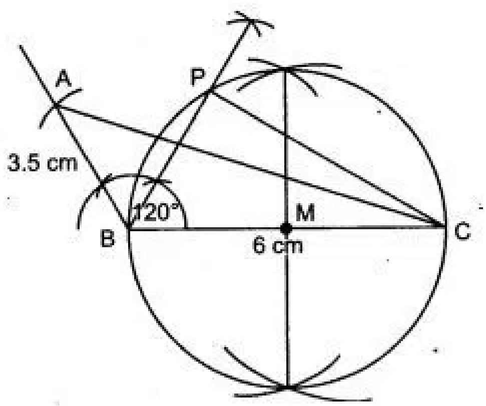
(3) Cut BA = 3·5 cm.

(4) Join A to C.

(5) Draw $\perp$ bisector MN of BC.

(6) Draw a circle O as centre and OC, OB radius.

(7) Draw angle bisector of $\angle$ ABC which intersect circle at P.



(ii) $\angle BCP = 30°$

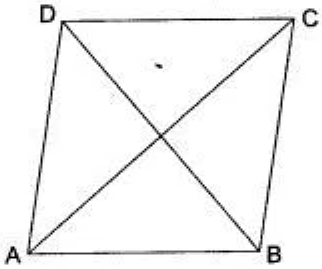
Question 12. The diagonals of a quadrilateral bisect each other at right angles. Show that the quadrilateral is a rhombus.

Solution : Since, the diagonals AC and BD of quadrilateral ABCD bisect each other at right angles.

$\therefore$ AC is the $\perp$ bisector of line segment BD

$\Rightarrow$ A and C both are equidistant from B and D

$\Rightarrow \quad AB = AD$ and $CB = CD \quad \ldots$(i)



Also, BD is the $\perp$ bisector of line segment AC

$\Rightarrow$ B and D both are equidistant from A and C

$\Rightarrow \quad AB = BC$ and $AD = DC \quad \ldots$(ii)

From (i) and (ii), we get

$$AB = BC = CD = AD$$

Thus, ABCD is a quadrilateral whose diagonals bisect each other at right angles and all four sides are equal.

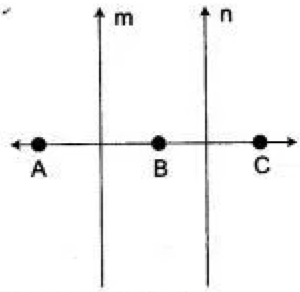
Hence, ABCD is a rhombus. Hence proved.

Question 13. What is the locus of points which are equidistant from the given non-collinear point A, B and C? Justify your answer.

Solution : Let A, B, C be three distinct points on a line l. Any point equidistant from A and B lies on the perpendicular bisector of AB. So, points equidistant from A and B lies on line m.

Similarly, points equidistant from B and C lies on line n which is the perpendicular bisector of BC.

Thus, any point equidistant from A, B and C must be common to both the lines m and n.



But $m \perp AB$ and $n \perp BC$.

$\therefore m \perp AC$ and $n \perp AC$

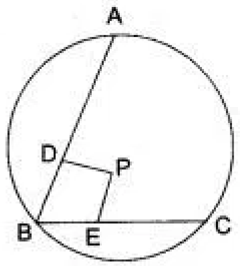
$\Rightarrow \quad m \parallel n$

So, no points is common to both m and n.

Hence, the required locus is the null set ϕ.

Question 14. Find the locus of points which are equidistant from three non-collinear points.

Solution : Let A, B and C be three non-collinear points. Join AB and BC. Let P be a moving point. Since, P is equidistant from A and B, it follows that P lies on the perpendicular bisector of AB.



Again P is equidistant from B and C. So, P lies on the perpendicular bisector of BC.

Thus, P is the point of intersection of the perpendicular bisector of AB and BC. So, P coincides with the centre of the circle passing through three given non-collinear points. Hence, the required locus is the centre of the circle passing through three given non-collinear points.

Question 15. Show that the locus of the centres of all circles passing through two given points A and B, is the perpendicular bisector of the line segment AB.

Solution : Let P and Q be the centres of two circles S and S′, each passing through two given points A and B. Then,

$$PA = PB$$

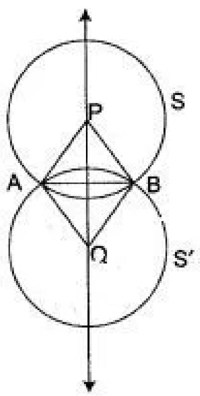
[Radii of the same circle]

$\Rightarrow$ P lies on the perpendicular bisector of AB ...(i)

Again, $QA = QB$

[Radii of the same circle]

$\Rightarrow$ Q lies on the perpendicular bisector of AB ...(ii)



From (i) and (ii), it follows that P and Q both lies on the perpendicular bisector of AB.

Hence, the locus of the centres of all the circles passing through A and B is the perpendicular bisector of AB.

Question 16. Using ruler and compasses construct:
(i) a triangle ABC in which AB = 5.5 cm, BC = 3.4 cm and CA = 4.9 cm.
(ii) the locus of point equidistant from A and C.
(iii) a circle touching AB at A and passing through C.

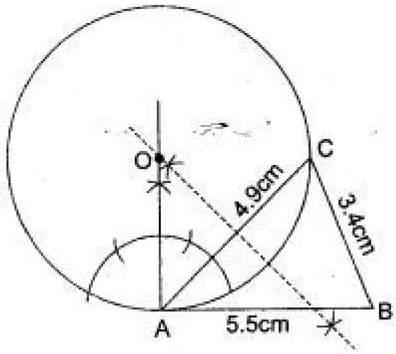
Solution. Steps of construction :

(i) Draw AC = 4·9 cm, draw AB = 5·5 cm and AC = 4·9 cm.

(ii) Draw bisector $l \perp AC$.

(iii) Draw $AO \perp AB$.

(iv) Intersection of AO and L is centre of circle.



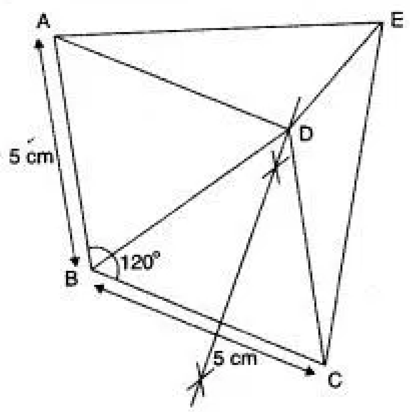
Question 17. Using only a ruler and compass construct $\angle ABC = 120°$, where AB = BC = 5 cm.

(i) Mark two points D and E which satisfy the condition that they are equidistant from both BA and BC.

(ii) In the above figure, join AD, DC, AE and EC. Describe the figures :

(a) AECB, (b) ABD, (c) ABE.

Solution : (i) and (ii)



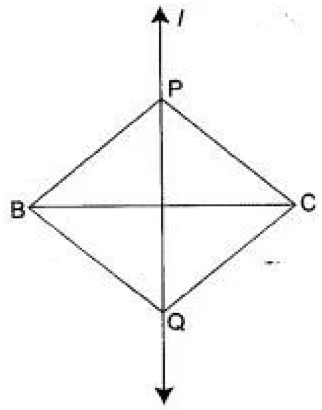
(a) A quadrilateral

(b) A triangle

(c) A triangle. Ans.

Question 18. ΔPBC and ΔQBC are two isosceles triangles on the same base BC but on the opposite sides of line BC. Show that PQ bisects BC at right angles.

Solution : Given : Two Δ^S PBC and QBC on the same base BC but on the opposite sides of BC such that PB = PC and QB = QC.



To prove : PQ bisects BC and is $\perp$ to BC.

Proof : Since, the locus of points equidistant from two given points is the perpendicular bisector of the segment joining them. Therefore, ΔPBC is isosceles

$\Rightarrow$ PB = PC

$\Rightarrow$ P lies on the perpendicular bisector of BC

Δ QBC is isosceles $\Rightarrow$ QB = QC

$\Rightarrow$ Q lies on the perpendicular bisectors of BC

$\therefore$ PQ is the perpendicular bisector of BC

Hence, PQ bisects BC at right angles.

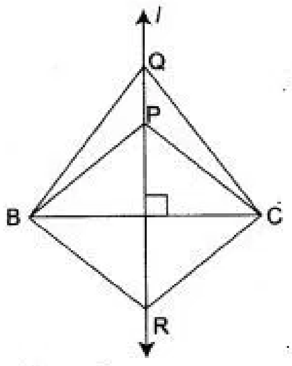
Hence proved.

Question 19. Δ PBC, Δ QBC and Δ RBC are three isosceles triangles on the same base BC. Show that P, Q and R are collinear.

Solution : Given : Three isosceles triangles PBC, QBC and RBC on the same base BC such that PB = PC, QB = QC and RB = RC.

To prove : P, Q, R are collinear.

Proof : Let l be the perpendicular bisector of BC. Since, the locus of points equidistant from B and C is the perpendicular of the segment joining them. Therefore,



Δ PBC is an isosceles

$\Rightarrow$ PB = PC

$\Rightarrow$ P lies on l ...(i)

Δ QBC is isosceles

$\Rightarrow$ QB = QC

$\Rightarrow$ Q lies on l ...(ii)

ΔRBC is an isoscles

⇒ RB = RC

⇒ R lies on l ...(iii)

From (i), (ii) and (iii), it follows that P, Q and R lie on L.

Hence, P, Q and R are collinear.

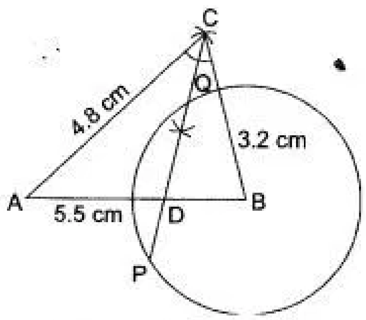
Hence proved.

Question 20. Without using set squares or protractor construct:
(i) Triangle ABC, in which AB = 5.5 cm, BC = 3.2 cm and CA = 4.8 cm.
(ii) Draw the locus of a point which moves so that it is always 2.5 cm from B.
(iii) Draw the locus of a point which moves so that it is equidistant from the sides BC and CA.
(iv) Mark the point of intersection of the loci with the letter P and measure PC.

Solution : (i) Draw a triangle by given measurements.

(ii) The locus of a point which moves so that it is always 2·5 cm from B is a circle as shown in the figure. Ans.

(iii) The locus of a point is bisector of ∠ ACB. Ans.

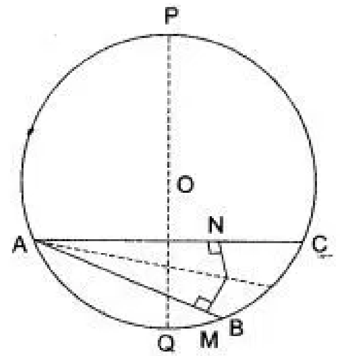


(iv) The circle and bisector intersect in two points PD = 0·9 cm and PC = 3·4 cm. Ans.

Question 21. Use ruler and compasses only for this question. Draw a circle of radius 4 cm and mark two chords AB and AC of the circle of length f 6 cm and 5 cm respectively.
(i) Construct the locus of points, inside the circle, that are equidistant from A and C. Prove your construction.
(ii) Construct the locus of points, inside the circle, that are equidistant from AB and AC.

Solution : (i) Draw PQ, the perpendicular bisector of chord AC. PQ is the required locus, which is the diameter of the circle.

Reason : We know each point on the perpendicular bisector of AB is equidistant from A and B. Also the perpendicular bisector of a chord passes through the centre of the circle and any chord passing through the centre of the circle is its diameter.



$\therefore$ PQ is the diameter of the circle.

(ii) Chords AB an AC intersects at M and N is a moving point such that LM = LN, where LM $\perp$ AB and

$$LN \perp AC$$

In right Δ ALN and Δ ALB

	$\angle ANL = \angle ABL$	(90° each)
	$AL = AL$	(Common)
	$NL = BL$	[Given]
$\therefore$	$\Delta ALN = \Delta ALB$	[R.H.S.]
Hence	$\angle MAL = \angle BAL$	c.p.c.t.

Thus, L lies on the bisector of $\angle BAC$.

Hence proved.

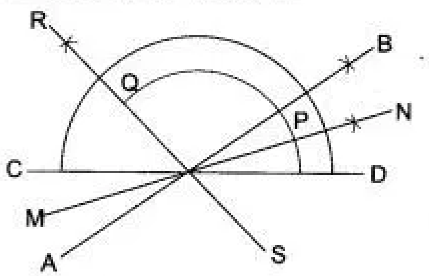
Question 22. Draw two intersecting lines to include an angle of 30°. Use ruler and compass to locate points, which are equidistant from these lines and also 2 cm away from these points of intersection. How many such points exist ?

Solution : AB and CD are two intersecting lines at an angle of 30°. Their point of intersection is O.

Draw MON and ROS, the bisector of angles between AB and CD. On ON, locate a point P such that OP = 2 cm.

On OR locate a point Q such that OQ = 2 cm.

Since, P and Q are on the angle bisectors of angles between AB and CD, hence each of P and Q is equidistant from AB and CD.

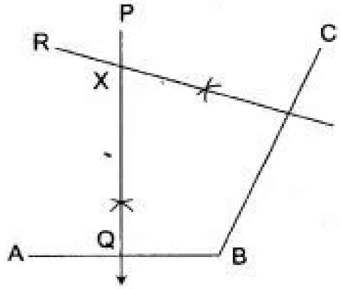


Also, OP = OQ = 2 cm

Hence, P and Q are the required points. Ans.

Question 23. How will you find a point equidistant from three given points A, B, C which are not in the same straight line?

Solution : (i) The locus of points equidistant from three given points A, B & C is the straight line PQ, which bisects AB at right angles.



(ii) Similarly, the locus of points equidistant from B and C is the straight line RS which bisects BC at right angles.

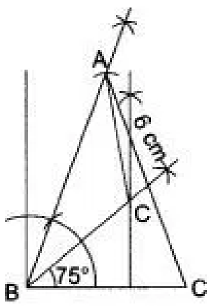
Hence, the point common to PQ and RS must satisfy both conditions; that is to say, X, the point of intersection of PQ and RS will be equidistant from A, B and C. Ans.

Question 24. Without using set squares or protractor.

(i) Construct a Δ ABC, given BC = 4 cm, angle B = 75° and CA = 6 cm.

(ii) Find the point P such that PB = PC and P is equidistant from the side BC and BA. Measure AP.

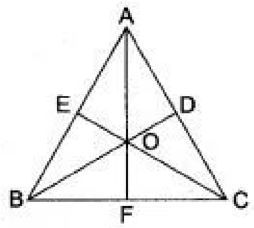
Solution : (i) Draw BC = 4 cm. Draw BA at B such that $\angle$ ABC = 75°. Cut CA = 6 cm. Then Δ ABC is the required Δ.



(ii) Draw single bisector of $\angle$ B. Draw $\perp$ bisector of BC. Their point of intersection (P) is the requisite point.

AP $\doteq$ 3·9 cm. Ans.

Question 25. In Fig. AB = AC. BD and CE are the bisectors of $\angle ABC$ and $\angle ACB$ respectively such that BD and CE intersect each other at O. AO produced meets BC at F. Prove that AF is the right bisector of BC.



Solution : Given : A ΔABC in which AB = AC. BD, the bisector of $\angle ABC$ meets CE, the bisector of $\angle ACB$ at O. AO produced meets BC at F.

To prove : AF is the right bisector of BC.

Proof : We have, AB = AC

$\Rightarrow$ A lies on the right bisector of BC ...(i)

and $\angle ABC = \angle ACB$

Now, $\angle ABC = \angle ACB$

$$\Rightarrow \frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$$

$$\Rightarrow \angle OBC = \angle OCB$$

[$\because$ BD and CE are bisector of $\angle B$ and $\angle C$ respectively]

$$\Rightarrow OB = OC$$

[$\because$ Sides opposite to equal angles are equal]

$\Rightarrow$ O lies on the right bisector of BC ...(ii)

From (i) and (ii), we obtain

$\Rightarrow$ A and O both lie on the right bisector of BC.

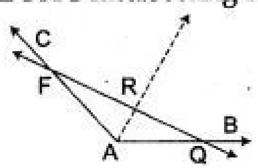
$\Rightarrow$ AO is the right bisector of BC

Hence, AF is the right bisector of BC.

Hence proved.

Question 26. Given : $\angle BAC$, a line intersects the arms of $\angle BAC$ in P and Q. How will you locate a point on line segment PQ, which is equidistant from AB and AC ? Does such a point always exist ?

Solution : Since, locus of points equidistant from AB and AC is the bisector of $\angle BAC$. Draw the bisector of $\angle BAC$ intersecting PQ at R.



Since, R is on the bisector, so it is equidistant from AB and AC.

Yes, such a point always exists as there will be definitely a point where angular bisector and line will intersect.

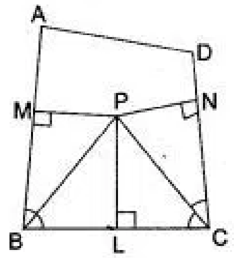
Hence, R is the required point. Ans.

Question 27. The bisectors of $\angle B$ and $\angle C$ of a quadrilateral ABCD intersect in P. Show that P is equidistant from the opposite sides AB and CD.

Solution : Given : A quadrilateral ABCD in which bisectors of $\angle B$ and $\angle C$ meet in P. $PM \perp AB$ and $PN \perp CD$.

To prove : $PM = PN$

Construction : Draw $PL \perp BC$



Proof : Since, P lies on the bisector of $\angle B$

$\therefore$ P is equidistant from BC and BA

$\Rightarrow \quad PL = PM \quad$ (i)

Also, P lies on the bisector of $\angle C$ [Given]

$\therefore$ P is equidistant from CB and CD

$\Rightarrow \quad PL = PN \quad$ ...(i)

From (i) and (ii), we have

$PL = PM$

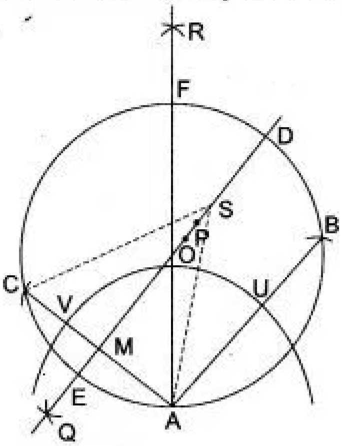
and $PL = PN$

$\Rightarrow \quad PM = PN.$ Hence proved.

Question 28. Use ruler and compasses only for this question. Draw a circle of radius 4 cm and mark two chords AB and AC of the circle of length 6 cm and 5 cm respectively.
(i) Construct the locus of points, inside the circle, that are equidistant from A and C. Prove your construction.
(ii) Construct the locus of points, inside the circle, that are equidistant from AB and AC.

Solution : Draw a circle of radius 4 cm whose centre is O. Take a point A on the circumference of this circle.

With A as centre and radius 6 cm draw an arc to cut the circumference at B. Join AB.



Then AB is the chord of the circle of length 6 cm.

With A as centre and radius 5 cm draw another arc to cut the circumference at C. Join AC then AC is the chord of the circle of length 5 cm.

With A as centre and a suitable radius, draw two arcs on opposite sides of AC.

With C as centre and the same radius, draw two arcs on opposite sides of AC to intersect the former arcs at P and Q.
Join PQ and produce to cut the circle at D and E.
Join DE. Then chord DE is the locus of points inside the circle that Ls equidistant from A and C.
As chord DE passes through (he centre O of the circle, it is a diameter. To prove the construction take any point S inside the circle on DE.

Question 29. Use ruler and compasses only for the following questions:

Construct triangle BCP, when CB = 5 cm, BP = 4 cm, $\angle PBC = 45°$.

Complete the rectangle ABCD such that :

(i) P is equidistant from AB and BC and

(ii) P is equidistant from C and D. Measure and write down the length of AB.

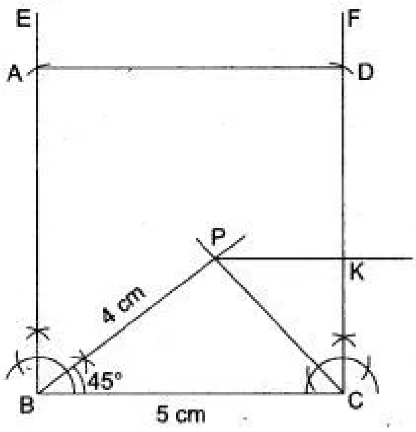
Solution :

Given : BC = 5 cm, BP = 4 cm and $\angle$ PBC = 45°

Steps of construction :

1. Construct ΔBCP with BC = 5 cm, BP = 4 cm and $\angle$ PBC = 45°.

2. Draw perpendiculars BE and CF and B and C respectively.



3. Draw perpendicular from on CF meeting CF in K.

4. Cut CD from CF, such that CK = KD.

5. Cut BA from BE, such that BA = CD.

6. Join AD.

Hence, ABCD is the required rectangle and AB = 5·7 cm. Ans.

Question 30. Ruler and compass only may be used in this question. All construction lines and arcs must be clearly shown, and be of sufficient length and clarity to permit assessment.
(i) Construct Δ ABC, in which BC = 8 cm, AB = 5 cm, $\angle$ ABC = 60°.

(ii) Construct the locus of points inside the triangle which are equidistant from BA and BC.

(iii) Construct the locus of points inside the triangle which are equidistant from B and C.

(iv) Mark as P, the point which is equidistant from AB, BC and also equidistant from B and C.

(v) Measure and record the length of PB.

Solution : (i) Stepts of Construction :

1. Draw a line segment BC = 8 cm.

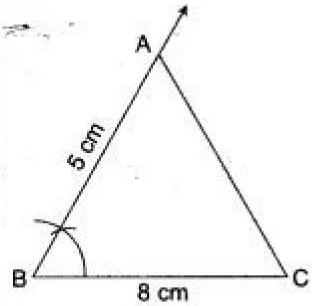
2. Make $\angle$ CBX = 60°.

3. Set off BA = 5 cm, along BX.

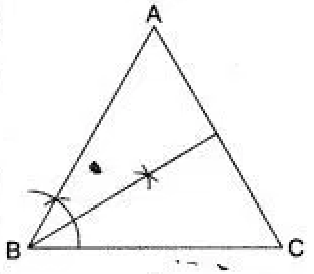
4. Join CA.

Then, Δ ABC is the required triangle.

(ii) We know that the locus of point equidistant from two inter-secting straight lines consist of a pair of straight lines that bisect the angles between the given straight lines.

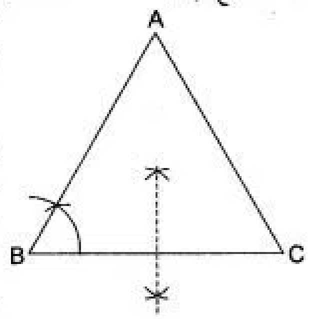


Therefore, in this case is the angle bisector of angle B. It is shown in the adjoining figure.



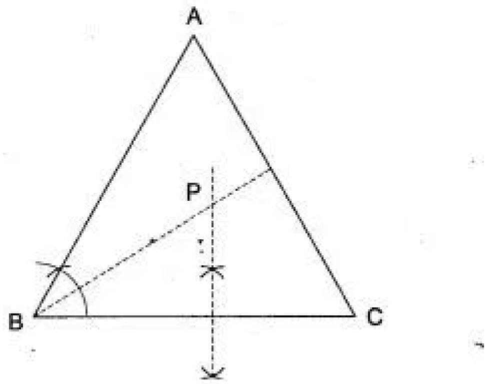
(iii) We know that the locus of a point equidistant from two fixed points is the right bisector of the straight line joining the two fixed points.

Therefore, in this case the right bisector of side BC of Δ ABC. It is shown in the given figure.



(iv) The point P, is the point in intersecting of angle bisector of $\angle$ ABC and the right bisector of BC.

It is shown in the following figure.



(v) On measuring, we find the length of PB $= 3$ cm. Ans.

Question 31. Ruler and compasses only may be used in this question. All construction lines and arcs must be clearly shown, and be of sufficient length and clarity to permit assessment.

(i) Construct a Δ ABC, in which BC $= 6$ cm, AB $= 9$ cm and $\angle$ ABC $= 60°$.

(ii) Construct the locus of the vertices of the triangles with BC as base, which are equal in area to Δ ABC.

(iii) Mark the point Q, in your construction, which would make Δ QBC equal in area to ΔABC, and isosceles.

(iv) Measure and record the length of CQ.

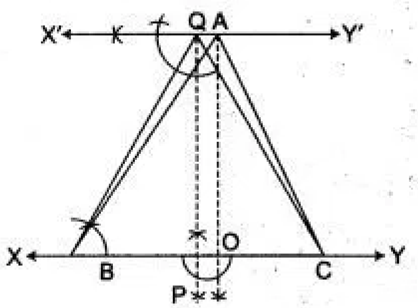
Solution : Steps of Construction :

(i) (1) Mark a horizontal line XY on your paper and take BC $= 6$ cm on it.

(2) Construct $\angle$ ABC $= 60°$ with arm AB $= 9$ cm.

(3) Join A and C to get the required ΔABC.

(ii) (1) Draw AD $\perp$ BC.



(2) Construct a line X'Y', perpendicular to AD, parallel to XY and passing through A.

(3) X'Y' is the required locus of the vertices of Δ^S with base BC and area to Δ ABC.

[$\because$ Δ^S having same base and height an equal in area]

(iii) (1) Draw right bisector PQ of BC, meeeting X'Y' in Q.

(2) Then Q is the point such that Δ QBC is an isosceles triangle and area (Δ QBC) = area (Δ ABC).

(iv) On measuring, we find CQ = 8·4 cm. Ans.

Question 32. Given $\angle$ BAC (Fig.), determine the locus of a point which lies in the interior of $\angle$ BAC and equidistant from two lines AB and AC.

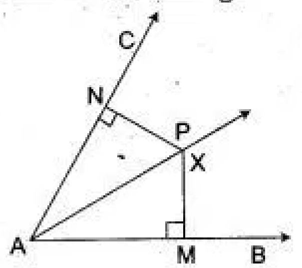
Solution : Given : $\angle$BAC and an interior point P lying in the interior of $\angle$BAC such that PM = PN.

Construction : Join AP and produced it to X.

Proof : In right Δs APM and APN we have

$$PM = PN \quad \text{[Given]}$$
$$AP = AP \quad \text{[Common]}$$

So, by RHS criterion of congruence



$$\Delta\, APM = \Delta\, APN$$
$$\Rightarrow \quad \angle PAM = \angle PAN$$

[$\because$ Corresponding parts of congruent triangle are equal]

$\Rightarrow$ AP is the bisector of $\angle$ BAC

$\Rightarrow$ P lies on the bisector of $\angle$ BAC

Hence, the locus of P is the bisector of $\angle$ BAC.

Now, we shall show that every point on the bisector of $\angle$ BAC is equidistant from AB and AC.

So, let P be a point on the bisector AX of $\angle$ BAC and PM $\perp$ AB and PN $\perp$ AC. Then, we have to prove that PM = PN.

In Δs PAM and PAN, we have

$$\angle PAM = \angle PAN$$

[$\because$ AX is the bisector of $\angle$ A]

$\angle PMA = \angle PNA$

[Each equal to 90°]

and $AP = AP$ [Common]

So, by AAS criterion of congruence

$\Delta PAM \cong \Delta PAN$

$\Rightarrow PM = PN$

[$\because$ Corresponding parts of congruent triangles are equal]

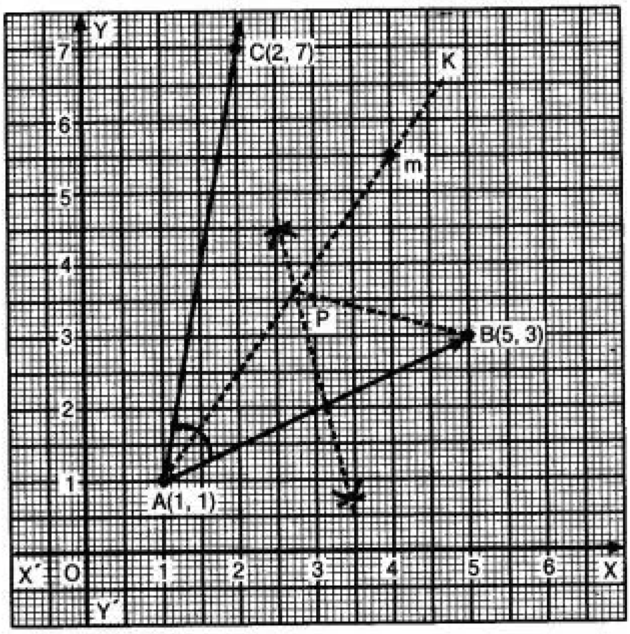
Hence, the locus of point P is the ray AX which is the bisector of $\angle BAC$. Ans.

Graphical Depiction

Question 1. Use graph paper for this question. Take 2 cm = 1 unit on both axis.
(i) Plot the points A (1, 1), B (5, 3) and C (2, 7);
(ii) Construct the locus of points equidistant from A and B;
(iii) Construct the locus of points equidistant from AB and AC;
(iv) Locate the point P such that PA = PB and P is equidistant from AB and AC;
(v) Measure and record the length PA in cm.

Solution :

(i) Plot the points A (1, 1), B (5, 3) and C (2, 7) as shown.



(ii) Join AB. Draw right bisector l of AB. Then, l is the locus of points equidistant from A and B.

(iii) Join AC. Draw bisector m of $\angle CAB$. Then, m is the locus of the points equidistant from AB and AC.

(iv) The point of intersection P of right bisector of AB and angle bisector of $\angle CAB$ is the point such that PA = PB and P is equidistant from AB and AC.

(v) On measuring PA = 2·5 cm. Ans.