

CBSE Test Paper 03

CH-2 Polynomials

1. The degree of the polynomial $(x^3 - 2)(x^2 - 11)$ is
 - a. 0
 - b. 5
 - c. 3
 - d. 2
2. A polynomial of degree 3 in x has at most
 - a. 3 terms
 - b. 1 term
 - c. 5 terms
 - d. 4 terms
3. If $(x+1)$ and $(x-1)$ are factors of $px^3 + x^2 - 2x + q$ then value of p and q are
 - a. $p = -1, q = 2$
 - b. $p = 2, q = -1$
 - c. $p = 2, q = 1$
 - d. $p = -2, q = -2$
4. If $P(x) = x^3 - 1$, then the value of $P(1) + P(-1)$ is
 - a. 2
 - b. 1
 - c. -2

d. 0

5. One of the zeroes of the polynomial $2x^2 + 7x - 4$ is

a. -2

b. $\frac{1}{2}$

c. $-\frac{1}{2}$

d. 2

6. Fill in the blanks:

Degree of the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is _____ .

7. Fill in the blanks:

If $p(x) = x^2 - 2\sqrt{2}x + 1$, then $p(2\sqrt{2})$ is equal to _____.

8. Is $\sqrt{3}x^2 - 2x$ a polynomial? Justify your answer.

9. Evaluate the following: $104^3 + 96^3$

10. Simplify the following products: $(\frac{1}{2}a - 3b)(3b + \frac{1}{2}a)(\frac{1}{4}a^2 + 9b^2)$

11. Factorise $25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30zx$ by using suitable identity.

12. If $x = \frac{3}{2}$ is a zero of the polynomial $2x^2 + kx - 12$, find the value of k.

13. If $a + b + c = 9$ and $a^2 + b^2 + c^2 = 35$, find the value of $a^3 + b^3 + c^3 - 3abc$.

14. Find the remainder when the polynomial $f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$ is divided by $x + 2$

15. What must be subtracted from $4x^4 - 2x^3 - 6x^2 + x - 5$ so that the result is exactly divisible by $2x^2 + x - 1$?

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Solution

1. (b) 5

Explanation:

$$\begin{aligned} & (x^3 - 2)(x^2 - 11) \\ &= x^3(x^2 - 11) - 2(x^2 - 11) \\ &= x^5 - 11x^3 - 2x^2 + 22 \end{aligned}$$

Here the highest power is 5.

Therefore, the degree is 5.

2. (a) 3 terms

Explanation: 3 terms of not more than the power of 3

3. (b) $p = 2$, $q = -1$

Explanation:

$$\text{Given : } f(x) = px^3 + x^2 - 2x + q$$

If $x + 1$ is a factor of $f(x)$. Then

$$f(-1) = 0$$

$$p(-1)^3 + (-1)^2 - 2(-1) + q = 0$$

$$-p + 1 + 2 + q = 0$$

$$-p + q = -3$$

$$p - q = 3. \quad \dots\dots(i)$$

Also, if $x - 1$ is a factor of $f(x)$, then

$$p(1)^3 + (1)^2 - 2(1) + q = 0$$

$$p + 1 - 2 + q = 0$$

$$p + q = 1. \quad \dots\dots(ii)$$

Adding eq.(i) and (ii), we get

$$2p = 4$$

$$p = 2$$

Subtracting eq.(ii) from eq.(i), we get

$$-2q = 2$$

$$q = -1$$

Therefore, $p = 2$, $q = -1$

4. (c) -2

Explanation:

$$P(x) = x^3 - 1,$$

then the value of $P(1) + P(-1)$

$$= (1)^3 - 1 + (-1)^3 - 1$$

$$= 1 - 1 - 1 - 1 = 1 - 3 = -2$$

5. (b) $\frac{1}{2}$

Explanation:

$$2x^2 + 7x - 4$$

$$= 2x^2 + 8x - x - 4$$

$$= 2x(x + 4) - 1(x + 4)$$

$$= (2x - 1)(x + 4)$$

$$2x - 1 = 0 \text{ and } x + 4 = 0$$

$$x = \frac{1}{2} \text{ and } x = -4$$

Therefore, one zero of the given polynomial is $\frac{1}{2}$

6. 4

7. 1

8. $\sqrt{3}x^2 - 2x$

In each term of this expression, the exponent of the variable x is a whole number.
Hence, it is a polynomial.

9. We have,

$$\begin{aligned} 104^3 + 96^3 &= (100 + 4)^3 + (100 - 4)^3 \\ &= 2(100^3 + 3 \times 100 \times 4^2) [\because (a + b)^3 + (a - b)^3 = 2(a^3 + 3ab^2)] \\ &= 2(1000000 + 300 \times 16) \\ &= 2(1000000 + 4800) \\ &= 2(1004800) \\ &= 2009600 \end{aligned}$$

10. We have,

$$\begin{aligned} &(\frac{1}{2}a - 3b)(3b + \frac{1}{2}a)(\frac{1}{4}a^2 + 9b^2) \\ &= (\frac{1}{2}a - 3b)(\frac{1}{2}a + 3b)(\frac{1}{4}a^2 + 9b^2) \\ &= [(\frac{1}{2}a)^2 - (3b)^2][\frac{1}{4}a^2 + 9b^2] [\because (a - b)(a + b) = a^2 - b^2] \\ &= [\frac{1}{4}a^2 - 9b^2][\frac{1}{4}a^2 + 9b^2] \\ &= (\frac{1}{4}a^2)^2 - (9b^2)^2 [(a - b)(a + b) = a^2 - b^2] \\ &= \frac{1}{16}a^4 - 81b^4 \end{aligned}$$

11. $25x^2 + 4y^2 + 9z^2 - 20xy - 12yz + 30zx$

$$\begin{aligned} &\Rightarrow (5x)^2 + (2y)^2 + (3z)^2 - 2 \times 5x \times 2y - 2 \times 2y \times 3z + 2 \times 3z \times 5x \\ &= (5x)^2 + (-2y)^2 + (3z)^2 + 2(5x)(-2y) + 2(-2y)(3z) + 2(3x)(5x) \\ &= (5x - 2y + 3z)^2 [\because a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a + b + c)^2] \end{aligned}$$

12. Let $p(x) = 2x^2 + kx - 12$

Since, $x = \frac{3}{2}$ is a zero of the polynomial.

$$\therefore P\left(\frac{3}{2}\right) = 0 \Rightarrow 2\left(\frac{3}{2}\right)^2 + k\left(\frac{3}{2}\right) - 12 = 0$$

$$\Rightarrow \frac{9}{2} + \frac{3k}{2} - 12 = 0 \Rightarrow \frac{3k}{2} = \frac{24-9}{2}$$

$$\Rightarrow 3k = 15 \Rightarrow k = 5$$

Hence, the value of k is 5.

13. Given: $a + b + c = 9$ and $a^2 + b^2 + c^2 = 35$

We know that,

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$\Rightarrow a^3 + b^3 + c^3 - 3abc = [a + b + c][(a^2 + b^2 + c^2) - (ab + bc + ca)] \dots\dots(i)$$

Now,

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$\Rightarrow (9)^2 = 35 + 2(ab + bc + ca)$$

$$\Rightarrow 81 = 35 + 2(ab + bc + ca)$$

$$\Rightarrow 2(ab + bc + ca) = 81 - 35 = 46$$

$$\Rightarrow ab + bc + ca = \frac{46}{2} = 23$$

Substituting the values in (i), we get,

$$a^3 + b^3 + c^3 - 3abc = 9(35 - 23)$$

$$= 9 \times 12$$

$$= 108$$

14. By remainder theorem, when $f(x)$ is divided by $(x + 2)$, the remainder is equal to $f(-2)$.

$$\text{Now, } f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$$

$$\Rightarrow f(-2) = 2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2$$

$$\Rightarrow f(-2) = 2 \times 16 - 6 \times (-8) + 2 \times 4 + 2 + 2 = 32 + 48 + 8 + 2 + 2 = 92$$

Hence, required remainder = 92.

15. By division algorithm, when $p(x) = 4x^4 - 2x^3 - 6x^2 + x - 5$ is divided by $q(x) = 2x^2 + x - 1$, the remainder is a linear expression.

So, let $r(x) = ax + b$ be subtracted from $p(x)$ so that the result is divisible by $q(x)$.

$$\text{Let } f(x) = p(x) - r(x) = p(x) - (ax + b)$$

$$\text{or, } f(x) = (4x^4 - 2x^3 - 6x^2 + x - 5) - (ax + b)$$

$$\text{or, } f(x) = 4x^4 - 2x^3 - 6x^2 + x(1 - a) - 5 - b$$

We have,

$$q(x) = 2x^2 + x - 1 = 2x^2 + 2x - x - 1 = 2x(x + 1) - 1(x + 1) = (x + 1)(2x - 1)$$

Clearly, $x + 1$ and $2x - 1$ are factors of $q(x)$. Therefore, $f(x)$ will be divisible by $q(x)$ if $x + 1$ and $2x - 1$ are factors of $f(x)$

$$\text{i.e., } f(-1) = 0 \text{ and } f\left(\frac{1}{2}\right) = 0$$

$$\Rightarrow 4 \times (-1)^4 - 2 \times (-1)^3 - 6 \times (-1)^2 + (-1)(1 - a) - 5 - b = 0$$

$$\text{and, } 4\left(\frac{1}{2}\right)^4 - 2 \times \left(\frac{1}{2}\right)^3 - 6 \times \left(\frac{1}{2}\right)^2 + \frac{1}{2}(1 - a) - 5 - b = 0$$

$$\Rightarrow 4 + 2 - 6 - 1 + a - 5 - b = 0 \text{ and } \frac{1}{4} - \frac{1}{4} - \frac{3}{2} + \frac{1}{2} - \frac{a}{2} - 5 - b = 0$$

$$\Rightarrow a - b - 6 = 0 \text{ and } -\frac{a}{2} - b - 6 = 0$$

$$\Rightarrow a - b = 6 \text{ and } a + 2b = -12$$

$$\Rightarrow (a - b) - (a + 2b) = 6 - (-12) \Rightarrow -3b = 18 \Rightarrow b = -6$$

Putting $b = -6$ in $a - b = 6$, we have

$$a - (-6) = 6 \Rightarrow a + 6 = 6 \Rightarrow a = 0$$

Putting the values of a and b in $r(x) = ax + b$, we get

$$r(x) = 0 \times x - 6 = -6$$

Hence, $p(x)$ is divisible by $q(x)$, if $r(x) = -6$ is subtracted from $p(x)$.