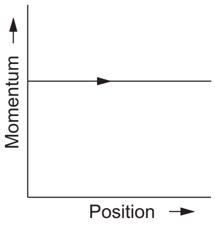
5

Linked-Comprehension-Type Questions

Each passage in this chapter is followed by multiple-choice questions, which have to be answered on the basis of the passage. Each question has four choices (a, b, c and d), out of which ***only one*** *is true.*

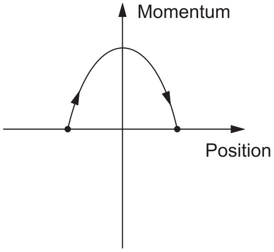
5.1 General Physics



- Phase-space diagrams are useful tools in analyzing all kinds of dynamical problems. They are specially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one dimension. For such systems, phase space is a plane in which position is plotted along the horizontal axis and momentum is plotted along the vertical axis. The phase-space diagram is $x(t)$ vs $p(t)$ curve in this plane. The arrow on the curve indicates the time flow. For example, the phase-space diagram for a particle moving with a constant velocity is a straight line as shown in the figure. We use the sign convention in which the position or the momentum upwards (or to the right) is positive and downward (or to the left) is negative.

1. The phase-space diagram for a ball thrown vertically up from the ground is

(a)



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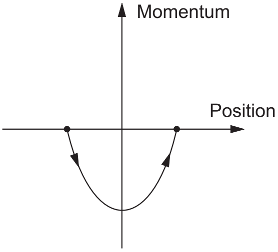

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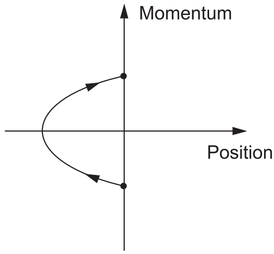

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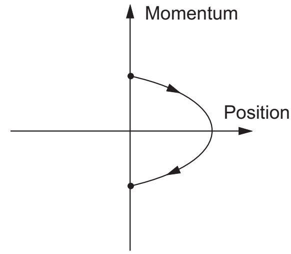
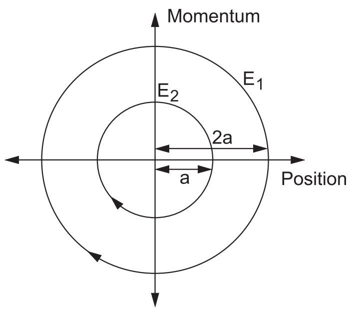
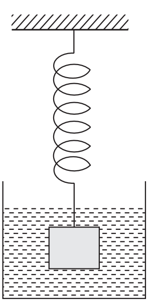


2. The phase-space diagram for simple harmonic motion is a circle centred at the origin. In the figure, the two circles represent the same oscillator but with different initial conditions and E_1 and E_2 are the total mechanical energies respectively. Then

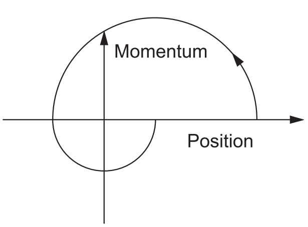


(a) $E_1 = \sqrt{2}E_2$
(b) $E_1 = 2E_2$
(c) $E_1 = 4E_2$
(d) $E_1 = 16E_2$

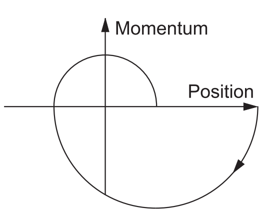
3. Consider the spring–mass system, with the mass submerged in water, as shown in the figure. The phase-space diagram for one cycle of this system is



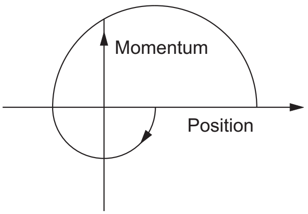
(a)



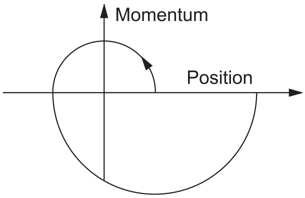
(b)



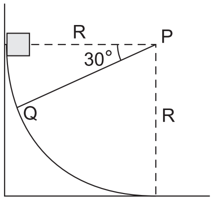
(c)



(d)



- A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q as shown in the figure is 150 J. [Take $g = 10$ m s^{-2}.]



4. The speed of the block when it reaches the point Q is

(a) 5 m s^{-1} (b) 10 m s^{-1}
(c) $10\sqrt{3}$ m s^{-1} (d) 20 m s^{-1}

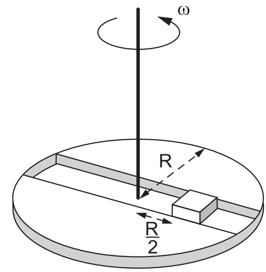
5. The magnitude of normal reaction that acts on the block at the point Q is

(a) 7.5 N (b) 8.6 N
(c) 11.5 N (d) 22.5 N

- A frame of reference that is accelerated with respect to an inertial frame of reference is called a noninertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a noninertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on a rotating disc and the force F_{in} experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega},$$

where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.



Now consider a smooth slot along a diameter of a disc of radius R rotating counterclockwise with a constant angular speed ω about its vertical axis through its centre. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis ($\vec{\omega} = \omega\hat{k}$). A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at $t = 0$ and is constrained to move only along the slot.

6. The distance r of the block at time t is

(a) $\frac{R}{4}(e^{2\omega t} + e^{-2\omega t})$ (b) $\frac{R}{2}\cos 2\omega t$

(c) $\frac{R}{2}\cos\omega t$ (d) $\frac{R}{4}(e^{\omega t} + e^{-\omega t})$

7. The net reaction of the disc on the block is

(a) $-m\omega^2 R\cos\omega t\hat{j} - mg\hat{k}$ (b) $m\omega^2 R\sin\omega t\hat{j} - mg\hat{k}$

(c) $\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$ (d) $\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\hat{k}$

- Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

8. The ratio x_1/x_2 equals

(a) 2 (b) $\frac{1}{2}$

(c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

9. When the disc B is brought in contact with the disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

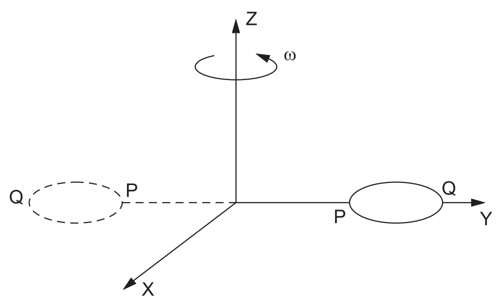
(a) $\dfrac{2I\omega}{3t}$ (b) $\dfrac{9I\omega}{2t}$

(c) $\dfrac{9I\omega}{4t}$ (d) $\dfrac{3I\omega}{2t}$

10. The loss of kinetic energy during the above process is

(a) $\dfrac{I\omega^2}{2}$ (b) $\dfrac{I\omega^2}{3}$

(c) $\dfrac{I\omega^2}{4}$ (d) $\dfrac{I\omega^2}{2t}$



- The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of its mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick as shown in the figure. When the disc–stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.

  Now consider two similar systems as shown in the figures ahead. In Figure I, the disc is with its face vertical and parallel to the xz-plane. In Figure II, the disc is with its face making an angle of 45° with the xy-plane and its horizontal diameter is parallel to the x-axis. In both the cases, the disc is welded at point P and the systems are rotated with a constant angular speed ω about the z-axis.

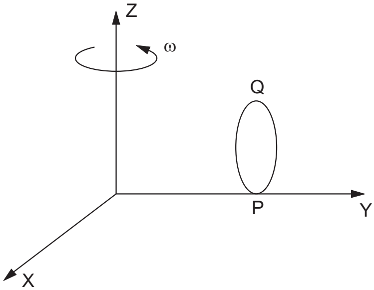


Fig. I

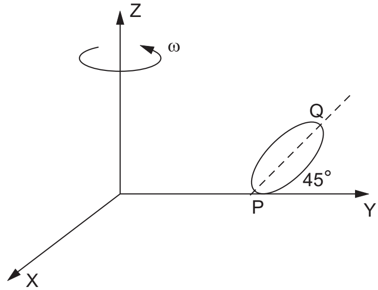


Fig. II

11. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is vertical in both the figures.

(b) It is vertical in Figure I, and it is at 45° to the *xz*-plane and lies in the plane of the disc in Figure II.

(c) It is horizontal in Figure I, and it is at 45° to the *xz*-plane and normal to the plane of the disc in Figure II.

(d) It is vertical in Figure I, and it is at 45° to the *xz*-plane and normal to the plane of the disc in Figure II.

12. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(a) It is $\sqrt{2}\,\omega$ in both the figures.

(b) It is ω in Figure I and $\frac{\omega}{\sqrt{2}}$ in Figure II.

(c) It is ω in Figure I and $\sqrt{2}\,\omega$ in Figure II.

(d) It is ω in both the figures.

- One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process, the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

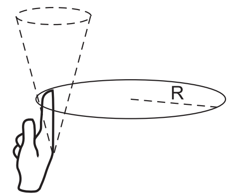


Fig. 1

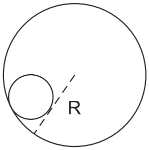


Fig. 2

13. The minimum value of ω_0 below which the ring will drop down is

(a) $\sqrt{\dfrac{2g}{\mu(R-r)}}$ (b) $\sqrt{\dfrac{g}{\mu(R-r)}}$

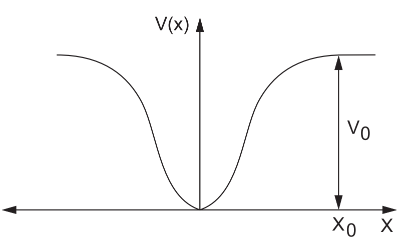
(c) $\sqrt{\dfrac{3g}{2\mu(R-r)}}$ (d) $\sqrt{\dfrac{g}{2\mu(R-r)}}$

14. The total kinetic energy of the ring is

(a) $\dfrac{3}{2}M\omega_0^2(R-r)^2$ (b) $\dfrac{1}{2}M\omega_0^2(R-r)^2$

(c) $M\omega_0^2(R-r)^2$ (d) $M\omega_0^2 R^2$

- When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\dfrac{m}{k}}$, as can be seen easily by using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4 (\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure).



15. If the total energy of the particle is E, it will perform periodic motion only if

(a) $E < 0$ (b) $E > 0$

(c) $V_0 > E > 0$ (d) $E > V_0$

16. For periodic motion of small amplitude A, the time period T of this particle is proportional to

(a) $A\sqrt{\frac{m}{\alpha}}$ (b) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$

(c) $A\sqrt{\frac{\alpha}{m}}$ (d) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

17. The acceleration of this particle for $|x| > X_0$ is

(a) proportional to V_0 (b) proportional to $\frac{V_0}{mX_0}$

(c) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (d) zero

- When a liquid medicine of density ρ is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

18. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is

(a) $2\pi rt$ (b) $2\pi RT$

(c) $\frac{2\pi r^2 T}{R}$ (d) $\frac{2\pi R^2 T}{r}$

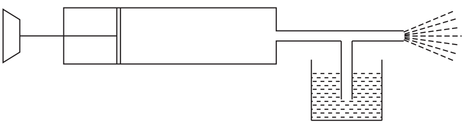
19. If $r = 5 \times 10^{-4}$ m, $\rho = 10^3$ kg m^{-3}, $g = 10$ m s^{-2}, $T = 0.11$ N m^{-1}, the radius of the drop when it detaches from the dropper is approximately

(a) 1.4×10^{-3} m (b) 3.3×10^{-3} m

(c) 2.0×10^{-3} m (d) 4.1×10^{-3} m

20. After the drop detaches, its surface energy is

(a) 1.4×10^{-6} J (b) 2.7×10^{-6} J
(c) 5.4×10^{-6} J (d) 8.1×10^{-6} J



- A spray gun is shown in the figure, where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. One end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.

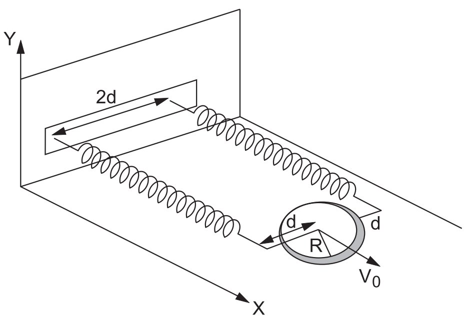
21. If the piston is pushed at a speed of 5 mm s^{-1}, the air comes out of the nozzle with a speed of

(a) 0.1 m s^{-1} (b) 1 m s^{-1}
(c) 2 m s^{-1} (d) 8 m s^{-1}

22. If the density of air is ρ_a and that of the liquid is ρ_l then for a given piston the speed (i.e., volume per unit time) at which the liquid is sprayed will be proportional to

(a) $\sqrt{\dfrac{\rho_a}{\rho_l}}$ (b) $\sqrt{\rho_a \rho_l}$

(c) $\sqrt{\dfrac{\rho_l}{\rho_a}}$ (d) ρ_l



- A uniform thin cylindrical disc of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disc symmetrically on either side at a distance d from the centre. The axle is massless, and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disc is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disc rolls without slipping with velocity $\vec{v_0} = v_0\hat{i}$. The coefficient of friction is μ.

23. The net external force acting on the disc when its centre of mass is at a distance x with respect to its equilibrium position is

(a) $-kx$ (b) $-2kx$

(c) $-\frac{2kx}{3}$ (d) $-\frac{4kx}{3}$

24. The centre of mass of the disc undergoes simple harmonic motion with angular frequency ω equal to

(a) $\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{2k}{M}}$

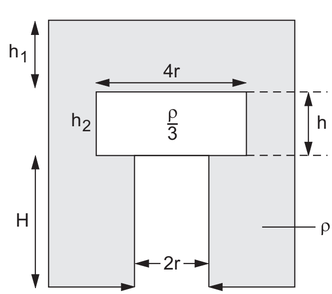
(c) $\sqrt{\frac{2k}{3M}}$ (d) $\sqrt{\frac{4k}{3M}}$

25. The maximum value of v_0 for which the disc will roll without slipping is

(a) $\mu g\sqrt{\frac{M}{k}}$ (b) $\mu g\sqrt{\frac{M}{2k}}$

(c) $\mu g\sqrt{\frac{3M}{k}}$ (d) $\mu g\sqrt{\frac{5M}{2k}}$

- A wooden cylinder of diameter $4r$, height h and density $\rho/3$ is kept on a hole of diameter $2r$ of a tank filled with water of density ρ, as shown in the figure. The height of the base of the cylinder from the base of the tank is H.



26. If the water level starts decreasing slowly then when it (the water) is at a height h_1 above the cylinder, the block just starts moving up. Then the value of h_1 is

(a) $\frac{2h}{3}$ (b) $\frac{5h}{4}$

(c) $\frac{5h}{3}$ (d) $\frac{5h}{2}$

27. The cylinder is prevented from moving up by applying a force and the water level is further decreased. Then the height of the water level (h_2 in the figure) for which the cylinder remains in original position without application of force is

(a) $\frac{h}{3}$ (b) $\frac{4h}{9}$

(c) $\frac{2h}{3}$ (d) h

28. If the height h_2 of the water level is further decreased then

(a) the cylinder remains at its original position and will not move up

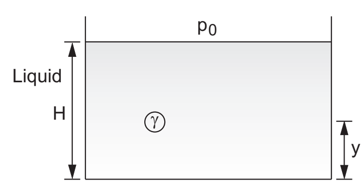
(b) for $h_2 = \frac{h}{3}$, the cylinder again starts to move up

(c) for $h_2 = \frac{h}{5}$, the cylinder again starts to move up

(d) for $h_2 = \frac{h}{4}$, the cylinder again starts to move up

5.2 Heat and Thermodynamics

- A small spherical monatomic ideal gas bubble $\left(\gamma = \frac{5}{3}\right)$ is trapped inside a liquid of density ρ_l (see the adjacent figure). Assume that the bubble does not exchange any heat with the liquid. The bubble contains n moles of the gas. The temperature of the gas when the bubble is at the bottom is T_0. The height of the liquid is H and the atmospheric pressure is p_0 (neglect the surface tension).



1. As the bubble moves upwards, besides the buoyant force, which of the following forces is/are acting on it?

(a) Only the force of gravity

(b) The force of gravity and the force due to the pressure of the liquid

(c) The force of gravity, the force due to the pressure of the liquid, and the force due to the viscosity of the liquid

(d) The force of gravity and the force due to the viscosity of the liquid

2. When the gas bubble is at a height y from the bottom, its temperature is

(a) $T_0\left(\dfrac{p_0+\rho_1 gH}{p_0+\rho_1 gy}\right)^{2/5}$ (b) $T_0\left[\dfrac{p_0+\rho_1 g(H-y)}{p_0+\rho_1 gH}\right]^{2/5}$

(c) $T_0\left(\dfrac{p_0+\rho_1 gH}{p_0+\rho_1 gy}\right)^{3/5}$ (d) $T_0\left[\dfrac{p_0+\rho_1 g(H-y)}{p_0+\rho_1 gH}\right]^{3/5}$

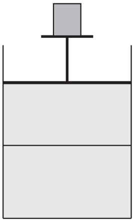
3. The buoyancy force acting on the gas bubble is (assume R to be the molar gas constant)

(a) $\rho_1 nRgT_0\dfrac{(p_0+\rho_1 gH)^{2/5}}{(p_0+\rho_1 gy)^{7/5}}$

(b) $\dfrac{\rho_1 nRgT_0}{(P_0+\rho_1 gH)^{2/5}[p_0+\rho_1 g(H-y)]^{3/5}}$

(c) $\rho_1 nRgT_0\dfrac{(p_0+\rho_1 gH)^{3/5}}{(p_0+\rho_1 gy)^{8/5}}$

(d) $\dfrac{\rho_1 nRgT_0}{(p_0+\rho_1 gH)^{3/5}[p_0+\rho_1 g(H-y)]^{2/5}}$



- In the figure, a container is shown to have a movable (without friction) piston on the top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R$ and $C_p = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$.

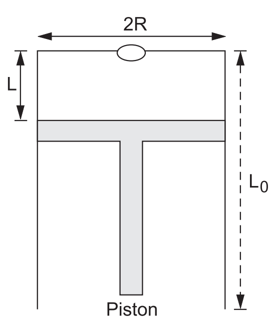
4. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(a) 550 K (b) 525 K
(c) 513 K (d) 490 K

5. Now consider the partition to be free to move without friction so that the pressure of gas in both the compartments is the same. The total work done by the gases till the time they achieve equilibrium will be

(a) $250R$ (b) $200R$
(c) $100R$ (d) $-100R$

- A fixed thermally conducting cylinder has its radius R and height L_0. The cylinder is open at its bottom and has a small hole at its top. A piston of mass M is held at a distance L from the top surface as shown in the figure. The atmospheric pressure is p_0.



6. The piston is pulled out slowly and held at a distance $2L$ from the top. The pressure in the cylinder between its top and the piston will then be

(a) p_0 (b) $\frac{p_0}{2}$

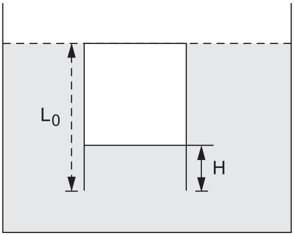
(c) $\frac{p_0}{2} + \frac{Mg}{\pi R^2}$ (d) $\frac{p_0}{2} - \frac{Mg}{\pi R^2}$

7. While the piston is at a distance $2L$ from the top, the hole at the top is sealed. The piston is then released to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(a) $\left(\dfrac{2p_0\pi R^2}{\pi R^2 p_0 + Mg}\right)(2L)$ (b) $\left(\dfrac{p_0\pi R^2 - Mg}{\pi R^2 p_0}\right)(2L)$

(c) $\left(\dfrac{p_0\pi R^2 + Mg}{\pi R^2 p_0}\right)(2L)$ (d) $\left(\dfrac{p_0\pi R^2}{\pi R^2 p_0 - Mg}\right)(2L)$

8. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder, as shown in the figure. The density of water is ρ. In equilibrium, the height H of the water column in the cylinder satisfies



(a) $\rho g(L_0 - H)^2 + p_0(L_0 - H) + L_0 p_0 = 0$
(b) $\rho g(L_0 - H)^2 - p_0(L_0 - H) - L_0 p_0 = 0$
(c) $\rho g(L_0 - H)^2 + p_0(L_0 - H) - L_0 p_0 = 0$
(d) $\rho g(L_0 - H)^2 - p_0(L_0 - H) + L_0 p_0 = 0$

5.3 Sound Waves

- Two waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling in a pipe placed along the x-axis.

1. Find the number of times the intensity is maximum in the time interval of 1 s.

(a) 4 (b) 6
(c) 8 (d) 10

2. Find the wave velocity of the louder sound.

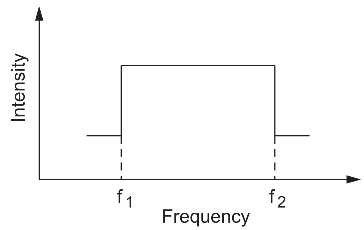
(a) $100\ \mathrm{m\ s^{-1}}$ (b) $192\ \mathrm{m\ s^{-1}}$
(c) $200\ \mathrm{m\ s^{-1}}$ (d) $96\ \mathrm{m\ s^{-1}}$

3. Find the number of times $y_1 + y_2 = 0$ at $x = 0$ in 1 s.

(a) 100 (b) 46

(c) 191 (d) 96

- Two trains A and B are moving with speeds 20 m s^{-1} and 30 m s^{-1} respectively in the same direction on the same straight track with B ahead of A. The engines are at the front ends. The engine of A blows a long whistle. Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz as shown in the figure. The spread in the frequency (highest frequency–lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m s^{-1}.



4. The speed of sound of the whistle is

(a) 340 m s^{-1} for the passengers in A and 310 m s^{-1} for the passengers in B

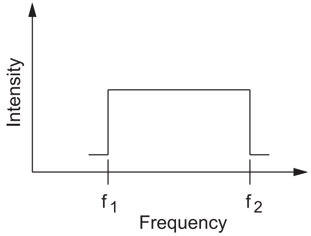
(b) 360 m s^{-1} for the passengers in A and 310 m s^{-1} for the passengers in B

(c) 310 m s^{-1} for the passengers in A and 360 m s^{-1} for the passengers in B

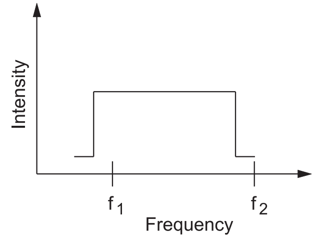
(d) 340 m s^{-1} for the passengers in both the trains

5. The distribution of the intensity of sound of the whistle as heard by the passengers in train A is best represented by

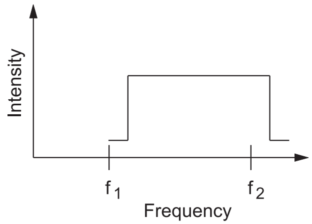
(a)



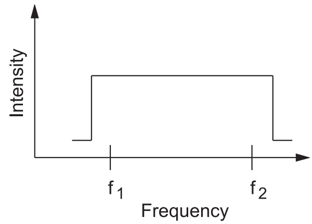
(b)



(c)



(d)

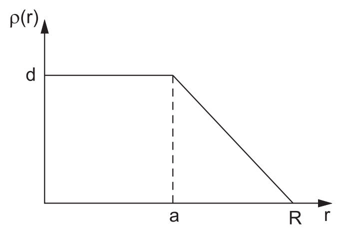


6. The spread of frequency as heard by the passengers in train B is

(a) 310 Hz (b) 330 Hz
(c) 350 Hz (d) 390 Hz

5.4 Electricity and Magnetism

- The nuclear charge (Ze) is nonuniformly distributed within a nucleus of radius R. The charge density, i.e., charge per unit volume, $\rho(r)$ is dependent only on the radial distance r from the centre of the nucleus, as shown in the figure. The electric field is only along the radial direction.



1. The electric field at $r = R$ is

(a) independent of a (b) directly proportional to a
(c) inversely proportional to a (d) none of these

2. For $a = 0$, the value of d (maximum value of ρ as shown in the figure) is

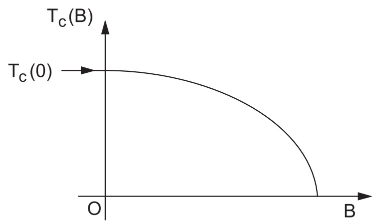
(a) $\dfrac{3Ze}{4\pi R^3}$ (b) $\dfrac{3Ze}{\pi R^3}$

(c) $\dfrac{4Ze}{3\pi R^3}$ (d) $\dfrac{Ze}{3\pi R^3}$

3. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies that

(a) $a = 0$ (b) $a = \frac{R}{2}$

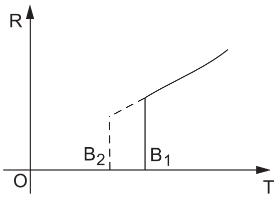
(c) $a = R$ (d) $a = \frac{2R}{3}$



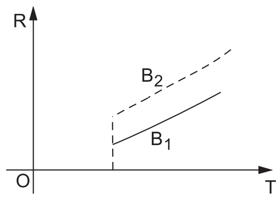
- The electrical resistance of certain materials known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of $T_C(B)$ on B is shown in the figure.

4. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

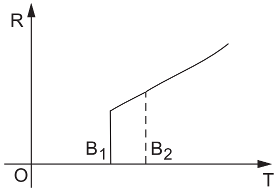
(a)



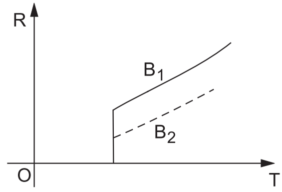
(b)



(c)



(d)



5. A superconductor has $T_C(0) = 100$ K. When a magnetic field of 7.5 tesla is applied, its T_C decreases to 75 K. For this material, one can definitely say that when

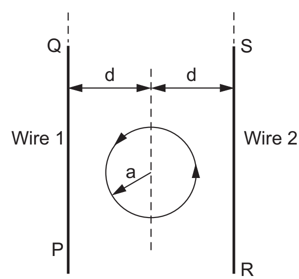
(a) $B = 5$ tesla, $T_C(B) = 80$ K

(b) $B = 5$ tesla, $75\text{ K} < T_C(B) < 100\text{ K}$

(c) $B = 10$ tesla, $75\text{ K} < T_C(B) < 100\text{ K}$

(d) $B = 10$ tesla, $T_C(B) = 70$ K

- The figure shows a circular loop of radius a with two long parallel wires (Wire 1 and Wire 2), all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wire are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above.



6. When $d \approx a$, but the wires are not touching the loop, it is found that the net magnetic field at the axis of the loop is zero at a height h above the loop.

(a) The currents in Wire 1 and Wire 2 flow in the directions PQ and RS respectively and $h \approx a$.

(b) The currents in Wire 1 and Wire 2 flow in the directions PQ and SR respectively and $h \approx a$.

(c) The currents in Wire 1 and Wire 2 flow in the directions PQ and SR respectively and $h \approx 1.2a$.

(d) The currents in Wire 1 and Wire 2 flow in the directions PQ and RS respectively and $h \approx 1.2a$.

7. Consider $d >> a$ and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are flowing in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop)

(a) $\dfrac{\mu_0 I^2 a^2}{d}$ (b) $\dfrac{\mu_0 I^2 a^2}{2d}$

(c) $\dfrac{\sqrt{3}\,\mu_0 I^2 a^2}{d}$ (d) $\dfrac{\sqrt{3}\,\mu_0 I^2 a^2}{2d}$

- A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let N be the number density of free electrons, each of mass m. When electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural frequency ω_P which is called the plasma frequency. To sustain the oscillations, a time-varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_P, all the free electrons are set to resonate together and all the energy is reflected. This is the explanation of high reflectivity of metals.

8. Taking the electronic charge e and the permittivity ε_0, use dimensional analysis to determine the correct expression for ω_0.

(a) $\sqrt{\dfrac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\dfrac{m\varepsilon_0}{Ne}}$

(c) $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\dfrac{m\varepsilon_0}{Ne^2}}$

9. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N = 4 \times 10^{27}$ m^{-3}. Take $\varepsilon_0 = 10^{-11}$ and mass $m = 10^{-30}$, where these quantities are in proper SI units.

(a) 800 nm (b) 600 nm

(c) 300 nm (d) 200 nm

- A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transmitted to a place 20 km away. The power can be transmitted either directly through a cable of large current-carrying capacity, or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is large energy dissipation. In the method using transformers, the dissipation is much smaller. By this method, a step-up transformer is used at the plant end so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor of unity. All the currents and voltages mentioned are rms values.

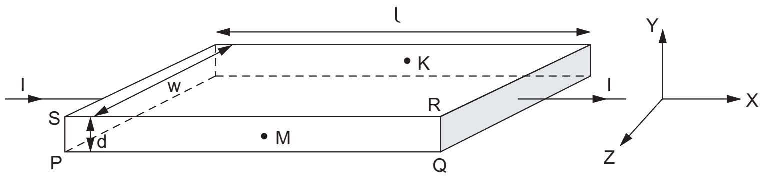
10. If the direct transmission method is used with a cable of resistance $0.4\ \Omega\ \text{km}^{-1}$, the power dissipation (in %) during transmission is

(a) 20 (b) 30
(c) 40 (d) 50

11. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers' end has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(a) 200 : 1 (b) 150 : 1
(c) 100 : 1 (d) 50 : 1

- In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are l, w and d respectively. A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface PQRS and appearance of equal and opposite charges on the face opposite PQRS. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



12. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2 respectively. Two points K and M are symmetrically located on the opposite faces parallel to the xy-plane (see figure) of each strip. V_1 and V_2 are the potential differences between K and M in strips 1 and 2 respectively. Then for a given current I flowing through each of them in a given magnetic field strength B, the correct statement(s) is/are:

(a) If $w_1 = w_2$ and $d_1 = 2d_2$ then $V_2 = 2V_1$
(b) If $w_1 = w_2$ and $d_1 = 2d_2$ then $V_1 = V_2$
(c) If $w_1 = 2w_2$ and $d_1 = d_2$ then $V_2 = 2V_1$
(d) If $w_1 = 2w_2$ and $d_1 = d_2$ then $V_2 = V_1$

13. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2 respectively. Strip 1 is placed in a magnetic field B_1 and strip 2 is placed in a magnetic field B_2, both along the positive y-direction. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2 respectively. Assuming that the current I is the same for both the strips, the correct option(s) is/are:

(a) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$
(b) If $B_1 = B_2$ and $n_1 = 2n_2$ then $V_2 = V_1$
(c) If $B_1 = 2B_2$ and $n_1 = n_2$ then $V_2 = 0.5V_1$
(d) If $B_1 = 2B_2$ and $n_1 = n_2$ then $V_2 = V_1$

- A point charge Q is moving in a circular orbit of radius R in the xy-plane with an angular speed ω. This can be considered as being equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from O to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

14. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(a) $\frac{BR}{4}$ (b) $\frac{BR}{2}$

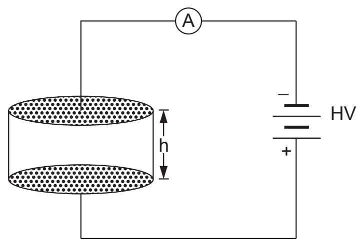
(c) BR (d) $2BR$

15. The change in the magnetic dipole moment associated with the orbit at the end of the time interval during which the magnetic field changes is

(a) $-\gamma BQR^2$ (b) $-\frac{\gamma}{2}BQR^2$

(c) $\frac{\gamma}{2}BQR^2$ (d) γBQR^2

- Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a lightweight and soft material and coated with a conducting material are placed on the bottom plate. Each ball has a radius $r << h$. Now a high-voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate is at $-V_0$. Due to their conducting surfaces, the balls will get charged, become equipotential with the plate and be repelled by it. The balls will eventually collide with the top plate where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel-plate capacitor. Assume that there are no collisions between the balls, and the interaction between them is negligible. (Ignore gravity.)



16. Which one of the following statements is correct?

(a) The balls will bounce back to the bottom plate carrying the opposite charge they went up with.

(b) The balls will execute SHM between the two plates.

(c) The balls will bounce back to the bottom plate carrying the same charge they went up with.

(d) The balls will stick to the top plate and remain there.

17. The average current in the steady state registered by the ammeter in the circuit will be

(a) proportional to $V_0^{1/2}$ (b) proportional to V_0^2
(c) proportional to V_0 (d) zero

- Consider a simple RC circuit as shown in Figure 1.

Process 1: In the circuit the switch S is closed at $t = 0$ and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time $T >> RC$). In the process, some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2: In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T >> RC$. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1. These two processes are depicted in Figure 2.

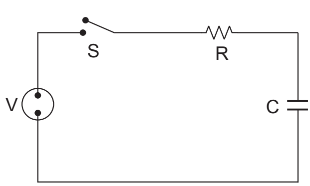


Fig. 1

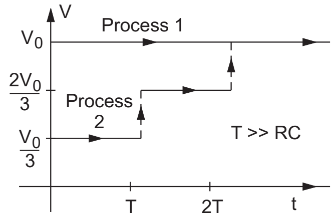


Fig. 2

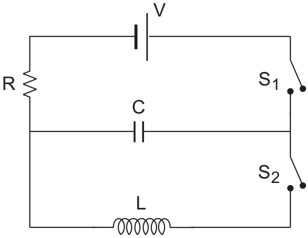
18. In Process 1, the energy stored in the capacitor, E_C, and heat dissipated across the resistance, E_D, are related by

(a) $E_C = \frac{1}{2}E_D$ (b) $E_C = 2E_D$
(c) $E_C = E_D$ (d) $E_C = E_0 \ln 2$

19. In Process 2, total energy dissipated across the resistance, E_D, is

(a) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (b) $E_D = \frac{1}{2}CV_0^2$
(c) $E_D = 3CV_0^2$ (d) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

- A capacitor of capacitance C can be charged (with the help of a resistance R) by a voltage source V, by closing the switch S_1 while keeping the switch S_2 open. The capacitor can be connected in series with an inductor L by closing switch S_2 and opening S_1.



20. Initially the capacitor was uncharged. Then the switch S_1 is closed and S_2 is kept open. If the time constant of this circuit is τ then

(a) after the time interval τ, charge on the capacitor is $CV/2$

(b) work done by the voltage source will be half of the heat dissipated when the capacitor is fully charged

(c) after time interval 2τ, charge on the capacitor is $CV(1 - e^{-2})$

(d) after time interval 2τ, charge on the capacitor is $CV(1 - e^{-1})$

21. After the capacitor gets fully charged, S_1 is opened and S_2 is closed so that the inductor is connected in series with the capacitor. Then

(a) at $t = 0$, the energy stored in the circuit is purely in the form of magnetic energy

(b) at any time $t > 0$, the current in the circuit is in the same direction

(c) at $t > 0$, there is no exchange of energy between the inductor and the capacitor

(d) at any time $t > 0$, the instantaneous current in the circuit may be $V\sqrt{\dfrac{C}{L}}$

22. If the total charge stored in the LC circuit is Q_0 then for $t \geq 0$,

(a) the charge on the capacitor is $Q = Q_0 \cos\left(\dfrac{\pi}{2} + \dfrac{1}{\sqrt{LC}}\right)$

(b) the charge on the capacitor is $Q = Q_0 \cos\left(\dfrac{\pi}{2} - \dfrac{1}{LC}\right)$

(c) the charge on the capacitor is $Q = -LC\dfrac{d^2Q}{dt^2}$

(d) the charge on the capacitor is $Q = -\dfrac{1}{\sqrt{LC}}\dfrac{d^2Q}{dt^2}$

- Modern trains are based on the Maglev technology in which a train is magnetically levitated and runs on its EDS Maglev system. There are coils on both sides of each wheel. Due to the motion of the train, a current is induced in the coil of the track, which levitates it. This is in accordance with Lenz's law. If the train comes down then according to Lenz's law, a repulsive force increase due to which the train gets uplifted and if it goes much higher then there is a net downward force due to gravity. The advantage of a Maglev train is that there is no friction between the train and the track, thereby reducing the power consumption and enabling the train to attain a very high speed. The major disadvantage of the Maglev train is that as it slows down, the electromagnetic forces decrease and it becomes difficult to keep the train levitated and as it moves forward, according to Lenz's law, there is an electromagnetic drag force.

23. What is the advantage of this system?

(a) There is no friction hence no power loss.
(b) No electric power is used.
(c) Gravitation force is zero.
(d) By Lenz's law, the train experiences a drag.

24. What is the major disadvantage of this system?

(a) The train experiences an upward force according to Lenz's law.
(b) The friction force creates a drag on the train.
(c) Retardation is caused.
(d) By Lenz's law, the train experiences a drag.

25. Which force causes the train to elevate upward?

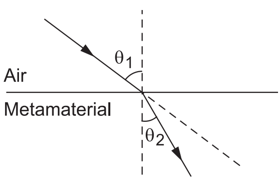
(a) Electrostatic force (b) Time-varying electric field
(c) Magnetic force (d) Induced electric field

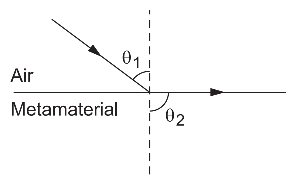
5.5 Ray Optics and Wave Optics

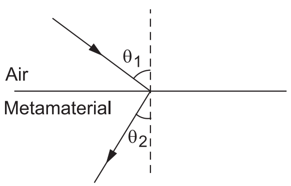
- Most materials have the refractive index $n > 1$. So, when a light ray from air enters a naturally occurring material then by Snell's law $\left(\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}\right)$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of a medium is given by the relation $n = \frac{c}{v} = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of the electromagnetic wave in vacuum, v is its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

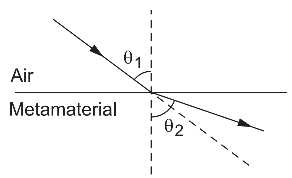
In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Materials with a negative refractive index can now be artificially prepared and are called metamaterials. They exhibit significantly different optical behaviour without violating any physical law. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to the normal materials, the frequency of light remains unchanged upon refraction even in metamaterials.

1. For the light incident from air on a metamaterial, the appropriate ray diagram is

(a)


(b)


(c)


(d)


2. Choose the correct statement.

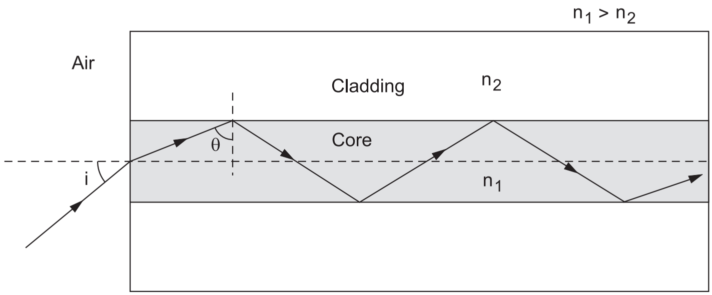
(a) The speed of light in the metamaterial is $v = c|n|$.

(b) The speed of light in the metamaterial is $v = \frac{c}{|n|}$.

(c) The speed of light in the metamaterial is $v = c$.

(d) The wavelength of light in the metamaterial (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, when λ_{air} is the wavelength of light in air.

- Light guidance in an optical fibre can be understood by considering a structure comprised of a thin solid glass cylinder of refractive index n_1, surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.



3. For the two structures, namely S_1 with $n_1 = \frac{\sqrt{45}}{4}$ and $n_2 = \frac{3}{2}$, and S_2 with $n_1 = \frac{8}{5}$ and $n_2 = \frac{7}{5}$ and taking the refractive index of water to be $\frac{4}{3}$ and that of air to be 1, choose the correct statement(s).

(a) The NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$.

(b) The NA of S_1 immersed in a liquid of refractive index $\frac{16}{\sqrt{15}}$ is the same as that of S_2 immersed in water.

(c) The NA of S_1 placed in air is the same as that of S_2 immersed in a liquid of refractive index $\frac{4}{\sqrt{15}}$.

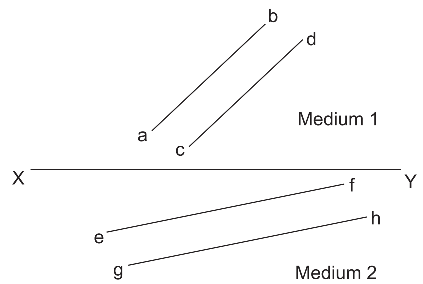
(d) The NA of S_1 placed in air is the same as that of S_2 placed in water.

4. If two structures of the same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

(a) $\dfrac{NA_1 NA_2}{NA_1 + NA_2}$ (b) $NA_1 + NA_2$

(c) NA_1 (d) NA_2

- The figure shows a surface XY separating two transparent media, Medium 1 and Medium 2. The lines *ab* and *cd* represent wavefronts of light travelling in Medium 1 and incident on XY. The lines *ef* and *gh* represent wavefronts of the light in Medium 2 after refraction.



5. Light travels as a

(a) parallel beam in each medium

(b) convergent beam in each medium

(c) divergent beam in each medium

(d) divergent beam in one medium and convergent beam in the other medium

6. The phases of the light wave at *c*, *d*, *e* and *f* are ϕ_c, ϕ_d, ϕ_e and ϕ_f respectively. It is given that $\phi_c \neq \phi_f$.

(a) ϕ_c cannot be equal to ϕ_d

(b) ϕ_d can be equal to ϕ_e

(c) $(\phi_d - \phi_f)$ is equal to $(\phi_c - \phi_e)$

(d) $(\phi_d - \phi_c)$ is not equal to $(\phi_f - \phi_e)$

7. The speed of light is

(a) the same in the two media

(b) greater in Medium 1 than in Medium 2

(c) greater in Medium 2 than in Medium 1

(d) different at *b* and *d*

5.6 Modern Physics

- The key feature of Bohr theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr quantization condition.

1. A diatomic molecule has moment of inertia I. By Bohr quantization condition, its rotational energy in the nth level ($n = 0$ is not allowed) is

(a) $\dfrac{1}{n^2}\left(\dfrac{h^2}{8\pi^2 I}\right)$ (b) $\dfrac{1}{n}\left(\dfrac{h^2}{8\pi^2 I}\right)$

(c) $n\left(\dfrac{h^2}{8\pi^2 I}\right)$ (d) $n^2\left(\dfrac{h^2}{8\pi^2 I}\right)$

2. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\dfrac{4}{\pi} \times 10^{11}$ Hz. Then, the moment of inertia of CO molecule about its centre of mass is close to (take $h = 2\pi \times 10^{-34}$ J s)

(a) 2.76×10^{-46} kg m^2 (b) 1.87×10^{-46} kg m^2

(c) 4.67×10^{-47} kg m^2 (d) 1.17×10^{-47} kg m^2

3. In a CO molecule, the distance between C (mass = 12 amu) and O (mass = 16 amu), where 1 amu = $\dfrac{5}{3} \times 10^{-27}$ kg, is close to

(a) 2.4×10^{-10} m (b) 1.9×10^{-10} m

(c) 1.3×10^{-10} m (d) 4.4×10^{-11} m

- The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Considering the decay of a neutron as a two-body decay process, it was predicted that the kinetic energy of the electrons should be a constant. But it was observed experimentally that the kinetic energy of the electron has a continuous spectrum. Considering a three-body decay process, i.e., $n \to p + e^- + \overline{\nu}_e$, around 1930, Pauli explained the observed energy spectrum of the electron. If one assumes the antinutrino ($\overline{\nu}_e$) to be massless and possessing negligible energy and the neutron to be at rest, and applies the principle of conservation of momentum and energy, the maximum kinetic energy of the electron is found to be 0.8×10^6 eV. The kinetic energy of the proton is only the recoil energy.

4. If the antinutrino had a mass 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

(a) $0 \leq K \leq 0.8 \times 10^6$ eV (b) $3.0 \text{ eV} \leq K \leq 0.8 \times 10^6$ eV
(c) $2.0 \text{ eV} \leq K \leq 0.8 \times 10^6$ eV (d) $0 \leq K \leq 0.8 \times 10^6$ eV

5. What is the maximum energy of the antineutrino?

(a) Zero (b) Much less than 0.8×10^6 eV
(c) Nearly 0.8×10^6 eV (d) Much larger than 0.8×10^6 eV

- The mass of any nucleus ${}^{A}_{Z}X$ is less than the sum of the masses of $(A - Z)$ neutrons and Z protons in the nucleus. The energy corresponding to this mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two lighter nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also, two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below.

${}^{1}_{1}H$	1.007825 u	${}^{2}_{1}H$	2.014102 u	${}^{3}_{1}H$	3.016050 u	${}^{4}_{2}He$	4.002603 u
${}^{6}_{3}Li$	6.015123 u	${}^{7}_{3}Li$	7.016004 u	${}^{70}_{30}Zn$	69.925325 u	${}^{82}_{34}Se$	81.916709 u
${}^{152}_{64}Gd$	151.919803 u	${}^{206}_{82}Pb$	205.974455 u	${}^{209}_{83}Bi$	208.980388 u	${}^{210}_{84}Po$	209.982876 u

6. Which of the following statements is correct?

(a) The nucleus ${}^{6}_{3}Li$ can emit an alpha particle.
(b) The nucleus ${}^{210}_{84}Po$ can emit a proton.
(c) Deuterons and alpha particles can undergo complete fusion.
(d) The nuclei ${}^{70}_{30}Zn$ and ${}^{82}_{34}Se$ can undergo complete fusion.

7. The kinetic energy (in keV) of the alpha particle released when the nucleus ${}^{210}_{84}Po$ at rest undergoes alpha decay is

(a) 5319 (b) 5422 (c) 5707 (d) 5818

- When a particle is restricted to move along x-axis between $x = 0$ and $x = a$, where a is of nanometre dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to the linear momentum as $E = \frac{p^2}{2m}\cdot$ Thus, the energy of the particle can be described by a quantum number n, taking values $n = 1, 2, 3, \ldots$, corresponding to the number of loops in the standing wave.

  Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. Take $h = 6.6 \times 10^{-34}$ J s and $e = 1.6 \times 10^{-19}$ C.

8. The allowed energy of the particle for a particular value of n is proportional to

(a) a^{-2} (b) $a^{-3/2}$ (c) a^{-1} (d) a^2

9. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to

(a) 0.8 meV (b) 8 meV
(c) 80 meV (d) 800 meV

10. The speed of the particle that can take discrete values, is proportional to

(a) $n^{-3/2}$ (b) n^{-1} (c) $n^{1/2}$ (d) n

- Scientists are working hard to develop a nuclear fusion reactor. Nuclei of heavy hydrogen, ${}^2_1H$, known as deuteron and denoted by D can be thought of as a candidate for such a reactor. The D–D reaction is ${}^2_1H + {}^2_1H \rightarrow {}^3_2He + n + \text{energy}$. In the core of the fusion reactor, a gas of heavy hydrogen is fully ionized into deuteron nuclei and electrons. This collection of ${}^2_1H$ nuclei and electrons is known as plasma. The nuclei move randomly in the reactor core and occasionally come close enough for nuclear fusion to take place. Usually, the temperatures in the reactor core are too high and no material wall can be used to confine the plasma. Special techniques are used which confine the plasma for a time t_0 before the particles fly away from the core. If n is the number density (number/volume) of deuterons, the product nt_0 is called Lawson number. In one of the criteria, a reaction is termed successful if Lawson number is greater than 5×10^{14} s cm^{-3}.

It may be helpful to use the following: Boltzmann constant $k = 8.6 \times 10^{-5}$ eV K^{-1}; $\dfrac{e^2}{4\pi\varepsilon_0} = 1.44 \times 10^{-9}$ eV m.

11. In the core of a nuclear fusion reactor, the gas becomes plasma because of the

(a) strong nuclear force acting between the deuterons
(b) Coulomb force acting between the deuterons
(c) Coulomb force acting between the deuteron–electron pair
(d) high temperature

12. Assume that two deuteron nuclei in the core of a fusion reactor at temperature T are moving towards each other, each with a kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also, neglect any interaction with other particles in the core. The minimum temperature T required for them to reach a separation of 4×10^{-15} m is in the range

(a) 1.0×10^9 K $< T < 2.0 \times 10^9$ K (b) 2.0×10^9 K $< T < 3.0 \times 10^9$ K
(c) 3.0×10^9 K $< T < 4.0 \times 10^9$ K (d) 4.0×10^9 K $< T < 5.0 \times 10^9$ K

13. The results of calculations for four different designs of a fusion reactor using D–D reaction are given below. Which of these is the most promising based on Lawson criterion?

(a) Deuteron density = 2.0×10^{12} cm^{-3},
confinement time = 5.0×10^{-3} s
(b) Deuteron density = 8.0×10^{14} cm^{-3},
confinement time = 9.0×10^{-1} s
(c) Deuteron density = 4.0×10^{23} cm^{-3},
confinement time = 1.0×10^{-11} s
(d) Deuteron density = 1.0×10^{24} cm^{-3},
confinement time = 4.0×10^{-12} s

Answers

5.1 General Physics

1. d	**2.** c	**3.** b	**4.** b	**5.** a	**6.** d
7. c	**8.** c	**9.** a	**10.** b	**11.** a	**12.** d
13. b	**14.** c	**15.** c	**16.** b	**17.** d	**18.** c
19. a	**20.** b	**21.** c	**22.** a	**23.** d	**24.** d
25. c	**26.** c	**27.** b	**28.** a		

5.2 Heat and Thermodynamics

1. d	**2.** b	**3.** b	**4.** d	**5.** d	**6.** a
7. d	**8.** c				

5.3 Sound Waves

1. a	**2.** c	**3.** d	**4.** b	**5.** a	**6.** a

5.4 Electricity and Magnetism

1. a	**2.** b	**3.** c	**4.** a	**5.** b	**6.** c
7. b	**8.** c	**9.** b	**10.** b	**11.** a	**12.** a, d
13. a, c	**14.** b	**15.** b	**16.** a	**17.** b	**18.** c
19. d	**20.** c	**21.** d	**22.** c	**23.** a	**24.** d
25. d					

5.5 Ray Optics and Wave Optics

1. c	**2.** b	**3.** a, c	**4.** d	**5.** a	**6.** c
7. b					

5.6 Modern Physics

1. d	**2.** b	**3.** c	**4.** d	**5.** c	**6.** c
7. a	**8.** a	**9.** b	**10.** d	**11.** d	**12.** a
13. b					

Hints and Solutions

5.1 General Physics

1. During upward motion the momentum and displacement both are positive and as position increases, momentum decreases.

2. Energy $E \propto A^2$, where A is the amplitude.

$$\therefore \frac{E_1}{E_2} = \left(\frac{2a}{a}\right)^2 \Rightarrow E_1 = 4E_2.$$

3. Total energy $E = \dfrac{p^2}{2m} + \dfrac{1}{2}Kx^2$.

However, $E = \dfrac{1}{2}KA(t)^2$, where $A(t)$ is the amplitude

$$\therefore \quad \frac{1}{2}KA(t)^2 = \frac{p^2}{2m} + \frac{1}{2}Kx^2$$

or $p^2 + mK^2x^2 = mKA(t)^2$.

Hence the equation of motion is a circle in phase space whose radius is decreasing due to the friction of the liquid.

4. From work–energy principle: $W_g + W_f = \Delta KE$.

So, $mgR \sin 30° + W_f = \dfrac{1}{2} mv^2$

$$\Rightarrow \quad (1\text{ kg})(10\text{ m s}^{-2})(40\text{ m})\left(\frac{1}{2}\right) - 150\text{ J} = \frac{1}{2}(1\text{ kg})v^2$$

$$\Rightarrow \quad v^2 = (10\text{ m s}^{-1})^2 \Rightarrow v = 10\text{ m s}^{-1}.$$

5. The centripetal force $\left(\dfrac{mv^2}{R}\right)$ is provided by $(\mathcal{N} - mg\cos 60°)$, so the normal reaction, $\mathcal{N} = mg\cos 60° + \dfrac{mv^2}{R} = 7.5\text{ N}$.

6. Centripetal acceleration, $a = \omega^2 r$.

$$\therefore \quad v\frac{dv}{dr} = r\omega^2$$

$$\Rightarrow \int_0^v v\,dv = \int_{R/2}^r \omega^2\,r dr \Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}}.$$

$$\therefore \quad \int_{R/2}^{r} \frac{dr}{\sqrt{r^2 - R^2/4}} = \omega \int_0^t dt.$$

Let $r = \frac{R}{2}\sec\theta$, $dr = \frac{R}{2}\sec\theta\tan\theta\, d\theta$

$$\Rightarrow \quad \int_{R/2}^{r} \frac{\frac{R}{2}\sec\theta\tan\theta}{\frac{R}{2}\tan\theta} d\theta = \omega \int dt$$

$$\Rightarrow \quad \omega t = \ln\left(2r + \frac{\sqrt{4r^2 - R^2}}{R}\right)$$

$$\Rightarrow \quad r = \frac{R}{4}(e^{\omega t} + e^{-\omega t}).$$

7. Term $2m(\vec{v}_{\text{rot}} \times \vec{\omega})$ gives the normal reaction from the edge ($\mathscr{N}_e$)

$$v_{\text{rot}} = \frac{dr}{dt} = \frac{R}{4}(e^{\omega t} - e^{-\omega t})\,\omega.$$

$$\therefore \quad \mathscr{N}_e = 2m\frac{R\omega}{4}(e^{\omega t} - e^{-\omega t})\cdot\omega.$$

$$\Rightarrow \quad \vec{\mathscr{N}_e} = \frac{mR\omega^2}{2}(e^{\omega t} - e^{-\omega t})\hat{j}.$$

Normal reaction from the bottom of the slot = $mg\hat{k}$.

$\therefore$ net reaction from the slot is

$$\vec{R} = \frac{mR\omega^2}{2}(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}.$$

8. $\frac{1}{2}I(2\omega)^2 = \frac{1}{2}kx_1^2;\ \frac{1}{2}(2I)\omega^2 = \frac{1}{2}kx_2^2$

$$\Rightarrow \quad \frac{x_1}{x_2} = \sqrt{2}.$$

9. Conserving angular momentum

$$I(2\omega) + 2I(\omega) = (I + 2I)\,\omega'.$$

$\therefore$ common angular velocity $\omega' = \frac{4}{3}\omega$.

Now, angular impulse = (average frictional torque) × time

$\Rightarrow \quad \tau \cdot t$ = change in angular momentum

$$= 2I\left(\frac{4}{3}\omega - \omega\right) = \frac{2I\omega}{3}$$

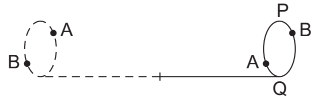
$$\Rightarrow \quad \tau = \frac{2I\omega}{3t}.$$

10. Initial kinetic energy $= \frac{1}{2}I(2\omega)^2 + \frac{1}{2}(2I)\omega^2 = 3I\omega^2$.

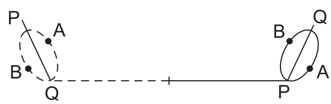
Final kinetic energy $= \frac{1}{2}(I+2I)\left(\frac{4}{3}\omega\right)^2$.

$\therefore$ loss in kinetic energy $= 3I\omega^2 - \frac{8}{3}I\omega^2 = \frac{I\omega^2}{3}$.

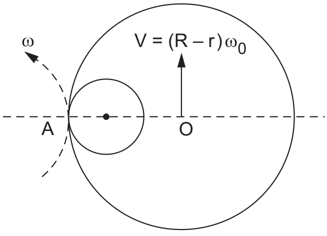
11. As shown in the figure, when the system as a whole turns by 180°, the disc also turns by 180° about its vertical axis. Hence the instantaneous axis that passes through the centre of mass is vertical in both the cases (a) and (b).



12. As shown in the adjoining figure, when the system as a whole is turned by 180°, the disc also turns by 180° about the vertical axis. Hence for both the cases (a) and (b), the angular speed about the instantaneous axis that passes through the centre of mass is ω.



13.
$$N = M\omega^2(R-r).$$
$$F_{max} = \mu M\omega^2(R-r) \le Mg$$
$$\Rightarrow \quad \omega^2 \le \frac{g}{\mu(R-r)}.$$
$$\therefore \omega_{min} = \sqrt{\frac{g}{\mu(R-r)}}.$$



14. $\omega R = \omega_0(R-r) \Rightarrow \omega = \omega_0\left(\frac{R-r}{R}\right)$.

Rotational kinetic energy $= \frac{1}{2}I\omega^2$

$$= \frac{1}{2}(2MR^2)\omega_0^2\left(\frac{R-r}{R}\right)^2 = M\omega_0^2(R-r)^2.$$

15. For an oscillating system, the kinetic energy must be zero periodically for a finite value of x. Also, $E < V_0$.

16. $V(x) = \alpha x^4$. $\quad \therefore [\alpha] = ML^{-2}T^{-2}$.

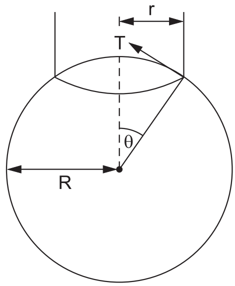
The only expression which has the dimension M^0L^0T is (b).

17. For $x > X_0$, V is constant.

Hence, force = 0 and therefore acceleration = 0.

18. Vertical upward force due to surface tension = $(2\pi r)(T \sin \theta)$

$= 2\pi r T\left(\frac{r}{R}\right) = 2\pi r^2 T/R.$



19. $2\pi r^2 T/R = mg = \left(\frac{4}{3}\pi R^3\right)\rho g.$

20. Surface energy $= 4\pi R^2 T.$

21. From continuity equation,

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow \quad \pi(20)^2 \times 5 = \pi(1)^2 V_2$$

$$\Rightarrow \quad V_2 = 2000 \text{ mm s}^{-1} = 2 \text{ m s}^{-1}.$$

22. $\frac{1}{2}\rho_a V_a^2 = \frac{1}{2}\rho_l V_l^2$. For a given value of V_a, $V_l \propto \sqrt{\frac{\rho_a}{\rho_l}}$.

23. For translation: $2kx - f = Ma_{CM}$. ...(i)

For rotation: torque due to friction (f)

$$fR = I_{CM}\,\alpha = \frac{MR^2}{2}\frac{a_{CM}}{R}$$

$$\Rightarrow \quad f = Ma_{CM/2}. \qquad \text{...(ii)}$$

Adding equations (i) and (ii),

$$2kx = \frac{3}{2}Ma_{CM} \Rightarrow a_{CM} = \frac{4}{3}\frac{kx}{M}.$$

$\therefore$ net external force on the disc is

$Ma_{CM} = \frac{4}{3}kx$ (–ve sign indicates retardation).

24. The equation for angular motion is

torque $= I\alpha = -2kx\cdot R$

$$\Rightarrow \quad \alpha = -\frac{2kR}{I}x = -\frac{2kR}{\frac{3MR^2}{2}}(R\theta)$$

$$\Rightarrow \quad \alpha = -\frac{4k}{3M}\theta = -\omega^2\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{4k}{3M}}.$$

25. From equation (ii), $f = \frac{1}{2}Ma_{CM} = \mu N = \mu Mg$

$$\Rightarrow \quad Ma_{CM} = 2\mu Mg = \frac{4}{3}kx$$

$$\Rightarrow \quad x = \frac{3}{2}\left(\frac{\mu Mg}{k}\right).$$

At this value of x, friction (static) is limiting and slipping just starts. At this position the velocity v_0 can be obtained from energy-conservation:

$$\frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega^2 = 2\left(\frac{1}{2}kx^2\right). \qquad \text{...(iii)}$$

Take $I = \frac{1}{2}MR^2$, $\omega = \frac{v_0}{R}$ and $x = \frac{3}{2}\frac{\mu Mg}{k}$ to solve (iii), from which we get

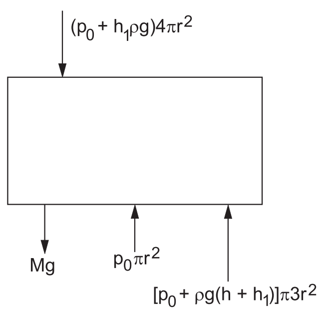
$$v_0 = \mu g\sqrt{\frac{3M}{k}}.$$

26. $M = \pi(4r^2)h\,\frac{\rho}{3}g.$

For equilibrium,

$$(p_0 = \rho g h_1)\pi 4r^2 + Mg$$
$$= (p_0 + \rho g(h + h_1)) \cdot \pi \cdot 3r^2 + p_0\pi r^2.$$

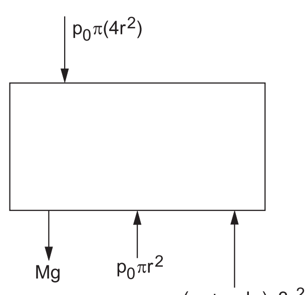
Simplifying, $h_1 = \frac{5}{3}h.$



27. $p_0\pi 4r^2 + Mg = p_0\pi r^2 + (\rho g h_1 + p_0)\pi 3r^2.$

Substituting M and simplifying,

$$h_1 = 4h/9.$$



28. The cylinder will not move up if $h_2 < \frac{4}{9}h.$

5.2 Heat and Thermodynamics

1. In addition to the buoyant force, the weight (mg) downward and the viscous force against the motion (downward) act together.

2. $p_1^{1-\gamma}T_1^{\gamma} = p_2^{1-\gamma}T_2^{\gamma}$.

   $T_1 = T_0$, $p_1 = p_0 + \rho_l gH$, and

   $p_2 = p_0 + \rho_l g(H-y)$.

3. Buoyant force $= \rho_l Vg = \rho_l g\,\dfrac{nRT_2}{p_2}$

   and $\rho_2 = \rho_0 + \rho_l g(H-y)$

   $$\Rightarrow \quad T_2 = T_0\left[\frac{p_0 + \rho_l g(H-y)}{p_0 + \rho_l gH}\right]^{2/5}.$$

4. Let the final temperature of the gases $= T$.

   Heat expelled by the lower compartment (nC_v, ΔT)

   $$= 2\left(\frac{3}{2}R\right)(700 - T).$$

   Heat absorbed by the upper compartment

   $$(nC_p\Delta T) = 2\left(\frac{7}{2}R\right)(T - 400).$$

   For equilibrium,

   $7\mathrm{T} - 2800 = 2100 - 3T$

   $\Rightarrow \quad T = 490$ K.

5. $\Delta W_1 + \Delta U_1 = \Delta Q_1$ and $\Delta W_2 + \Delta U_2 = \Delta Q_2$.

   For the total system, $\Delta Q_1 + \Delta Q_2 = 0$

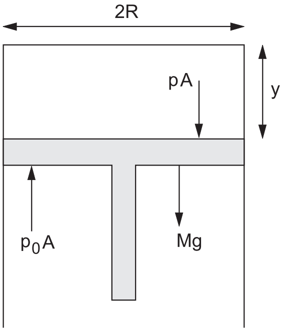
   $\Rightarrow \quad \frac{7}{2}R(T-400) = \frac{5}{2}R(700 - T) \Rightarrow T = 525$ K.

   Work done by gas 1, $\Delta W_1 = nR\Delta T = 2R(525 - 400) = 250R$.

   Work done by gas 2, $\Delta W_2 = nR\Delta T = 2R(525 - 700) = -350R$.

   $\Rightarrow$ net work $= \Delta W_1 + \Delta W_2 = -100R$.

6. With the hole open, the space between the top and the piston is exposed to the atmosphere so the pressure $= p_0$.



7. For equilibrium,

$$Mg + p\pi R^2 = p_0\pi R^2$$

$\Rightarrow$ $p\pi R^2 = p_0\pi R^2 - Mg.$

For isothermal expansion,

$$p_0(\pi R^2 \cdot 2L) = p(\pi R^2 \cdot y)$$

$$\Rightarrow \quad y = \left(\frac{p_0\pi R^2}{p_0\pi R^2 - Mg}\right)2L.$$

8. $\pi R^2 p_0 L_0 = p(L_0 - H)\pi R^2$

$\Rightarrow$ $p = p_0 L_0/(L_0 - H).$

For equilibrium,

$$p = p_0 + \rho g(L_0 - H).$$

Equating, we get

$$\rho g(L_0 - H)^2 + p_0(L_0 - H) - L_0 p_0 = 0.$$

5.3 Sound Waves

1. Beat frequency = difference in frequency.

Here, $f_1 = \dfrac{100\pi}{2\pi} = 50\ \text{s}^{-1}$, $f_2 = \dfrac{92\pi}{2\pi} = 46\ \text{s}^{-1}$.

$\Rightarrow$ required number $= f_1 - f_2 = 4.$

2. Wave velocity, $v = \dfrac{\omega}{k}$.

$$\therefore \quad v_1 = \frac{100\pi}{0.5\pi} = 200\ \text{m s}^{-1},$$

$$v_2 = \frac{92\pi}{0.46\pi} = 200\ \text{m s}^{-1}.$$

$\Rightarrow$ $v_1 = v_2 = 200\ \text{m s}^{-1}.$

3. At $x = 0$

$$y_1 + y_2 = A\cos 100\pi t + A\cos 92\pi t = 0,$$

$\therefore$ $\cos(100\pi t) = -\cos(92\pi t) = \cos[(2n+1)\pi - (92\pi t)]$

$\Rightarrow$ $100\pi t = (2n+1)\pi - 92\pi t$

$$\Rightarrow \quad t = \frac{2n+1}{192}.$$

$$\therefore \quad \Delta t = t_{n+1} - t_n = \frac{2}{192}.$$

Hence, the required number $= \dfrac{1}{\Delta t} = \dfrac{192}{2} = 96.$

4. The speed of sound depends on the frame of reference. Thus the speed of sound relative to train A is (340 + 20) m s^{-1} = 360 m s^{-1}. Similarly, relative to train B, speed of sound = (340 – 30) m s^{-1} = 310 m s^{-1}.

5. As there is no relative motion between the trains and the passengers in both the trains, the distribution of intensity of sound of the whistle will be uniform. Hence, the best graph for this situation must be (a).

6. According to Doppler effect, apparent frequency

$$f' = \frac{v \pm v_o}{v + v_s} f.$$

Here v = 340 m s^{-1}, v_o(train B) = 30 m s^{-1},

v_s(source ≡ train A) 20 m s^{-1} and f = true frequency.

∴ lowermost frequency,

$$f_1' = \left(\frac{340-30}{340-20}\right)(800 \text{ Hz}) = 775 \text{ Hz}.$$

Similarly, the uppermost frequency,

$$f_2' = \left(\frac{340-30}{340-20}\right)(1120 \text{ Hz}) = 1085 \text{ Hz}.$$

⇒ frequency spread = 310 Hz.

5.4 Electricity and Magnetism

1. From Gauss's theorem

$$E \cdot 4\pi R^2 = \frac{Q}{\varepsilon_0} = \frac{Ze}{\varepsilon_0}$$

⇒ $E = \dfrac{Ze}{4\pi\varepsilon_0 R^2}$, hence independent of a.

2. Charge density as a function of r is

$$\rho(r) = d\left(1 - \frac{r}{R}\right).$$

Total charge enclosed

$$Q = Ze = \int_0^R \rho(r) \cdot 4\pi r^2 dr = \frac{\pi d R^3}{3}.$$

$$\therefore \quad d = \frac{3Ze}{\pi R^3}.$$

3. Linear dependence of the electric field within the nucleus with radial distance is possible only if the charge density is uniform throughout, so $a = R$.

From Gauss's theorem,

$$E \cdot 4\pi r^2 = \frac{Ze}{\varepsilon_0} = \frac{\frac{4}{3}\pi r^3 d}{\varepsilon_0}$$

$$\Rightarrow \qquad E = \left(\frac{1}{3\varepsilon_0} d\right) r.$$

$$\therefore \qquad E \propto r.$$

4. Critical temperature decreases with the increase in magnetic field. As shown in option (a), magnetic field B_2 is larger than B_1 (given) and the critical temperature is lower.

5. For $B = 0$, $T_C = 100$ K;
$B = 7.5$ T, $T_C = 75$ K;
$B = 5$ T, $75 \text{ K} < T_C < 100$ K.

6. The net magnetic field at the given points will be zero, if

$$|\vec{B}_{\text{wires}}| = |\vec{B}_{\text{ring}}|.$$

Due to the ring,

$$B_{\text{ring}} = \frac{\mu_0 I a^2}{2(a^2+h^2)^{3/2}}, \text{ vertically up the plane.}$$

Due to the wires,

$$B_{\text{wires}} = 2\frac{\mu_0 I}{2\pi\sqrt{a^2+h^2}} \times \frac{a}{\sqrt{a^2+h^2}} = \frac{\mu_0 I a}{\pi(a^2+h^2)}$$

$$\Rightarrow \qquad \frac{\mu_0 I a}{\pi(a^2+h^2)} = \frac{\mu_0 I a^2}{2(a^2+h^2)^{3/2}}$$

$$\Rightarrow \qquad h = a\sqrt{\frac{\pi^2}{4} - 1} \approx 1.2a.$$

For the net field to be zero, the magnetic field due to straight currents will be directed vertically down the plane, so the current flows in the directions PQ and SR.

7. Magnetic field at the midpoint of the two straight wires, $B = \frac{\mu_0 I}{\pi d}$.

Magnetic moment of the loop, $m = I\pi a^2$.

Magnitude of torque, $|\vec{\tau}| = |\vec{m} \times \vec{B}|$

$$= mB \sin 150° = \frac{1}{2} mB$$

$$= \frac{\mu_0 I^2 a^2}{2d}.$$

8. Check dimensionally: $\sqrt{\dfrac{Ne^2}{m\varepsilon_0}} = T^{-1} = [\omega_0]$.

9. $\omega = 2\pi\nu = 2\pi\dfrac{c}{\lambda}$.

10. Input power = 600 kW, voltage = 4000 V.

Total line resistance = $(0.4\ \Omega\ \text{km}^{-1})(20\ \text{km}) = 8\ \Omega$.

$\because$ $P = VI \Rightarrow (600\ \text{kV})(10^3\ \text{A}) = (4000\ \text{V})I \Rightarrow I = 150\ \text{A}$.

$\therefore$ energy loss due to heat dissipation,

$P = I^2R = (150\ \text{A})^2(8\ \Omega) = 180000\ \text{W} = 180\ \text{kW}$.

$\therefore$ % loss = $\left(\dfrac{180\ \text{kW}}{600\ \text{kW}}\right) \times 100\% = 30\%$.

11. Given: $\dfrac{N_\text{p}}{N_\text{s}} = \dfrac{1}{10}$ in the step-up transformer.

Now, $\dfrac{V_\text{s}}{V_\text{p}} = \dfrac{N_\text{s}}{N_\text{p}} \Rightarrow \dfrac{V_\text{s}}{4000\ \text{V}} = \dfrac{10}{1} \Rightarrow V_\text{s} = 40000\ \text{V}$.

Hence, the required ratio in the step-down transformer

$$\frac{N_\text{p}}{N_\text{s}} = \frac{V_\text{p}}{V_\text{s}} = \frac{40,000}{200} = 200:1.$$

12. $qvB = qE = q\dfrac{V_\text{M} - V_\text{K}}{W}$.

$\therefore$ $V_\text{M} - V_\text{K} = WvB$.

But current $I = neAv = ne(Wd)v$

$\Rightarrow$ $Wv = \dfrac{I}{ned}$.

$\therefore$ $V_\text{M} - V_\text{K} = \left(\dfrac{I}{ned}\right)B$.

For $w_1 = w_2,\ d_1 = 2d_2$; $V_2 = 2V_1$.

For $w_1 = 2w_2,\ d_1 = d_2$; $V_1 = V_2$.

13. $V_\text{M} - V_\text{K} = \left(\dfrac{I}{ned}\right)B$.

For $n_1 = 2n_2,\ B_1 = B_2 \Rightarrow V_2 = 2V_1$.

For $B_1 = 2B_2,\ n_1 = n_2 \Rightarrow V_1 = 2V_2$.

or $V_2 = 0.5V_1$.

14. Let E = magnitude of the induced electric field, so

$$E(2\pi R) = \frac{d\phi_B}{dt} = \pi R^2\frac{dB}{dt} = \pi R^2 B$$

$$\Rightarrow \quad E = \frac{RB}{2}.$$

15. Change in angular momentum,

$$\Delta L = \int \tau\, dt = \int EQRdt = \frac{RB}{2} QR \times 1 = \frac{QBR^2}{2}.$$

Now, change in the magnitude of the dipole moment,

$$\Delta\mu = \gamma \Delta L$$
$$= -\frac{\gamma}{2} BQR^2 \text{ (considering direction).}$$

16. The balls get negatively charged after hitting the top plate and get attracted towards the positively charged bottom plate. The motion executed will be periodic but not SHM.

17. As the balls keep on carrying charge from one plate to the other, current will continue to flow even in steady state.

While at the bottom plate, $V_0 = \dfrac{Kq}{r}$

$$\Rightarrow \quad q = \frac{V_0 r}{K}.$$

In the cavity of the cylinder, electric field,

$$E = [V_0 - (-V_0)]/h = \frac{2V_0}{h}.$$

$$\left(\therefore \text{ time to reach the other plate, } t = \sqrt{\frac{2h}{a}} = \frac{1}{V_0}\sqrt{\frac{Kmh}{r}}. \right)$$

Acceleration, $a = \dfrac{F}{m} = \dfrac{qE}{m} = \dfrac{2rV_0^2}{mK}.$

If there are n balls,

average current $= \dfrac{nq}{t} = n\left(\dfrac{V_0 r}{K}\right) V_0 \sqrt{\dfrac{r}{Kmh}} = \dfrac{nr}{K}\sqrt{\dfrac{r}{Kmh}}\, V_0^2.$

$$\Rightarrow \quad I_{av} \propto V_0^2.$$

18. $E_C = \dfrac{1}{2} CV_0^2.$

Energy delivered by the cell $= Q_0 V_0 = (CV_0)(V_0) = CV_0^2.$

$$E_D = CV_0^2 - E_C = \frac{1}{2} CV_0^2$$

$$\Rightarrow \quad E_C = E_D.$$

19. If the capacitor is charged from V_i to V_f ($V_f > V_i$), heat dissipated is

$$\Delta E = W_{battery} - \Delta U$$
$$= C(V_f - V_i) \cdot V_f - \frac{1}{2} C(V_f^2 - V_i^2)$$
$$= \frac{1}{2} C(V_f - V_i)^2.$$

Total heat dissipated in the two processes taken together,

$$\begin{aligned}E_D &= (\Delta E)_1 + (\Delta E)_2 + (\Delta E)_3\\ &= \frac{1}{2}C\left(\frac{V_0}{3}-0\right)^2 + \frac{1}{2}C\left(\frac{2V_0}{3}-\frac{V_0}{3}\right)^2 + \frac{1}{2}C\left(V_0-\frac{2V_0}{3}\right)^2\\ &= \frac{1}{6}CV_0^2.\end{aligned}$$

20. Instantaneous charge on the capacitor during charging, $Q = Q_0\left(1-e^{-t/\tau}\right)$, where $Q_0 = CV$ = steady state charge.
At $t = 2\tau$, $Q = CV(1-e^{-2})$.

21. With S_1 open and S_2 closed, LC oscillation in the circuit gets set in, for which $Q = Q_0 \cos \omega t$, where $\omega^2 = \frac{1}{LC}$, $Q_0 = CV$.

Now, $I = -\frac{dQ}{dt} = CV\,\omega \sin \omega t = CV\frac{1}{\sqrt{LC}}\sin \omega t.$

$\therefore\ I_{max} = V\sqrt{\frac{C}{L}}.$

22. Instantaneous charge, $Q = Q_0 \cos \omega t$.

Differentiating, $\frac{d^2Q}{dt^2} = -\omega^2 Q_0 \cos \omega t = -\omega^2 Q$

$\Rightarrow\ Q = -\frac{1}{\omega^2}\frac{d^2Q}{dt^2} = -LC\frac{d^2Q}{dt^2}.$

23. Absence of friction, hence no power loss.

24. According to Lenz's law, drag is experienced by the train.

25. The upward elevation is caused by the induced electric field.

5.5 Ray Optics and Wave Optics

1. A metamaterial has negative refractive index.

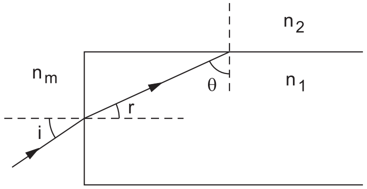
Now, $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1.$

$\because\ n_2$ is negative, so θ_2 is negative.

2. $n = \frac{c}{v}$, for metamaterial. $\therefore\ v = \frac{c}{|n|}.$

3. For total internal reflection, $\theta \geq C$.

$\Rightarrow\ 90° - r \geq C$



$\Rightarrow$ $\sin(90° - r) \geq \sin C$

$\Rightarrow$ $\cos r \geq \sin C$.

From Snell's law,

$$n_m \sin i_m \equiv n_1 \sin r$$

and $\sin C = \dfrac{n_2}{n_1}$

$$\Rightarrow \quad \sin^2 i_m = \frac{n_1^2}{n_m^2}(1 - \cos^2 r) = \frac{n_1^2}{n_m^2}(1 - \sin^2 C)$$

$$\Rightarrow \quad \text{NA} = \sin i_m = \sqrt{\frac{n_1^2}{n_m^2}\left(1 - \frac{n_2^2}{n_1^2}\right)} = \frac{\sqrt{n_1^2 - n_2^2}}{n_m}.$$

Putting the proper values, the correct options are found to be (a) and (c).

4. For total internal reflection to take place in both the structures, the numerical aperture should be the least one for the combined structure. Hence, the correct option is (d).

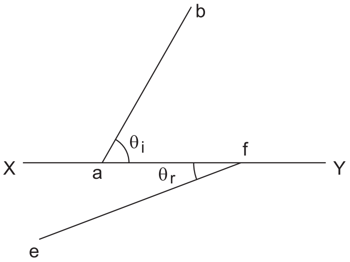
5. The normal to the wavefront represents the direction of the rays. In both the media the wavefronts are parallel, so light travels as parallel beam in each case.

6. We know that a wavefront is represented as a surface with identical phase during wave propagation. Hence,

$$\phi_c = \phi_d \text{ and } \phi_e = \phi_f$$

$\Rightarrow$ $\phi_d - \phi_f = \phi_c - \phi_e$.

7. The angle between the incident wavefront (ab) and the refracting surface (XY) is the angle of incidence (θ_i). Similarly, θ_r is the angle of refraction between XY and the refracted wave front ef.



From Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow \frac{c_0}{c_1}\sin\theta_1 = \frac{c_0}{c_2}\sin\theta_2,$$

where c's represent the speed of light.

$$\Rightarrow \quad \frac{1}{c_1}\sin\theta_1 = \frac{1}{c_2}\sin\theta_2 \Rightarrow \frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2}.$$

$\because$ $\theta_1 > \theta_2$, hence $c_1 > c_2$.

Hence, the speed of light is greater in Medium 1.

5.6 Modern Physics

1. According to Bohr quantum condition, angular momentum $L = \frac{nh}{2\pi}$.

Hence, rotational kinetic energy $= \frac{L^2}{2I} = \left(\frac{nh}{2\pi}\right)^2 \frac{1}{2I} = n^2 \frac{h^2}{8\pi^2 I}$.

2. Rotational KE for the CO molecule in the nth state is $E_n = n^2 \frac{h^2}{8\pi^2 I}$.

Hence for excitation from ground state ($n = 1$) to the first excited state ($n = 2$), we have

$$\frac{4h}{\pi} \times 10^{11} = \frac{(2^2 - 1^2)h^2}{8\pi^2 I}.$$

Simplifying, the moment of inertia of CO molecule about its centre of mass,

$$I = \left(\frac{3h}{8\pi}\right)\left(\frac{10^{-11}}{4}\right) \text{kg m}^2 = 1.87 \times 10^{-46} \text{ kg m}^2.$$

3. Moment of inertia $I = \frac{m_1 m_2}{m_1 + m_2} r^2$.

$$\therefore \qquad r = \sqrt{\frac{I(m_1 + m_2)}{m_1 m_2}}.$$

Substituting the values,

$$r = \sqrt{\frac{(1.87 \times 10^{-46} \text{ kg m}^2)(12 \text{ amu} + 16 \text{ amu})}{(12 \text{ amu})(16 \text{ amu})}}$$

$$= \sqrt{\frac{(1.87 \times 10^{-46})(28)}{(12 \times 16) \times \frac{5}{3} \times 10^{-27}}} \text{ m} = 1.3 \times 10^{-10} \text{ m}.$$

4. $0 \leq (\text{KE})_{\beta^-} \leq Q - (\text{KE})_p - (\text{KE})_{\bar{\nu}}$

$\Rightarrow 0 \leq (\text{KE})_{\beta^-} < Q.$

5. $Q = (0.8)(10^6 \text{ eV})$.

$(\text{KE})_p + (\text{KE})_{\beta^-} + (\text{KE})_{\bar{\nu}} = Q.$

$(\text{KE})_p$ is nearly zero, when $(\text{KE})_{\beta^-}$ is zero.

So, $(\text{KE})_{\bar{\nu}} = Q - (\text{KE})_p \simeq Q (\simeq 0.8 \times 10^6 \text{ eV})$.

6. For the nuclear reaction ${}^{6}_{3}\mathrm{Li} \to {}^{4}_{2}\mathrm{He} + {}^{2}_{1}\mathrm{H}$,

$$\frac{Q}{c^2} = 6.01523 - 4.002603 - 2.014102 = -0.001582.$$

This being negative, α-decay is not possible.

For ${}^{210}_{84}\mathrm{Po} \to {}^{1}_{1}\mathrm{H} + {}^{209}_{83}\mathrm{Bi}$,

$$\frac{Q}{c^2} = 209.9828766 - 1.007825 - 208.980388 = -0.005337 < 0.$$

This reaction is also not possible.

For ${}^{2}_{1}\mathrm{H} + {}^{4}_{2}\mathrm{He} \to {}^{6}_{3}\mathrm{Li}$,

$$\frac{Q}{c^2} = 2.014102 + 4.002603 - 6.015123 = 0.001627 > 0.$$

This reaction is possible [option (c)].

For ${}^{70}_{30}\mathrm{Ze} + {}^{82}_{34}\mathrm{Se} \to {}^{152}_{64}\mathrm{Ge}$,

$$\frac{Q}{c^2} = 69.925325 + 81.916709 - 151.919803 = -0.077769 < 0.$$

This reaction is not possible.

7. For ${}^{210}_{84}\mathrm{Po} \to {}^{4}_{2}\mathrm{He} + {}^{206}_{82}\mathrm{Pb}$,

$$Q = (209.982876 - 4.002603 - 205.97455)\,c^2 = 5.422 \text{ MeV}.$$

Conserving linear momentum,

$$\sqrt{2K_1(4)} = \sqrt{2K_2(206)}$$

$$\Rightarrow \quad 4K_1 = 206K_2.$$

$$\therefore \quad K_1 = \frac{103}{2}K_2.$$

But, $K_1 + K_2 = 5.422$ MeV

$$\Rightarrow \quad K_1 + \frac{2}{103}K_1 = 5.422 \text{ MeV}$$

$$\Rightarrow \quad \frac{105}{103}K_1 = 5.422 \text{ MeV}$$

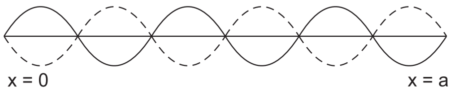
$$\Rightarrow \quad K_1 = 5.319 \text{ MeV} = 5319 \text{ keV}.$$

8. If n segments are contained in length a then $a = n\left(\frac{\lambda}{2}\right)$

$\Rightarrow \quad \lambda = \dfrac{2a}{n} = \dfrac{h}{p}$, so the linear momentum $p = n\dfrac{h}{2a}$.

$\Rightarrow$ Kinetic energy, $E = \dfrac{p^2}{2m} = \dfrac{n^2h^2}{8ma^2}$.

$\therefore$ $E \propto a^{-2}$.



9. For the ground state, $n = 1$,

$$E_1 = \frac{h^2}{8\,ma^2} = \frac{(6.6 \times 10^{-34}\text{ J s})^2}{8(1 \times 10^{-30}\text{ kg})\,(6.6 \times 10^{-9}\text{ m})^2} = 0.125 \times 10^{-20}\text{ J}$$

$$= \frac{0.125 \times 10^{-20}}{e}\text{ eV} \approx 8\text{ meV}.$$

10. $\because$ $p = mv = n\dfrac{h}{2a}$,

$\therefore$ velocity, $v = n\dfrac{h}{2ma}$.

$\therefore$ $v \propto n$.

11. Fusion is a nuclear reaction (as in the sun) which requires a very high temperature.

12. Mean translational kinetic energy per molecule = $\dfrac{3}{2}$ kT, where $k\left(=\dfrac{R}{N}\right)$ is the Boltzmann constant.

Now, $2\left(\dfrac{3}{2}\,kT\right) = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{e^2}{d}\right)$.

Solving, we get $T = 1.4 \times 10^9$ K, which exists in the temperature range of $1.0 \times 10^9\text{ K} < T < 2.0 \times 10^9$ K.

13. As given, the Lawson number $nt_0 > 5 \times 10^{14}\text{ s cm}^{-3}$.

The deuteron density $n = 8 \times 10^{14}\text{ cm}^{-3}$ and the confinement time $t_0 = 9 \times 10^{-1}$ s satisfies the requirement.

❑