Surface Area & Volume of Solids

Surface Area of Right Circular Cylinders

We come across many objects in our surroundings which are cylindrical, i.e., shaped like a **cylinder**, for example, pillars, rollers, water pipes, tube lights, cold-drink cans and LPG cylinders. This three-dimensional figure is found almost everywhere.

We can easily make cylindrical containers using metal sheets of any length and breadth. Say we have to make an open metallic cylinder (as shown below) of radius 14 cm and height 40 cm. How will we calculate the dimensions of the metal required for making this specific cylinder?

We will do so by calculating the surface area of the required cylinder. This surface area will be equal to the area of metal sheet required to make the cylinder.

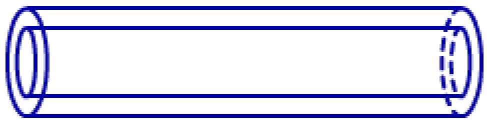
Knowledge of surface areas of three-dimensional figures is important in finding solutions to several real-life problems involving them. In this lesson, we will learn the formulae for the surface area of a right circular cylinder. We will also solve examples using these formulae.

Features of a right circular cylinder

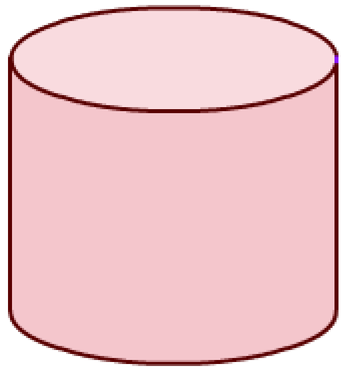
1. A right circular cylinder has two plane surfaces circular in shape.
2. The curved surface joining the plane surfaces is the lateral surface of the cylinder.
3. The two circular planes are parallel to each other and also congruent.
4. The line joining the centers of the circular planes is the axis of the cylinder.
5. All the points on the lateral surface of the right circular cylinder are equidistant from the axis.
6. Radius of circular plane is the radius of the cylinder.

Two types of cylinders are given below.

1. Hollow cylinder: It is formed by the lateral surface only. Example: A pipe



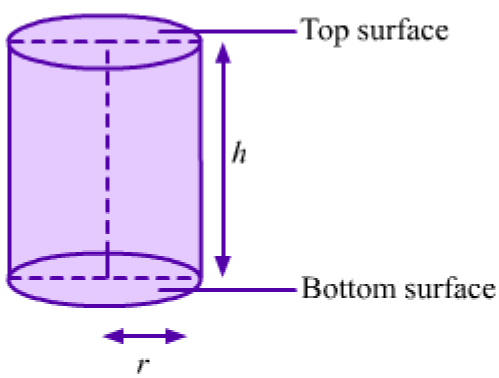
2. Solid cylinder: It is the region bounded by two circular plane surfaces with the lateral surface. Example: A garden roller



Solid cylinder

Formulae for the Surface Area of a Right Circular Cylinder

Consider a cylinder with base radius *r* and height *h*.



The formulae for the surface area of this cylinder are given as follows:

Curved surface area of the cylinder = $2\pi rh$

Area of two circular faces of cylinder = $2\pi r^2$

Total surface area of the cylinder = $2\pi r(r + h)$

Note: We take π as a constant and its value as 22/7 or 3.14.

Here, curved (or lateral) surface area refers to the area of the curved surface excluding the top and bottom surfaces. Total surface area refers to the sum of the areas of the top and bottom surfaces and the area of the curved surface.

Did You Know?

Pi

- Pi is a mathematical constant which is equal to the ratio of the circumference of a circle to its diameter.
- It is an irrational number represented by the Greek letter ‘π’ and its value is approximately equal to 3.14159.
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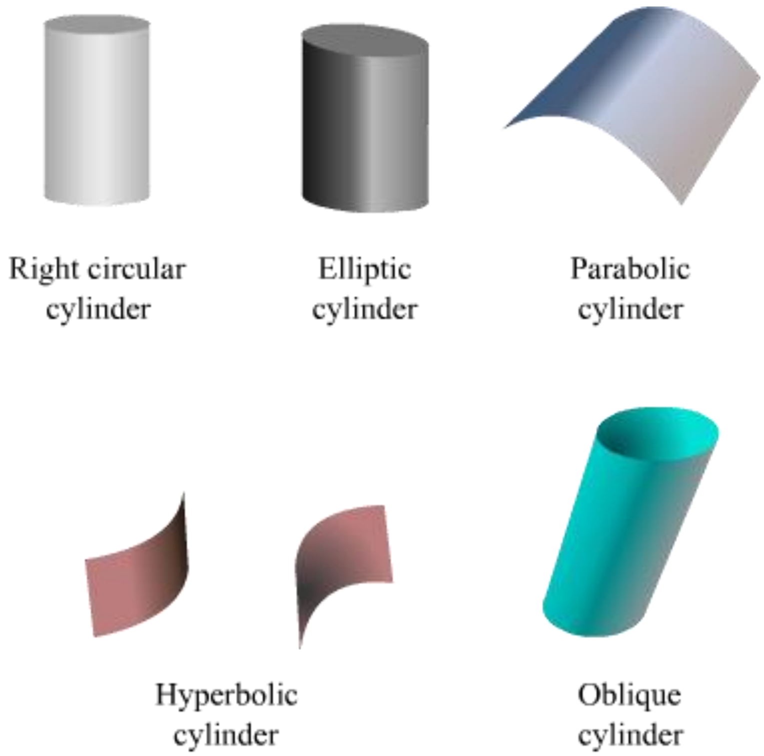
- William Jones (1706) was the first to use the Greek letter to represent this number.
- Pi is also called ‘Archimedes’ constant’ or ‘Ludolph’s constant’.
- Pi is a ‘transcendental number’, which means that it is not the solution of any finite polynomial with whole numbers as coefficients.
- Suppose a circle fits exactly inside a square; then, pi = $4\times\frac{\text{Area of the circle}}{\text{Area of the square}}$

Know Your Scientist

William Jones (1675−1749) was a Welsh mathematician who is primarily known for his proposal to use the Greek letter ‘π’ for representing the ratio of the circumference of a circle to its diameter. His book ***Synopsis Palmariorum Matheseos*** includes theorems on differential calculus and infinite series. In this book, π is used as an abbreviation for perimeter.

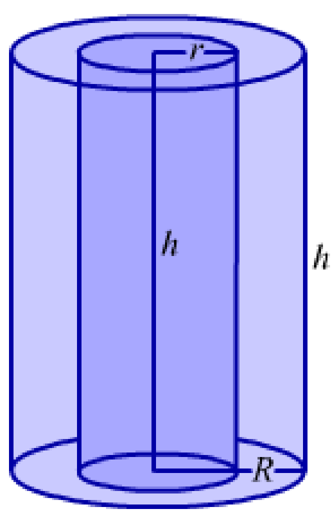
Whiz Kid

There are many types of cylinders—right circular cylinder (whose base is circular), elliptic cylinder (whose base is an ellipsis or oval), parabolic cylinder, hyperbolic cylinder, imaginary elliptic cylinder, oblique cylinder (whose top and bottom surfaces are displaced from each other), etc.



Formulae for the Surface Area of a Right Circular Hollow Cylinder

Consider a hollow cylinder of height *h* with external and internal radii *R* and *r* respectively,



Here, curved surface area, CSA = External surface area + Internal surface area

$$= 2\pi Rh + 2\pi rh$$
$$= 2\pi h\left(R + r\right)$$

Total surface area, TSA = Curved surface area + 2 × Base area

$$= 2\pi h\left(R + r\right) + 2 \times \left[\pi R^2 - \pi r^2\right]$$
$$= 2\pi h\left(R + r\right) + 2\pi\left(R + r\right)\left(R - r\right)$$
$$= 2\pi\left(R + r\right)\left(R - r + h\right)$$

Here, thickness of the hollow cylinder = $R - r$.

Example Based on the Surface Area of a Right Circular Cylinder

Solved Examples

Easy

Example 1:

The curved surface area of a right circular cylinder of height 7 cm is 44 cm^2. Find the diameter of the base of the cylinder.

Solution:

Let r be the radius and h be the height of the cylinder.

It is given that:

h = 7 cm

Curved surface area of the cylinder = 44 cm^2

So, $2\pi rh$ = 44 cm^2

$$\Rightarrow 2 \times \frac{22}{7} \times r \times 7 \text{ cm} = 44 \text{ cm}^2$$
$$\Rightarrow r = \frac{44 \times 7}{2 \times 22 \times 7} \text{ cm}$$
$$\Rightarrow \therefore r = 1 \text{ cm}$$

Thus, diameter of the base of the cylinder = $2r$ = 2 cm

Example 2:

The radii of two right circular cylinders are in the ratio 4 : 5 and their heights are in the ratio 3 : 1. What is the ratio of their curved surface areas?

Solution:

Let the radii of the cylinders be $4r$ and $5r$ and their heights be $3h$ and h.

Let S_1 be the curved surface area of the cylinder of radius $4r$ and height $3h$.

$\therefore S_1 = 2\pi \times 4r \times 3h = 24\pi rh$

Let S_2 be the curved surface area of the cylinder of radius $5r$ and height h.

$\therefore S_2 = 2\pi \times 5r \times h = 10\pi rh$

Now,

$$\frac{S_1}{S_2} = \frac{24\pi rh}{10\pi rh} = \frac{12}{5}$$
$$\Rightarrow S_1 : S_2 = 12 : 5$$

Thus, the curved surface areas of the two cylinders are in the ratio 12 : 5.

Medium

Example 1:

Find the height and curved surface area of a cylinder whose radius is 14 dm and total surface area is 1760 dm^2.

Solution:

Radius (r) of the cylinder = 14 dm

Let the height of the cylinder be h.

Total surface area of the cylinder = 1760 dm^2

So, $2\pi r (r + h) = 1760$ dm^2

$$\Rightarrow 2\times\frac{22}{7}\times 14(14+h)\,\text{dm} = 1760\ \text{dm}^2$$

$$\Rightarrow 14+h = \frac{1760\times 7}{2\times 22\times 14}\,\text{dm}$$

$$\Rightarrow 14+h = 20\ \text{dm}$$

$$\Rightarrow \therefore h = (20-14)\,\text{dm} = 6\ \text{dm}$$

Thus, the height of the cylinder is 6 dm.

Now, curved surface area of the cylinder = $2\pi rh$

$$= (2\times\frac{22}{7}\times 14\times 6)\ \text{dm}^2$$

$$= 528\ \text{dm}^2$$

Example 2:

There are ten identical cylindrical pillars in a building. If the radius of each pillar is 35 cm and the height is 12 m, then find the cost of plastering the surface of all the pillars at the rate of Rs 15 per m^2.

Solution:

Radius (r) of one pillar $= 35\ \text{cm} = \frac{35}{100}\ \text{m} = 0.35\ \text{m}$

Height (h) of one pillar = 12 m

∴ Curved surface area of one pillar = $2\pi rh$

$$= \left(2\times\frac{22}{7}\times 0.35\times 12\right)\ \text{m}^2$$

$$= 26.4\ \text{m}^2$$

⇒ Curved surface area of ten pillars = 10 × 26.4 m^2 = 264 m^2

Cost of plastering 1 m^2 of surface = Rs 15

⇒ Cost of plastering 264 m^2 of surface = Rs (15 × 264) = Rs 3960

Therefore, the cost of plastering the ten pillars of the building is Rs 3960.

Hard

Example 1:

A cylindrical road roller is of diameter 175 cm and length 1.5 m. It has to cover an area of 0.33 hectare on the ground. How many complete revolutions must the roller take to cover the ground? (1 hectare = 10000 m^2)

Solution:

Diameter of the cylindrical roller $= 175 \text{ cm} = \frac{175}{100} \text{ m} = \frac{7}{4} \text{ m}$

∴ Radius (r) of the cylindrical roller = $\frac{7}{8}$ m

Length (h) of the cylindrical roller = 1.5 m

Area covered by the roller in one complete revolution = Curved surface area of the roller

= $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times \frac{7}{8} \times 1.5\right) \text{ m}^2$$
$$= 8.25 \text{ m}^2$$

Area of the ground to be covered = 0.33 hectare = 0.33 × 10000 m^2 = 3300 m^2

∴ Number of complete revolutions $= \frac{\text{Area of the ground covered by the roller}}{\text{Area covered by the roller in one revolution}}$

$$= \frac{3300 \text{m}^2}{8.25 \text{m}^2}$$
$$= 400$$

Thus, the roller must take 400 complete revolutions to cover the ground.

Example 2:

The internal diameter, thickness and height of a hollow cylinder are 20 cm, 1 cm and 25 cm respectively. What is the total surface area of the cylinder?

Solution:

Internal diameter of the cylinder = 20 cm

∴ Internal radius (r) of the cylinder $= \frac{20}{2} \text{ cm} = 10 \text{ cm}$

Thickness of the cylinder = 1 cm

∴ External radius (R) of the cylinder = (10 + 1) cm = 11 cm

Height (h) of the cylinder = 25 cm

Internal curved surface area of the cylinder = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 10 \times 25\right) \text{ cm}^2$$
$$= \frac{11000}{7} \text{cm}^2$$

External curved surface area of the cylinder = $2\pi Rh$

$$= \left(2 \times \frac{22}{7} \times 11 \times 25\right) \text{ cm}^2$$
$$= \frac{12100}{7} \text{cm}^2$$

The two bases of the cylinder are ring-shaped. Therefore, their area is given as follows:

Area of base = $\pi (R^2 - r^2)$

$$= \left[\frac{22}{7}\left(11^2 - 10^2\right)\right] \text{ cm}^2$$
$$= \left(\frac{22}{7} \times 21\right) \text{ cm}^2$$
$$= 66 \text{ cm}^2$$

So, total surface area of the cylinder = Internal CSA + External CSA + 2 × Area of base

$$= \left(\frac{11000}{7} + \frac{12100}{7} + 2 \times 66 \right) \text{ cm}^2$$

$$= \left(\frac{23100}{7} + 132 \right) \text{ cm}^2$$

$$= (3300 + 132) \text{ cm}^2$$

$$= 3432 \text{ cm}^2$$

Volume of a Right Circular Cylinder

Water tanks like the ones shown below are a common enough sight.

Clearly, these tanks are cylindrical or shaped like a cylinder. The choice of this shape for a water tank (and many other storage containers) is because a cylinder provides a large volume for a little surface area.

Also, this shape can withstand much more pressure than a cube or a cuboid, which makes it easy to transport. Another example of a cylindrical storage container is the LPG cylinder.

The amount of space occupied by a water tank is the same as the volume of the tank. So, to find the capacity or the amount of space occupied by a tank, we need to find the volume of the tank.

In this lesson, we will learn the formula to calculate the volume of a right circular cylinder and solve some examples using the same.

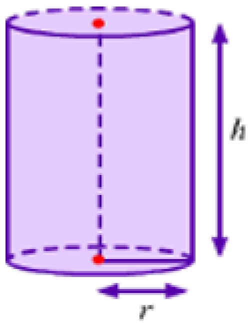
Did You Know?

LPG tanks are cylinder-shaped so that they can withstand the pressure inside them. If these tanks were square or rectangular in shape, then an increase in pressure inside them would cause the tanks to reform themselves so as to gain a rounded shape.

This, in turn, could result in leakage at the corners. Actual LPG tanks are designed to have no corners.

Formula for the Volume of a Right Circular Cylinder

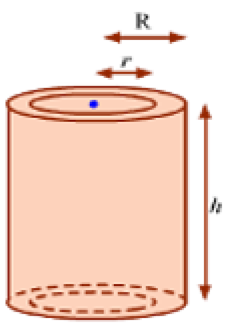
Consider a solid cylinder with *r* as the radius of the circular base and *h* as the height.



The formula for the volume of this right circular solid cylinder is given as follows:

Volume of the solid cylinder = Area of base × Height

Volume of the solid cylinder = $\pi r^2 h$



Consider a hollow cylinder with internal and external radii as *r* and *R* respectively, and height as *h*.

The formula for the volume of this right circular hollow cylinder is given as follows:

Volume of the hollow cylinder = $\pi (R^2 - r^2) h$

In right prisms, top and base surfaces are congruent and parallel while lateral faces are perpendicular to the base. Thus, their volumes can also be calculated in the same manner as that of right cylinders.

Volume of the right prism = Area of base × Height

Did You Know?

The volume of a pizza (which is always cylindrical in shape) is hidden in its name itself. If we take the radius of a pizza to be ‘z’ and its thickness to be ‘a’, then its volume is ‘$\pi z^2 a$’ or ‘**pi.z.z.a**’.

Solved Examples

Easy

Example 1:

A cylindrical tank can hold 11000 L of water. What is the radius of the base of the tank if its height is 3.5 m?

Solution:

Let r be the radius of the base of the cylindrical tank.

Height (h) of the tank = 3.5 m

Volume of the tank = 11000 L = 11 m^3 (∵ 1000 L = 1 m^3)

Volume of a cylinder = $\pi r^2 h$

In this case, we have

$$\pi r^2 h = 11\ \text{m}^3$$
$$\Rightarrow \left(\frac{22}{7} \times r^2 \times 3.5\ \text{m}\right) = 11\ \text{m}^3$$
$$\Rightarrow 11\, r^2 = 11\ \text{m}^2$$

$\Rightarrow r = 1$ m

Thus, the radius of the base of the cylindrical tank is 1 m.

Example 2:

What is the height of a cylinder whose volume is 6.16 m^3 and the diameter of whose base is 28 dm?

Solution:

Diameter of the base of the cylinder = 28 dm

∴ Radius (r) of the base $=\left(\frac{28}{2}\right)$ dm

$$=14 \text{ dm}$$
$$=\left(\frac{14}{10}\right) \text{ m} \qquad \left(\because 1 \text{ dm} = \frac{1}{10} \text{ m}\right)$$
$$=1.4 \text{ m}$$

Volume of the cylinder = 6.16 m^3

$$\Rightarrow \pi r^2 h = 6.16 \text{ m}^3$$
$$\Rightarrow \frac{22}{7} \times (1.4 \text{ m})^2 \times h = 6.16 \text{ m}^3$$
$$\Rightarrow h = \left[\frac{6.16 \times 7}{22 \times (1.4)^2}\right] \text{m}$$

⇒ h = 1 m

Thus, the height of the cylinder is 1 m.

Example 3:

The external diameter, thickness and length of a cylindrical water pipe are 22 cm, 1 cm, and 8 m respectively. What amount of material went into making this pipe?

Solution:

External diameter of the hollow cylindrical pipe = 22 cm

∴ External radius, R = $\left(\frac{22}{2}\right) \text{cm} = 11 \text{ cm}$

Thickness of the pipe = 1 cm

∴ Internal radius, r = (11 − 1) cm = 10 cm

Length (h) of the pipe = 8 m = (8 × 100) cm = 800 cm (∵ 1 m = 100 cm)

∴ Volume of the material used to make the pipe $= \pi\left(R^2 - r^2\right)h$

$$= \left[\frac{22}{7} \times \left(11^2 - 10^2\right) \times 800\right] \text{cm}^3$$

$$= \left[\frac{22}{7} \times 21 \times 800\right] \text{cm}^3$$

$$= 52800 \text{ cm}^3$$

Thus, 52800 cm^3 of material was used to make the water pipe.

Medium

Example 1:

The diameter and height of a solid metallic cylinder are 21 cm and 25 cm respectively. If the mass of the metal is 8 g per cm^3, then find the mass of the cylinder.

Solution:

Diameter of the cylinder = 21 cm

∴ Radius (r) of cylinder = $\left(\frac{21}{2}\right) \text{cm}$

Height (h) of the cylinder = 25 cm

To find the mass of the metallic cylinder, we have to first find the volume of the cylinder.

Volume of the cylinder $= \pi r^2 h$

$$= \left[\frac{22}{7} \times \left(\frac{21}{2}\right) \times \left(\frac{21}{2}\right) \times 25\right] \text{cm}^3$$

$$= 8662.5 \text{ cm}^3$$

Mass of 1 cm^3 of the metal = 8 g

∴ Mass of 8662.5 cm^3 of the metal= (8662.5 × 8) g

$$= 69300 \text{ g}$$

$$= \left(\frac{69300}{1000}\right) \text{kg} \qquad \left(\because 1 \text{ g} = \frac{1}{1000} \text{kg}\right)$$

$$= 69.3 \text{ kg}$$

Thus, the mass of the cylinder is 69.3 kg.

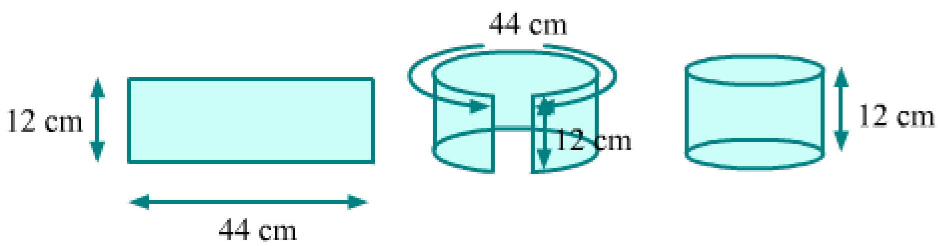
Example 2:

A rectangular sheet of paper is folded to form a cylinder of height 12 cm. If the length and breadth of the sheet are 44 cm and 12 cm respectively, then find the volume of the cylinder.

Solution:

Height (*h*) of the cylinder = 12 cm

Let *r* be the radius of the cylinder. We can find this value from the circumference of the base of the cylinder. As shown in the figure, this circumference is nothing but the length of the sheet.



So, circumference of the base of the cylinder = 44 cm

$$\Rightarrow 2\pi r = 44$$
$$\Rightarrow r = \frac{44}{2\pi}$$
$$\Rightarrow r = \frac{44 \times 7}{2 \times 22}$$

$$\Rightarrow r = 7cm$$

Now, volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 12 \text{ cm}^3$$
$$= 1848 \text{ cm}^3$$

Hard

Example 1:

The inner and outer diameters of a cylindrical iron pipe are 54 cm and 58 cm respectively and its length is 5 m. What is the mass of the pipe if 1 cm^3 of iron has a mass of 8 g?

Solution:

Inner diameter of the hollow cylindrical iron pipe = 54 cm

∴ Inner radius, $r = \left(\frac{54}{2}\right) \text{cm} = 27 \text{ cm}$

Outer diameter of the pipe = 58 cm

∴ Outer radius, $R = \left(\frac{58}{2}\right) \text{cm} = 29 \text{ cm}$

Length (h) of the pipe = 5 m = (5 × 100) cm = 500 cm

∴ Volume of the pipe $= \pi\left(R^2 - r^2\right)h$

$$= \left[\frac{22}{7} \times \left(29^2 - 27^2\right) \times 500\right] \text{cm}^3$$
$$= \left[\frac{22}{7} \times 112 \times 500\right] \text{cm}^3$$
$$= 176000 \text{ cm}^3$$

Mass of 1 cm^3 of iron = 8 g

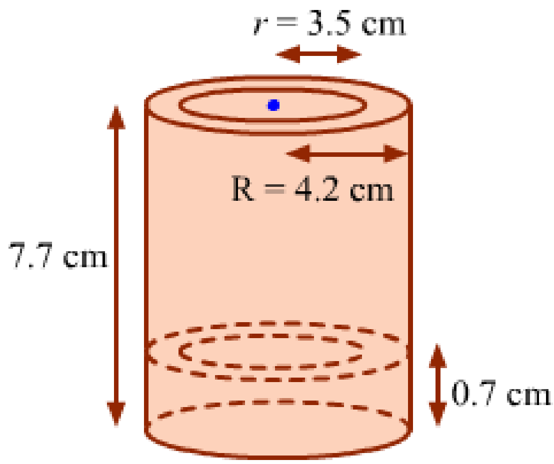
∴ Mass of 176000 cm^3 of iron = (8 × 176000) g

$$= \left(\frac{8 \times 176000}{1000}\right) \text{kg} \qquad \left(\because 1 \text{ g} = \frac{1}{1000} \text{kg}\right)$$
$$= 1408 \text{ kg}$$

Thus, the mass of the hollow cylindrical iron pipe is 1408 kg.

Example 2:

The internal and external radii of a cylindrical juice can (as shown in the figure) are 3.5 cm and 4.2 cm respectively. The total height of the can is 7.7 cm. The thickness of the base (i.e., a solid cylinder) is 0.7 cm. If the mass of the material used in the can is 3 g per cm^3, then find the mass of the can.



Solution:

To find the mass of the juice can, we need to first find its volume.

The juice can shown in the figure contains two cylinders. One is a solid cylinder (i.e., the base of the can) and the other is a hollow cylinder (i.e., the cylindrical part that stands on the base).

External radius (R) of the hollow cylinder = 4.2 cm

Internal radius (r) of the hollow cylinder = 3.5 cm

Thickness (h) of the base = 0.7 cm (i.e., the height of the solid cylinder)

Total height (H) of the juice can = 7.7 cm

∴ Height (h') of the hollow cylinder = (7.7 − 0.7) cm = 7 cm

Volume of the juice can = Volume of the solid base + Volume of the hollow cylinder on the base

$$= \pi R^2 h + \pi\left(R^2 - r^2\right)h'$$

$$= \pi\left[R^2 h + \left(R^2 - r^2\right)h'\right]$$

$$= \frac{22}{7}\left[(4.2)^2 \times 0.7 + \left\{(4.2)^2 - (3.5)^2\right\} \times 7\right]\text{cm}^3$$

$$= \frac{22}{7}(12.348 + 5.39 \times 7)\text{cm}^3$$

$$= \frac{22}{7}(12.348 + 37.73)\text{cm}^3$$

$$= \left(\frac{22}{7} \times 50.078\right)\text{cm}^3$$

$$= 157.388 \text{ cm}^3$$

Mass of the material per cm^3 = 3 g

∴ Mass of the material used in the container = (3 × 157.388) g

= 472.164 g

Thus, the mass of the juice can is 472.164 g.

Example 3:

A well 3.5 m in diameter and 20 m deep is dug in a rectangular field of dimensions 20 m × 14 m. The earth taken out is spread evenly across the field. Find the level of earth raised in the field.

Solution:

Length (l) of the field = 20 m

Breadth (b) of the field = 14 m

Diameter (d) of the well = 3.5 m

∴ Radius (r) of the well= $\frac{3.5}{2}\text{m}$

Depth (h) of the well = 20 m

Volume of the dug out earth = $\pi r^2 h$

Now, the area of the field on which the dug out earth is spread is given by the difference between the area of the entire field and the area of the field covered by the cross-section of the well.

$\Rightarrow l \times b - \pi r^2$

Let H be the level of earth raised in the field.

Volume of earth spread in the field = Volume of the dug out earth

$$\Rightarrow \left(l \times b - \pi r^2\right)H = \pi r^2 h$$
$$\Rightarrow \left(20 \times 14 - \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2}\right)H = \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} \times 20$$
$$\Rightarrow \left(280 - \frac{269.5}{28}\right)H = \frac{5390}{28}$$
$$\Rightarrow \left(280 - \frac{77}{8}\right)H = \frac{385}{2}$$
$$\Rightarrow \frac{2163}{8}H = \frac{385}{2}$$
$$\Rightarrow H = \frac{385}{2} \times \frac{8}{2163}$$

$$\Rightarrow H = 0.71197 \text{ m} \approx 0.712 \text{ m}$$

Therefore, the level of earth in the field is raised by about 0.712 m.

Surface Areas of Cones

Surface Area of a Right Circular Cone

Traffic cones, conical tents, party hats, ice cream cones are some examples of objects shaped like a **cone**. The knowledge of the surface area of a cone is essential in the manufacture of such conical objects. Take, for example, the following case.

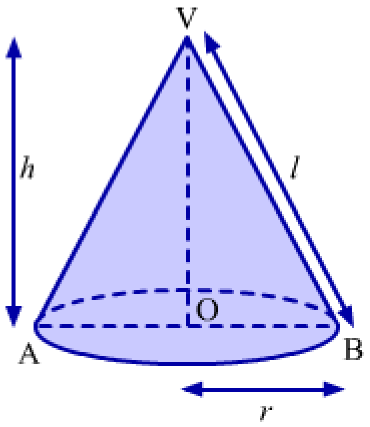
X Ltd. is a company that organizes adventures trips. It has a contract with Y Ltd., a company that manufactures tents. Y Ltd. uses canvas to make the specific conical tents ordered by X Ltd. Now, the area of canvas required to make one such conical tent is exactly equal to the surface area of the conical tent.

Thus, Y Ltd is able to order the required amount of canvas from the market to prepare the tents according to the specifications.

This is just one of the many examples from real life involving the concept of surface area. In this lesson, we will learn the formulae for the surface area of a right circular cone. We will also apply the formulae in solving a few examples.

Formulae for the Surface Area of a Right Circular Cone

Consider a cone with a base radius *r*, height *h* and slant height *l*.



The fixed point V is the **vertex** of the cone and the fixed line VO is the **axis** of the cone.

The length of line segment joining the vertex to the centre O of the base is called the **height of the base** and the length of the line segment joining the vertex to any point on the circular edge of the base is called the **slant height** of the cone.

The relation between the height, radius and slant height of the cone is: $\boldsymbol{l^2 = r^2 + h^2}$.

The formulae for the surface area of the given cone are given as follows:

Curved surface area of the cone = $\boldsymbol{\pi r l}$

Total surface area of the cone = π*r* (*l* + *r*)

Here, curved (or lateral) surface area refers to the area of the curved surface excluding the base, and total surface area refers to the sum of the area of the base and the area of the curved surface.

Did You Know?

A cone is the shape obtained by rotating a right triangle around one of its two shorter sides.

Example Based on the Surface Area of a Right Circular Cone

Did You Know?

A cone is a three-dimensional geometric figure that does not have uniform or congruent cross-sections.

Largest Cone Cut Out from a Cylinder

Solved Examples

Easy

Example 1:

The curved surface area of a cone is 1914 cm^2 and its base radius is 21 cm. Find

i)the slant height of the cone.

ii)the total surface area of the cone.

Solution:

i) Radius (r) of the cone = 21 cm

Curved surface area of the cone = 1914 cm^2

Let the slant height of the cone be l.

$\therefore \pi rl = 1914\ cm^2$

$$\Rightarrow \left(\frac{22}{7} \times 21 \times l\right) = 1914$$
$$\Rightarrow l = \frac{1914}{66} \text{cm}$$
$$\Rightarrow \therefore l = 29 \text{ cm}$$

Thus, the slant height of the cone is 29 cm.

ii)Total surface area of the cone = π*r* (*l* + *r*)

$$= \left[\frac{22}{7} \times 21 \times (29+21)\right] \text{ cm}^2$$
$$= \frac{22}{7} \times 21 \times 50 \text{ cm}^2$$
$$= 3300 \text{ cm}^2$$

Example 2:

The total surface area of a cone is 33264 cm^2 and its base radius and slant height are in the ratio 3 : 5. Find the slant height of the cone.

Solution:

Let the radius and slant height of the cone be 3*x* and 5*x* respectively.

Total surface area of the cone = 33264 cm^2

$$\Rightarrow \pi r (l + r) = 33264 \text{cm}^2$$
$$\Rightarrow \frac{22}{7} \times 3x(5x + 3x) = 33264 \text{cm}^2$$
$$\Rightarrow 24x^2 = \frac{33264 \times 7}{22} \text{ cm}^2$$
$$\Rightarrow x^2 = \frac{33264 \times 7}{22 \times 24} \text{cm}^2$$
$$\Rightarrow x^2 = 441 \text{ cm}^2$$
$$\Rightarrow \therefore x = \sqrt{441} \text{ cm} = 21 \text{ cm}$$

So, slant height of the cone = 5*x* = 5 × 21 cm = 105 cm

Medium

Example 1:

The height and radius of the base of a conical tomb are 8 m and 6 m respectively. Find the cost of whitewashing the outer surface of the tomb at the rate of Rs 2000 per 50 m^2.

Solution:

Radius (r) of the conical tomb = 6 m

Height (h) of the base of the conical tomb = 8 m

Let the slant height of the conical tomb be l.

We know that $l^2 = r^2 + h^2$

$\Rightarrow l^2 = (6^2 + 8^2)\ m^2$

$\Rightarrow l^2 = (36 + 64)\ m^2$

$\Rightarrow l^2 = 100\ m^2$

$\Rightarrow \therefore l = \sqrt{100}\ \text{m} = 10\ \text{m}$

∴ Curved surface area of the conical tomb = $\pi r l$

$= \left(\frac{22}{7} \times 6 \times 10\right)\ \text{m}^2$

$= 188.57\ \text{m}^2$

Cost of whitewashing 50 m^2 of surface = Rs 2000

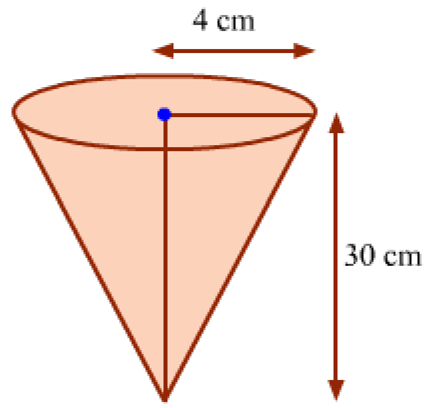
⇒ Cost of whitewashing 1 m^2 of surface = $\text{Rs}\ \frac{2000}{50} = \text{Rs}\ 40$

⇒ Cost of whitewashing 188.57 m^2 of surface = 188.57 × Rs 40 = Rs 7542.80

Thus, the cost of whitewashing the outer surface of the tomb is Rs 7542.80.

Example 2:

A corncob (which is shaped like a cone) is of length 30 cm and the radius of its broadest end is 4 cm. If about 5 grains are present per square centimetre of the cob, then approximately how many grains are there on the entire cob?



Solution:

Total grains on the cob = Curved surface area of the cob × Number of grains per cm^2

Radius (r) of the cob = 4 cm

Height (h) of the base of the cob = 30 cm

Let the slant height of the cob be l.

We know that $l^2 = r^2 + h^2$

$\Rightarrow l^2 = (4^2 + 30^2)\ cm^2$

$\Rightarrow l^2 = (16 + 900)\ cm^2$

$\Rightarrow l^2 = 916\ cm^2$

$\Rightarrow \therefore l = \sqrt{916}\ cm = 30.26\ cm$

Curved surface area of the cob = $\pi r l$

$$= \left(\frac{22}{7} \times 4 \times 30.26\right)\ cm^2$$
$$= 380.4\ cm^2$$

Total number of grains = 380.4 × 5 = 1902

Thus, there are about 1902 grains on the entire cob.

Hard

Example 1:

A cone and a cylinder have the same radius and height. If the ratio of the radius to height is 5 : 12, then find the ratio of the curved surface area of the cone to that of the cylinder.

Solution:

The cylinder and the cone have the same radius and height. Let *r* be this radius and *h* be the height.

It is given that:

$$\frac{r}{h} = \frac{5}{12}$$

⇒ $r = 5x$ and $h = 12x$, where x is any constant

Slant height (l) of the cone $= \sqrt{r^2 + h^2}$

$$= \sqrt{(5x)^2 + (12x)^2}$$
$$= \sqrt{25x^2 + 144x^2}$$
$$= \sqrt{169}x$$
$$= 13x$$

$$\therefore \frac{\text{Curved surface area of the cone}}{\text{Curved surface area of the cylinder}} = \frac{\pi rl}{2\pi rh}$$

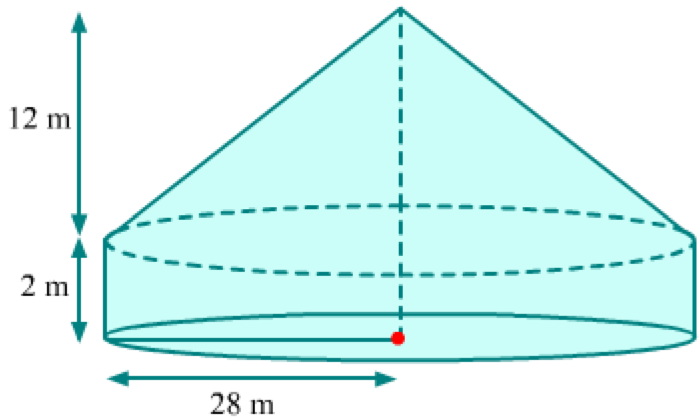
$$= \frac{13x}{2 \times 12x}$$
$$= \frac{13}{24}$$

Hence, the curved surface areas of the cone and the cylinder are in the ratio 13 : 24.

Example 2:

A cylindrical tent of height 2 m and radius 28 m is surmounted by a right circular cone. If the total height of the tent is 14 m and the cost of papering is Rs 3 per square metre, then calculate the total money spent in papering the inner side of the tent.

Solution:



Radius (r) of the cylindrical part = 28 m

Height (h) of the cylindrical part = 2 m

∴ Curved surface area of the cylindrical part = $2\pi rh$

$$= \left(2 \times \frac{22}{7} \times 28 \times 2\right) \text{ m}^2$$
$$= 352 \text{ m}^2$$

Radius of the base of the conical part = Radius of the cylindrical part = 28 m

Let H and l be respectively the height and slant height of the conical part.

H = (14 − 2) m = 12 m (âˆµ Total height of the tent = 14 m)

We know that $l^2 = r^2 + H^2$

$\Rightarrow l^2 = (28^2 + 12^2) \text{ m}^2$

$\Rightarrow l^2 = (784 + 144) \text{ m}^2$

$\Rightarrow l^2 = 928 \text{ m}^2$

$\Rightarrow \therefore l = \sqrt{928}\ \text{m} = 30.46\ \text{m}$

∴ Curved surface area of the conical part = π*rl*

$= \frac{22}{7} \times 28 \times 30.46\ \text{m}^2$
$= 2680.48\ \text{m}^2$

Total surface area = Sum of the curved surface areas of the cylindrical and conical parts

= (352 + 2680.48) m^2

= 3032.48 m^2

Cost of papering 1 m^2 of surface = Rs 3

⇒ Cost of papering 3032.48 m^2 of surface = Rs 3 × 3032.48 = Rs 9097.44

Thus, the total money spent in papering the inner side of the tent is Rs 9097.44.

Volume of Cone

Volume of a Right Circular Cone

Ice creams are loved by one and all. Take a look at the one shown.

Clearly, what is shown above is an ice cream cone, i.e., ice cream inside a crisp conical wafer. The amount of ice cream present in the cone is equal to the volume of the cone. In other words, the number of cubic units of ice cream that will exactly fill the cone is the volume of the cone.

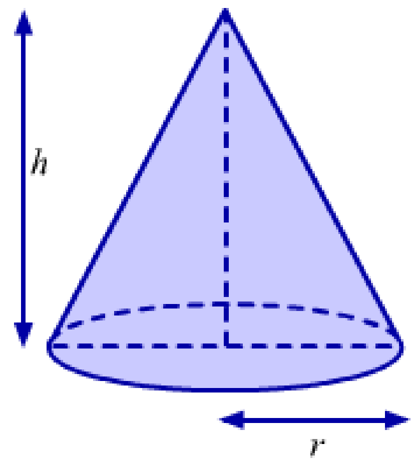
In this lesson, we will learn the formula for the volume of a right circular cone and solve examples using the same.

Did You Know?

A waffle maker named Ernest Hamwi is credited by a few to be the inventor of the ice cream cone. He is said to have come up with the idea in 1904 to help an ice cream vendor who had run out of dishes to serve ice cream.

Formula for the Volume of a Right Circular Cone

Consider a cone of radius *r* and height *h*.



The formula for the volume of this right circular cone is given as follows:

Volume of the cone = $\frac{1}{3}\pi r^2 h$

Using the above formula, we can find the cubic units of ice cream that exactly fill a cone.

Let us say the radius and height of an ice cream cone are 3.5 cm and 9 cm respectively. Then,

Volume of the cone = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 3.5\times 3.5\times 9\ \text{cm}^3 = 115.5\ \text{cm}^3$

Thus, the amount of ice-cream that exactly fills the cone is 115.5 cm^3.

Did You Know?

For a cone and a cylinder with the same base radius and height, the volume of the cone is one-third that of the cylinder.

Solved Examples

Easy

Example 1:

The height and slant height of a conical funnel are 21 cm and 29 cm respectively. How many litres of water can the funnel hold?

Solution:

The amount of water that the funnel can hold is equal to the volume of the funnel.

Height (h) of the funnel = 21 cm

Slant height (l) of the funnel= 29 cm

Let the radius of the circular base of the funnel be r.

Now, $l^2 = r^2 + h^2$

$\Rightarrow (29\text{ cm})^2 = r^2 + (21\text{ cm})^2$

$\Rightarrow 841\text{ cm}^2 = r^2 + 441\text{ cm}^2$

$\Rightarrow r^2 = (841 - 441)\text{ cm}^2$

$\Rightarrow r^2 = 400\text{ cm}^2$

$\Rightarrow r = \sqrt{400}\text{ cm} = 20\text{ cm}$

Volume of the funnel $= \frac{1}{3}\pi r^2 h$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 20 \times 20 \times 21\right)\text{cm}^3$$
$$= 8800\text{ cm}^3$$
$$= \frac{8800}{1000}\text{L} \qquad \left(\because 1\text{ cm}^3 = \frac{1}{1000}\text{L}\right)$$
$$= 8.8\text{ L}$$

Thus, the funnel can hold 8.8 L of water.

Example 2:

If A, B and C are respectively the height, volume and curved surface area of a cone, then prove that $3B(\pi A^3 + 3B) = C^2A^2$.

Solution:

It is given that A, B and C are respectively the height, volume and curved surface area of the cone.

Let r and l be the radius and slant height of the cone.

Now,

$$B = \frac{1}{3}\pi r^2 A \qquad \ldots(1)$$

$$C = \pi r l \qquad \ldots(2)$$

$$l = \sqrt{r^2 + A^2} \qquad \ldots(3)$$

We have to prove $3B(\pi A^3 + 3B) = C^2A^2$. Let us take the LHS of this equation.

$$\begin{aligned}
&3B\left(\pi A^3 + 3B\right) \\
&= 3\pi BA^3 + 9B^2 \\
&= 3\pi\left(\frac{1}{3}\pi r^2 A\right)\times A^3 + 9\left(\frac{1}{3}\pi r^2 A\right)^2 && \text{(Using equation 1)} \\
&= \pi^2 r^2 A^4 + \pi^2 r^4 A^2 \\
&= \pi^2 r^2 A^2\left(A^2 + r^2\right) \\
&= \pi^2 r^2 A^2 \times l^2 && \text{(Using equation 3)} \\
&= \pi^2 r^2 l^2 \times A^2 \\
&= C^2 A^2 && \text{(Using equation 2)} \\
&= \text{RHS}
\end{aligned}$$

Medium

Example 1:

The radius and slant height of a cone are in the ratio 3 : 5. If the volume of the cone is 12936 m^3, then find the radius, height and slant height of the cone.

Solution:

Let the radius (r) and slant height (l) of the cone be $3x$ and $5x$ respectively.

Let the height of the cone be h.

We know that $l^2 = r^2 + h^2$

$\Rightarrow (5x)^2 = (3x)^2 + h^2$

$\Rightarrow h^2 = 25x^2 - 9x^2$

$\Rightarrow h^2 = 16x^2$

$\Rightarrow h = \sqrt{16x^2} = 4x$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

It is given that the volume of the cone is 12936 m^3.

So, $\frac{1}{3}\pi r^2 h = 12936 \text{ m}^3$

$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times (3x)^2 \times (4x) = 12936 \text{ m}^3$$
$$\Rightarrow \frac{1}{3} \times \frac{22}{7} \times 9 \times 4 \times x^3 = 12936 \text{ m}^3$$
$$\Rightarrow x^3 = \frac{12936 \times 3 \times 7}{22 \times 9 \times 4} \text{m}^3$$
$$\Rightarrow x^3 = 343 \text{ m}^3$$
$$\Rightarrow x^3 = (7 \text{ m})^3$$

$$\Rightarrow x = 7m$$

Now, $r = 3x = (3 \times 7)$ m = 21 m

$h = 4x = (4 \times 7)$ m = 28 m

$l = 5x = (5 \times 7)$ m = 35 m

Thus, the radius, height and slant height of the cone are 21 m, 28 m and 35 m respectively.

Example 2:

If the radii and heights of two cones are in the ratios 2 : 3 and 5 : 4 respectively, then find the ratio of the volumes of the cones.

Solution:

Let r_1 and h_1 be the radius and height of one cone.

Let r_2 and h_2 be the radius and height of the other cone.

It is given that the radii of the cones are in the ratio 2 : 3.

$$\therefore \frac{r_1}{r_2} = \frac{2}{3}$$

It is given that the heights of the cones are in the ratio 5 : 4.

$$\therefore \frac{h_1}{h_2} = \frac{5}{4}$$

Ratio of the volumes of the cones $= \dfrac{\text{Volume of the first cone}}{\text{Volume of the second cone}}$

$$= \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right)$$

$$= \left(\frac{2}{3}\right)^2 \times \left(\frac{5}{4}\right)$$

$$= \frac{4}{9} \times \frac{5}{4}$$

$$= \frac{5}{9}$$

Thus, the volumes of the two cones are in the ratio 5 : 9.

Hard

Example 1:

Find the volume (in terms of π) of the solid figure obtained when a right triangle with sides 8 cm, 15 cm and 17 cm is revolved about the side

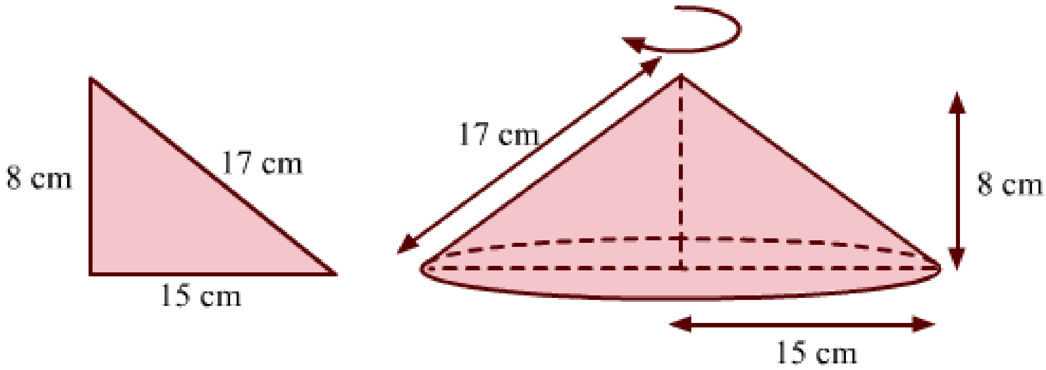
i) 8 cm.

ii) 15 cm.

Solution:

i) The sides of the given right triangle are 8 cm, 15 cm and 17 cm.

If this right triangle is revolved about the side 8 cm, then we will obtain a solid figure as is shown.



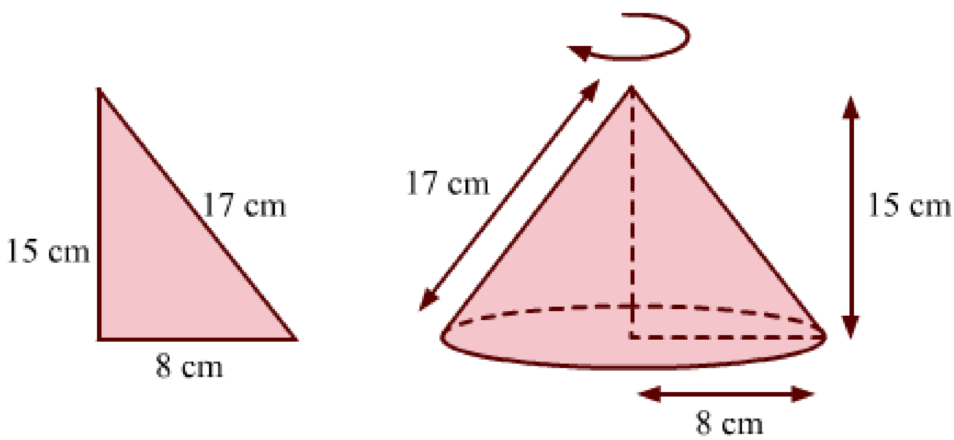
The solid figure so obtained is a cone.

The radius (*r*)and height (*h*) of the cone are 15 cm and 8 cm respectively.

∴ Volume of the cone formed $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 15 \times 15 \times 8 \text{ cm}^3$
$= 600\pi \text{ cm}^3$

ii) If the same right triangle is revolved about the side 15 cm, then we will obtain the following solid figure.



Again, the solid figure so obtained is a cone.

The radius (*r*)and height (*h*) of the cone are 8 cm and 15 cm respectively.

∴ Volume of the cone formed $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 8 \times 8 \times 15 \text{ cm}^3$$
$$= 320\pi \text{ cm}^3$$

Example 2:

The surface area of a sphere of radius 5 cm is five times the curved surface area of a cone of radius 4 cm. Find the

i) height of the cone.

ii) volume of the cone.

Solution:

i) Let r_1 be the radius of the sphere and r_2 be the radius of the cone.

Let h the height and l the slant height of the cone.

It is given that r_1 = 5 cm and r_2 = 4 cm

According to the question, we have:

Surface area of the sphere = 5 × Curved surface area of the cone

$$\Rightarrow 4\pi r_1^2 = 5 \times \pi r_2 l$$
$$\Rightarrow 4 \times (5 \text{ cm})^2 = 5 \times 4 \text{ cm} \times l$$

$$\Rightarrow l = 5cm$$

We know that $l = \sqrt{r_2^{\,2} + h^2}$

$$\Rightarrow (5 \text{ cm})^2 = (4 \text{ cm})^2 + h^2$$
$$\Rightarrow h^2 = 25 \text{ cm}^2 - 16 \text{ cm}^2$$
$$\Rightarrow h^2 = 9 \text{ cm}^2$$

$$\Rightarrow h = \sqrt{9}\, cm = 3cm$$

Thus, the height of the cone is 3 cm.

ii) Volume of the cone $= \frac{1}{3}\pi r_2^{\,2} h$

$$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3 \text{ cm}^3$$
$$= 50.29 \text{ cm}^3$$

Surface Areas of a Sphere and a Hemisphere

What images come to your mind when the word '**sphere**' is mentioned? The light hollow ball used in table tennis, the leather ball used in cricket, the inflatable balls used in the games of football and basketball and the heavy metallic shots used for shot-putting are all examples of the perfectly round three-dimensional shape called sphere.

Now, if you were to cut each of the objects mentioned above along its diameter, then you would obtain the three-dimensional figure called hemisphere.

As its name indicates, a hemisphere is the half of a sphere. Any sphere when cut along the diameter yields two equal hemispheres.

A sphere has only a curved surface; so, in its case, the total surface area is the same as the area of its curved surface.

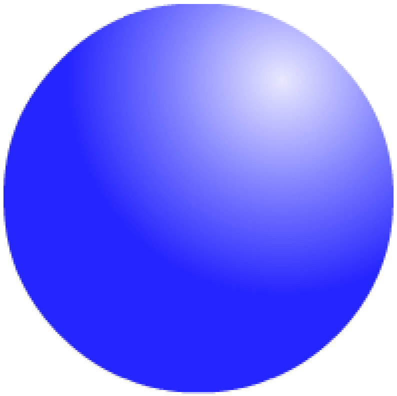
This, however, is not the case with a hemisphere. Consider the whole watermelon and the half of the same shown below.

Clearly, the whole watermelon has only a curved exterior, but what about its half? Observe how the half of the watermelon has both a curved exterior and a flat surface.

So, in case of a hemisphere the total surface area is different from the area of its curved surface.

Sphere and hemisphere

A **sphere** is a solid described by the rotation of a semi-circle about a fixed diameter.

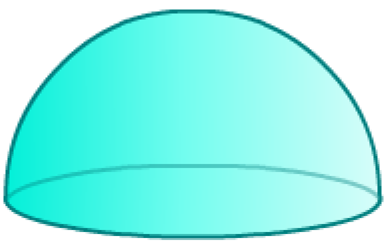


Sphere

Properties of a sphere:

1. A sphere has a centre.
2. All the points on the surface of the sphere are equidistant from the centre.
3. The distance between the centre and any point on the surface of the sphere is the radius of the sphere.

Hemisphere: A plane through the centre of the sphere divides it into two equal parts each is called a hemisphere.

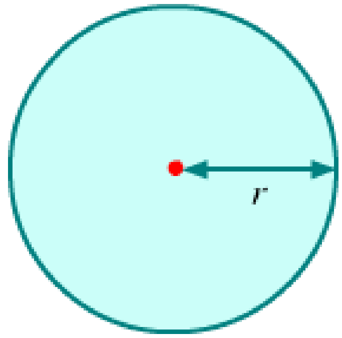


Hemisphere

In this lesson, we will learn the formulae for the surface areas of a sphere and a hemisphere. We will also solve problems using the same.

Formula for the Surface Area of a Sphere

Consider a sphere of radius *r*.



The formula for the surface area (curved or total) of this sphere is given as follows:

Surface area of a sphere = 4πr^2

As mentioned before, the total surface area of a sphere is the same as its curved surface area since a sphere has only a curved surface.

Did You Know?

Among all geometric shapes, a sphere has the smallest surface area for a given volume. Take, for example, bubbles and water droplets. Their spherical shape enables them to hold as much air as possible with the least surface area.

Example Based on the Surface Area of a Sphere

Solved Examples

Easy

Example 1:

What is the radius of a globe whose surface area is 1256 cm^2? (Use π = 3.14)

Solution:

Let the radius of the globe be *r*.

Surface area of a sphere = $4\pi r^2$

It is given that the surface area of the globe is 1256 cm^2.

So, $4\pi r^2 = 1256\ cm^2$

$\Rightarrow 4 \times 3.14 \times r^2 = 1256\ cm^2$

⇒ $12.56 \times r^2 = 1256$ cm^2

$$\Rightarrow r^2 = \left(\frac{1256}{12.56}\right) \text{cm}^2$$

$$\Rightarrow r^2 = 100 \text{ cm}^2$$

$$\Rightarrow \therefore r = \left(\sqrt{100}\right) \text{ cm} = 10 \text{ cm}$$

Thus, the radius of the globe is 10 cm.

Medium

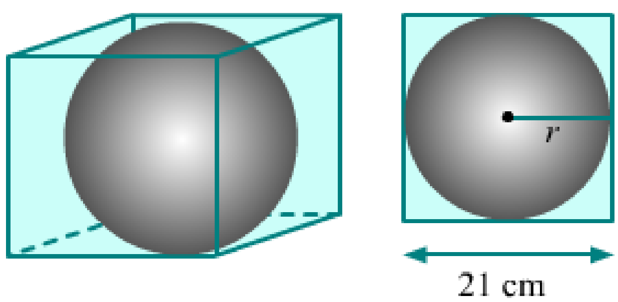
Example 1:

Find the surface area of the largest sphere that can be inscribed in a cube of edge 21 cm.

Solution:

Edge of the cube = 21 cm

Suppose the largest sphere that can be inscribed in this cube has a radius *r*.



This sphere will touch all the six walls of the cube. Therefore, the diameter of the sphere will be equal to the edge of the cube.

So, $2r = 21$ cm

$$\Rightarrow r = \left(\frac{21}{2}\right) \text{cm}$$

Now, surface area of the required sphere = $4\pi r^2$

$$= \left(4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\right) \text{ cm}^2$$
$$= 1386 \text{ cm}^2$$

Example 2:

The diameter of a football is approximately 20/19 times the diameter of a volleyball. What is the ratio of their surface areas?

Solution:

Let the diameter of the volleyball be *x*.

∴ Diameter of the football = $\frac{20}{19}x$

Now, radius (r_1) of the volleyball = x/2

Radius (r_2) of the football $= \frac{10}{19}x$

Required ratio $= \frac{\text{Surface area of the football}}{\text{Surface area of the volleyball}}$

$$= \frac{4\pi r_2^2}{4\pi r_1^2}$$
$$= \left(\frac{r_2}{r_1}\right)^2$$
$$= \left(\frac{\frac{10}{19}x}{\frac{x}{2}}\right)^2$$
$$= \left(\frac{20}{19}\right)^2$$
$$= \frac{400}{361}$$

Thus, the surface areas of the football and the volleyball are in the ratio 400 : 361.

Hard

Example 1:

If the diameter of a sphere is increased by 25%, then what will be the percentage increase in its curved surface area?

Solution:

Let *r* be the radius and *S* be the curved surface area of the sphere.

$\therefore S = 4\pi r^2$

Percentage increase in diameter = 25

∴ Increase in diameter = 25% of 2*r* = $\left(\frac{25}{100} \times 2r\right) = \frac{r}{2}$

⇒ Increased diameter = $2r + \frac{r}{2} = \frac{5r}{2}$

∴ Increased radius = $\frac{5r}{4}$

Let *S* ' be the new curved surface area of the sphere.

$\therefore S' = 4\pi\left(\frac{5r}{4}\right)^2 = \frac{25\pi r^2}{4}$

Now, increase in curved surface area $= S' - S$

$= \frac{25\pi r^2}{4} - 4\pi r^2$

$= \frac{9\pi r^2}{4}$

∴ Percentage increase in curved surface area $= \frac{S' - S}{S} \times 100$

$$= \frac{\frac{9\pi r^2}{4}}{4\pi r^2} \times 100$$

$$= \frac{900}{16}$$

$$= 56.25$$

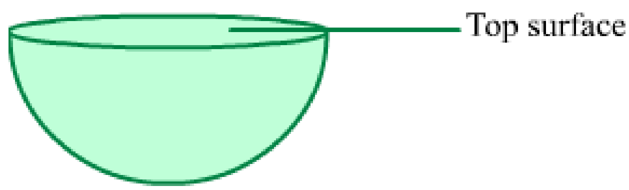
Thus, the curved surface area of the sphere will increase by 56.25%.

Formulae for the Surface Area of a Hemisphere

A hemisphere is a three dimensional solid having two faces, one edge and no vertex.

Since a hemisphere is obtained by cutting a sphere along its diameter, the radius of a hemisphere is the same as that of the sphere from which it is cut.

Consider a hemisphere with radius *r*.



The formulae for the surface area of this hemisphere are given as follows:

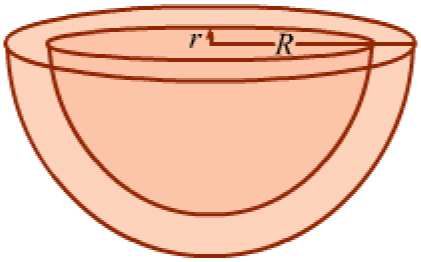
Curved surface area of the hemisphere = $2\pi r^2$

Total surface area of the hemisphere = $3\pi r^2$

Here, curved (or lateral) surface area refers to the area of the curved surface excluding the area of the top surface, and total surface area refers to the sum of the area of the curved surface and the area of the top surface.

Formula for the Surface Area of a Hollow Hemisphere Sphere

Let *R* and *r* be the outer and inner radii of the hollow hemisphere.



Here, curved surface area = Outer surface area + Inner surface area

$$= 2\pi R^2 + 2\pi r^2$$
$$= 2\pi \left(R^2 + r^2\right)$$

The total surface area = curved surface area + Area at the base

$$= 2\pi \left(R^2 + r^2\right) + \pi \left(R^2 - r^2\right)$$
$$= \pi \left(2R^2 + 2r^2 + R^2 - r^2\right)$$
$$= \pi \left(3R^2 + r^2\right)$$

Solved Examples

Easy

Example 1:

If the total surface area of a hemisphere is 462 cm^2, then find its radius.

Solution:

Let the radius of the hemisphere be *r*.

Total surface area of the hemisphere is given by the formula $3\pi r^2$.

It is given that the total surface area of the hemisphere is 462 cm^2.

So, 462 cm^2 = $3\pi r^2$

$$\Rightarrow 462 \text{ cm}^2 = 3 \times \frac{22}{7} \times r^2$$
$$\Rightarrow r^2 = \frac{462 \times 7}{3 \times 22} \text{cm}^2$$
$$\Rightarrow r^2 = 49 \text{ cm}^2$$
$$\Rightarrow \therefore r = \sqrt{49} \text{ cm} = 7 \text{ cm}$$

Thus, the radius of the hemisphere is 7 cm.

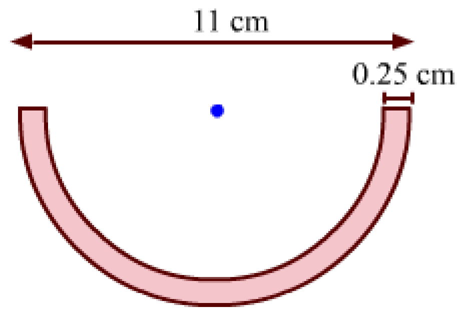
Medium

Example 1:

A hemispherical steel bowl is 0.25 cm thick. The outer diameter of the bowl is 11 cm. Calculate the cost of tin-plating the inner surface of the bowl at the rate of Rs 16 per 100 cm^2.

Solution:

The figure according to the given specifications can be made as follows:



Outer diameter of the hemispherical bowl = 11 cm

∴ Outer radius of the hemispherical bowl = $\frac{11}{2}$ cm $= 5.5$ cm

Thickness of the hemispherical bowl = 0.25 cm

∴ Inner radius (r) of the hemispherical bowl = (5.5 − 0.25) cm = 5.25 cm

Inner curved surface area of the hemispherical bowl **=** $2\pi r^2$

$$= (2 \times \frac{22}{7} \times 5.25 \times 5.25)\ \text{cm}^2$$
$$= 173.25\ \text{cm}^2$$

Cost of tin-plating 100 cm^2 of surface = Rs 16

∴ Cost of tin-plating 1 cm^2 of surface = Rs $\frac{16}{100}$

⇒ Cost of tin-plating 173.25 cm^2 of surface = $\text{Rs}\left(173.25\times\frac{16}{100}\right)=\text{Rs } 27.72\approx\text{Rs } 27.80$

Thus, the cost of tin-plating the inner surface of the hemispherical bowl is Rs 27.80.

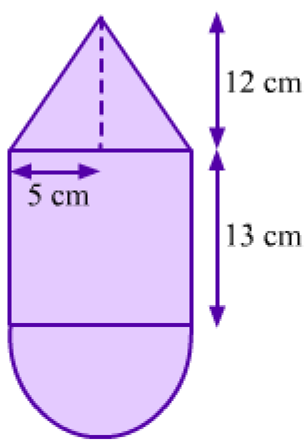
Hard

Example 1:

A toy is in the shape of a right circular cylinder with a hemisphere at one end and a cone at the other. The height and radius of the cylindrical part are 13 cm and 5 cm respectively. Calculate the curved surface area of the toy if the height of the conical part is 12 cm.

Solution:

The figure according to the given specifications can be made as follows:



Radius (r) of the cylinder = 5 cm

Height (H) of the cylinder = 13 cm

Height (h) of cone = 12 cm

Radii of the cone and the hemisphere = Radius of the cylinder = 5 cm

Now, slant height (l) of the cone $=\sqrt{r^2+h^2}$

$$= \sqrt{5^2 + 12^2} \text{ cm}$$
$$= \sqrt{25 + 144} \text{ cm}$$
$$= \sqrt{169} \text{ cm}$$
$$= 13 \text{ cm}$$

∴ Surface area of the toy = CSA of the hemisphere + CSA of the cylinder + CSA of the cone

$$= 2\pi r^2 + 2\pi rH + \pi rl$$
$$= \pi r(2r + 2H + l)$$
$$= \frac{22}{7} \times 5 \times (2 \times 5 + 2 \times 13 + 13) \text{ cm}^2$$
$$= \frac{22}{7} \times 5 \times 49 \text{ cm}^2$$
$$= 770 \text{ cm}^2$$

Thus, the curved surface area of the toy is 770 cm^2.

Volumes of Spheres and Hemispheres

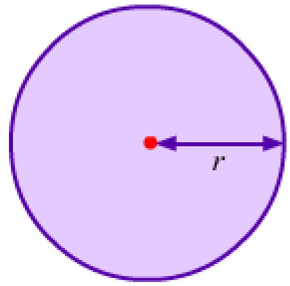
Consider the basketball shown.

Clearly, the basketball, (or, for that matter, any ball) is spherical or shaped like a sphere. Being inflatable, a basketball acquires its shape on being filled with air. The amount of air inside a basketball filled to its capacity helps us ascertain the volume of the ball.

In this lesson, we will learn the formulae for the volumes of spheres and hemispheres, and solve problems using the same.

Formula for the Volume of a Sphere

Consider a solid sphere of radius *r*.



The formula for the volume of this solid sphere is given as follows:

Volume of the solid sphere = $\frac{4}{3}\pi r^3$

Using the above formula, we can calculate the amount of air in a basketball filled to its capacity.

Suppose we have a basketball with radius 18 cm. Then,

Volume of the basketball = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 18 \times 18 \times 18 \text{ cm}^3 = 24438.86 \text{ cm}^3$

Thus, the amount of air filled inside the basketball is 24438.86 cm^3.

Example Based on the Volume of a Sphere

Solved Examples

Easy

Example 1:

Find the radius of a sphere if its volume is $179\frac{2}{3}\text{ cm}^3$.

Solution:

Volume of a sphere = $\frac{4}{3}\pi r^3$

It is given that the volume of the given sphere is $179\frac{2}{3}\text{ cm}^3$.

Let the radius of the given sphere be *r*.

So,

$$\frac{4}{3}\pi r^3 = 179\frac{2}{3}\text{cm}^3$$

$$\Rightarrow \frac{4}{3}\times\frac{22}{7}r^3 = \frac{539}{3}\text{cm}^3$$

$$\Rightarrow r^3 = \frac{539\times 7\times 3}{4\times 22\times 3}\text{cm}^3$$

$$= \left(\frac{7}{2}\right)^3 \text{cm}^3$$

$$\Rightarrow r = \frac{7}{2}cm = 3.5cm$$

Thus, the radius of the sphere is 3.5 cm.

Medium

Example 1:

Find the volume of a sphere if its surface area is 154 cm^2.

Solution:

Surface area of a sphere = $4\pi r^2$

It is given that the surface area of the given sphere is 154 cm^2.

Let the radius of the given sphere be *r*.

So,

$$4\pi r^2 = 154$$

$$\Rightarrow 4\times\frac{22}{7}\times r^2 = 154$$

$$\Rightarrow r^2 = \frac{154\times 7}{22\times 4}$$

$$\Rightarrow r^2 = \frac{49}{4}$$

$\Rightarrow r = \frac{7}{2} cm$

Now, volume of a sphere = $\frac{4}{3}\pi r^3$

Therefore, volume of the given sphere = $\frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \text{cm}^3$

= 179.67 cm^3

Example 2:

The diameter of Earth is about 20/19 times that of Venus. What is the ratio of their volumes?

Solution:

Volume of a sphere = $\frac{4}{3}\pi r^3$

Let the diameter of Venus be *x*.

∴ Diameter of Earth = $\frac{20}{19}x$

Now, radius (r_1) of Venus = x/2

And, radius (r_2) of Earth = $\frac{10}{19}x$

Ratio of the volumes of the two planets $= \frac{\text{Volume of Earth}}{\text{Volume of Venus}}$

$$= \frac{\frac{4}{3}\pi r_2^3}{\frac{4}{3}\pi r_1^3}$$

$$= \left(\frac{r_2}{r_1}\right)^3$$

$$= \left(\frac{\frac{10}{19}x}{\frac{x}{2}} \right)^3$$

$$= \left(\frac{20}{19} \right)^3$$

$$= \frac{8000}{6859}$$

Thus, the ratio of the volumes of Earth and Venus is 8000 : 6859.

Hard

Example 1:

Find the number of spherical lead shots each 2.1 cm in diameter which can be obtained from a rectangular solid of lead with dimensions 66 cm × 42 cm × 21 cm.

Solution:

Let x number of spherical lead shots be obtained from the given solid of lead.

Volume of a cuboid = $l \times b \times h$

∴ Volume of lead in the given rectangular solid = (66 × 42 × 21) cm^3

Diameter of a lead shot = 2.1 cm

∴ Radius of a lead shot = $\frac{2.1}{2} \text{ cm} = 1.05 \text{ cm}$

Volume of a sphere = $\frac{4}{3}\pi r^3$

∴Volume of a lead shot = $\frac{4}{3} \times \frac{22}{7} \times (1.05)^3 \text{ cm}^3$

⇒ Volume of x number of lead shots = $\frac{4}{3} \times \frac{22}{7} \times x \times (1.05)^3 \text{ cm}^3$

According to the question, we have:

Volume of *x* number of lead shots = Volume of lead in the rectangular solid

So,

$$\frac{4}{3}\times\frac{22}{7}\times x\times(1.05)^3 = 66\times 42\times 21$$
$$\Rightarrow x = \frac{66\times 42\times 21\times 3\times 7}{4\times 22\times (1.05)^3}$$

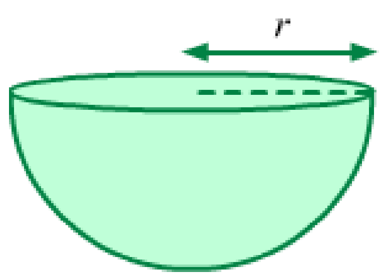
$$\Rightarrow x = 12000$$

Thus, 12000 spherical lead shots can be formed from the given solid of lead.

Formula for the Volume of a Hemisphere

On cutting a solid spherical object into two equal parts, we obtain two solid hemispheres. The radius of each hemisphere so obtained is the same as that of the sphere.

Consider a hemisphere of radius *r*.



Since hemispheres are obtained by cutting a sphere in half, the volume of each resultant hemisphere is equal to half of that of the sphere.

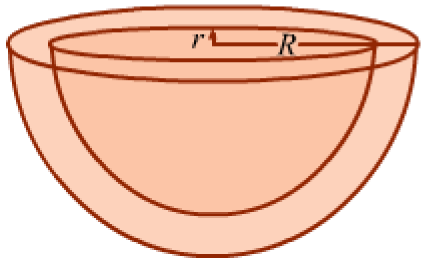
The formula for the volume of this solid hemisphere is arrived at as follows:

Volume of the solid hemisphere $= \frac{1}{2}\times\frac{4}{3}\pi r^3$

$$= \frac{2}{3}\pi r^3$$

Formula for the Volume of a Hollow Hemisphere

Let *R* and *r* be the outer and inner radii of the hollow hemisphere.



Volume of a hollow hemisphere = Volume of outer hemisphere − Volume of inner hemisphere

$$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$$
$$= \frac{2}{3}\pi\left(R^3 - r^3\right)$$

Solved Examples

Easy

Example 1:

How many litres of milk can a hemispherical bowl of diameter 21 cm hold?

Solution:

Diameter of the hemispherical bowl = 21 cm

∴ Radius (r) of the hemispherical bowl= $\frac{21}{2}$ cm

Volume of the hemispherical bowl = $\frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \text{ cm}^3$$
$$= 2425.5 \text{ cm}^3$$
$$= \frac{2425.5}{1000} \text{ L} \qquad \left(\because 1 \text{ cm}^3 = \frac{1}{1000} \text{ L}\right)$$
$$= 2.4255 \text{ L}$$
$$\approx 2.43 \text{ L}$$

Thus, the hemispherical bowl can hold approximately 2.43 L of milk.

Medium

Example 1:

A hemispherical bowl is made of one-centimetre-thick steel. The inside radius of the bowl is 6 cm. Find the volume of steel used in making the bowl.

Solution:

Inner radius (*r*) of the hemispherical bowl = 6 cm

Outer radius (*R*) of the bowl = (6 + 1) cm = 7 cm (∵ Steel used has thickness of 1 cm)

Volume of the inner hemisphere = $\frac{2}{3}\pi r^3$

Volume of the outer hemisphere = $\frac{2}{3}\pi R^3$

∴ Volume of steel used $= \frac{2}{3}\pi\left(R^3 - r^3\right)$

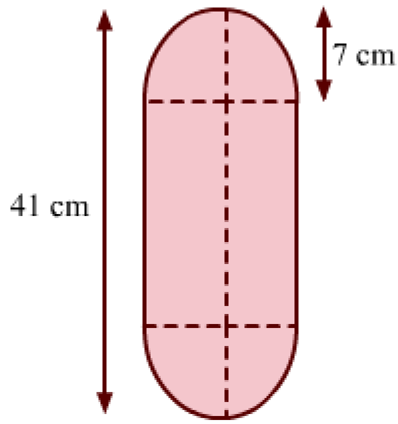
$$= \frac{2}{3} \times \frac{22}{7} \times \left[(7)^3 - (6)^3\right] \text{ cm}^3$$
$$= \frac{2}{3} \times \frac{22}{7} \times (343 - 216) \text{ cm}^3$$
$$= \frac{2}{3} \times \frac{22}{7} \times 127 \text{ cm}^3$$
$$= 266.095 \text{ cm}^3$$

Thus, the volume of steel used in making the bowl is 266.095 cm^3.

Hard

Example 1:

A solid is in the form of a cylinder with hemispherical ends as is shown in the figure. Find the volume of the solid.



Solution:

It is given that:

Radius (r) of the cylinderical part of the solid = 7 cm

Height (h) of the same = [41 − (2 × 7)] cm = (41 − 14) cm = 27 cm

Also, radius of each hemispherical part is the same as that of the cylinderical part.

∴ Volume of the solid = Volume of the cylinderical part + Volumes of the hemispherical parts

$$= \pi r^2 h + 2\left(\frac{2}{3}\pi r^3\right)$$
$$= \pi r^2\left(h + \frac{4r}{3}\right)$$
$$= \frac{22}{7} \times 7 \times 7 \times \left(27 + \frac{4 \times 7}{3}\right) \text{cm}^3$$
$$= \frac{22}{7} \times 7 \times 7 \times \frac{109}{3} \text{cm}^3$$
$$= 5595.33 \text{ cm}^3$$

Conversion Of Solids From One Shape Into Another

In our daily life, we come across various shapes of objects. For example, we see the objects of wax (candle) in various shapes like cylindrical, conical, circular etc. Have you ever thought how these objects are made?

First of all, the wax is melted and after that the liquid wax is poured into containers which have the special shape in which we want to mould the wax. Then after cooling the wax we get the desired shape.

We also convert solids from one shape into another. For example, a metallic wire of cylindrical shape is melted and recast into a spherical shape; earth dug out from a well is uniformly distributed to form an embankment around it.

The main concept in such conditions is that the amount of material before the conversion remains the same as the amount of material after the conversion or we can say that the volume before the conversion and after the conversion remains constant.

Let us consider a situation where we have a ball which is made up of wax. We measured the radius of the ball and it came out to be 3 cm. Now, we want to make a candle of length 16 cm from the wax ball. What should be the radius of the mould so that no wax is wasted?

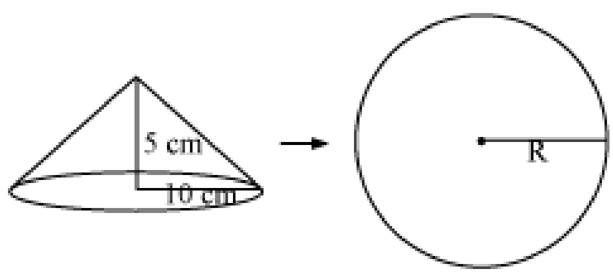
We use this concept for solving the problems related to conversion of solids. Let us discuss some examples based on the above idea.

Example 1:

A solid right circular metallic cone of radius 10 cm and height 5 cm is melted and recast into a sphere. Find the radius of the sphere.

Solution:

Let us consider the above information geometrically.



It is given that radius of cone, r = 10 cm

Height of cone, h = 5 cm

Let R be the radius of sphere.

Since the cone is melted to form a sphere, Volume of metal before conversion = volume of metal after conversion

∴ Volume of cone = Volume of sphere

$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$$

$$4 \times R^3 = (10)^2 \times 5 = 500$$

$$R^3 = 125 = (5)^3$$

$R = 5$ cm

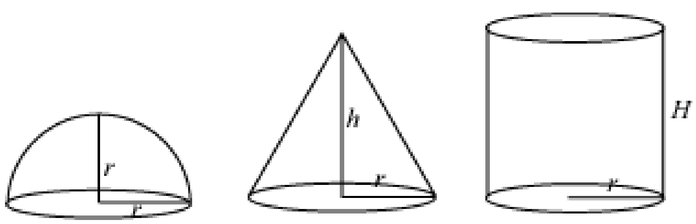
Thus, the radius of the sphere is 5 cm.

Example 2:

If the volumes of a solid hemisphere, a solid right circular cone and a solid right circular cylinder of same base are equal, then find the ratio of their heights.

Solution:

Let us consider the above information by drawing figures.



All the three solids have equal base, i.e. their radii are equal.

Let *r* be the radius of the above three figures. The height of the hemisphere will also be *r*. Let the height of the cone and cylinder be *h* and *H* respectively.

But it is given that,

Volume of hemisphere = Volume of cone = Volume of cylinder

$$\frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 h = \pi r^2 H$$

$\frac{2}{3}r = \frac{1}{3}h = H$

$2r = h = 3H$

The LCM of 2, 1, and 3 is 6.

Now, let us write

$2r = h = 3H$ = 6*x*

⇒ $2r = 6x,\ h = 6x,\ 3H = 6x$

Hence, $r = 3x,\ h = 6x$ and $H = 2x$

Now, *r*: *h*: *H* = 3*x*: 6*x*: 2*x*

r: *h*: *H* = 3: 6: 2

∴ Ratio of their heights = 3: 6: 2

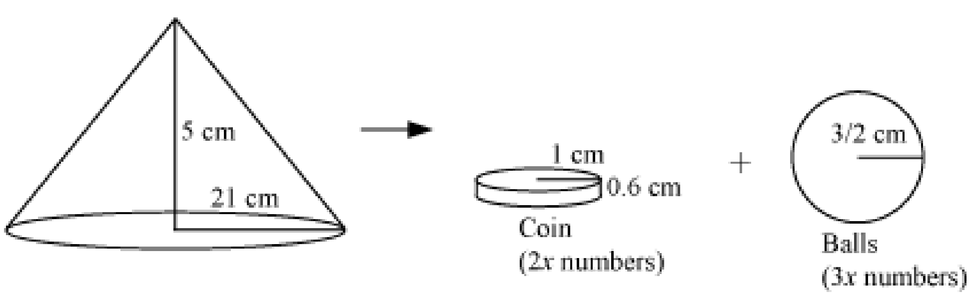
Example 3:

A solid right circular metallic cone of height 5 cm and radius 21 cm is melted to form some coins of diameter 2 cm and width 6 mm and some spherical balls of diameter 3 cm. If the ratio of the number of coins to the number of balls is 2:3, then find the number of coins and balls.

Solution:

For the cone,

Height, *h* = 5 cm and radius, *r* = 21 cm



For cylindrical coin,

Diameter = 2 cm

So, radius, r_1 = 1 cm

Height, h_1 = 6 mm = 0.6 cm

For spherical balls,

Diameter = 3 cm

So, radius, r_2 = 3/2 cm

Let number of spherical coins = 2*x* and number of spherical balls = 3*x*

Metallic cone is melted and recast to form coins and balls.

∴ Volume of coins + Volume of balls = Volume of Cone

$$2x\left(\pi r_1^2 h_1\right) + 3x\left(\frac{4}{3}\pi r_2^3\right) = \frac{1}{3}\pi r^2 h$$

$$2x\left(1^2 \times 0.6\right) + 4x\left(\frac{3}{2}\right)^3 = \frac{1}{3}(21)^2 \times 5$$

$$1.2x + \frac{27}{2}x = 735$$

$$\frac{12}{10}x + \frac{27}{2}x = 735$$

$$\frac{147}{10}x = 735$$

$$x = 735 \times \frac{10}{147} = 50$$

∴ Number of coins = 2*x* = 2 × 50 = 100

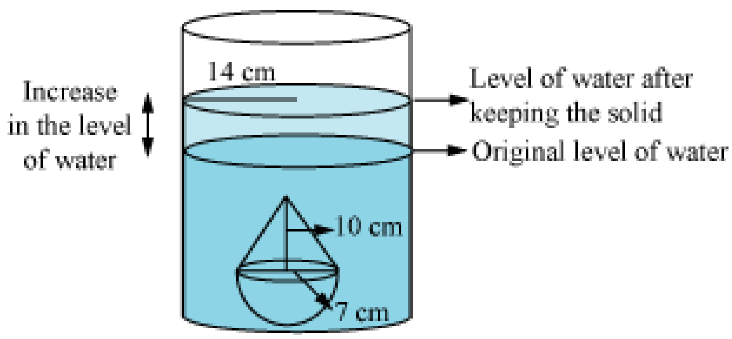
And number of spherical balls = 3*x* = 3 × 50 = 150

Example 4:

A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 10 cm and the diameter of the base is 14 cm. The toy is placed in a cylinder which is half filled with water. Find the height of increase in water level in the cylinder, if the radius of the cylinder is 14 cm.

Solution:

Let us draw the figure for the given information.



For the toy,

Diameter of the hemisphere = 14 cm

Radius of hemisphere, r = 7 cm (Also radius for cone)

Height of the cone, h = 10 cm

For the cylinder,

Radius, R = 14 cm

Let height of increase in the level of water = H

When the toy is kept in the cylindrical vessel, the water will rise uniformly taking the shape of a cylinder, i.e. the increase in the level of water will be in the shape of a cylinder of radius 14 cm.

Volume of water raised in the cylinder = Volume of the toy

$$\pi R^2 H = \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$R^2 H = \frac{1}{3} r^2 (2r + h)$$

$$(14)^2 \times H = \frac{1}{3} \times (7^2) \times (2 \times 7 + 10)$$

$$196H = 392$$

$H = 2$ cm

Thus, the water level will become 2 cm high after keeping the toy in the cylindrical vessel.

Volume Of Combination Of Solids

We come across many figures in our daily life that are made up of two or more solid figures. Let us consider such an example.

A company produces metallic solid toys that are in the shape of a cylinder with one hemisphere and one cone stuck to their opposite ends. The length of the entire toy is 30 cm; the diameter of the cylinder is 14 cm, while the height of the cone is 10 cm.

Can we find out how much metal should the company order to make 200 toys of this type?

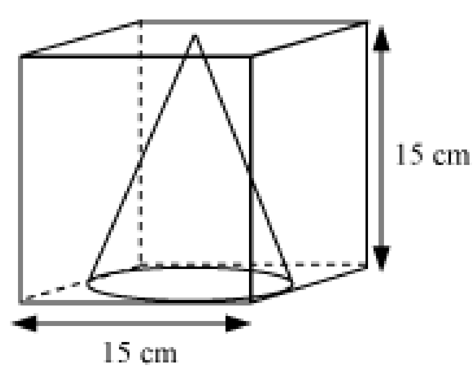
In this way, we can find the volume of any solid figure which is formed by combining two or more basic solids. Let us now look at some more examples.

Example 1:

A largest cone is to be taken out from a cube of edge 15 cm. Find the volume of the remaining portion. (Use π = 3.14)

Solution:

The figure can be drawn as follows:



It is given that length of the cube, *l* = 15 cm

The base of the cone is a circle whose diameter is equal to the length of the edge of the cube.

∴ Radius of cone, *r* = 15/2 cm

The height of the cone would be equal to the height of cube.

∴ Height of cone, *h* = 15 cm

Volume of the remaining portion = Volume of Cube − Volume of Cone

$= l^3 - \frac{1}{3}\pi r^2 h$

$= (15)^3 - \frac{1}{3} \times 3.14 \times \frac{15}{2} \times \frac{15}{2} \times 15$

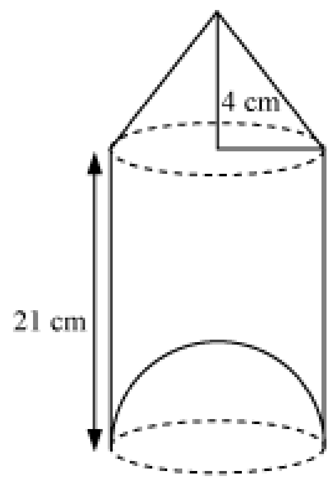
= 3375 − 883.125

= 2491.875 cm^3

Thus, the volume of the remaining portion is 2491.875 cm^3.

Example 2:

A plastic toy is in the following shape. The diameter of the cylindrical shape is 7 cm, but the bottom of the toy has a hemispherical raised portion. The top of the toy is a cone of same base. If the height of the cylinder is 21 cm and cone is 4 cm, find the amount of air inside the toy. (Use π = 22/7)



Solution:

Diameter of the cylinder = 7 cm

Radius of cylinder, *r* = 7/2 cm

r = 7/2, is also the radius for cone and hemisphere.

Height of the cylinder, *H* = 21 cm

Height of the cone, *h* = 4 cm

∴ Volume of the toy = Volume of cylinder + Volume of Cone − Volume of Hemisphere

$$=\pi r^2 H + \frac{1}{3}\pi r^2 h - \frac{2}{3}\pi r^3$$

$$=\frac{1}{3}\pi r^2 (3H + h - 2r)$$

$$= \frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\left(3\times 21+4-2\times\frac{7}{2}\right)$$

$$= \frac{1}{3}\times 11\times\frac{7}{2}\times 60$$

= 770 cm^3

Thus, the volume of air inside the given toy is 770 cm^3.

Example 3:

A right triangle, whose perpendicular sides are 30 cm and 40 cm, is made to revolve about its hypotenuse. Find the volume of figure so obtained in terms of π.

Solution:

Let ABD be a right-angled triangle, such that AB = 30 cm and AD = 40 cm.

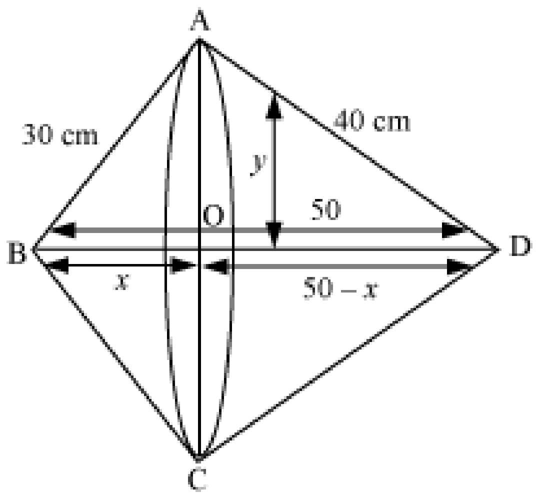
Using Pythagoras theorem, we have

$BD^2 = AB^2 + AD^2$

$BD^2 = 30^2 + 40^2 = 900 + 1600 = 2500$

$BD = \sqrt{2500} = 50$ cm

After revolution, we have the following figure.



Let OB = x and OA = y

Using Pythagoras theorem in triangle AOB, we obtain

$OA^2 + OB^2 = AB^2$

$x^2 + y^2 = 30^2$

$y^2 = 900 - x^2$ … (1)

Using Pythagoras theorem in triangle AOD, we obtain

$OA^2 + OD^2 = AD^2$

$y^2 + (50 - x)^2 = 40^2$

$900 - x^2 + 2500 + x^2 - 100x = 1600$ {using equation (1)}

$100x = 1800$

$x = 18$

On putting the value of x in equation (1), we obtain

$y^2 = 900 - x^2 = 900 - 18^2 = 900 - 324 = 576$

$y = \sqrt{576} = 24$ cm

Here, y is the radius of the cone's ABC and ADC.

Now, height of the cone ABC, $x = 18$ cm

Height of the cone ADC, $50 - x = 32$ cm

Volume of the double cone = Volume of cone ABC + Volume of cone ADC

$= \frac{1}{3}\pi y^2 x + \frac{1}{3}\pi y^2 (50 - x)$

$= \frac{1}{3}\pi y^2 (x + 50 - x)$

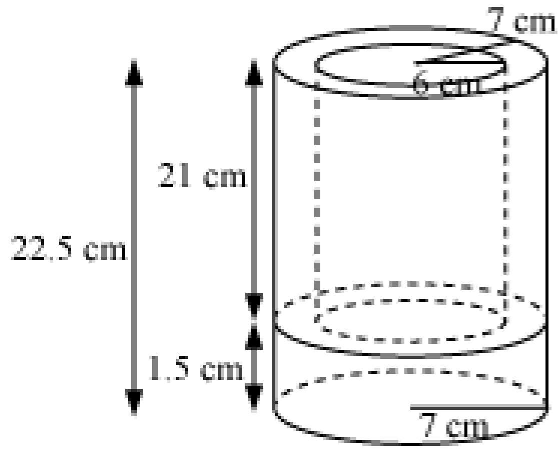
$= \frac{1}{3}\pi \times 24 \times 24 \times 50$

$= 9600\pi$ cm^3

Thus, the volume of the figure so obtained is 9600π cm^3.

Example 4:

An iron container is cylindrical in shape as shown in the figure. The inner and outer diameters are 12 cm and 14 cm respectively. The container has a solid base of width 1.5 cm and the total height of the container is 22.5 cm. If the mass of 1 cm^3 of iron is 8 gm, find the weight of the container.



Solution:

The iron container consists of two cylinders.

1. A solid cylinder and
2. A hollow cylinder

External and internal diameters of the hollow cylinder are 14 and 12 cm respectively.

Hence, internal radius, r = 6 cm and external radius R = 7 cm

Height of the solid cylinder, h = 1.5 cm

∴ Height of the hollow cylinder, H = 22.5 − 1.5 = 21 cm

Volume of the iron used in the container = Volume of solid cylinder + Volume of hollow cylinder

$= \pi R^2 h + \pi\left(R^2 - r^2\right)H$

$= \frac{22}{7} \times 7 \times 7 \times 1.5 + \frac{22}{7}\left(7^2 - 6^2\right) \times 21$

= 231 + 858

= 1089 cm^3

Mass of 1 cm^3 of iron = 8 gm

Mass of 1089 cm^3 of iron = 1089 × 8 = 8712 gm = 8.712 kg

Thus, the weight of the container is 8.712 kg.