Sample Question Paper - 19
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

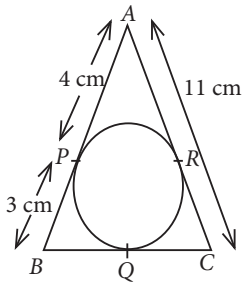
General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Solve the following quadratic equation for x : $x^2 - 2ax - (4b^2 - a^2) = 0$

2. In figure, ΔABC is circumscribing a circle, find the length of BC.



3. The sum of the first n terms of an A.P. is $3n^2 + 6n$. Find the n^{th} term of this A.P.

OR

The ninth term of an A.P. is –32 and the sum of its eleventh and thirteenth terms is –94. Find the common difference of the A.P.

4. Find the class marks of classes 20-25 and 45-55.

5. A cone and a cylinder have the same radii but the height of the cone is 3 times that of the cylinder. Find the ratio of their volumes.

6. The frequency of the class succeeding the modal class in the following frequency distribution is

Class interval	10–15	15–20	20–25	25–30	30–35	35–40	40–45
Frequency	3	7	16	12	9	5	3

OR

Find the mean of 1, 2, 3, 4, ..., n.

SECTION - B

7. Construct a pair of tangents to a circle of radius 3 cm from a point which is at a distance of 5 cm from its centre.

8. Find the value of m so that the quadratic equation $mx\,(5x - 6) + 9 = 0$ has two equal roots.

OR

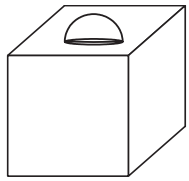
A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hr less for the same journey. Find the speed of the train.

9. The sum of 4^{th} and 8^{th} terms of an A.P. is 24 and sum of its 6^{th} and 10^{th} terms is 44. Find the sum of first ten terms of the A.P.

10. The angles of elevation of the top and bottom of a tower which is at a building from two points at a distance of 4 m and 9 m from the base of the building and in the same straight line with it are 60° and 30° respectively. Find the height of the tower.

SECTION - C

11. In the given figure, there is a decorative block, made up of two solids - a cube and a hemisphere. The base of the block is a cube of side 6 cm and the hemisphere fixed on the top has a diameter of 3.5 cm. Find the total surface area of the block. $\left(\text{Use } \pi = \frac{22}{7}\right)$



OR

A toy is in the form of a cone of base radius 3.5 cm mounted on a hemisphere of base diameter 7 cm. If the total height of the toy is 15.5 cm, find the total surface area of the toy. $\left(\text{Use } \pi = \frac{22}{7}\right)$

12. Prove that tangents drawn at the ends of a diameter of a circle are parallel.

Case Study - 1

13. In view of CORONA virus pandemic, a company decided to provide health cover of ₹ 5,00,000 to each of its 100 employees. To estimate the total expenditure, company call a health insurance agent and ask to provide the quotation for the same. The agent first make the Table 1 (given below) for the distribution of ages of 100 employees and then make Table 2 (given below) for the quotation.

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
Number of employees	2	6	24	45	78	89	92	98	100

Table 1

Investment (in ₹)	8000-12000	12000-16000	16000-20000	20000-24000	24000-28000	28000-32000
Number of employees	3	15	20	25	30	7

Table 2

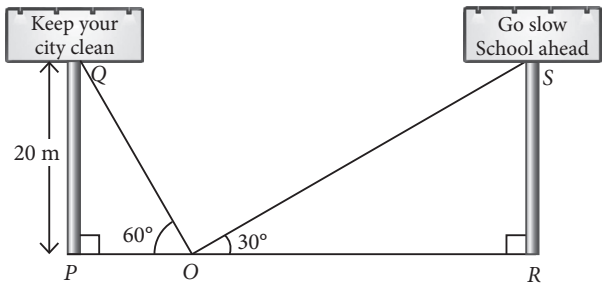
Based on the above information, answer the following questions.

(i) If the minimum age of employee in the company is 15 years, then find the average age of the employees.

(ii) Find the average expenditure on each employee.

Case Study - 2

14. Two hoardings are put on two poles of equal heights standing on either side of the road. From a point between them on the road, the angle of elevation of the top of poles are 60° and 30° respectively. Height of the each pole is 20 m.



Based on the above information, answer the following questions. (Use $\sqrt{3} = 1.73$).

(i) Find the length of *RO*.

(ii) If the angle of elevation made by pole *PQ* is 45°, then find the length of *PO*.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. We have, $x^2 - 2ax - (4b^2 - a^2) = 0$

$\Rightarrow x^2 - 2ax - 4b^2 + a^2 = 0$

$\Rightarrow (x^2 - 2ax + a^2) - 4b^2 = 0 \Rightarrow (x - a)^2 - (2b)^2 = 0$

$\Rightarrow (x - a + 2b)(x - a - 2b) = 0$

$\Rightarrow x - a + 2b = 0$ or $x - a - 2b = 0$

$\Rightarrow x = a - 2b$ or $x = a + 2b$

2. The lengths of tangents drawn from an external point to the circle are equal.

$\therefore$ $AP = AR = 4$ cm, $BP = BQ = 3$ cm

and $CQ = CR = CA - AR = 11 - 4 = 7$ cm

$\therefore$ $BC = BQ + CQ = 3 + 7 = 10$ cm

3. We have, $S_n = 3n^2 + 6n$

$\therefore$ $S_{n-1} = 3(n - 1)^2 + 6(n - 1)$

$= 3(n^2 + 1 - 2n) + 6n - 6$

$= 3n^2 + 3 - 6n + 6n - 6 = 3n^2 - 3$

Hence, n^{th} term of A.P. is $a_n = S_n - S_{n-1}$

$= (3n^2 + 6n) - (3n^2 - 3) = 6n + 3$

OR

Let the first term be a and d be the common difference of the A.P.

Given, $a_9 = -32 \Rightarrow a + 8d = -32$ ...(i)

Also, $a_{11} + a_{13} = -94$

$\Rightarrow a + 10d + a + 12d = -94 \Rightarrow 2a + 22d = -94$

$\Rightarrow a + 11d = -47$ ...(ii)

Subtracting (ii) from (i), we have

$-3d = 15 \Rightarrow d = -5$

4. The class mark of class 20–25 $= \frac{20+25}{2} = 22.5$

The class mark of class 45–55 $= \frac{45+55}{2} = 50$

5. Let the radius and the height of the cylinder are r and h respectively.

So, the radius of the cone is r and height of the cone is $3h$.

$\therefore$ Volume of the cylinder $= \pi r^2 h$

Volume of the cone $= \frac{1}{3}\pi r^2 3h = \pi r^2 h$

So, the required ratio $= \frac{\pi r^2 h}{\pi r^2 h} \Rightarrow 1:1$

6. The modal class is 20-25 as it has maximum frequency. So, the class succeeding the modal class is 25-30 with frequency 12.

OR

Sum of the numbers 1, 2, 3,...,$n = n\frac{(n+1)}{2}$

$\therefore$ Mean $= \frac{n(n+1)}{2} \div n = \frac{n+1}{2}$

7. Steps of construction :

Step-I : Draw a circle with centre O and radius 3 cm.

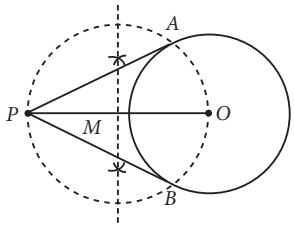
Step-II : Take a point P such that $OP = 5$ cm. Join OP.

Step-III : Draw the perpendicular bisector of line segment OP which intersect the line OP at point M.

Step-IV : Taking M as centre and OM as radius draw a circle intersecting the given circle at A and B.

Step-V : Join PA and PB.

Thus, PA and PB are the required tangents.



8. Given, $mx(5x - 6) + 9 = 0$

$\therefore$ $5mx^2 - 6mx + 9 = 0$

For equal roots, discriminant, $D = 0$

$\therefore$ $(-6m)^2 - 4 \cdot 5m \cdot 9 = 0$

$\Rightarrow 36m^2 - 180m = 0 \Rightarrow 36m(m - 5) = 0$

$\Rightarrow m = 5$ $(\because m \neq 0)$.

OR

Let the speed of the train be x km/hr.

According to question,

$\frac{360}{x} - \frac{360}{x+5} = 1 \Rightarrow \frac{(x+5-x)360}{x(x+5)} = 1$

$\Rightarrow 1800 = x^2 + 5x \Rightarrow x^2 + 5x - 1800 = 0$

$\Rightarrow x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow x(x + 45) - 40(x + 45) = 0 \Rightarrow (x - 40)(x + 45) = 0$

$\Rightarrow x = 40$ $(\because$ Speed can't be negative)

Hence, the speed of the train is 40 km/hr.

9. Let a be the first term and d be the common difference of the A.P.

n^{th} term of the A.P. is $a_n = a + (n - 1)d$

Now, we have $a_4 + a_8 = 24$

$\Rightarrow \quad a + 3d + a + 7d = 24$

$\Rightarrow \quad 2a + 10d = 24 \Rightarrow a + 5d = 12$ ...(i)

Also, $a_6 + a_{10} = 44$

$\Rightarrow \quad a + 5d + a + 9d = 44$

$\Rightarrow \quad 2a + 14d = 44 \Rightarrow a + 7d = 22$ ...(ii)

On subtracting (i) from (ii), we get

$2d = 10 \Rightarrow d = 5$

On substituting the value of d in (i), we get

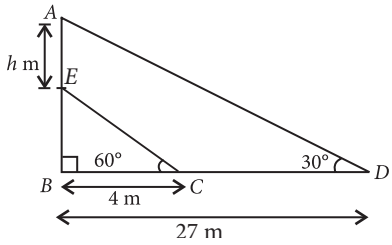
$a + 5(5) = 12 \Rightarrow a = -13$

Thus, the A.P. is $-13, -8, -3, \ldots$

$\therefore$ Sum of first 10 terms, $S_{10} = \frac{10}{2}\left[2a + (10-1)d\right]$

$= 5[2(-13) + 9 \times 5] = 5(-26 + 45) = 5 \times 19 = 95$

10. Let the height of the tower be $AE = h$ m height of building $ED = x$ m and C, D are the observation points.



In ΔEBC, $\tan 60° = \frac{x}{4} \Rightarrow x = 4\sqrt{3}$ m

In ΔABD, $\tan 30° = \frac{h+x}{27} \Rightarrow h + x = \frac{27}{\sqrt{3}} = 9\sqrt{3}$ m

$\Rightarrow h + 4\sqrt{3} = 9\sqrt{3} \Rightarrow h = 5\sqrt{3}$

Hence, the height of the tower is $5\sqrt{3}$ m.

11. Total surface area of the block

= Surface area of cube + Curved surface area of hemisphere – Area of base of hemisphere

Surface area of cube = $6a^2 = 6 \times 6^2 = 216$ cm^2

Curved surface area of hemisphere – Area of base of hemisphere = $2\pi r^2 - \pi r^2 = \pi r^2$

$= \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} = 9.625$ cm^2

$\therefore$ Required total surface area of the block

$= 216 + 9.625 = 225.625$ cm^2

OR

Radius (r) of cone = Radius (r) of hemisphere

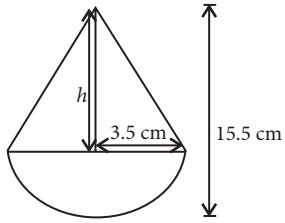
= 3.5 cm

Height (H) of toy = 15.5 cm

$\therefore$ Height (h) of cone

= 15.5 – 3.5 = 12 cm

Hence, slant height (l) of cone = $\sqrt{h^2 + r^2}$



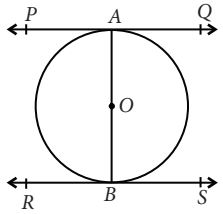
$= \sqrt{(12)^2 + (3.5)^2} = \sqrt{144 + 12.25} = 12.5$ cm

$\therefore$ Total surface area of toy = $\pi rl + 2\pi r^2$

$= \frac{22}{7} \times 3.5(12.5) + 2 \times \frac{22}{7}(3.5)^2$

$= \frac{22 \times 3.5}{7}(12.5 + 7) = \frac{1501.5}{7} = 214.5$ cm^2

12. Given : A circle $C(O, r)$ with diameter AB and let PQ and RS be the tangents drawn to the circle at point A and B.



To prove : $PQ \parallel RS$

Proof : Since tangent at a point to a circle is perpendicular to the radius through the point of contact.

i.e., $OA \perp PQ$ and $OB \perp SR$

i.e., $AB \perp PQ$ and $AB \perp RS$

$\Rightarrow \quad \angle PAB = 90°$ and $\angle ABS = 90° \Rightarrow \angle PAB = \angle ABS$

$\Rightarrow \quad PQ \parallel RS$ [$\because \angle PAB$ and $\angle ABS$ are alternate interior angles]

13. (i)

Age (in years)	**No. of employees (f_i)**	x_i	$x_i - a = d_i$	$f_i\, d_i$
15-20	2	17.5	–20	–40
20-25	6–2 = 4	22.5	–15	–60
25-30	24 – 6 = 18	27.5	–10	–180
30-35	45 – 24 = 21	32.5	–5	–105
35-40	78 – 45 = 33	37.5 = a	0	0
40-45	89 – 78 = 11	42.5	5	55
45-50	92 – 89 = 3	47.5	10	30
50-55	98 – 92 = 6	52.5	15	90
55-60	100 – 98 = 2	57.5	20	40
Total	100			–170

$\therefore$ Average age = $a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 37.5 + \left(\frac{-170}{100}\right)$

= 37.5 – 1.70 = 35.8 $\approx$ 36 years (approx)

(ii)

Investment	x_i	f_i	$d_i = x_i - a$	$f_i\, d_i$
8000-12000	10000	3	–8000	–24000
12000-16000	14000	15	–4000	–60000
16000-20000	18000 = a	20	0	0
20000-24000	22000	25	4000	100000
24000-28000	26000	30	8000	240000
28000-32000	30000	7	12000	84000
Total		100		340000

$\therefore$ Average expenditure = $a + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

$= 18000 + \dfrac{340000}{100}$

= 18000 + 3400 = ₹ 21400

14. (i) In ΔORS, we have

$$\tan 30° = \frac{RS}{OR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{20}{OR} \Rightarrow OR = 20\sqrt{3} \text{ m}$$

(iii) In ΔOPQ,

If $\angle POQ = 45°$, then

$$\tan 45° = \frac{PQ}{PO} \Rightarrow 1 = \frac{20}{PO}$$

$\Rightarrow$ $PO = 20$ m.