CHAPTER 12

Circle and Tangents

Introduction

When we look around us we see objects having different shapes. For example, coins, bangles, wheels of a cycle, etc. All the items mentioned above have some common property.

Edge of a coin

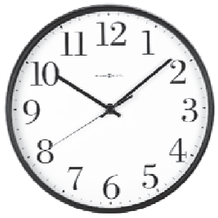
Figure - 1

Edge of a bangle

Figure - 2

Edge of a wheel

Figure - 3



Edge of a clock

Figure - 4

The edges of all of these items seem circular. We can find many more items or objects similar to these items. Can you quickly think of six other similar objects? Balls, marbles, drops of water and many other similar things are spherical.

Ball

Figure - 5

Marbles

Figure - 6

These objects (figures–5 and 6) are different from a circle and from the other figures given above. Discuss among your classmates and write the differences between things which are similar to coins and things that resemble a football.

In this chapter we will look at the properties of surfaces of things like coins, that is, circular surfaces.

What is a circle?

Take a point "O" on paper and take another point "P", 5 *cm* away from O.

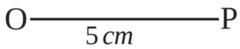


Figure - 1 (i)

Can we take some more points which are at a distance of 5 *cm* from the point O? How can we locate these points? How many such points are there?

Take a compass and open it 5 *cm* wide. Put the tip of the compass on point O and mark point at a distance of 5 *cm* from O. On joining all points on paper which are at a distance of 5 *cm* from O, we will get the shape shown in figure 1(ii). This type of closed figure drawn on a plane is known as a circle. Circle is a group of points in a plane which are located at a fixed distance from a fixed point and which form a closed shape. Point O is called the center of the circle. The distance from the center to any point on the circle is called radius. Can we locate a point on a wheel or clock or bangle or coin, etc. from which the distance upto the tip is equal?

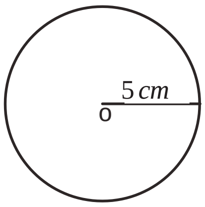


Figure - 1 (ii)

Try These

Write true or false giving reasons and examples.

1. Circle has multiple radii.
2. All radii of a circle are same.

Chord

Draw a circle on a paper and take two points on its perimeter. Two points, P and Q are shown in figure-2. On joining these two points we get a line segment PQ and this line segment is a chord of the circle. Can you imagine how many such lines are there whose end points lie on the circle? You will find there are infinite such chords.

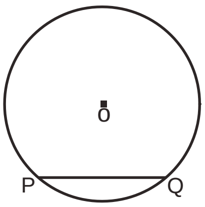


Figure - 2

Try These

1. Identify the chords in the figures given below.

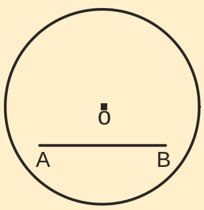


Figure - 3

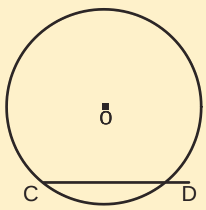


Figure - 4

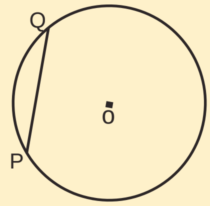


Figure - 5

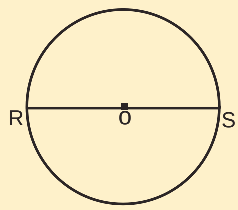


Figure - 6

2. Are all chords of same length?
3. Which is the longest chord?

Longest Chord of a Circle

See figure-7 which shows a circle whose center is O and in which various chords AB, CD, EF and MN, etc. Observe the lengths of these chords.

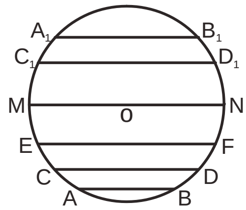


Figure - 7

Between AB and MN, which chord is longer?

Between CD and MN, which chord is longer?

Similarly, which is the longer chord among EF and MN?

On comparing A_1B_1 and MN, we find that length of chord MN is more. Can you find any special property in chord MN which is not present in remaining chords?

Chord MN passes through the center of the circle. A chord which passes through the center of the circle is called diameter of circle. Given a circle, can you draw a chord which is longer than the diameter?

No, you will find that diameter is the greatest chord in a circle.

Think and Discuss

Can we draw more diameters besides MN in figure-7? If yes, then how many diameters can be drawn?

Arc of a circle

If we take any two points on the perimeter of the circle then they will divide the circle in two parts (figures-8, 9, 10).

In this circle one part is small and the other is big. The smaller part of the circle is said to be minor arc $\overset{\frown}{AMB}$ and the bigger part is said to be major arc $\overset{\frown}{ANB}$ (figure-9).

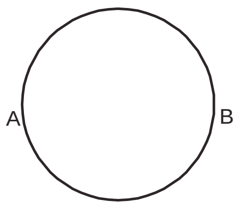


Figure - 8

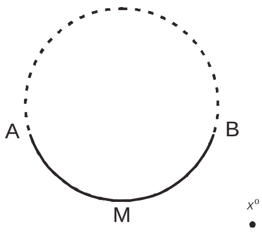


Figure - 9

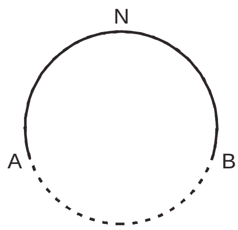


Figure - 10

In figure-8, if from point A we start moving in a circular path and reach back A, then distance travelled is perimeter of circle.

Segment of a Circle

Draw a chord AB in a circle. Can you tell the number of parts into which the internal part of the circle is divided by the chord (figure-11)? You can see that the chord divides the internal part of a circle in two parts. Area which lies within the chord and the arc is called segment of a circle. Area which is between chord and minor arc is called minor segment and area which is between chord and major arc is called major segment.

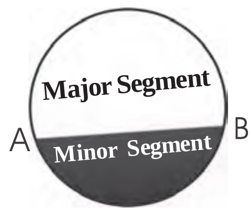


Figure - 11

Try These

Draw a circle on a paper and try to find relation between length of chord and corresponding minor segment by drawing various chords of different lengths.

We can see that when the length of a chord is less, then the area of minor sector will also be less.

Think and Discuss

1. Radius of a circle is 6 *cm*. The lengths of some its chords are 4*cm*, 6 *cm*, 10 *cm* and 8 *cm* respectively. Write the major segments corresponding to these chords in increasing order of area.
2. In the circle given above where the radius is 6 *cm*, when the length of chord is 12 *cm* then what kind of relation will you find between major segment and minor segment?

Sector

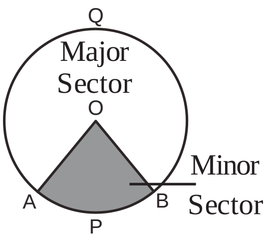


Figure - 12

Take two points A and B on a circle (see figure-12). Join the end points of arc AB with the center O. The area which lies between the two radii drawn from the end points of arc AB and the arc itself is called sector.

As in the case of segment, you will find that the area which is surrounded by minor arc and radii is called minor sector and area which is surrounded by major arc and radii is called major sector. OAPB is minor sector and OAQB is major sector.

Try These

Identify radius, chord, diameter, sector and segment in given figure and write in the table.

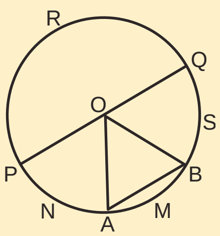


Figure - 13

Radius	Chord	Diameter	Arc	Sector	Segment

Congruent Circles

We saw that two figures which are able to completely cover each other are known as congruent figures.

Take two circles of equal radii and centers at O_1 and O_2 respectively. Take diameter AB in circle with center at O_1 and diameter CD in circle with center at O_2 (Figures-14, 15).

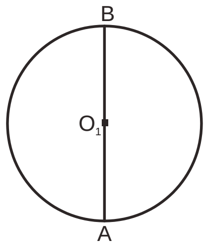


Figure - 14

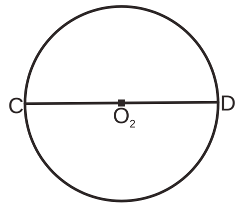


Figure - 15

Put the circles on one-another so that center O_1 lies exactly center O_2 and end points A and B of diameter AB lie on C and D respectively. You can see that one circle is completely covering another one and so we can say that both are congruent. Repeat this activity by drawing more pairs of circles with same radius.

You will find that circles with equal radius are congruent.

Subtended angle made by chord on center

We are given a line segment AB and a point O which does not lie on the line segment (Figure-16).

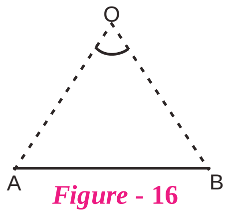


Figure - 16

Join O to A and B. $\angle AOB$ is said to be the angle subtended by line segment AB on point O.

We have a circle whose center is O and which has two chords are AB and CD (figure-17). Angles made by chord AB and CD are $\angle AOB$ and $\angle COD$ respectively. Can you tell which angle is greater, $\angle AOB$ and $\angle COD$? Can you see any relation between the length of chord and angle subtended by chord at the center? You can say that greater the length of chord greater is the angle subtended by it on the center.

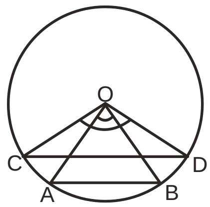


Figure - 17

Try These

Draw a circle with radius 5 *cm*. Draw pairs of chords of length 3, 5, 8, 10 and 6 *cm*. Measure the angles made by chord on center and write in given table.

Length of chord	3 *cm*	5 *cm*	6 *cm*	8 *cm*	10 *cm*
Angles subtended					

After filling the above table you will find that in a circle, equal chords subtend equal angles at the center.

Some theorems related to circles

We have learnt different methods of proving geometrical statements. Now we consider some statements about circles which reflect the properties of circles. Let us take the same statement which we mentioned above, that is, in a circle equal chords subtend equal angles at the center.

Theorem - 1.

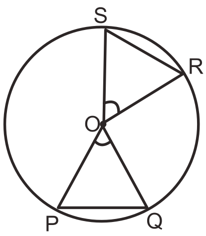


Figure - 18

Statement : Equal chord of a circle subtend equal angles at the center.

Given : A circle with center O where PQ and RS are two equal chords.

To Prove : $\angle POQ = \angle ROS$

Proof : In ΔPOQ and ΔROS

$OP = OR$ (Radii of same circle)

$OQ = OS$ (Radii of same circle)

$PQ = RS$ (Given)

Thus, $\Delta POQ \cong \Delta ROS$ (SSS Congruency)

$\therefore$ $\angle POQ = \angle ROS$ (Corresponding parts of congruent triangles)

Will the converse of this statement also be true, i.e. if angles made by chords at the center are equal, then the chords are equal. Let us prove this statement.

Theorem - 2.

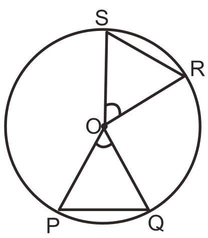


Figure - 19

Statement: If angles made by chords at the center of a circle are equal, then chords are also equal.

Given : A circle with center O with two chords PQ and RS and $\angle POQ = \angle ROS$

To Prove: $PQ = RS$

Proof : In ΔPOQ and ΔROS

$OP = OR$ (Radii of same circle)

$OQ = OS$ (Radii of same circle)

$\angle POQ = \angle ROS$ (Given)

Thus, $\Delta POQ \cong \Delta ROS$ (SAS Congruency)

$\therefore$ $PQ = RS$ (Corresponding parts of congruent triangles)

If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are also equal.

Example-1. In figure-20 if chords AB and BC are equal and $\angle AOB = 35°$ then find $\angle AOC$.

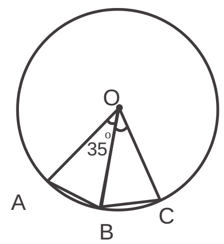


Figure - 20

Solution : $\angle AOB = \angle BOC$ (Equal chords in a circle subtend equal angles)

$\angle BOC = 35°$ $(\angle AOB = 35°\text{ given})$

Thus, $\angle AOC = \angle AOB + \angle BOC$

$= 35° + 35°$

$= 70°$

$\therefore$ $\angle AOC = 70°$

Example-2. A regular pentagon is drawn in a circle. What will be the angle subtended at the center of the circle by each side of the pentagon?

Solution : All five sides of a regular pentagon are equal and they make equal angles at the center.

Let each side of the regular pentagon make angle $x°$ at the center.

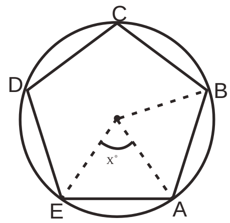


Figure - 21

Thus, $5x° = 360°$ (Why?)

$$x° = \frac{360°}{5}$$

$\therefore$ $x° = 72°$

Try These

1. A regular polygon is drawn inside a circle. Each side of the regular polygon makes an angle of 60° on center. Find the number of side of regular polygon.

Perpendicular from Center to Chord

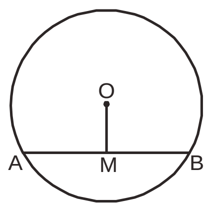


Figure - 22

On a piece of paper, draw a circle whose center is at O and where AB is a chord. Draw a perpendicular from center that meets AB at point M. What you can say about AM and BM?

Are they equal? How can we find out? Here, which mathematical arguments can we use? Can we use congruency of triangles?

Theorem - 3.

Statement: A perpendicular drawn from the center of a circle on to a chord bisects the chord.

Given : A circle with center at O and whose chord is AB and $OM \perp AB$

To Prove : AM = MB

Construction : Join O to A and B.

Proof : In ΔOMA and ΔOBM

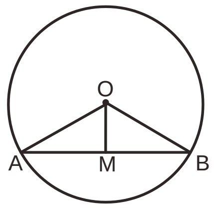


Figure - 23

$OA = OB$ (Radii of same circle)

$OM = OM$ (Common)

$\angle OMA = \angle OMB$ (Right angles)

$\Delta OMA \cong \Delta OMB$ (By Right angle – hypotenuse-side congruency)

Thus, $AM = MB$ (Corresponding parts of congruent triangles)

What is the converse of this theorem? Is the line drawn from the center which bisects the chord perpendicular to the chord?

Theorem - 4.

Statement : A line segment which joins the center of a circle and mid-point of a chord is perpendicular to the chord.

Given : A circle with center O. AB is chord and M is a mid-point of chord.

To Prove : $OM \perp AB$

Construction : Join O to A and B.

Proof : In ΔOMA and ΔOMB

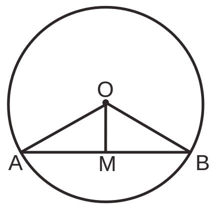


Figure - 24

$OA = OB$ (Radii of same circle)

$AM = MB$ (Given)

$OM = OM$ (Common side)

$\Delta OMA \cong \Delta OMB$ (SSS congruency)

Thus, $\angle OMA = \angle OMB$ (Corresponding parts of congruent triangles)

$\angle OMA + \angle OMB = 180°$ (By linear pair axiom)

$\angle OMA + \angle OMA = 180°$ $(\angle OMA = \angle OMB)$

$2\angle OMA = 180°$

$\angle OMA = 90°$

Thus, $OM \perp AB$

Let us solve some examples of circle by using these properties of circles.

Example.-3. If radius of a circle is 5 *cm* then find the length of a chord which is 3 *cm* away from center.

Solution : In ΔOAC, $OA = 5\ cm$ $OC = 3\ cm$

By Pythagoras theorem,

$$OA^2 = OC^2 + AC^2$$

$$5^2 = 3^2 + AC^2$$

$$AC^2 = 5^2 - 3^2$$

$$AC^2 = 25 - 9$$

$$AC^2 = 16$$

$$AC = 4$$

Thus, chord AB = 2 × AC = 8 *cm*

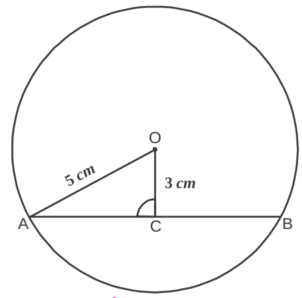


Figure - 25

Example-4. A chord which is 24 *cm* long is 5 *cm* away from center of the circle. Find the diameter of the circle.

Solution : OR = 5 *cm*, chord PQ = 24 *cm*

$$PR = \frac{1}{2} PQ \quad cm$$

$$= \frac{1}{2} \times 24$$

$$= 12 \quad cm$$

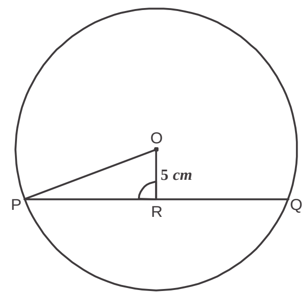


Figure - 26

Using Pythagoras theorem in ΔOPR

$$OP^2 = PR^2 + OR^2$$

$$= 12^2 + 5^2$$

$$= 144 + 25$$

$$= 169$$

$$OP = 13$$

So, diameter of circle $= 2 \times OP$

$$= 2 \times 13$$

$$= 26 \quad cm$$

Example-5. A line *l* intersects two concentric circles (circles with same center) on points A, B, C and D (See figure-27).
If AD = 18 *cm* and BC = 8 *cm* then find AB. Center of circle is O.

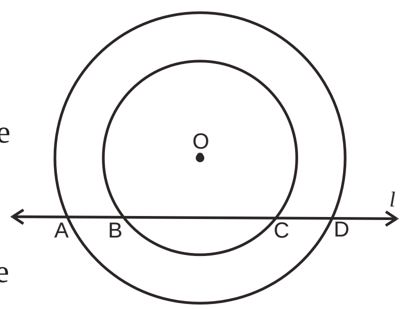


Figure - 27

Solution : Draw a perpendicular OM from center O on line *l* (See figure-28)

$$OM \perp BC$$

$\therefore$ $BM = MC$ ...... (i)

$BM + MC = BC$

$BM + BM = 8$

$2BM = 8$

$BM = 4\ cm$

Similarly, $OM \perp AD$

$AM = MD$ ...... (ii)

$AM + MD = AD$

$2AM = 18$

$AM = 9\ cm$

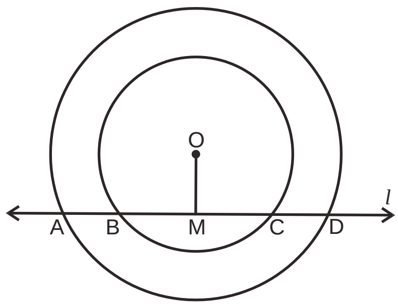


Figure - 28

So, $AB = AM - BM$

$= 9 - 4$

$= 5\ cm$

Example-6. Two chords of a circle PQ and RS are parallel and AB is perpendicular bisector of PQ. Prove that AB also bisects chord RS.

Solution : We know that bisector of a chord of circle passes through center of circle. Chord AB is perpendicular bisector of PQ.

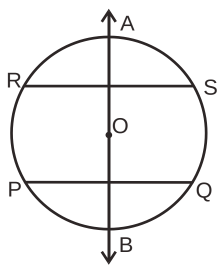


Figure - 29

$\therefore$ AB passes through center of circle-

$AB \perp PQ$ और $PQ \parallel RS \Rightarrow AB \perp RS$

Thus, $AB \perp RS$ and AB passes through center of circle.

$\therefore$ AB will be perpendicular bisector of chord RS.

Thus, AB bisects chord AB.

Try This

Find the length of the perpendicular drawn from the center of the circle of radius 5 *cm* on to a chord of length 6 *cm*.

Exercise - 1

1. Find the length of chord of a circle, if-

   (i) Radius = 13 *cm* and distance of chord from center = 12 *cm*

   (ii) Radius = 15 *cm* and distance of chord from center = 9 *cm*

2. Find the radius of a circle if length of chord and its distance from center are respectively:

   (i) 8 *cm* and 3 *cm* (ii) 14 *cm* and 24 *cm*

3. PQ is diameter of a circle (figure - 30). $MN \perp PQ$ and PQ=10 *cm* and PR=2 *cm* then find the length of MN.

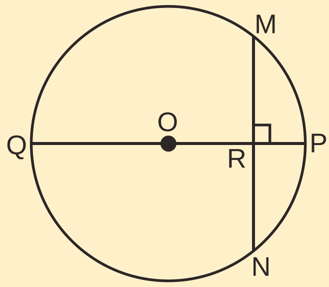


Figure - 30

4. In figure-31 chord AB = 18 *cm* and PQ is perpendicular bisector of chord AB which meets the chord on point M. If MQ = 3 *cm* then find the radius of circle.

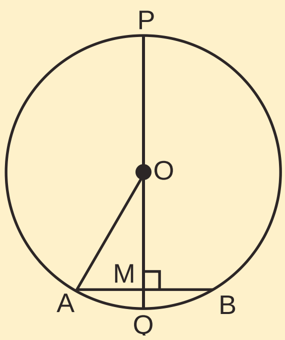


Figure - 31

5. A circle with center O and chords PQ and OR such that $\angle PQO = \angle OQR = 55^\circ$. Prove that PQ = QR..
6. AB and AC are two equal chords of a circle with center O. If $OD \perp AB$ and $OE \perp AC$ then prove that ΔADE is isosceles triangle.

Circle which passes through three non-collinear points.

Take a point P on paper. Draw a circle which is passes through point P. Can we draw one more circle which passes through point P? How many such circles can be drawn? (See figure-33)

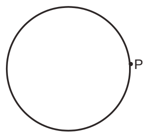


Figure - 32

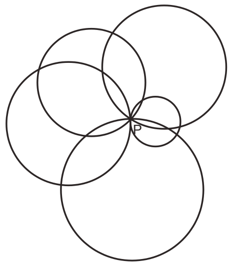


Figure - 33

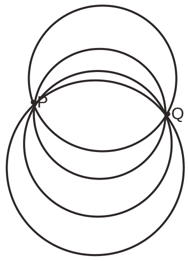


Figure - 34

You can see several circles can be drawn. Similarly, several circles can be drawn passing through two points P and Q (See figure-34). Can we draw a circle which passes through three non-collinear points?

Theorem - 5.

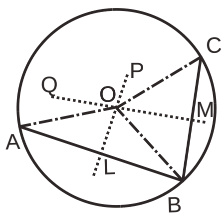


Figure - 35

Statement : One and only one circle can be drawn such that it passes through three non collinear points.

Given : A, B and C are three non-collinear points.

To Prove : One and only one circle can be drawn through the points A, B and C.

Construction : Connect points A to B and B to C. Draw PL and QM which are perpendicular bisectors of AB and BC, respectively. Let PL and QM intersect each other at point O. Join O to A, B and C.

Proof : Point O lies on PL which is perpendicular bisector of AB

$\therefore$ OA = OB .....(i) (Each point which lies on the perpendicular bisector of a line segment is at the same distance from the end points of the line segment)

Similarly, O lies on MQ which is perpendicular bisector of BC.

$\therefore$ OB = OC .....(ii)

OA = OB = OC = r (Assume) (PL and QM will intersect at the same point). O is the only point which will be equidistant from the points A, B and C.

Hence, only one circle passes through three non-collinear points.

We use this fact to draw a circle through all three vertices of a triangle. The circle is known as the incircle of the triangle and its center is called incenter.

Try This

Arc of a circle is given (See figure-36).

Complete the circle after finding its center.

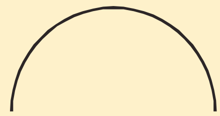


Figure - 36

Think and Discuss

Can we draw a circle which passes through three collinear points?

Chords and their distances from the center

Infinite chords can be drawn in a circle. Draw a circle of any radius. Draw parallel chords for this circle (see figure-37). Can you see any relation between lengths of chords and their distances from the center? Write the chords AB, CD and EF in descending order according to their distances from center. You will find that as the lengths of chords increase their distances from center decreases; thus, diameter is the longest chord in a circle whose distance from center is zero. Will the distances from center of chords be same if we take two equal chords? Let us verify this statement.

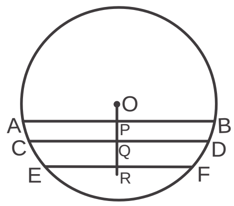


Figure - 37

Theorem - 6.

Statement : Equal chords of a circle (or congruent circles) are at the same distance from center (or from centers).

Given : A circle with center at O and two equal chords PQ and RS; OL and OM are perpendiculars from O to PQ and RS respectively.

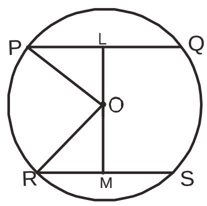


Figure - 38

To Prove : OL = OM

Construction : Join O to P and R.

Proof : PQ = RS (Given)

$$\frac{1}{2}PQ = \frac{1}{2}RS$$

PL = RM (Perpendicular drawn from center divides the chord into two equal parts)

OP = OR (Radii of a circle)

$\angle OLP = \angle OMR = 90°$ (By construction)

$\Delta OLP \cong \Delta OMR$ (By RHS congruency theorem)

$\therefore$ OL = OM (Corresponding parts of congruent triangles)

Try This

Chords which are equidistant from the center of a circle are equal. Prove.

Let us solve some examples by using the above results.

Example-7. Radius of a circle is 20 *cm*. Difference between two equal and parallel chords is 24 *cm*. Find the lengths of the chords.

Solution :

OM = ON .....(i) (Equal chords are equidistante from center)

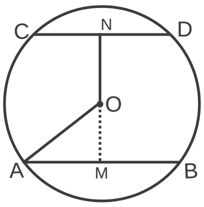


Figure - 39

MN = OM + ON

MN = OM + OM by (i)

24 = 2 OM

OM = 12 *cm*

OA = 20 *cm*

In ΔOAM

$$OA^2 = OM^2 + AM^2$$

$$AM^2 = OA^2 - OM^2$$

$$= 20^2 - 12^2$$

$$= 400 - 144$$

$$= 256$$

$$AM = 16$$

So, length of chord $AB = 2 \times AM$

$$= 2 \times 16$$

$$= 32\ cm$$

Example-8. Two parallel chords which are 6 cm and 8 cm long respectively lie on opposite sides of center of a circle. The distance between chords AB and CD is 7 cm. Find the radius of circle.

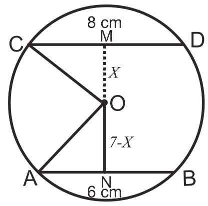


Figure - 40

Solution : Here, $AB = 6\ cm$

$$AN = \frac{1}{2}AB$$

$$= \frac{1}{2}\times 6$$

$$AN = 3\,cm$$

(Perpendicular drawn from center to chord bisects the chord)

Similarly $CD = 8\ cm$

$$CM = \frac{1}{2}CD$$

$$= \frac{1}{2}\times 8 = 4\ cm$$

In ΔOAN

$$OA^2 = ON^2 + AN^2$$

$$OA^2 = (7-x)^2 + 3^2 \quad (\because MN=7CM, \text{ let } OM=x \text{ then } ON=7-x)$$

In ΔOCM

$$OC^2=OM^2+CM^2$$

$$OC^2 = x^2 + 4^2$$

$\because$ $OA = OC$ (Radii of the same circle)

$\therefore$ $OA^2 = OC^2$

So, $(7-x)^2 + 3^2 = x^2 + 4^2$

$$x^2 - 14x + 58 = x^2 + 16$$

$$-14x = 16 - 58$$

$$14x = 42$$

$$x = 3\ cm$$

So, radius of circle

$$OA^2 = (7-x)^2 + 3^2$$

$$= (7-3)^2 + 3^2$$

$$= 16 + 9$$

$$= 25$$

$$OA = 5\ cm$$

Radius of circle $OA = 5\ cm$

Exercise - 2

1. Two chords AB and AC of a circle are equal. Prove that center of circle lies on the bisector of $\angle BAC$.

2. Two parallel chords which are 10 *cm* and 24 *cm* long respectively lie on opposite sides of center of a circle. The distance between chords is 17 *cm*. Find the diameter of circle.

3. Centre of a circle is O and PO is the angle bisector of $\angle APD$ (see figure-41). Prove that AB = CD.

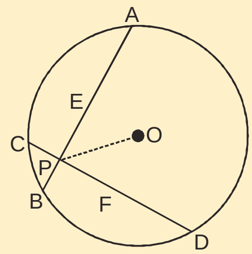


Figure - 41

4. O and C are centers of two circles whose radii are 13 *cm* and 3 *cm* respectively (see figure-42). If perpendicular bisector of OC meets on points A and B of the bigger circle then find the length of AB.

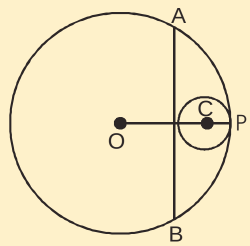


Figure - 42

5. Two equal chords AB and CD in a circle with center at O meet at right angle on point E. If P and Q are mid-points of chords AB and CD then prove that OPEQ is a square.

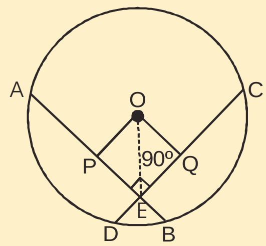


Figure - 43

Angle Subtended by Arc of a Circle at the Centre

If two points A and B lie on a circle then the circle is divided into two parts. Join the end point A and B of minor arc AB to the centre O. Angle $\angle AOB$ made by arc AB at the centre O is known as **central angle.** Again, take two points P and Q on the circle such that minor arc PQ made by them is greater than minor arc AB and makes an Angle $\angle POQ$ at the centre O (see figure-44). Can you see any relation between length of arc and angle subtended by arc at the centre? You can see in figure-44 that if the length of the arc is more, then angle subtended by it at the centre is also more.

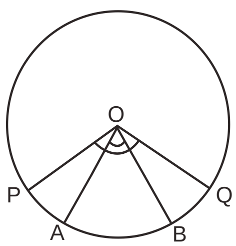


Figure - 44

Think and Discuss

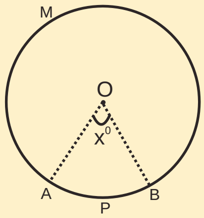


Figure - 45

If degree measure of angle of minor arc (figure-45) APB of a circle be $x°$ then degree measure of major arc will be $(360° - x°)$. Why?

Join the end points of an arc of a circle to any point on the remaining circumference of the circle as shown in figure-46. Then $\angle ACB$ is the angle which is subtended by arc APB at any point on the remaining circumference of the circle.

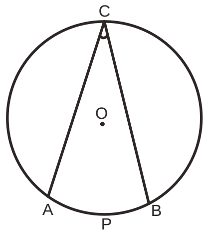


Figure - 46

Let us try to understand the relation between the angle subtended by an arc at the centre and that subtended on the remaining circumference of the circle.

Theorem - 7.

Statement : The angle subtended by an arc at the centre is double (twice) the angle subtended by it at any point on the remaining circumference of the circle.

Given : $\angle POQ$ subtended by an arc PQ at the centre of the circle and an angle $\angle PRQ$ at point R on the remaining circumference of the circle.

To Prove : $\angle POQ = 2\angle PRQ$

Construction : Join R to O and extend to point M.

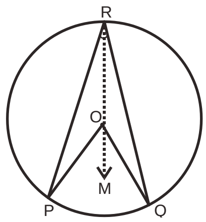


Figure - 47

Proof :

In ΔPOR

$OP = OR$ (Radii of a circle)

$\angle OPR = \angle ORP$ (Opposite angles on equal sides of a triangle are equal)

$\angle POM = \angle OPR + \angle ORP$ (Exterior angles theorem)

$\angle POM = 2\angle ORP$ ....(1)

In ΔQOR

$OQ = OR$ (Radii of a circle)

$\angle OQR = \angle ORQ$ (Opposite angles on equal sides of a triangle)

$\angle QOM = \angle ORQ + \angle OQR$ (Exterior angles theorem)

$\angle QOM = 2\angle ORQ$ ....(2)

So, $\angle POM + \angle QOM = 2\angle ORP + 2\angle ORQ$ ....on adding (1) and (2)

$$\angle POQ = 2(\angle ORP + \angle ORQ)$$

$$\angle POQ = 2\angle PRQ$$

Let us consider a situation when arc is a semi-circle.

Theorem - 8.

Statement : Angle subtended by the diameter of a circle at a point on the circumference is a right angle.

Given : $\angle LNM$ is subtended by a chord on the circle.

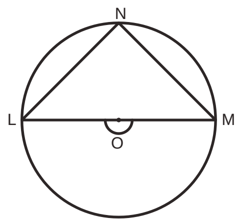


Figure - 48

To Prove : $\angle LNM = 90°$

Proof : $\angle LOM = 180°$ (Straight line)

$\angle LOM = 2\angle LNM$ (By theorem-7)

$\therefore$ $2\angle LNM = 180°$

Or $\angle LNM = \dfrac{180°}{2} = 90°$

So, we can say that angle subtended by the diameter at any point on the circumference is a right angle.

Example-9. In figure-49, O is centre of the circle and $\angle OPR = 30°$ and $\angle OQR = 40°$. Then find $\angle POQ$.

Solution : In ΔPOQ

$OP = OR$ (Radii of a circle)

$\therefore$ $\angle OPR = \angle ORP = 30°$ (Angles of isosceles triangle)

Similarly, in

$\angle OQR = \angle ORQ = 40°$

So, $\angle PRQ = \angle ORP + \angle ORQ$

$= 30° + 40°$

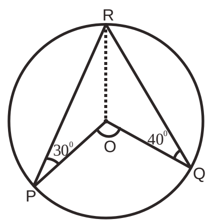


Figure - 49

$\angle PRQ = 70°$

$\therefore$ $\angle POQ = 2\angle PRQ$ (Angle subtended on centre is twice the angle subtended on remaining segment)

$\angle POQ = 2 \times 70° = 140°$

Example-10. In figure-50, AB is diameter and O is centre of given circle. If $\angle OAP = 50°$ then find $\angle OPB$.

Solution : In ΔAOP

$OA = OP$ (Radii of the same circle)

$\therefore \quad \angle OAP = \angle OPA = 50°$

$\angle APB = 90°$ (Angle subtended by diameter)

So, $\quad \angle APB = \angle OPA + \angle OPB$

$90° = 50° + \angle OPB$

$\therefore \quad \angle OPB = 40°$

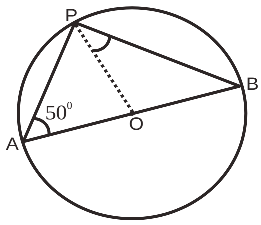


Figure - 50

Try These

Find the value of *x* in given figures-

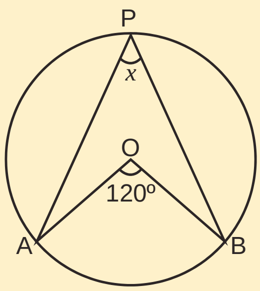


Figure - 51

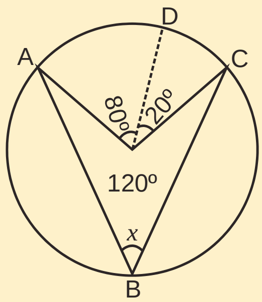


Figure - 52

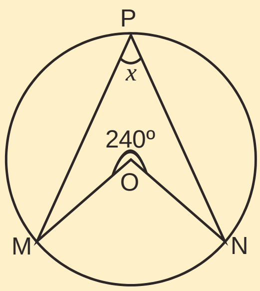


Figure - 53

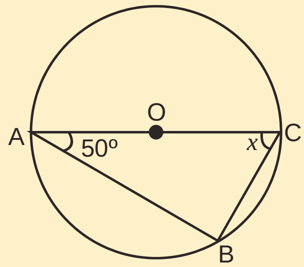


Figure - 54

Let us see the relation between angles which are present in the same segment of a circle.

Theorem - 9.

Statement : Angles in the same segment of a circle are equal.

Given : $\angle ACB$ and $\angle ADB$ which are in the same segment of circle where O is the centre.

To Prove : $\angle ACB = \angle ADB$

Proof : $\angle AOB = 2\angle ACB$ (Since the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\angle AOB = 2\angle ADB$

So, $\quad 2\angle ACB = 2\angle ADB$

$\angle ACB = \angle ADB$

So, we can say that which are in same segment of a circle are equal.

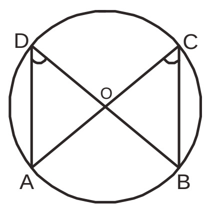


Figure - 55

Try These

Find the value of x and y in given figures-

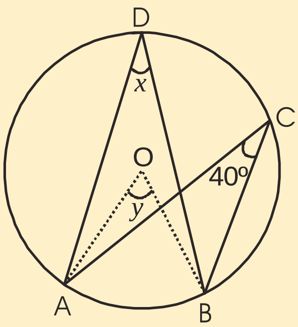


Figure - 56

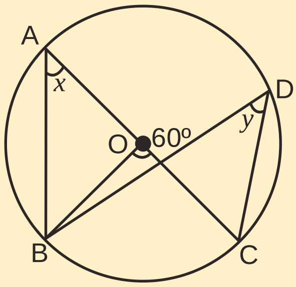


Figure - 57

Example-11. In figure-58, $\angle CAB = 25°$ and $\angle ADB = 35°$. Then find $\angle ABC$.

Solution : Here in figure

$\angle ADB = \angle ACB$ (Angle in same segment)

$\therefore \quad \angle ACB = 35°$

In ΔABC

$\angle ABC + \angle ACB + \angle CAB = 180°$

$\angle ABC + 35° + 25° = 180°$

$\angle ABC = 180° - 60°$

$\angle ABC = 120°$

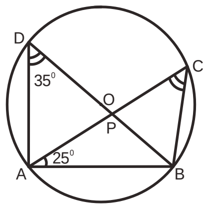


Figure - 58

Example-12. Prove that the circle drawn on any one of the equal sides of an isosceles triangle as diameter bisects the base.

Solution : ΔABC in which AB = AC and a circle is drawn by taking AB as diameter which intersects the side BC of triangle of D.

Since angle in a semi-circle is a right angle.

$\therefore \quad \angle ADB = 90°$

But, $\quad \angle ADB + \angle ADC = 180°$

$90° + \angle ADC = 180°$

$\angle ADC = 90°$

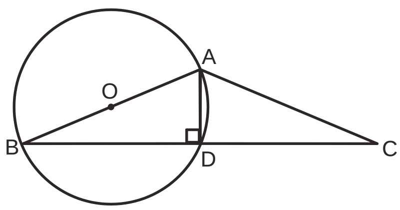


Figure - 59

In ΔADB and ΔADC

	AB = AC	(Given)
	AD = AD	(Common side)
And	$\angle ADB = \angle ADC = 90°$	
$\therefore$	$\Delta ADB \cong \Delta ADC$	(By RHS congruence)
	BD = DC	

Exercise-3

1. In figure-60, O is centre of circle, PQ is a chord. If $\angle PRQ = 50^{\circ}$ then find $\angle OPQ$.

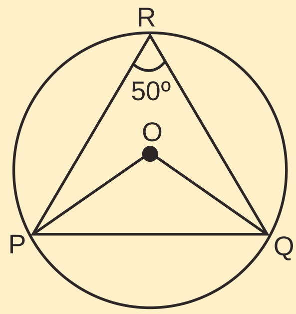


Figure - 60

2. In figure, find the value $\angle PBO$ if $\angle AOB = 50^{\circ}$ and $\angle PQB = 75^{\circ}$.

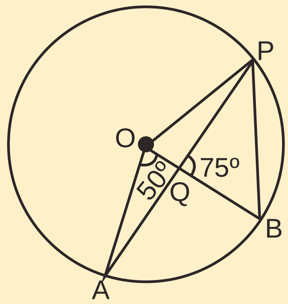


Figure - 61

3. If O is the centre of circle, find the value of x in figure.

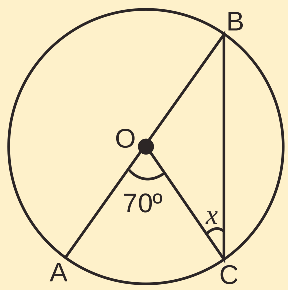


Figure - 62

4. If O is the centre of circle, find the value of x.

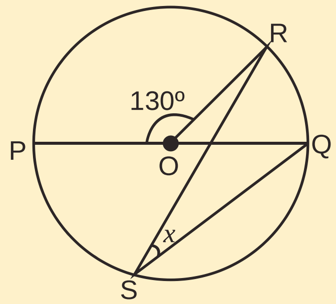


Figure - 63

5. If O is the centre of circle, then find the value of x in given figure.

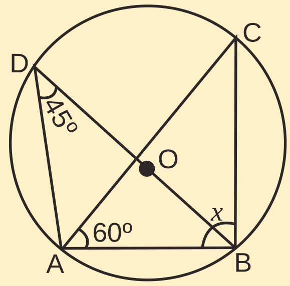


Figure - 64

6. Find the value of $\angle ABC$ in figure.

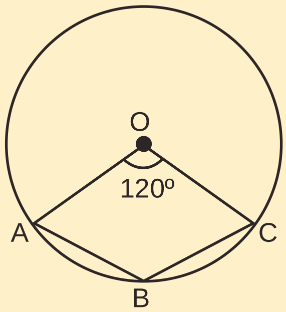


Figure - 65

7. Find the value of x in figure and prove that AD||BC

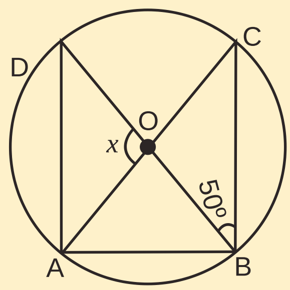


Figure - 66

8. In figure, O is centre of circle and $OD \perp AB$ if OD=5 cm then find the value of AC.

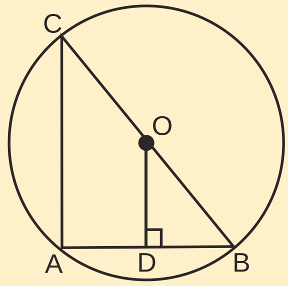
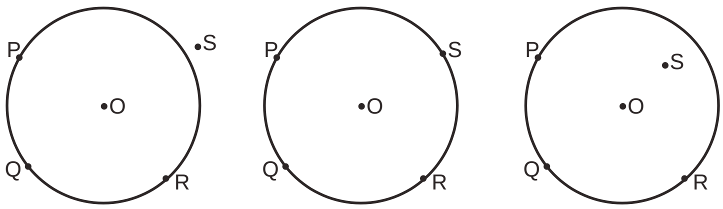


Figure - 67

Cyclic Quadrilateral

We saw that one and only one circle can be drawn through three non-collinear points. Can we draw a circle passing through four non-collinear points (of which no three are collinear)? If we draw a circle through three non-collinear points P, Q, R then the position of the fourth point S can be as follows (figure-68).

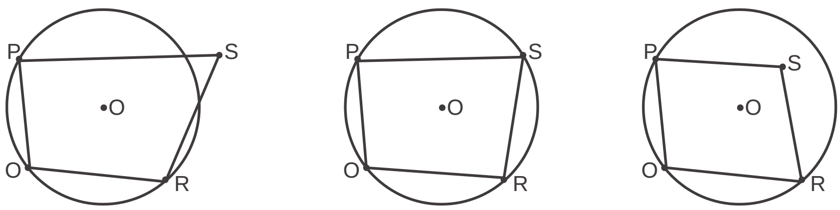


Position-1 Position-2 Position-3

Figure - 68

In position – 1, the point S is located outside the circle, in position – 2 it is located on the circle and in position – 3 it is located within the circle. Therefore, we can say that if we take 4 non-collinear points then it is possible for a circle to pass through them and it is also possible that all four do not lie on the circle.

In figure – 68, if we join P, Q, R and S we obtain a quadrilateral (see figure – 69).



Position-1 Position-2 Position-3

Figure - 69

All four vertices of the quadrilateral obtained in position – 2 are located on the circle. If all four vertices of a quadrilateral are located on a circle, it is known as cyclic circle. Does this quadrilateral have any special property not seen in other quadrilaterals?

Try These

Draw a circle of any radius. Take any four points on the circle and use them to form a quadrilateral. Measure the pairs of opposite angles and find their sum.

You will find that the sum of opposite angles of any quadrilateral whose 4 vertices lie on a circle is 180^{o}.

We will now try to find the logical proof of the above statement.

Theorem - 10.

Statement : The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Given : A cyclic quadrilateral ABCD.

To Prove : $\angle A + \angle C = \angle B + \angle D = 180°$

Construction : Join A and C to O.

Proof : We know the relation between the angle subtended by arc ABC at centre and at any point on remaining part of circle.

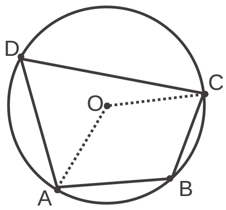


Figure - 70

Angle subtended by arc ABC at the center $\angle AOC = 2\angle ADC$ (i)

Angle subtended by arc CDA at the center $\angle COA = 2\angle ABC$ (ii)

So, $\angle AOC + \angle COA = 2(\angle ADC + \angle ABC)$

$$360° = 2(\angle D + \angle B)$$

$$\angle B + \angle D = \frac{360°}{2}$$

$$\angle B + \angle D = 180°$$

In quadrilateral ABCD,

$$\angle A + \angle C + \angle B + \angle D = 360°$$

$$\angle A + \angle C + 180° = 360°$$

$$\angle A + \angle C = 180°$$

So, we can say that the sum of either pair of opposite angles of a cyclic quadrilateral is 180°. Is converse of this theorem true as well? Yes, if the sum of any pair of opposite angles of a quadrilateral is 180°, then the quadrilateral is cyclic, that is, we can draw a circle passing through all four vertices of this quadrilateral.

Example-13. Find the value of x in figure-71.

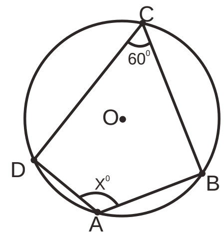


Figure - 71

Solution : $\angle A + \angle C = 180^o$ (Sum of opposite angles of cyclic quadrilateral is 180°)

$x + 60^o = 180^o$

$x = 120^o$

Example-14. Find the value of x in figure-72.

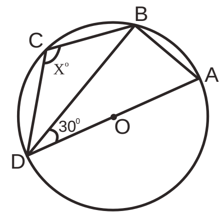


Figure - 72

Solution : $\angle ABD = 90^o$ (Angle subtended by diameter on any point on the circle)

$\angle ABD + \angle BDA + \angle DAB = 180^o$

$90^o + 30^o + \angle DAB = 180^o$

$\angle DAB = 180^o - 120^o$

$\angle DAB = 60^o$

$\angle DCB + \angle DAB = 180^o$ (Sum of opposite angles of cyclic quadrilateral)

$x + 60^o = 180^o$

$x = 120^o$

Example-15. P is a point on the side BC of a triangle ABC such that AB = AP. Through A and C, lines are drawn parallel to BC and PA respectively, so as to intersect at D (as shown in figure-73). Show that ABCD is a cyclic quadrilateral.

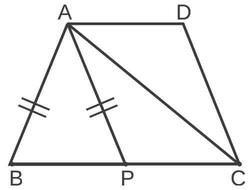


Figure - 73

Solution : In ΔABP

AB=AP (Given)

Thus, $\angle ABP = \angle APB$

(Opposite angles of equal sides)

AP II CD and AD II BC (Given)

So, APCD is a parallelogram.

$\angle APC = \angle ADC$

(Opposite angles of a parallelogram)

Since $\angle APB + \angle APC = 180^o$ (Linear pair axiom)

$\angle ABP + \angle ADC = 180^o$ ($\angle APB = \angle ABP$ and $\angle APC = \angle ADC$)

If the sum of either pair of opposite angles of a quadrilateral is 180^0 then the quadrilateral is cyclic.

Exercise - 4

1. AB||CD in given figure. If $\angle DAB = 80^o$ then find the remaining interior angles of the quadrilateral.

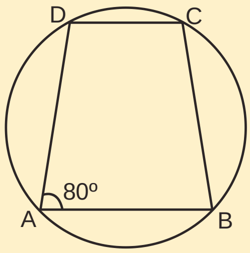


Figure - 74

2. Find $\angle QRS$ and $\angle QTS$ in the given figure.

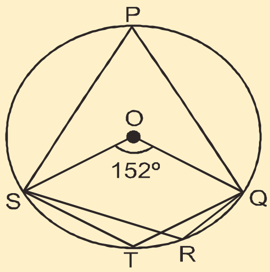


Figure - 75

3. ABCD is a cyclic quadrilateral in given figure whose side AB is diameter of the circle. If $\angle ADC = 150^o$ then find $\angle BAC$.

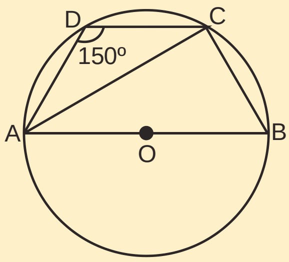


Figure - 76

4. Find the value of x in given figure.

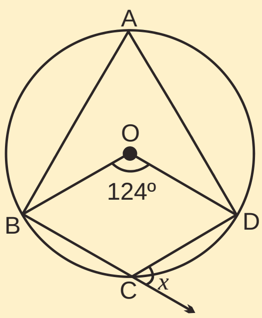


Figure - 77

5. PQ and RS are two parallel chords of a circle and lines RP and SQ intersect at point M (See figure-78). Prove that MP = MQ.

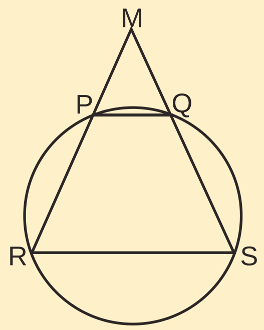


Figure - 78

6. PQ is diameter of semi-circle in given figure. If $\angle PQR = 60^o$ and $\angle SPR = 40^o$ then find the value of $\angle QPR$ and $\angle PRS$.

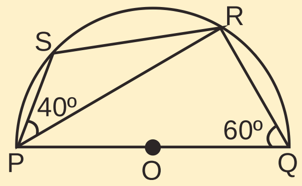


Figure - 79

7. If diagonals of a cyclic quadrilateral are diameters of the circle, then prove that the quadrilateral is a rectangle.

8. In figure-80 PQRS is a quadrilateral. If ∠P = ∠QRY then prove that PQRS is cyclic quadrilateral.

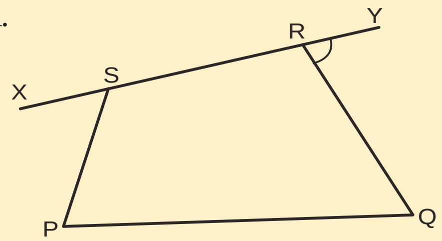


Figure - 80

9. If two non-parallel sides of a trapezium are equal, then prove that it is cyclic quadrilateral.

Tangents and Secant of a Circle

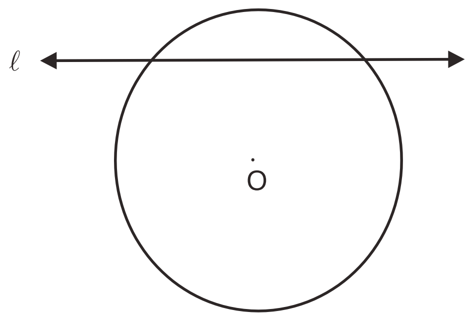


Figure - 81

Draw a circle and a line on paper as shown in figure-81. Now draw some lines parallel to *l*.

There are two common points A and B between line *m* and circle in given figure-82.

In the same way between line *l* and the circle there are two common points C and D. There is only one point E common between line *n* and the circle and there are no common points between line *p* and the circle.

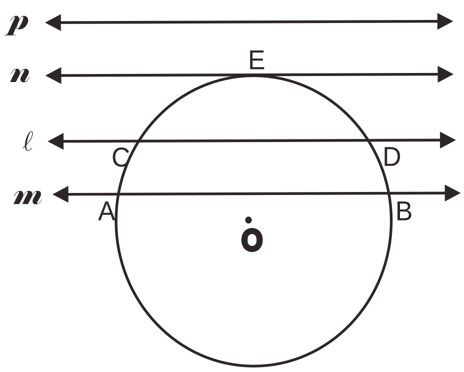


Figure - 82

We find that a line can be in three different positions relative to a circle.

In figure-83(i), line *l* does not intersect the circle so there are no common points between the line and the circle.

If figure-83(ii) line *l* intersects the circle at two different points so there are two common points P and Q between the line and the circle.

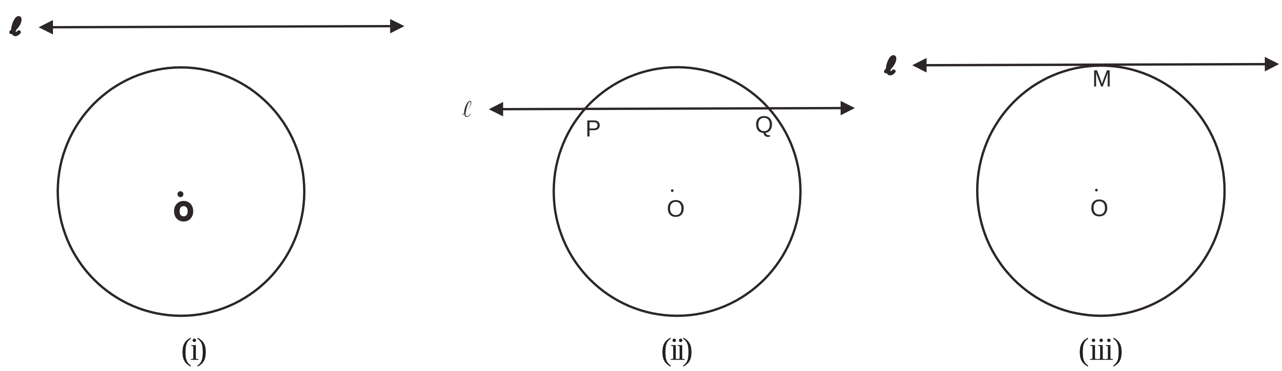


Figure - 83

In figure-83(iii) line *l* touches the circle at only one point so there is only one point M which is common to the line and the circle. In this situation, we say that *l* is tangent of circle and common point M is tangent point.

In figure-84 line *l* is intersecting the circle at two points P and Q. On keeping line *l* fixed at point P and rotating it in any direction continuously (see Q_1, Q_2, $Q_{3...}$) we will reach a condition where intersecting point Q becomes coincident with point P. In this case, we can call secant line as tangent to circle at point P and P as tangent contact point.

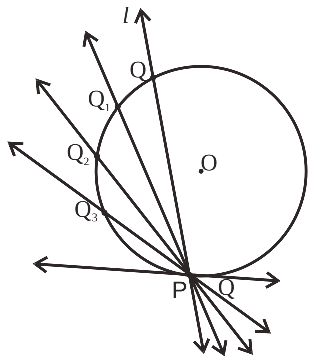


Figure - 84

The tangent to a circle is a special case of the secant, when the two end points of its corresponding chord coincide. Thus, tangent to a circle is that line which touches the circle at one point.

Think and Discuss

Only one tangent can be drawn at any particular point on a circle. Why?

Try These

Identify and write the names in your notebook of intersecting lines, secants and tangents in the following figure.

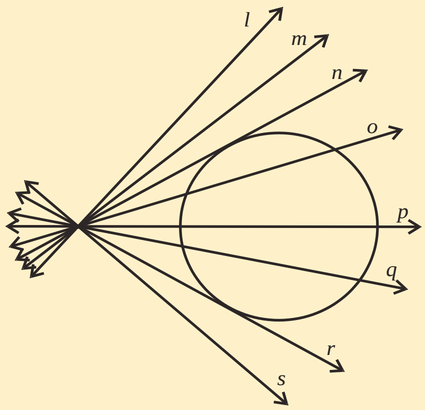


Figure - 85

Tangent line and radius passing through tangent contact point

The distance between a point and a line (when point is not on the line) is least when it is perpendicular. Will the distance from tangent to the centre be minimum, that is, is the radius passing through tangent point perpendicular at contact point?

Theorem - 11.

A radius of a circle meeting the contact point of tangent to the circle is perpendicular to the tangent.

Known : A circle with centre at O and having tangent AB meeting the circle at contact point, P.

To Prove : OP $\perp$ AB

Construction : Take point P on AB. Also take points Q, R, S on AB. Join all four points to the centre of the circle O.

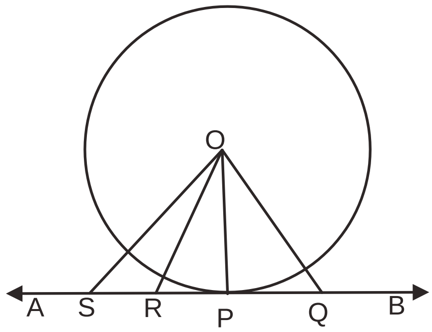


Figure - 86

Proof : In figure-86 we can see that points Q, R and S are located outside the circle and we know that distance between a point located outside the circle and the centre is more than the radius. Therefore, length of OP is least among OQ, OR and OS. Therefore, of all points on tangent AB, contact point P is at the least distance from the centre of the circle.

$\therefore$ OP $\perp$ AB

We use this fact to draw the tangent at any point on the circle when we know the centre of the circle.

How many tangents through a point located outside the circle

Take a point P outside the circle. Try to draw tangents to the circle passing through external point P (see figure – 87). You will find that two and only two tangents can be drawn on the circle from a point outside the circle. The distance between the external point P and the contact point of the tangent is known as length of the tangent. See figure – 87 and try to find some relation between PS and PT. Measure the lengths of PS and PT. You will find that PS = PT. Let us see the proof for this statement.

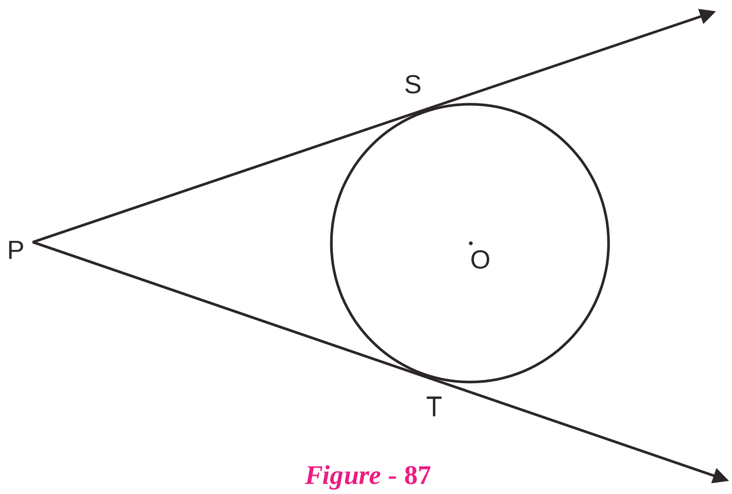


Figure - 87

Theorem - 12.

The lengths of tangents drawn from an external point to a circle are equal.

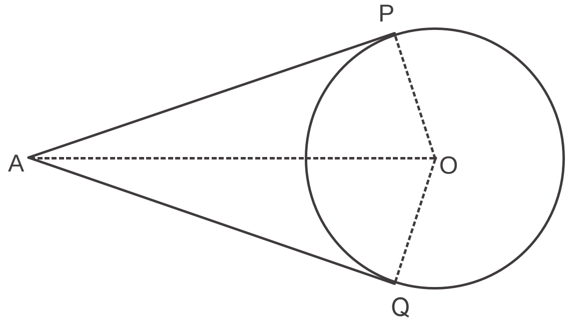


Figure - 88

Known : AP and AQ are two tangent segments to a circle drawn from the same external point A.

To Prove : AP = AQ

Construction : Join A, P, Q to the centre of the circle O.

Proof : In ΔOPA and ΔOQA

OP = OQ

(radii of the same circle)

OA = OA (common side)

$\angle APO = \angle AQO = 90°$ (radii passing through contact points are perpendicular to the tangent)

$\Delta OPA \cong \Delta OQA$ (angle-side congruency in right angle triangles)

Thus, AP = AQ (corresponding sides of congruent triangles)

In the proof of the above theorem, $\Delta OPA \cong \Delta OQA$ thus $\angle OAP = \angle OAQ$ We can say that the centre of the circle is located on the angle bisector of $\angle PAQ$ We can use this information to draw a circle which touches two intersecting lines. Especially, we can draw a circle that touches the three sides of a given triangle. The circle is known as incircle of the triangle and its centre is known as the incentre.

Example-16. In figure – 89, OP = 13 cm and the radius of the circle is 5 cm. Find the lengths of tangents PT and PS drawn on the circle from point P.

Solution : In ΔOPT

$\angle OTP = 90°$

In right angle triangle ΔOPT

$OP^2 = OT^2 + PT^2$

Or $13^2 = 5^2 + PT^2$

Or $PT^2 = 13^2 - 5^2$

$PT^2 = 169 - 25$

$PT^2 = 144$

PT = 12 *cm*

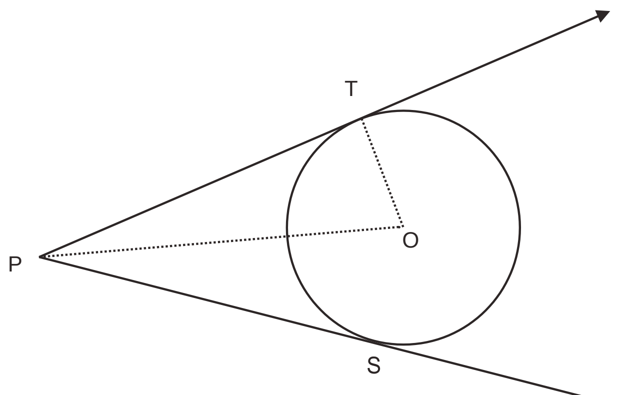


Figure - 89

We know that,

$$PS = PT$$

Thus, $PS = 12\ cm$

Thus, tangent $PT = PS = 12\ cm$

Example-17. In figure – 90, O is centre of the circle and PA and PB are tangents. If $\angle APB = 60°$ then find $\angle AOB$.

Solution : In quadrilateral AOPB

$$\angle OAP = \angle OBP = 90°$$

And $\angle OAP + \angle APB + \angle PBO + \angle AOB = 360°$

$$90° + 60° + 90° + \angle AOB = 360°$$

$$240° + \angle AOB = 360°$$

$$\angle AOB = 360° - 240°$$

$$\angle AOB = 120°$$

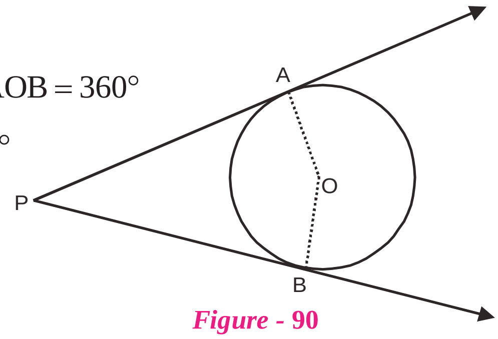


Figure - 90

Example-18. In figure-91, P, Q and R are points external to a circle with its centre at O. The lengths of tangents PA, QB and RC are 3 cm, 4 cm and 5 cm respectively. Find the perimeter of ΔPQR.

Solution : We know that lengths of tangent segments drawn from the same external point are equal.

$\therefore$ $PC = PA = 3\ cm$

$QA = QB = 4\ cm$

$RB = RC = 5\ cm$

$PQ = PA + AQ$

$PQ = 3 + 4 = 7\ cm$

$QR = QB + BR$

$QR = 4 + 5 = 9\ cm$

$PR = PC + CR$

$PR = 3 + 5 = 8\ cm$

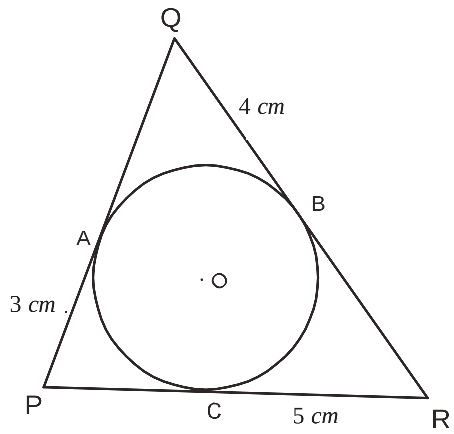


Figure - 91

Thus, perimeter of $\Delta PQR = PQ + QR + PR$

$= 7 + 9 + 8\ cm$

$= 24\ cm$

Example-19. Tangents TP and TQ are two tangents drawn from external point T on the circle with the centre at O. Prove that $\angle PTQ = 2\angle OPQ$.

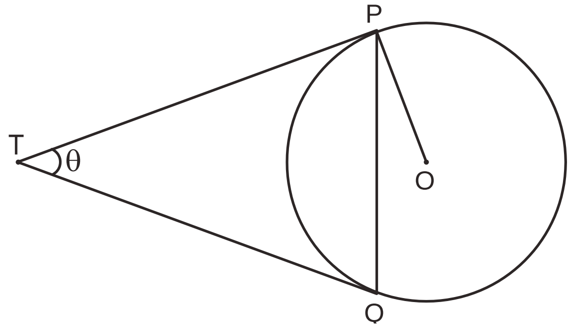


Figure - 92

Solution : Assume that $\angle PTQ = \theta$

$TP = TQ$ (from theorem-12)

Thus, ΔTPQ is an isosceles triangle where

$\angle TPQ + \angle TQP = 180^\circ - \theta$

$\therefore \quad \angle TPQ = \angle TQP = \frac{1}{2}(180^\circ - \theta)$

$\angle TPQ = 90^\circ - \frac{1}{2}\theta$

$\because \quad \angle OPT = 90^\circ$ है। (from theorem-11)

$\angle OPQ = \angle OPT - \angle TPQ$

$= 90^\circ - \left(90^\circ - \frac{1}{2}\theta\right)$

$= \frac{1}{2}\theta$

$= \frac{1}{2}\angle PTQ$

Thus, $\angle PTQ = 2\angle OPQ$ Hence Proved.

Segments of a Chord

PQ is a chord and R is a point on the chord located inside the circle. It is said that R internally divides the chord PQ into two segments PR and RQ. Similarly, if S is a point on line PQ located outside the circle then S is said to externally divide the chord into two segments SP and SQ.

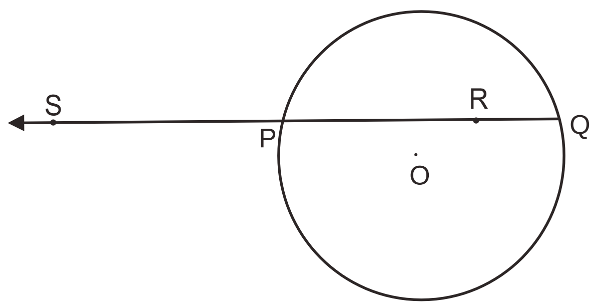


Figure - 93

Relation between tangent and secant

We have already seen the relation between two tangents drawn from the same external point. Is there any relation between the tangent and secant passing through the same external point?

Theorem - 13.

Statement : If PAB is a secant to a circle which intersects the circle at A and B and PT is a tangent to the circle then $PA \times PB = PT^2$

Given : Secant to the circle which intersects the circle at A and B and tangent PT to the circle.

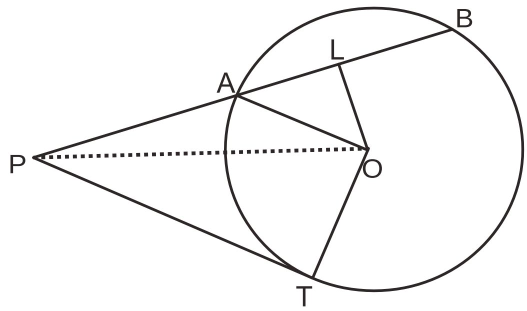


Figure - 94

To Prove : $PA \times PB = PT^2$

Construction : Draw OL perpendicular to AB. Join OP, OT and OA.

Proof : $PA \times PB = (PL - AL)(PL + LB)$

$= (PL - AL)(PL + AL)$

(Perpendicular to a chord from the centre of the circle divides it into two equal parts)

$= PL^2 - AL^2$

$= PL^2 - (OA^2 - OL^2)$ (From Pythagoras theorem)

$= PL^2 - OA^2 + OL^2$

$= PL^2 + OL^2 - OA^2$ (from Pythagoras theorem in triangle ΔOPL)

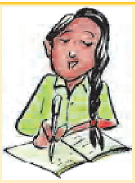
$= OP^2 - OA^2$

$= OP^2 - OT^2$ (OA = OT = radii)

$= PT^2$ (from Pythagoras theorem)

$\therefore \quad PA \times PB = PT^2$

Do This



If two chords of the same circle intersect each other internally or externally then the product of the segments of any one of the chords is equal to the product of the segments of the chord. That is, PA × PB = PC × PD.

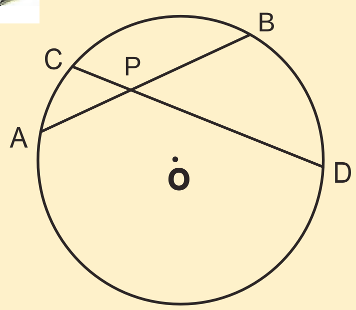


Figure - 95

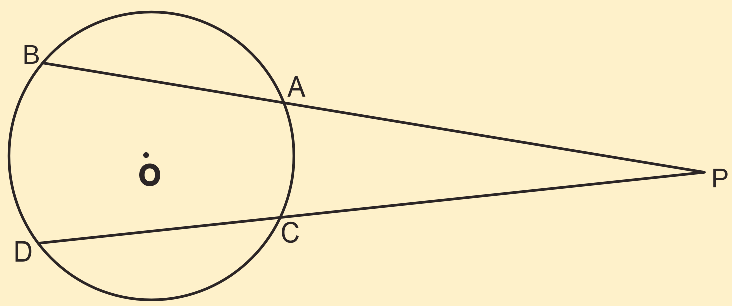


Figure - 96

Example-20. Let AB and CD be two chords of the circle, which intersect each other internally at point P. If PA = 2 *cm*, PB = 3 *cm* and PC = 4 *cm* then find the length of PD.

Solution : Given,

$PA = 2\ cm,\ PB = 3\ cm,$ and $PC = 4\ cm,$

Let $PD = x\ cm$

We know that $PA \times PB = PC \times PD$

$$2 \times 3 = 4 \times x$$

$$x = \frac{6}{4}$$

$$x = 1.5\ cm$$

$$PD = 1.5\ cm$$

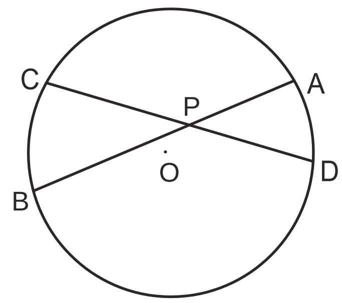


Figure - 97

Example-21. Chords PQ and RS intersect each other at point M which lies outside the circle. If MQ = 3 *cm*, MP = 8 *cm* and MS = 4 *cm* then find the lengths of MR and RS.

Solution : Given, $MQ = 3\ cm,\ MP = 8\ cm$ and $MS = 4\ cm$

Assume that $MR = x\ cm$

We know that : $MQ \times MP = MS \times MR$

$$3 \times 8 = 4 \times MR$$

$$MR = \frac{24}{4}$$

$$MR = 6\ cm$$

$$\text{Chord } RS = MR - MS$$

$$= 6 - 4$$

$$\text{Chord } = 2\ cm$$

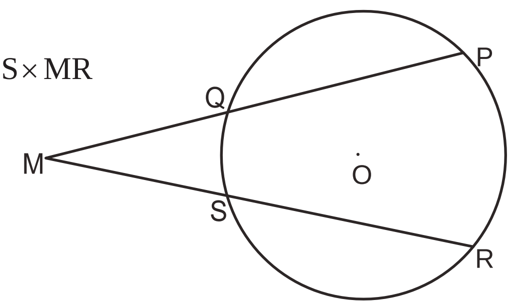


Figure - 98

Example-22. In figure – 99, if PA = 4 *cm* and PB = 9 *cm* then find the length of PT.

Solution : We know that $PA \times PB = PT^2$

$$4 \times 9 = PT^2$$

$$PT^2 = 36$$

$$PT = 6\ cm$$

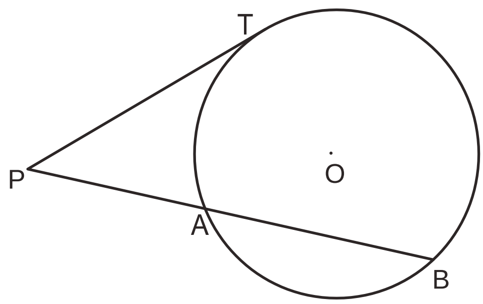


Figure - 99

Angle made by a chord and a tangent

Suppose we are given a circle with its centre at O and AB is a tangent at point P on the circle. Draw chord PQ from P. Take R on the major arc of the circle.

Major arc PRQ is called the alternate segment of the segment made by the chord PQ.

In figure – 100, if $\angle QPB = x°$ then $\angle OPQ = 90° - x°$ (Why?)

$\angle OPQ = \angle OQP = 90° - x°$ ($\because$ OP = OQ = radius)

In ΔPOQ

$$\angle POQ = 180° - \left[(90° - x°) + (90° - x°)\right]$$
$$= 180° - [180° - 2x°]$$
$$= 2x°$$
$$\angle PRQ = \frac{1}{2}\angle POQ$$
$$= \frac{1}{2} \times 2x°$$
$$= x°$$

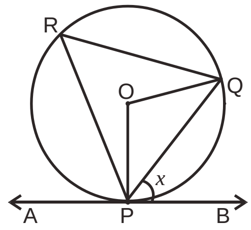


Figure - 100

Thus, we can say that "the angle between a tangent and chord at the contact point is equal to the angle made by that chord in the alternate segment."

This is also a theorem which is used to draw the tangent when we do not know the centre of the circle.

Example-23. In figure -101 PQ is tangent to the circle. If AOB is diameter of the circle and $\angle SAB = 50°$ then find $\angle ASP$.

Solution : $\angle BSQ = \angle SAB = 50°$

(from result of alternate segment)

$\angle ASB = 90°$

(angle formed by the diameter)

$\angle ABS + \angle ASB + \angle BAS = 180°$

$\angle ABS + 90° + 50° = 180°$

$\angle ABS = 40°$

$\because$ $\angle ASP = \angle ABS$

(from result of alternate segment)

$\therefore$ $\angle ASP = 40°$

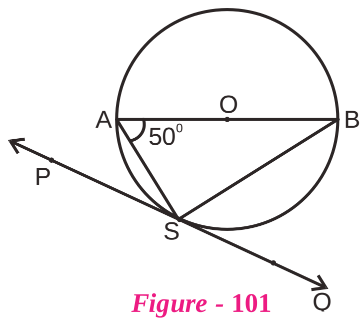


Figure - 101

Example-24. Tangent MN touches the circle at point P. Chord PQ is such that $\angle QPN = 52°$. Find $\angle POQ$ when O is the centre of the circle.

Solution : Let R be a point on the circumference of the circle. Join centre O to points P and Q. Similarly, join R to P and Q.

$\angle QPN = \angle PRQ = 52°$ (Since, the angle between a tangent and chord at the contact point is equal to the angle made by that chord in the alternate segment)

$\angle POQ = 2\angle PRQ$ (Angle at the centre is twice the angle formed on the circumference)

$\angle POQ = 2 \times 52°$

$\angle POQ = 104°$

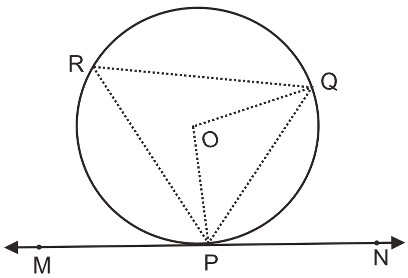


Figure - 102

Exercise 5

1. From a point P, which is at a distance of 10 cm from the centre of the circle, the length of tangent segment is 8 cm. Find the radius of the circle.

2. In figure – 103, $\angle POQ = 100°$, AP and AQ are tangents. Find the value of $\angle PAO$.

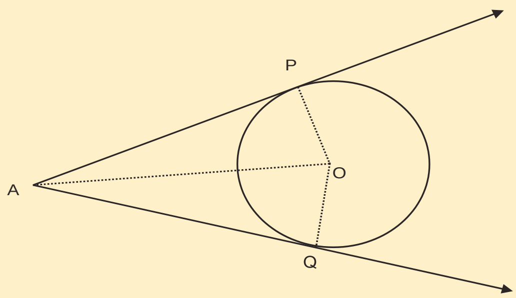


Figure - 103

3. Prove that the tangents drawn at the endpoints of a diameter of a circle are parallel to each other.

4. A circle touches all sides of quadrilateral ABCD. Prove that AB + CD = BC + DA.

5. Prove that the angle between two tangents drawn from the same external point is supplementary to the angle subtended at the centre by the chord which is formed by joining the contact points of the tangents.

6. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

What We Have Learnt

1. The set of all points on a plane which are equidistant from a given point is called circle.
2. Two equal chords in a circle subtend equal angles at the centre.
3. If the angles subtended by two chords at the center of a circle are equal then the chords are equal.
4. A perpendicular dropped from the centre of a circle on a chord bisects the chord.
5. The bisector from the centre of a circle on a chord is perpendicular to the chord.
6. One and only one circle can be drawn passing through three non-collinear points.
7. Two equal chords in a circle are equidistant from the centre.
8. The angle subtended by an arc at the centre is twice the angle subtended by the same arc at any other point on the circumference.
9. The angle subtended by the diameter on any point on the circumference is a right angle.
10. Angles in the same segment of a circle are equal to each other.
11. In a cyclic quadrilateral, the sum of any opposite pair of angles is equal to 180°.

12. If the sum of any pair of opposite angles of a quadrilateral is equal to 180^{O}, then the quadrilateral is cyclic.
13. The radius drawn from a tangent is perpendicular to the tangent.
14. The length of tangents to a circle drawn from the same external point is equal.
15. The angle between a tangent and chord at the contact point is equal to the angle made by that chord in the alternate segment

ANSWER KEY

Exercise - 1

1.	(i) 10 *cm*	(ii) 24 *cm*	
2.	(i) 5 *cm*	(ii) 25 *cm*	
3.	8 *cm*	4.	15 *cm*

Exercise - 2

2.	26 *cm*	4.	24 *cm*

Exercise - 3

1.	40^o	2.	80^o
3.	35^o	4.	25^o
5.	75^o	6.	120^o
7.	80^o	8.	10 *cm*

Exercise - 4

1. $\angle DCB = 100^o$, $\angle ABC = 80^o$, $\angle ADC = 100^o$
2. $\angle QRS = 104^o$, $\angle QTS = 104^o$
3. $\angle BAC = 60^o$
4. 62^o
6. $\angle QPR = 30^o$, $\angle PRS = 20^o$

Exercise - 5

1. 6 *cm*
2. 40^o